Set Theory Problems

SOLUTIONS

* (1) Formal as a Tux and Informal as Jeans

Describe the following sets in both formal and informal ways.

Formal Set Notation Description	*Informal English Description*
a) {2, 4, 6, 8, 10, …}	**The set of all positive even integers**
b) {…, -3, -1, 1, 3,…}	**The set of all odd integers**
c) $\{n \mid n = 2m \text{ for some } y \in \mathbb{N}\}$	**The set of all positive even integers** **(using the convention that 0 is not a natural number)**
d) $\{x \mid x=2n \text{ and } x=2k \text{ for some } n, k \in \mathbb{N}\}$	**The set of all positive multiples of 6**
e) $\{b \mid b \in \mathbb{Z} \text{ and } b=b+1\}$	**φ**
f) **{2, 20, 200}**	The set containing the numbers 2, 20, and 200
g) **$\{n \mid n \in \mathbb{Z} \text{ and } n > 42\}$**	The set containing all integers greater than 42
h) **$\{n \mid n \in \mathbb{Z} \text{ and } n < 42 \text{ and } n > 0\}$ $= \{n \mid n \in \mathbb{N} \text{ and } n < 42\}$**	The set containing all positive integers less than 42
i) **{`hello`}**	The set containing the string `hello`
j) **{`bba`, `bab`}**	The set containing the strings `bba` and `bab`
k) **φ = {}**	The set containing nothing at all
l) **{ε}**	The set containing the empty string

* (2) You Want Me to Write What?

Fill in the blanks with $\in$, $\notin$, $\subseteq$, $\supseteq$, =, or $\neq$.

Recall that $\mathbb{Z}$ is the set of all integers and φ is the empty set, {}.

a) 2 ___$\in$___ {2, 4, 6}

b) {2} ___$\subseteq$___ {2, 4, 6}

c) 1.5 ___$\notin$___ $\mathbb{Z}$

d) -1.5 ___$\notin$___ $\mathbb{Z}$

e) 15 ___$\in$___ $\mathbb{Z}$

f) -15 ___$\subseteq$___ $\mathbb{Z}$

g) φ ___$\subseteq$___ $\mathbb{Z}$

h) 54 ___$\in$___ {6, 12, 18, …}

i) 54 ___$\notin$___ {6, 12, 18}

j) {1, 3, 3, 5} ___=___ {1, 3, 5}

k) {-3, 1, 5} ___$\neq$___ {1, 3, 5}

l) {3, 1, 5} ___=___ {1, 3, 5}

* (3) Set Operations

Let A = {x, y} and B = {x, y, z}.

a) Is A a subset of B? **YES**

b) Is B a subset of A? **NO**

c) $A \cup B$ = **{x, y, z} = B**

d) $B \cup A$ = **{x, y, z} = B**

e) $A \cap B$ = **{x, y} = A**

f) $B \cap A$ = **{x, y} = A**

g) A x B = **{(x, x), (x, y), (x, z), (y, x), (y, y), (y, z)}**

h) B x A = **{(x, x), (x, y), (y, x), (y, y), (z, x), (z, y)}**

i) **P**(A) = **{φ, {x}, {x, y}}**

j) **P**(B) = **{φ, {x}, {y}, {z}, {x, y}, {x, z}, {y,z}, {x, y, z}}**

** (4) Does Order Matter?

Three important binary set operations are the union ($\cup$), intersection ($\cap$), and cross product (x).

A binary operation is called **commutative** if the order of the things it operates on doesn't matter.

For example, the addition (+) operator over the integers is commutative, because for all possible integers x and y, x + y = y + x.

However, the division (÷) operator over the integers is *not* commutative, since x ÷ y ≠ x ÷ y for all integers x and y. (Note it works for *some* integers x and y, specifically whenever x = y, but not for *every* possible integers x and y.)

a) Is the union operation commutative? (Does $A \cup B = B \cup A$ for all sets A and B?)

Yes. $A \cup B = \{x \mid x \in A \text{ or } x \in B\} = \{x \mid x \in B \text{ or } x \in A\} = B \cup A$

b) Is the intersection operation commutative?

Yes. $A \cap B = \{x \mid x \in A \text{ and } x \in B\} = \{x \mid x \in B \text{ and } x \in A\} = B \cap A$

c) Is the cross product operation commutative?

No. For a counterexample, see Problem (3g) and (3h) above.
Recall that order matters for pairs.

*** (5) Set Me Up

Consider the following sets: $A = \{\varphi\}$ $B = \{A\}$ $C = \{B\}$ $D = \{A, \varphi\}$
True (T) or False (F)?

a) $\varphi \in A$ **(T)**

b) $\varphi \subseteq A$ **(T)**

c) $\varphi \in B$ **(F)**

d) $\varphi \subseteq B$ **(T)**

e) $\varphi \in C$ **(F)**

f) $\varphi \subseteq C$ **(T)**

g) $\varphi \in D$ **(T)**

h) $\varphi \subseteq D$ **(T)**

i) $A \in B$ **(T)**

j) $A \subseteq B$ **(F)**

k) $A \in C$ **(F)**

l) $A \subseteq C$ **(F)**

m) $A \in D$ **(T)**

n) $A \subseteq D$**(T)**

o) $B \in A$ **(F)**

p) $B \subseteq A$ **(F)**

q) $B \in C$ **(T)**

r) $B \subseteq C$ **(F)**

s) $B \in D$ **(F)**

t) $B \subseteq D$ **(T)**

u) $C \in A$ **(F)**

v) $C \subseteq A$ **(F)**

w) $C \in B$ **(F)**

x) $C \subseteq B$ **(F)**

y) $C \in D$ **(F)**

z) $C \subseteq D$ **(F)**

Note: May be easier to think about the sets as:

$A = \{ \{\} \}$
$B = \{ \{\{\}\} \}$
$C = \{ \{\{\{\}\}\} \}$
$D = \{ \{\{\}\}, \{\}\}$