Comparison of Induction Machine Equivalent Circuit Models

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Abstract

Most textbooks on electric machines present a classical five-element constant impedance equivalent circuit for modeling polyphase induction machines operating under balanced polyphase sinusoidal steady-state conditions. Yet it is known for cage rotor machines that the rotor element impedances vary significantly with rotor frequency. Research is in progress to determine a better ac equivalent circuit model, balancing accuracy against practicality of use and data acquisition. This paper provides a qualitative assessment of several advanced machine models.

Introduction

For many years researchers have modeled the induction machine with a "standard" equivalent circuit containing 5 or 6 circuit elements whose values remain constant. This model is flawed in that the rotor element values vary with rotor speed and in that certain of the inductances saturate. Several researchers have used varied approaches to account for these discrepancies in more sophisticated models. This paper presents a summary of their methods with a comparison of the partial results obtained by one method.

Standard Equivalent Circuit

Most authors derive an equivalent circuit for an induction machine from the concept of a "rotating transformer." Consequently, they derive a five- or six-element circuit which is closely related to the physical system [3]. The relations given by a transformer analogy are accurate with two notable exceptions. First, since there are *air gaps* in the "core," the magnetic circuit has been *linearized*, which means that core saturation due to the mutual coupling between the windings is minimized. Second, the magnetic field due to the stator winding is sinusoidally distributed in space, which causes a sinusoidal distribution of the flux inside the stator. Further, since the windings of the stator are phase separated, the flux distribution also rotates at a speed proportional to the frequency of the applied sinusoidal voltage. It is this time-variation in the magnetic field which produces electromagnetic torque on the rotor. Typically, the rotor rotates at a speed slightly less than the *synchronous* speed of the flux wave, and this produces a frequency-shifted voltage source in the secondary rotor circuit.

In order to reflect the rotor quantities into the stator, this frequency shift must be accounted for. In order to account for the shift, the concept of slip is introduced. An expression for the slip of an induction machine is given in equation 1. A diagram of the equivalent circuit with the rotor elements reflected into the stator is shown in figure 1.

$$s = \frac{\omega_{sm} - \omega_{rm}}{\omega_{sm}} \tag{1}$$

ω_{sm} = speed of revolving field, in rad/s
ω_{rm} = speed of rotor, in rad/s

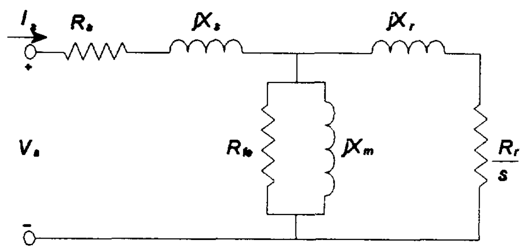

Figure 1. Standard 6-element per-phase wye equivalent circuit.

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where

R_s = stator winding resistance, in ohms
X_s = stator winding leakage reactance, in ohms
R_{fe} = core loss resistance, in ohms
X_m = magnetizing reactance, in ohms
X_r = rotor winding leakage reactance, in ohms, reflected into the stator
R_r = rotor winding leakage resistance, in ohms, reflected into the stator

Values for the above elements are typically found from two standard tests--the Blocked-Rotor Test and the Running-Light (No Load) Test.

Rogers' Double Cage Method

Another researcher in this area uses an equivalent circuit model which has two parallel branches in the rotor. The circuit is shown in figure 2 [4]. It is important to note that while this representation is more sophisticated, the element values are still constants. In order to find values for these elements, Rogers uses standard manufacturer's performance specifications as well as some assumptions about the machine's behavior [5]. It is assumed in this work that friction and windage losses are included as part of the load. Also note that the resistive core loss element has been removed from the circuit and is lumped with the mechanical losses of the machine. Finally, Rogers assumes that the winding losses are a fixed 75% of the total losses in the machine.

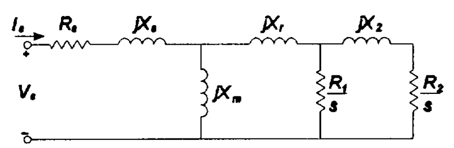

Figure 2. Rogers Equivalent Circuit

The given input data for this model are mostly nameplate ratings: stator voltage, output power, efficiency, power factor at full load, full-load slip, synchronous speed, inertia (including load), starting current, and torque ratio (the ratio of starting torque to rated torque).

Rogers derives the necessary relationships to solve for the circuit elements in terms of this input information. He also models saturation of the leakage reactances with an elaborate describing function given in equation (2).

$$DF = \begin{cases} 1 \; ; \; I \le I_{sat} \\ \dfrac{2}{\pi}\left[\sin^{-1}\left(\dfrac{I_{sat}}{I}\right) + \dfrac{1}{2}\sin\left(2\sin^{-1}\left(\dfrac{I_{sat}}{I}\right)\right)\right] \; ; \; I > I_{sat} \end{cases} \tag{2}$$

According to Rogers, the leakage reactance may be broken down into two parts--a saturable part, which is multiplied by the describing function (DF), and an unsaturable part, which represents the lower limit of the reactance as I approaches infinity. The saturation threshold current I_{sat} is determined iteratively from breakdown torque information.

Willis' SSFR Method

Willis adapts an approach used in synchronous machines to induction machines. The per-phase equivalent circuit is shown in figure 3 [2]. There is a striking similarity between this circuit and the one presented by Rogers, but there is considerable difference in the methods used to obtain values for the circuit elements.

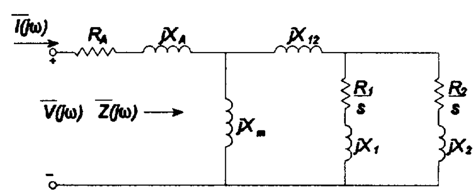

Figure 3. Willis Equivalent Circuit.

We determine the Stand-Still Frequency Response (SSFR) of the machine by exciting the stator over a range of frequency, with the rotor immobilized, measuring stator voltage, current, and power, and determining

$$Z(\omega) = R(\omega) + jX(\omega)$$

Two approaches may be used to find the values of the circuit elements. First, the operational impedance is found at the terminals by dividing the voltage by the current at each frequency point measured. Then the operational inductance is found. This inductance is plotted versus frequency and using an appropriate transfer function and an assumed circuit topology, the circuit values were derived using "software written for this purpose." [1]

Another method Willis mentions is Newton's method. He imposes equality constraints on the system at the measurement points. The only requirements are that there are enough constraints to match the number of unknowns, and that some initial guess for the circuit values is required. Further, Willis notes that is SSFR data is unavailable, this second method may be used along with readily available manufacturer's data [2].

Willis also accounts for saturation in his model, using an adaptation of a method used by other investigators [1]. His method only modifies the stator leakage inductance. The relationship is given in equation 3.

$$L_t = L_{tr} + \left[\frac{I_{tr}}{I_s} - 1\right]\left[\frac{L_{p0} - L_{tr}}{I_{tr}/I_{p0} - 1}\right] \qquad (4)$$

where

- L_t = stator leakage inductance, in henrys
- Is = stator current, in amps
- L_{tr} = stator leakage inductance at locked rotor, in henrys
- L_{po} = stator leakage inductance at pullout, in henrys
- I_{tr} = locked rotor stator current, in amps
- I_{po} = stator current at pullout torque, in amps

Cochran's Deep Bar Method

Cochran investigates advanced induction motor modeling by looking at the rotor bars from a magnetic circuits standpoint. He derives expressions for the rotor resistance and reactance in terms of rotor frequency. His equivalent circuit is shown in figure 4.

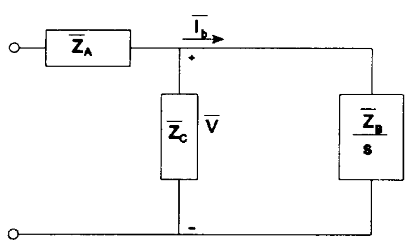

Figure 4. Cochran's Equivalent Circuit.

Although this circuit is presented for the simpler case of constant circuit values, it can also be used here if we let Z_b be a function of slip/rotor frequency. The expression is given in equation 4.

$$\bar{V} = \bar{I}_b R_{dc} \delta\, d_b \left[\frac{\sinh(2\delta d_b) + \sin(2\delta d_b)}{\cosh(2\delta d_b) + \cos(2\delta d_b)}\right] + j\bar{I}_b R_{dc} \delta\, d_b \left[\frac{\sinh(2\delta d_b) - \sin(2\delta d_b)}{\cosh(2\delta d_b) - \sin(2\delta d_b)}\right] \qquad (5)$$

where

- R_{dc} = DC resistance of the rotor bar, in ohms
- d_b = depth of rotor bar, in meters

$$\delta = \sqrt{\frac{4\pi^2 k f}{\rho\, 10^7}}$$

- k = ratio of slot width to bar width
- ρ = resistivity of the bar, in ohm-meters
- f = frequency of rotor bar current, in Hz

Note that the real part of this impedance is R(f) and the imaginary part is X(f). Also note that $f = f_s (s - 1)$, where f_s is synchronous mechanical frequency and the slip (s) is defined as usual:

$$s = \frac{\omega_s - \omega_r}{\omega_s} = \frac{f_s - f_r}{f_s}$$

This aspect of Cochran's discussion does not account for saturation of any of the inductances.

Results

Data for a 4-pole, 60 Hz, 460 Volt, 20 HP induction motor was available to implement Rogers' method. The developed torque (T_{dev}) was computed using Rogers' method and the standard equivalent circuit shown in figure 1 (sans R_{fe}). A plot comparing the two methods is given in figure 5.

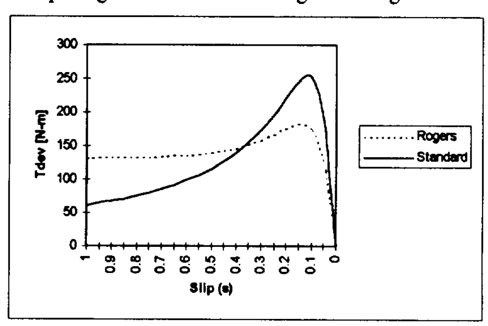

Figure 5. Comparison of T_{dev} for Rogers' method and the standard circuit.

Conclusions

There are certain trade-offs involved in creating any mathematical model. The more advanced models are not as intuitive as the simple five- or six-element equivalent circuit. Furthermore, the tests and methods used to find values for the elements in these circuits may be quite complex. On the other hand, the standard equivalent circuit is incapable of modeling some of the basic operating features of the machine, such as the deep-bar effect.

Rogers' and Willis' method use constant (independent of rotor frequency) R, L component values, and use rather complex expressions to deal with magnetic saturation. Cochran's method requires detailed information on the machine design, information that is not readily available to the application engineer. Our studies so far reveal that implementation of any of the models investigated requires resources not routinely available to practicing engineers. Our research objectives are to provide a simple, accurate model, requiring minimal machine data, that is straightforward to implement by application engineers. Research is underway to solve this problem.

References

[1] Willis, J.R., Brock, G.J., and Edmonds, J.S., "Derivation of Induction Motor Models from Standstill Frequency Response Tests," *IEEE Transactions on Energy Conversion*, December, 1989, 608-13.

[2] Willis, J.R., and Johnson, B.K., "Tailoring Induction Motor Analytical Models to Fit Known Motor Characteristics and Satisfy Particular Study Needs," *IEEE Transactions on Power Systems*, August, 1991, 959-65.

[3] Fitzgerald, A.E., Kingsley, C., and Umans, S.D., *Electric Machinery*, 5th Ed., New York: McGraw-Hill, 1990.

[4] Rogers, G.J., "Demystifying Induction Motor Behavior," *IEEE Computer Applications in Power*, January, 1994, 29-33.

[5] Rogers, G.J., "Induction Machine Modelling for Electromagnetic Transient Program," *IEEE Transactions on Energy Conversion*, December, 1987, 622-28.

[6] Cochran, P.L., *Polyphase Induction Motors: Analysis, Design, and Application*, New York: Marcel Dekker, Inc., 1989.