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Advances in Direct Time Integration Schemes for Dynamic Analysis

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The accurate solution of dynamic response in finite element analyses has been the subject of extensive research for the last few decades. In general, implicit schemes are used when the transient response can be obtained with a relatively small number of large time steps, typically of order 10^{-3} s, and explicit schemes are used when many time steps of small size need be used, typically of order 10^{-6} s. The most widely-used schemes in implicit solutions are the Newmark trapezoidal rule and alpha generalized method, and in explicit solutions the central difference method [1]. However, these schemes have some undesirable characteristics, and recently more effective methods have been proposed, which we want to expose briefly in this short article.

Implicit Time Integration: Bathe Method

The trapezoidal rule is unconditionally stable in linear analyses, and has the characteristics of no amplitude decay and a reasonable amount of period elongation. Hence, on first sight, the solution errors seem to have excellent qualities. However, in fact, the quality of no amplitude decay can cause major solution problems, because frequencies may be sampled that should be suppressed (for example, because they are an artifact of finite element modeling). In linear analysis this phenomenon can be easily and directly seen (an example is given below), and in nonlinear analysis, the phenomenon can also render the iterative solution difficult to converge. We illustrate the solution behaviors below.

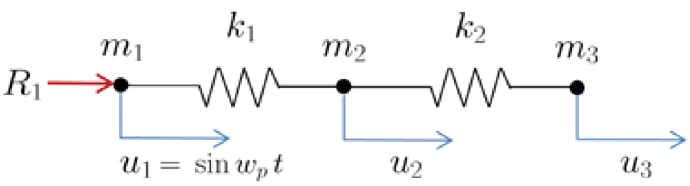


Figure 1: Model problem of three degrees of freedom spring system $k_1=10^7$, $k_2=1$, $m_1=0$, $m_2=1$, $m_3=1$, $\omega_p=1.2$

Figure 1 gives a simple two spring model solved [2,3]. While very simple, the model contains the essence of many practical finite element models. The stiff spring represents stiff components in a structural model, which may be largely due to modeling constraints with stiff elements, while the soft spring represents the rest of the model. The aim is to only solve for the response in the soft part of the structure, like in a mode superposition solution. The trapezoidal rule gives very large errors in this linear analysis, see Figures 2 and 3. The response prediction can be improved by introducing damping, numerical or physical, but then the question will always be how much damping to introduce when not knowing the desired response. The same holds when using the generalized alpha method.

A new scheme is the Bathe method, which combines the use of the trapezoidal rule and Euler backward method [1-3]. In the Bathe method, no parameter is (usually) set and the accuracy of solution is simply dependent on the size of the time step used. As the time step becomes smaller the accuracy increases. Figures 2 and 3 show that the method gives the desired response, just like obtained in a mode superposition solution including only the lowest mode response with the static correction. Further results are given in ref. [3] where it is also shown that the error in the reaction using the Newmark method is very large.

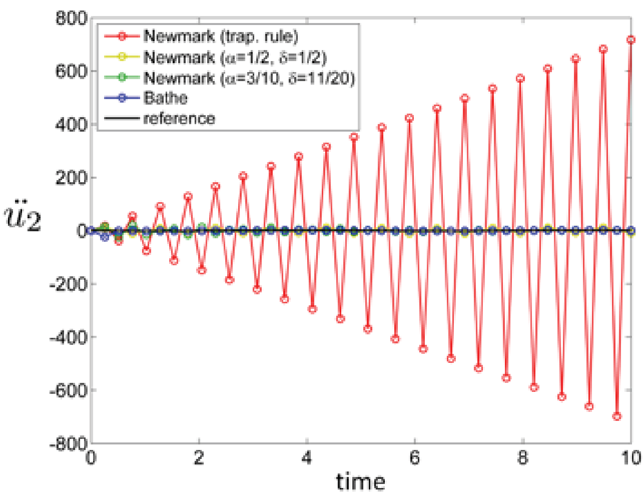


Figure 2: Acceleration of node 2 for various methods

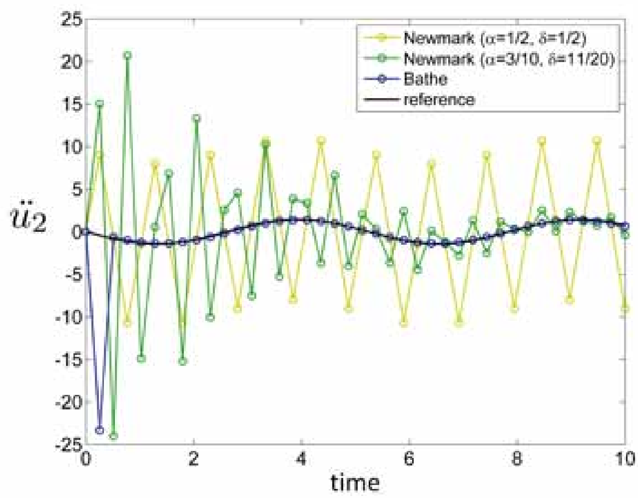


Figure 3: Acceleration of node 2 for various methods (the overshoot in the first time step of the Bathe method could be eliminated by using in the Newmark method $\delta = 3/4$, $\alpha = 1.0$ for the first step only).

There is also a parameter in the Bathe method on the size of the sub-step (but this parameter, changing the accuracy, is by far mostly used in its default value, see refs. 1-3). Hence the advantage of the Bathe method is that no parameter values need to be chosen.

While the Bathe method is about twice as expensive per time step (since two sub-steps are used), the higher accuracy in general allows to use less steps in linear response solutions. In nonlinear analysis the Bathe method is overall frequently more effective because it converges much better in the nonlinear iterations of the time steps, larger time steps can be employed, and the method remains stable when the Newmark and alpha generalized methods become unstable (unless sufficient damping is introduced).

The above observations are demonstrated in the solutions given in Figures 4 to 10.

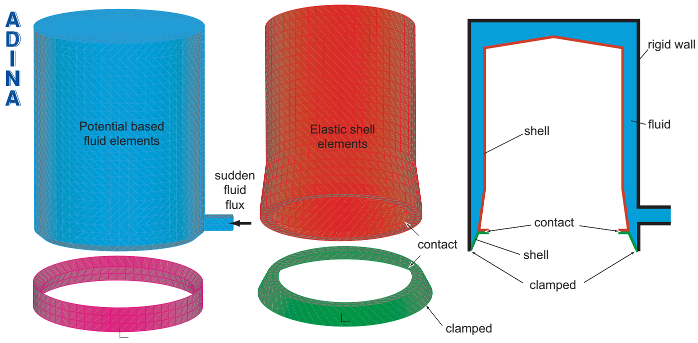
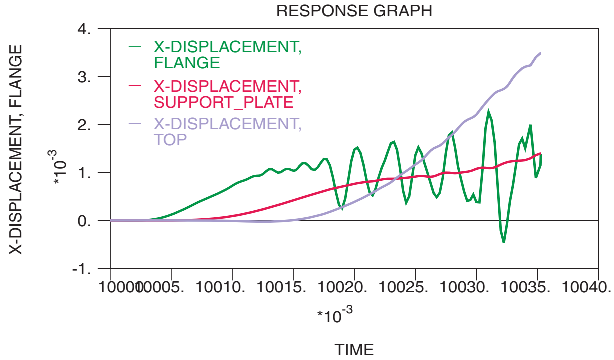


Figure 4: Schematic of the shell-fluid problem considered; results shown in Figures 5 - 8

Figure 4 shows the model considered, which consists of an elastic shell fully clamped at its base and a fluid surrounding it contained by an exterior rigid wall. Shell elements and subsonic potential based fluid elements are used to represent the media. The shell structure consists of two parts with frictional contact conditions between them. The model is subjected to a sudden fluid flux representing a pipe break. The resulting shock waves cause the internal parts of the model that are in contact to rapidly change status. For the implicit dynamic analysis of such problems usually the Newmark time integration is used. However, when contact conditions are included between internal parts, the contact surfaces repeatedly stick and slip, which results in rapid pressure pulses in the fluid. As a consequence, high frequency vibrations are observed. These high frequency oscillations are spurious in the Newmark method solution and grow with time. After a while, the solution becomes obviously very erroneous and may even diverge. The results using the Newmark method without damping are shown in Figure 5. Note the highly oscillatory response of the flange, the non-smooth contact status between the internal parts and the parasitic pressure distribution.

To overcome this problem, different techniques can be used, such as adding physical damping to the model (e.g. Rayleigh damping). In this case the damping will only be applied to the structure and the question is how much damping to introduce when physically it is negligible. Alternatively, the Newmark method can be used to introduce numerical damping. This reduces the numerical oscillations, but also reduces the physical response which should be solved for, and the question is how much numerical damping to introduce in order to obtain acceptable results.



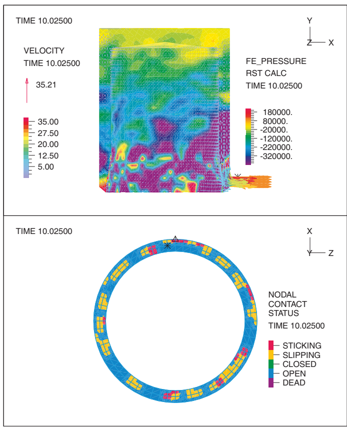
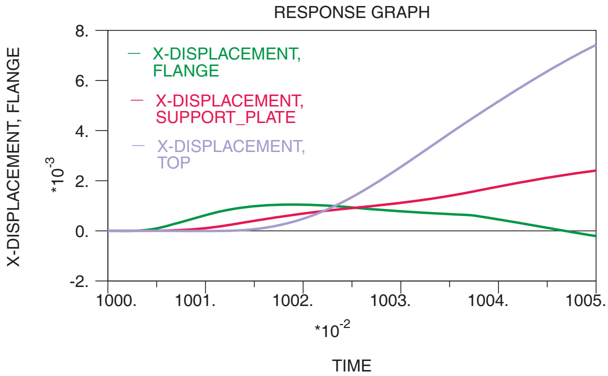


Figure 5: Predicted response using Newmark method, no damping (δ = 0.5, α = 0.25)

Figures 6 and 7 show that while the presence of physical damping or numerical damping improves the results using the Newmark method, to suppress all oscillations, the damping must be increased to high levels, which is not desirable. However, when using the Bathe method, no numerical parameter had to be adjusted and no artificial physical damping was introduced in the model, see Figure 8. The results achieved in this analysis led to the subsequent use of the Bathe method in the analyses of large finite element models.



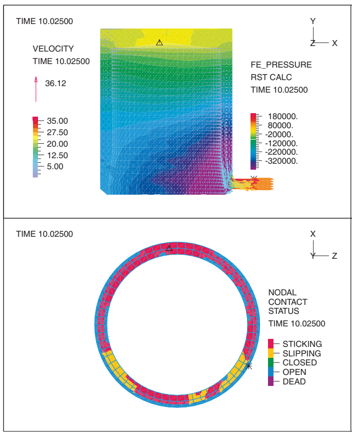
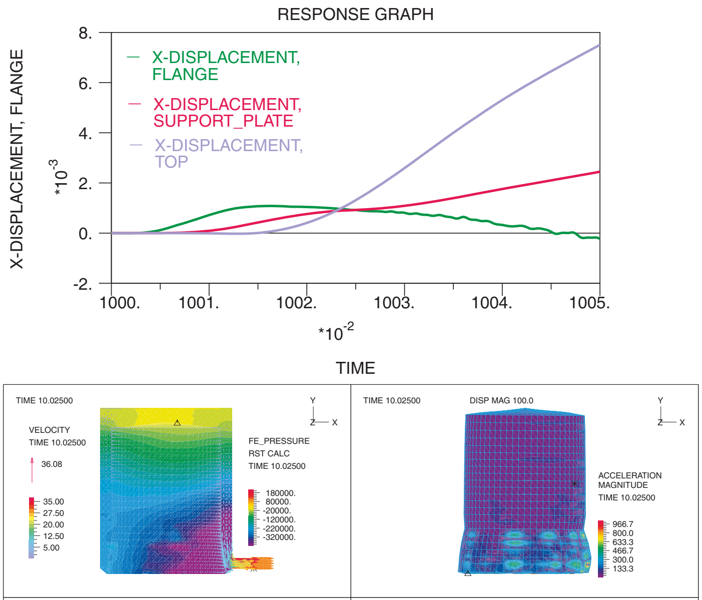


Figure 6: Predicted response using Newmark method with Rayleigh damping, with C = 0.001



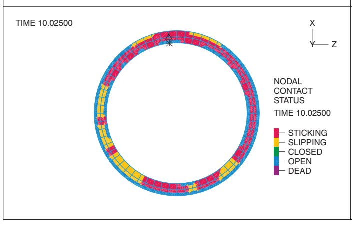
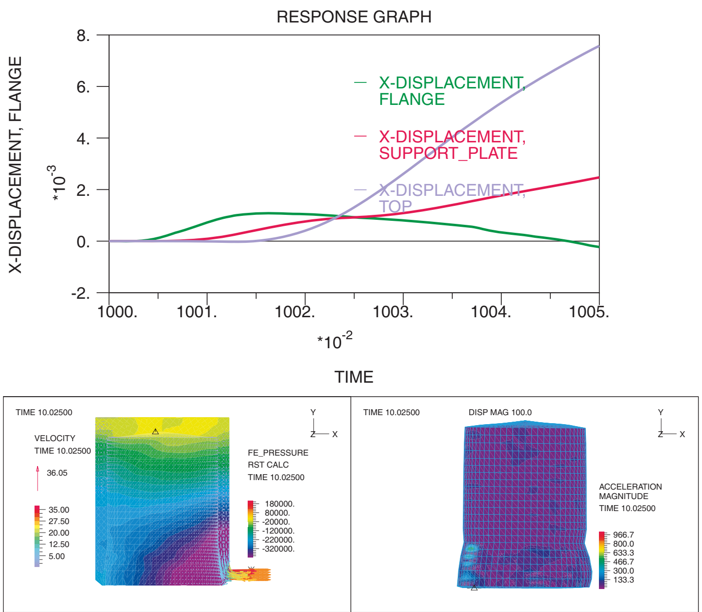


Figure 7: Predicted response using Newmark method with numerical damping, (δ = 0.6, α = 0.3025)



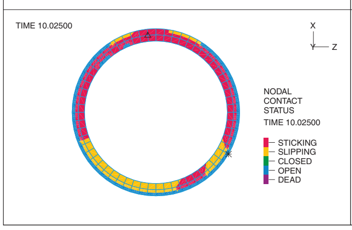


Figure 8: Predicted response using Bathe method, no physical damping

Another example solution pertains to the rotation of a heavy antenna structure, with focus on high accuracy of the antenna positioning and orientation. In this application, we see very large displacements over long time ranges in the transient analysis, and numerical stability can be difficult to achieve. Figure 9 shows the model of the antenna, which is rotated with various angular velocities using the classical trapezoidal rule and the Bathe method for time integration.

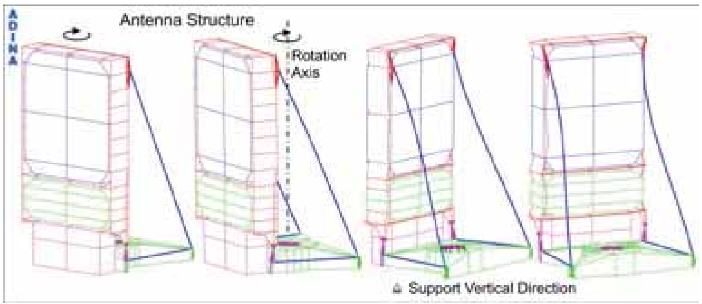


Figure 9: Antenna model in various rotational positions using Bathe Method

When using the Bathe method, the solution is obtained very accurately for many revolutions, whereas the Newmark time integration procedure fails before finishing the second revolution, see Figure 10 for the antenna rotation instability occurring in the solution. The numerical instability is also well seen when studying the axial forces in the antenna stabilizers, see Figure 10, and occurs quite suddenly. No physical damping, e.g. Rayleigh damping, is used in the model. This antenna rotation problem may be seen as an extension of the problem of a rotating stiff pendulum[2].

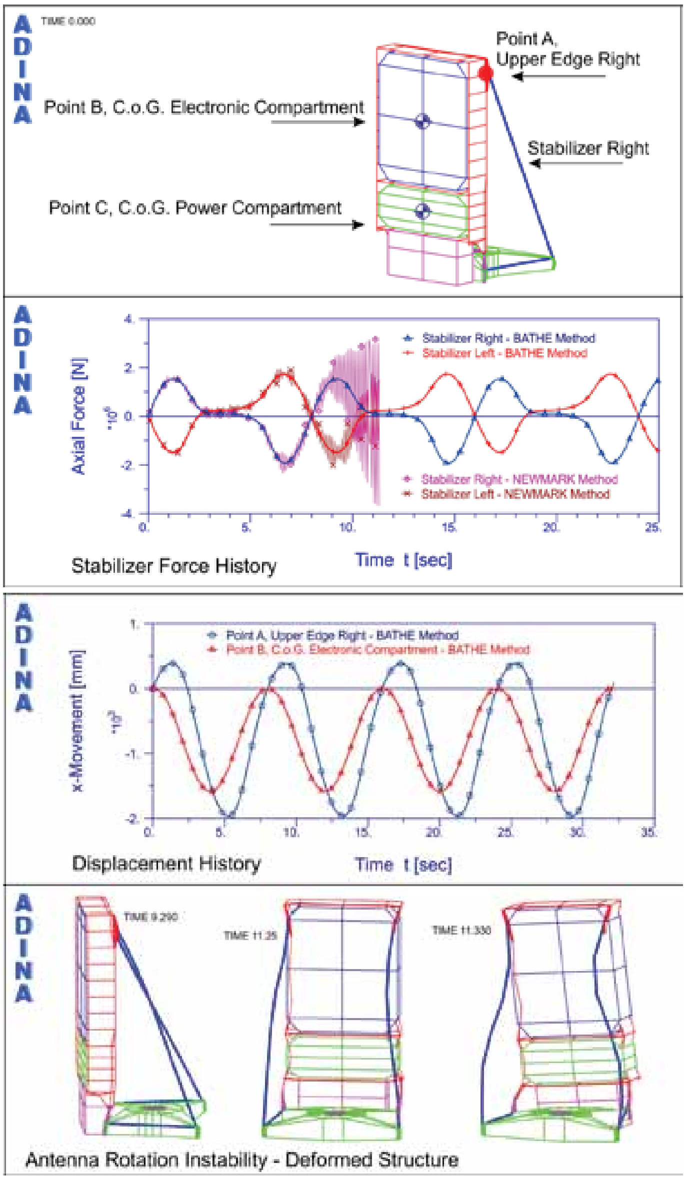


Figure 10: Predicted transient response of antenna using Newmark and Bathe Method

Although the above analyses focus on relatively simple problems, the mentioned solution phenomena are rather general and occur in many large-scale practical analyses of structures and fluid-structure interactions. In particular, considering contact problems, a spurious response of oscillatory nature can cause the nonlinear iterations not to converge.

While the above discussion refers to implicit integration, of course, explicit time integration is also widely used in practice. Using explicit integration, mostly wave propagation problems are considered, but structural vibration and even static problems are also solved.

Similar to the above observations regarding the trapezoidal rule, the predicted response obtained using the central difference method can show spurious oscillations in the high frequency modes [4]. These are frequencies and modes that cannot be represented by the chosen mesh. Ideally, any response in these modes would be automatically suppressed — but without loss of accuracy in the frequencies and modes that can be represented by the mesh.

Explicit Time Integration: Noh-Bathe Method

A new explicit time integration scheme, referred to as the Noh-Bathe method was developed with the same aim as for the implicit Bathe scheme [4]. The method automatically suppresses spurious high frequency response, without using any non-physical parameters, while accurately integrating those modes that can be spatially resolved. The computational cost of using the procedure is only slightly larger than the cost with the central difference method, when using the same mesh, but frequently coarser meshes can be used with the Noh-Bathe scheme.

Figures 11 and 12 show the analysis of the crushing of a tube. Figure 11 shows the deformations at three different times, and Figure 12 shows the acceleration-time solution curves of the impactor. We see that spurious oscillations are present in the central difference method solution, while the Noh-Bathe method solution does not show such oscillations.

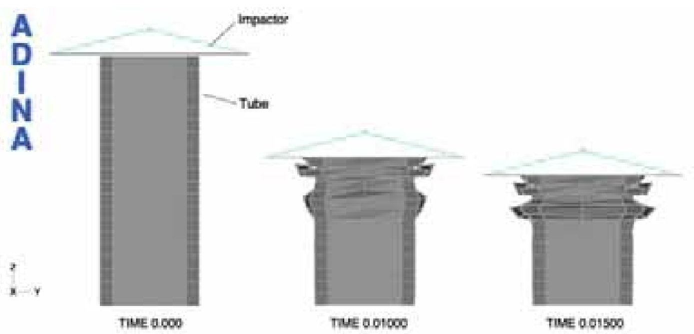


Figure 11: Tube-crush problem: Noh-Bathe method predicted deformations at t = 0.000, 0.010, and 0.015 s

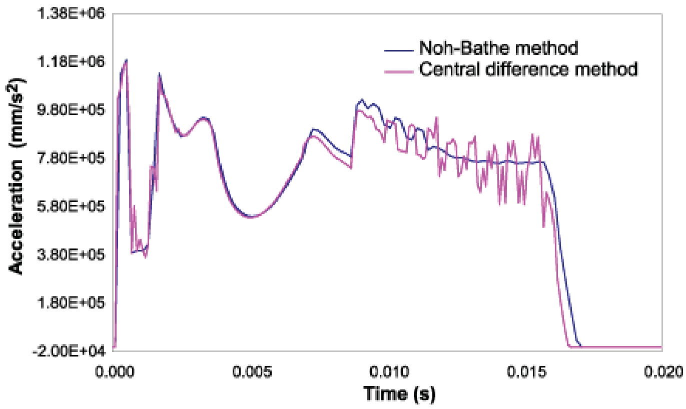


Figure 12: Impactor acceleration-time response for the tube

Further solutions of problems, algorithmic details and observations are given in the additional references [5-8].

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