REVIEW PROBLEMS: COMPLEXITY THEORY

- 1) What is the difference between computability theory and complexity theory?
- 2) All studied complexity classes are subsets of the class of decidable languages. Why?
- 3) Fill in the blanks with \subseteq , \supseteq or = :
 - a) $O(n^4 + 7n)$ $O(n^5 15n^2)$
 - b) O(log(n)) _____ O(n)
 - c) $O(34n^3)$ _____ $O(2n^3 14)$
 - d) $O(4^n)$ _____ $O(n^4)$
- 4) Define the class P in two different (but of course equivalent) ways. Give an example of a language in P.

5) Let $L = \{xy \mid x \text{ and } y \text{ are strings in } \Sigma^* \text{ and } y \text{ is the reverse of } x\}$. Prove that L is in P. 6) Let CONNECTED = $\{ \langle G \rangle \mid \text{Every node of undirected graph } G \text{ is reachable from every other } f \rangle$ node by a legal path}.

Show that CONNECTED is in P.

7) Define the class NP in two different (but of course equivalent) ways. Give an example of a language in NP.

8) How do we know that $P \subseteq NP$?

9) Show that P is closed under complement.

10) What is the P =? NP question? Why is it important?

11) What does it mean for a problem to be NP-complete? Name two NP-complete languages.

- 12) Let PRIMES = $\{x \mid x \text{ is a prime number}\}$
 - a) Show that PRIMES is in NP.

b) PRIMES was suspected for years to be unsolvable in polynomial time. Then, in 2004, someone proved PRIMES is in P. From part (a) above, you know that PRIMES is in NP. Why does this not solve the P =? NP question?

13) How does the idea of "reducibility" used in defining NP-completeness differ from the idea used in computability theory?

14) Let KNAPSACK = {<S, V, W> | S is a set of items, each with a value and a weight, and there is a way to choosing items with a total value of V and a total weight of W or less}.

Show that KNAPSACK is in NP.