INFINITE SERIES CONVERSION TESTS

- **Divergence Test**

If $\lim_{n\to\infty} a_n \neq 0$, then the series **diverges**.

Otherwise, the test is **inconclusive** (just because the limit is 0 doesn't *necessarily* mean the series converges).

- **Ratio Test**

Assume that for all n, $a_n > 0$.

Suppose that there exists r such that: $\lim_{n\to\infty} \left| \frac{a_{n+1}}{a_n} \right| = r.$

If ***r* < 1**, then the series **converges**.
If ***r* > 1**, then the series **diverges**.
If ***r* = 1**, the ratio test is **inconclusive**, and the series may converge or diverge.

- **Integral Test**

The infinite series can be compared to an integral to establish convergence or divergence.

Let $f : [1, \infty) \to \mathbb{R}_+$ be a positive and monotone decreasing function such that $f(n) = a_n$.

If… $\int_1^\infty f(x)\,dx = \lim_{t\to\infty} \int_1^t f(x)\,dx < \infty,$ then the series **converges**.

But if the integral **diverges**, then the series does so as well.

In other words, the infinite series ***a_n* converges *<u>if and only if</u>* the integral converges.**

- **Alternating Series Test (Leibniz Test)**

Assume that for all n, $a_n > 0$.

An <u>alternating series</u> is of the form: $\sum_{n=0}^{\infty} (-1)^n a_n$

If $\lim_{n\to+\infty} a_n = 0$ and a_n is monotone decreasing, then the alternating series **converges**.

- **Limit Comparison Test**

If $\{a_n\}, \{b_n\} > 0$ and the limit $\lim_{n\to\infty} \frac{a_n}{b_n}$ exists (non-infinite) and is not zero,

then $\sum_{n=1}^{\infty} a_n$ converges <u>if and only if</u> $\sum_{n=1}^{\infty} b_n$ converges.

- **Series Comparison Test**

Let $\sum a_n$ and $\sum b_n$ be series with non-negative terms.

If $a_n \leq b_n$ for all n sufficiently large, then:

1. If $\sum b_n$ converges, then $\sum a_n$ also converges.
2. If $\sum a_n$ diverges, then $\sum b_n$ also diverges.

Informally, if the "larger" series converges, so does the "smaller." If the "smaller" series divers, so does the "larger."