Set Theory Problems

* (1) Formal as a Tux and Informal as Jeans

Describe the following sets in both formal and informal ways.

Formal Set Notation Description	***Informal English Description***
a) $\{2, 4, 6, 8, 10, \ldots\}$	
b) $\{\ldots, -3, -1, 1, 3, \ldots\}$	
c) $\{n \mid n = 2m \text{ for some } y \in \mathbb{N}\}$	
d) $\{x \mid x=2n \text{ and } x=2k \text{ for some } n, k \in \mathbb{N}\}$	
e) $\{b \mid b \in \mathbb{Z} \text{ and } b=b+1\}$	
f)	The set containing the numbers 2, 20, and 200
g)	The set containing all integers greater than 42
h)	The set containing all positive integers less than 42
i)	The set containing the string `hello`
j)	The set containing the strings `bba` and `bab`
k)	The set containing nothing at all
l)	The set containing the empty string

* (2) You Want Me to Write What?

Fill in the blanks with $\in$, $\notin$, $\subseteq$, $\supseteq$, =, or $\neq$.

Recall that $\mathbb{Z}$ is the set of all integers and φ is the empty set, {}.

a) 2 _______ {2, 4, 6}

b) {2} _______ {2, 4, 6}

c) 1.5 _______ $\mathbb{Z}$

d) -1.5 _______ $\mathbb{Z}$

e) 15 _______ $\mathbb{Z}$

f) -15 _______ $\mathbb{Z}$

g) φ _______ $\mathbb{Z}$

h) 54 _______ {6, 12, 18, …}

i) 54 _______ {6, 12, 18}

j) {1, 3, 3, 5} _______ {1, 3, 5}

k) {-3, 1, 5} _______ {1, 3, 5}

l) {3, 1, 5} _______ {1, 3, 5}

* (3) Set Operations

Let A = {x, y} and B = {x, y, z}.

a) Is A a subset of B?

b) Is B a subset of A?

c) A ∪ B =

d) B ∪ A =

e) A ∩ B =

f) B ∩ A =

g) A x B =

h) B x A =

i) **P**(A) =

j) **P**(B) =

** (4) Does Order Matter?

Three important binary set operations are the union (∪), intersection (∩), and cross product (x).

A binary operation is called **commutative** if the order of the things it operates on doesn't matter.

For example, the addition (+) operator over the integers is commutative, because for all possible integers x and y, it's always true that x + y = y + x.

However, the division (÷) operator over the integers is *not* commutative, since x ÷ y ≠ x ÷ y for all integers x and y. (Note it works for *some* integers x and y, specifically whenever $x^2 = y^2$, but not for *every* possible integers x and y.)

a) Is the union operation commutative? (Does A ∪ B = B ∪ A for all sets A and B?)

b) Is the intersection operation commutative?

c) Is the cross product operation commutative?

*** (5) Set Me Up

Consider the following sets: $A = \{\varphi\}$ $B = \{A\}$ $C = \{B\}$ $D = \{A, \varphi\}$
True (T) or False (F)?

a) $\varphi \in A$

b) $\varphi \subseteq A$

c) $\varphi \in B$

d) $\varphi \subseteq B$

e) $\varphi \in C$

f) $\varphi \subseteq C$

g) $\varphi \in D$

h) $\varphi \subseteq D$

i) $A \in B$

j) $A \subseteq B$

k) $A \in C$

l) $A \subseteq C$

m) $A \in D$

n) $A \subseteq D$

o) $B \in A$

p) $B \subseteq A$

q) $B \in C$

r) $B \subseteq C$

s) $B \in D$

t) $B \subseteq D$

u) $C \in A$

v) $C \subseteq A$

w) $C \in B$

x) $C \subseteq B$

y) $C \in D$

z) $C \subseteq D$