Bounds and Finite Limits of Sequences

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Bounds

Def: A set or sequence S is **bounded from above** iff there exists a number M such that for all elements x in S, $x \leq M$.

This M is called *an* **upper bound** for S.

Def: A set or sequence S is **bounded from below** iff $\exists$ m s.t. $\forall x \in S$, $x \geq m$.

This m is called *a* **lower bound** for S.

Infimum = **inf** = **Greatest Lower Bound** = **GLB** = "best" lower bound

Supremum = **sup** = **Least Upper Bound** = **LUB** = "best" upper bound

Finite Limits

Def: A sequence $\{a_n\}$ has a **finite limit** L iff for all $\varepsilon > 0$, there exists a natural number N (which is a function of ε) such that for all $n > N$, $|a_n - L| < \varepsilon$.

Equivalent definition of a finite limit in math-speak:

Def: A sequence $\{ a_n \}$ has a **finite limit** L if $\forall \varepsilon > 0, \exists N(\varepsilon) \in \mathbb{N}$ s.t. $\forall n > N, | a_n - L | < \varepsilon$

"Almost all" of the terms of the series = *All but a finite number* of terms of the series ("Almost all" the terms of $\{a_n\}$ have some property whenever there exists a number N such that for all $n>N$, a_n has this property.

Equivalent definition of a finite limit using "almost all" terminology:

Def: A sequence $\{a_n\}$ has a **finite limit** L iff for all $\varepsilon > 0$, almost all the terms of $\{a_n\}$ satisfy $|a_n - L| < \varepsilon$.