Functions and Their Limits

104003 Differential and Integral Calculus I
Technion International School of Engineering 2010-11
Tutorial Summary – January 2, 2011 – Kayla Jacobs

Domain, Image, Range

Notation: Function **f: Domain → Range**

Domain: Set of "input" values for which the function is defined.
Image : The set of "output" values which the function returns.
Range = Co-Domain = Target: Any set (usually nice) containing the image;
may be equal to the image *or* a larger set containing the image.

Increasing and Decreasing Functions

Monotonically *in*creasing function: $x_1 \le x_2 \Leftrightarrow f(x_1) \le f(x_2)$

Monotonically *de*creasing function: $x_1 \le x_2 \Leftrightarrow f(x_1) \ge f(x_2)$

1-to-1, Onto

f: A → B is…

- **Injective (1-to-1)**: If $f(x_1) = f(x_2)$, then $x_1 = x_2$. Equivilently: If $x_1 \ne x_2$, then $f(x_1) \ne f(x_2)$.
  (Every element in B is mapped to by at most one element of A.)
- **Surjective (onto)**: For all y in B, there is an x in A such that y = f(x).
  (Every element in B has one or more matching elements in A)
- **Bijective (1-to-1 *and* onto)**

Inverses

If y = f(x) is a bijective (1-to-1 *and* onto) function,
then there exists an **<u>inverse function</u>** $\mathbf{f^{-1}}$ such that $f^{-1}(f(x)) = f^{-1}(y) = x$

A function is bijective iff it is invertible (has an inverse).

Elementary Operations and Functions

The 5 Elementary Operations: $+$, $-$, $\times$, $\div$, composition [composition: $f (g(x)) = (f \circ g)(x)$]

Elementary Functions: The functions we get from:
c *(const)* x a^x sin(x) arcsin(x) $\log_a(x)$
… and their combinations through the elementary operations

Definition of a Finite Limit of Function as x → a

$\lim_{x\to a} f(x) = L$ iff: for all ε > 0, there exists a δ >0 such that for all x≠a,

$$0 < |x - a| < \delta \quad \Leftrightarrow \quad |f(x) - L| < \varepsilon$$

Algebra of Limits:

Let $\lim_{x\to a} f(x) = L$ and $\lim_{x\to a} g(x) = K$. Then:

- **Sum:** $\lim_{x\to a}\big(f(x) + g(x)\big) = L + K$
- **Product:** $\lim_{x\to a}\big(f(x) * g(x)\big) = L * K$
- **Quotient:** $\lim_{x\to a}(f(x)/g(x)) = L/K$ (if g and K are both non-0)
- **Multiplication by a constant:** $\lim_{x\to a}\big(c * f(x)\big) = c * L$ (where c is any constant)