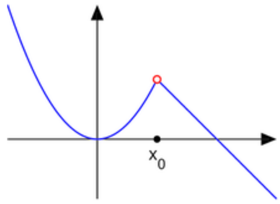
Discontinuities and Derivatives

104003 Differential and Integral Calculus I
Technion International School of Engineering 2010-11
Tutorial Summary – January 23, 2011 – Kayla Jacobs

Discontinuities

- **Removable Discontinuity at x_0**

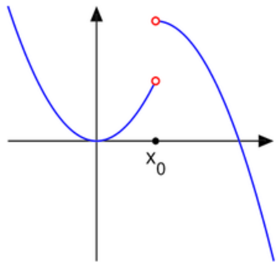
  Limit exists at x_0, BUT either:
  - **value of function at x_0 is undefined**
  - **value of function at x_0 is different from the limit at x_0**



Example of a **removable discontinuity**, where the value of the function is different from the limit

- **Discontinuity of the 1st Kind ("jump" discontinuity) at x_0**

  Both 1-sided limits at x_0 exist, BUT are unequal



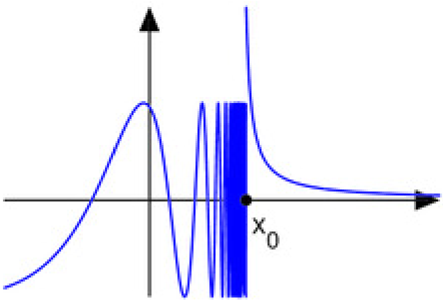
Example of a **jump discontinuity (discontinuity of the 1st kind)**

- **Discontinuity of the 2nd Kind at x_0**

  One or both 1-sided limits don't exist

  Remember: a "limit" of infinity doesn't exist!

  Examples: $\sin(1/x)$ at $x = 0$
  $\tan(x)$ at $x = \pi/2$



Example of a **discontinuity of the 2nd kind**
Here, *both* 1-sided limits don't exist
(though it would have been enough for just one to not exist)

Derivatives

Def: Let f be a function defined in the region of point x_0.
f is **differentiable** at x_0 if:

$$f'(x_0) = \lim_{h \to 0} \frac{f(x_0 + h) - f(x_0)}{h}$$

… exists and is finite.

Intuitive definitions:

- Slope of tangent line of function
- Rate of change of function

Practical examples:

- **Velocity** = derivative of **position** (with respect to time)
- **Acceleration** = derivative of **velocity** (with respect to time)

(See “Delightful Differentiation” handout for a reference sheet of helpful derivative facts.)