Discontinuities

- **Removable Discontinuity at** \( x_0 \)
  
  Limit exists at \( x_0 \), BUT either:
  - value of function at \( x_0 \) is undefined
  - value of function at \( x_0 \) is different from the limit at \( x_0 \)

- **Discontinuity of the 1st Kind ("jump" discontinuity) at** \( x_0 \)
  
  Both 1-sided limits at \( x_0 \) exist, BUT are unequal

- **Discontinuity of the 2nd Kind at** \( x_0 \)
  
  One or both 1-sided limits don’t exist

  *Remember: a “limit” of infinity doesn’t exist!*

  **Examples:**
  - \( \sin \left( \frac{1}{x} \right) \) at \( x = 0 \)
  - \( \tan(x) \) at \( x = \pi/2 \)
Derivatives

Def: Let $f$ be a function defined in the region of point $x_0$. $f$ is \textbf{differentiable} at $x_0$ if:

$$f'(x_0) = \lim_{h \to 0} \frac{f(x_0 + h) - f(x_0)}{h}$$

... exists and is finite.

Intuitive definitions:

• Slope of tangent line of function
• Rate of change of function

Practical examples:

• \textbf{Velocity} = derivative of \textbf{position} (with respect to time)
• \textbf{Acceleration} = derivative of \textbf{velocity} (with respect to time)

(See “Delightful Differentiation” handout for a reference sheet of helpful derivative facts.)