Delightful Differentiation

104003 Differential and Integral Calculus I

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Tutorial Handout – January 23, 2011 – Kayla Jacobs

General differentiation rules

Linearity:

$$(cf)' = cf'$$

$$(f+g)' = f' + g'$$

Product rule:

$$(fg)' = f'g + fg'$$

Reciprocal rule:

$$\left(\frac{1}{f}\right)' = \frac{-f'}{f^2}, \qquad f \neq 0$$

Quotient rule:

$$\left(\frac{f}{g}\right)' = \frac{f'g - fg'}{g^2}, \qquad g \neq 0$$

Chain rule:

$$(f \circ g)' = (f' \circ g)g'$$

Derivative of inverse function:

$$(f^{-1})' = \frac{1}{f' \circ f^{-1}}$$

for any differentiable function *f* of a real argument, with real values, when the compositions and inverses exist

Generalized power rule:

$$(f^g)' = f^g\left(g' \ln f + \frac{g}{f}f'\right)$$

Derivatives of simple functions

$$c' = 0$$

$$x' = 1$$

$$(cx)' = c$$

$$|x|' = \frac{x}{|x|} = \operatorname{sgn} x, \qquad x \neq 0$$

$$(x^c)' = cx^{c-1} \qquad \text{where both } x^c \text{ and } cx^{c-1} \text{ are defined}$$

$$\left(\frac{1}{x}\right)' = \left(x^{-1}\right)' = -x^{-2} = -\frac{1}{x^2}$$

$$\left(\frac{1}{x^c}\right)' = \left(x^{-c}\right)' = -cx^{-(c+1)} = -\frac{c}{x^{c+1}}$$

$$\left(\sqrt{x}\right)' = \left(x^{\frac{1}{2}}\right)' = \frac{1}{2}x^{-\frac{1}{2}} = \frac{1}{2\sqrt{x}}, \qquad x > 0$$

Derivatives of exponential and logarithmic functions

$$(c^x)' = c^x \ln c, \qquad c > 0$$

$$(e^x)' = e^x$$

$$(\log_c x)' = \frac{1}{x \ln c}, \qquad c > 0, c \neq 1$$

$$(\ln x)' = \frac{1}{x}, \qquad x \neq 0$$

$$(\ln |x|)' = \frac{1}{x}$$

$$(x^x)' = x^x(1 + \ln x)$$

$$\frac{d}{dx}[\ln(f(x))] = \frac{f'(x)}{f(x)}$$

$$\frac{d}{dx} \log_b(x) = \frac{d}{dx}\frac{\ln(x)}{\ln(b)} = \frac{1}{x \ln(b)} = \frac{\log_b(e)}{x}.$$

Derivatives of trigonometric functions

$$(\sin x)' = \cos x \qquad (\sin^{-1} x)' = \frac{1}{\sqrt{1-x^2}}$$

$$(\cos x)' = -\sin x \qquad (\cos^{-1} x)' = -\frac{1}{\sqrt{1-x^2}}$$

$$(\tan x)' = \sec^2 x = \frac{1}{\cos^2 x} \qquad (\tan^{-1} x)' = \frac{1}{1+x^2}$$

$$(\sec x)' = \sec x \tan x \qquad (\sec^{-1} x)' = \frac{1}{|x|\sqrt{x^2-1}}$$

$$(\csc x)' = -\csc x \cot x \qquad (\csc^{-1} x)' = -\frac{1}{|x|\sqrt{x^2-1}}$$

$$(\cot x)' = -\csc^2 x = \frac{-1}{\sin^2 x} \qquad (\cot^{-1} x)' = -\frac{1}{1+x^2}$$

Derivatives of hyperbolic functions

$$(\sinh x)' = \cosh x = \frac{e^x + e^{-x}}{2} \qquad (\operatorname{arsinh} x)' = \frac{1}{\sqrt{x^2+1}}$$

$$(\cosh x)' = \sinh x = \frac{e^x - e^{-x}}{2} \qquad (\operatorname{arcosh} x)' = \frac{1}{\sqrt{x^2-1}}$$

$$(\tanh x)' = \operatorname{sech}^2 x \qquad (\operatorname{artanh} x)' = \frac{1}{1-x^2}$$

$$(\operatorname{sech} x)' = -\tanh x \operatorname{sech} x \qquad (\operatorname{arsech} x)' = -\frac{1}{x\sqrt{1-x^2}}$$

$$(\operatorname{csch} x)' = -\coth x \operatorname{csch} x \qquad (\operatorname{arcsch} x)' = -\frac{1}{x\sqrt{1+x^2}}$$

$$(\coth x)' = -\operatorname{csch}^2 x \qquad (\operatorname{arcoth} x)' = -\frac{1}{x^2-1}$$