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Project Title: Encoding Topological Information with Non-Abelian Berry's Phases

Over the last twenty years, physicists' understanding of the quantum properties of matter has been revolutionized by a series of profound theoretical advances and exciting experimental discoveries. Among these new findings are the fractional and spin quantum hall effects, non-abelian statistics, and topological quantum algorithms; these phenomena and others are united by their common topological and geometric underpinnings. For example, the existence of fractional statistics in two dimensions is possible only because a loop around a particle in a two-dimensional space is not homotopic to the trivial cycle. Even though physicists have learned much over the last few years, however, a solid theoretical understanding of these topological phenomena has eluded us. Physicists are aware of the existence of specific phenomena that exhibit interesting geometric properties, but yet lack methods to characterize these so-called “topological orders” in generality. Developing such methods is of great importance to theorists and experimentalists alike, given the rich variety of phenomena rooted in topologically ordered systems and the applications of their properties to real-world problems such as fault-tolerant quantum computing. This fall, I am pursuing a UROP with Prof. Xiao-Gang Wen in the physics department at MIT, which will aim to patch our insufficient theoretical understanding of topological orders by studying so-called non-abelian Berry's phases.

The notion of a Berry's phase is actually one rooted in classical, not quantum, mechanics. Given a manifold with some Riemannian structure (e.g. the state space of a classical particle, the space of solutions to the Schrodinger equation for a given Hamiltonian), the parallel transport of a tangent vector over a loop on the manifold is, in general, non-trivial. From this observation, it can be shown that when a physical system is subjected to a cyclic, adiabatic process (so that the adiabatic theorem holds and the ground state is parallel transported along the parameter space), the state of the system is modulated by a global phase, called the Berry's phase, that depends only on the geometry of the path taken. It has long been known that the Berry's phase can detect “holes” or singularities in the parameter space. It is natural then, to consider the generalization of the Berry's phase concept to the transport of an arbitrary vector space, in the hope that one might be able to encode more topological information in the process. After this generalization, the Berry's phase actually becomes a unitary operator known as the non-abelian Berry's phase.

Previous work by Prof. Wen and others have demonstrated the capability of the non-abelian Berry's phase to encode greater topological information in the case of abelian fractional quantum hall states on the torus. My UROP will focus on generalizing this method and applying it to a variety of physical systems. One immediate target is to prove the conjecture that the non-abelian Berry's phase is enough to completely characterize all the topological orders of abelian fractional quantum hall states on Riemann surfaces of sufficiently high genus. Intuitively, one would expect this to be true since the fundamental group of a high-genus surface is very large; that is, there are many topologically distinct adiabatic cycles that one can study for such surfaces. More ambitiously, I will attempt to generalize this discussion to non-abelian fractional quantum hall states, and eventually, I will try to use this method to explain and classify topological orders in generic Hamiltonians. These ideas, however, are only starting points. The structures exhibited by systems supporting topological orders are rich and varied. Possible directions for future research are innumerable, given that we know topological orders exist on surfaces of dimension larger than two, in physical systems completely unrelated to the quantum hall effect.

My motivation for undertaking this project is threefold. First, the project naturally connects to much of my coursework. In the fall semester, I am taking 8.324 and 8.333, MIT's courses in advanced quantum field theory and statistical mechanics, courses that both discuss the influence of topology in physics. In addition, I am taking 8.871, which is a special topics class taught by Prof. Frank Wilczek specifically about topological and geometric phenomena in quantum mechanics. Finally, I am taking 18.965, a course about the geometry of manifolds, which provides much of the mathematical framework I will need for my project. By studying the fractional quantum hall effect, then, I will directly connect my new physical and mathematical knowledge to problems of immediate theoretical interest. Second, I want to accumulate additional experience in theoretical physics research. At this point, I have committed myself to a career in theoretical physics. There is no better way, therefore, to improve my knowledge and research ability than by becoming directly involved in a theoretical physics project. Finally, as with all my endeavors, I am motivated by a curiosity of the physics itself. Even though our understanding of topological order is largely incomplete, I find the subject beautiful and worthy of further study. This particular subfield of physics is fast-paced and very new; I hope to discover a piece of the fundamental theory and thereby bring myself to the forefront of this exciting frontier.