On a new segment-to-segment contact algorithm

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Abstract

A new contact algorithm is presented which satisfies both stability and the contact patch test. The segment-to-segment algorithm involves a contact pressure interpolation and an accurate integration of the contact constraints over the surfaces of the contacting bodies. Numerical integration is carried out over sub-segments based on the element topologies of both contacting surfaces. The algorithm is applicable to both linear and quadratic element surface interpolations.

Keywords: Contact algorithm; Finite element solution; Stability; Patch test

1. Introduction

To guarantee stability and optimal convergence, contact formulations, like other mixed formulations, should satisfy an ellipticity and an inf–sup condition [1,2]. Furthermore, the contact algorithm should satisfy a contact patch condition, which describes its ability to represent a state of constant normal traction between two flexible contacting bodies. However, a review of the literature indicates that current contact algorithms do not satisfy both, the stability and contact patch conditions [3].

In this paper, we present a new contact algorithm, which satisfies both requirements. We classify the algorithm as a segment-to-segment procedure since it involves an accurate integration of the contact constraints over the surfaces of the contacting bodies, not just using values at the nodes. We describe the solution approach using 2D conditions but the theory is directly applicable to 3D conditions as well.

2. Contact formulation

Consider a system consisting of two bodies in contact (Fig. 1). Assuming infinitesimally small displacements, a linear elastic material and frictionless conditions, the contact problem can be expressed as a constrained minimization problem

$$\min_{\mathbf{v}\in K} [\Pi_A(\mathbf{v}) + \Pi_B(\mathbf{v})] \quad (1)$$

where $\mathbf{v}$ represents any admissible displacement, Π_I denotes the total potential of body I not accounting for contact effects, and K represents the set of functions satisfying the no-penetration contact constraint

$$K = \{\mathbf{v} \mid \mathbf{v} \in V;\ g(\mathbf{v}) \geq 0 \text{ on } \Gamma_C\} \quad (2)$$

where g is the gap,

$$V = \left\{\mathbf{v} \mid \mathbf{v} \in H^1;\ \mathbf{v} = \mathbf{0} \text{ on } \Gamma_D\right\} \quad (3)$$

and H^1 is the usual Sobolev space.

Using a Lagrange multiplier to enforce the contact constraint, and assuming contact, the minimization problem is

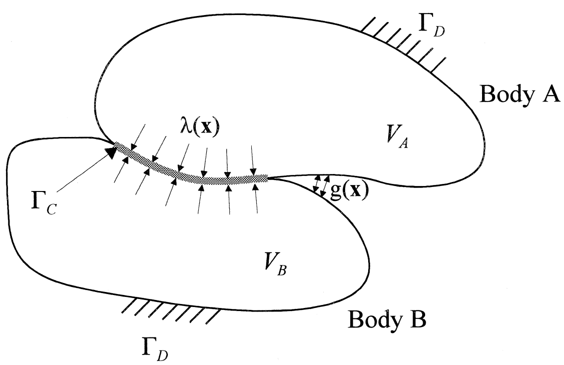

Fig. 1. Two bodies in contact.

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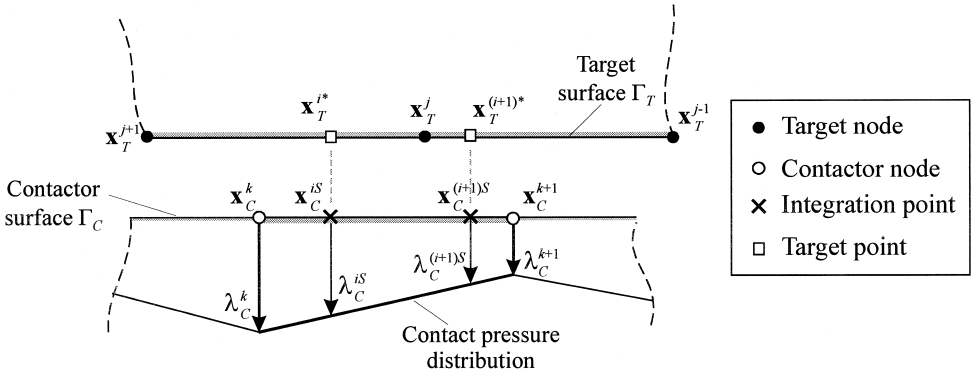

Fig. 2. Schematic of new contact algorithm.

converted to an unconstrained saddle point problem involving the following functional

$$\Pi_L(\mathbf{v}, \lambda) = \Pi_A(\mathbf{v}) + \Pi_B(\mathbf{v}) + \Pi_C(\mathbf{v}, \lambda) \tag{4}$$

where

$$\Pi_C(\mathbf{v}, \lambda) = \int_{\Gamma_C} \lambda g(\mathbf{v})\, \mathrm{d}\Gamma_C \tag{5}$$

and λ is the contact pressure which can only be zero or positive.

The variational form of the contact problem can be obtained by extremizing Eq. (4) with respect to the field variables $\mathbf{v}$ and λ. Note that the constraint function method can be used to solve the contact problem without the need for distinguishing between active and inactive contact constraints [1].

3. New contact algorithm

The algorithm involves a master–slave approach. One of the surfaces, Γ_C, is assumed to be the contactor, and the other, Γ_T, is the target as shown in Fig. 2. The contact constraint is evaluated at the integration points (not necessarily the nodes) along Γ_C. Let the superscript i denote an integration point. For a point with coordinates $\mathbf{x}_C^{iS}$, the displacement $\mathbf{v}_C^{iS}$ can be interpolated from the nodal displacements on Γ_C as follows:

$$\mathbf{v}_C^{iS} = \sum_k h_C^{ik} \mathbf{v}_C^k \tag{6}$$

where h_C^{ik} is the interpolation function (evaluated at point i) relating the displacement of the contactor point to the displacements of the contactor nodes. For each integration point on the contactor surface Γ_C the displacement of the target point on Γ_T is interpolated as follows:

$$\mathbf{v}_T^{i*} = \sum_j h_T^{ij} \mathbf{v}_T^j \tag{7}$$

We then assume that the discretized Lagrange multiplier space Q_h is

$$Q_h = \left\{ \lambda_h \mid \lambda_h \in H^{-1/2},\ \lambda_h|_{\hat{K}} \in P_i^j(\hat{K}) \right\} \tag{8}$$

where P_i^j denotes a polynomial of degree j, with C^i-continuity between elements, and $\hat{K}$ is a reference contact segment. The polynomial degree j must be less than or equal to that of the element interpolation, and the segments $\hat{K}$ are defined on Γ_C . Thus, the Lagrange multiplier value at integration point i is obtained as follows:

$$\lambda_C^{iS} = \sum_k H_C^{ik} \lambda_C^k \tag{9}$$

where the λ_C^k are the independent (usually nodal) multipliers on Γ_C and the interpolation function values H_C^{ik} depend on the polynomial degree and inter-element continuity of the contact pressure field.

The contact integral of Eq. (5) is then converted to a summation over the integration points (see Fig. 2)

$$\Pi_C = \sum_i \lambda_C^{iS} w^i [(\mathbf{v}_C^{iS} - \mathbf{v}_T^{i*}) \cdot \mathbf{N}^i + g_0^{iS}] \tag{10}$$

where w^i is the integration weight factor, $\mathbf{N}^i$ is the unit normal vector to measure the gap, and g_0^{iS} is the initial gap width; all given at integration point i.

It is important that we select a numerical quadrature rule that accurately evaluates the contact integral. This expression is piecewise continuous with possible discontinuities occurring at the nodes of *either* contact surfaces. Accordingly, any integration scheme involving integration points that are dictated by only one of the two surfaces cannot exactly evaluate Eq. (5) regardless of the number of integration points used. If, however, the integration intervals are based on 'sub-segments' corresponding to any two neighboring nodes regardless of their surface of origin, an exact evaluation is possible. This accurate integration feature enables the algorithm to pass the patch test for both linear and quadratic elements.

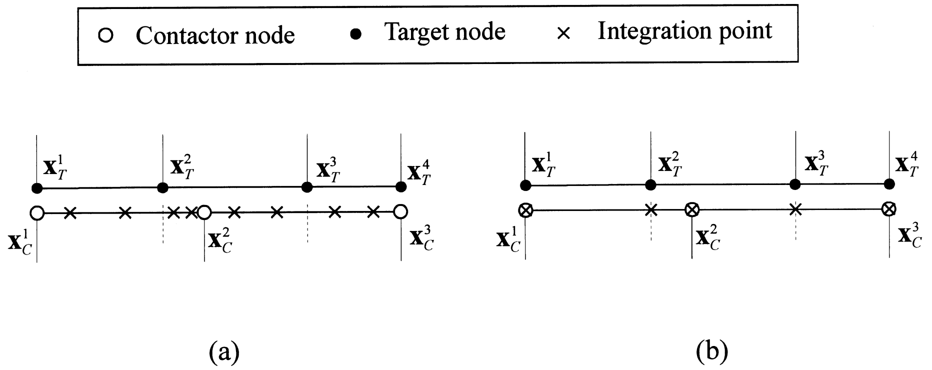

Fig. 3. Location of integration points based on: (a) Gaussian quadrature, and (b) trapezoidal rule.

Hence, the algorithm involves two main steps. In the first, the sub-segment boundaries are determined by projecting the nodes of the target surface onto the contactor surface (only the edge nodes need to be projected for quadratic and higher order elements). In the second step, the contact expression on each sub-segment is integrated using Gaussian or Newton–Cotes integration rules as shown in Fig. 3.

4. Stability and patch conditions for contact algorithms

Contact algorithms should satisfy the stability and patch conditions. Stability is represented by an ellipticity and an inf–sup condition. Satisfying the ellipticity condition depends on the use of appropriate finite elements and boundary conditions, not on the contact formulation. The inf–sup condition for contact problems can be represented as follows [3]

$$\inf_{\lambda_h \in Q_h} \sup_{\mathbf{v}_h \in V_h} \frac{\int_{\Gamma_C} \lambda_h g(\mathbf{v}_h)\, d\Gamma_C}{\|\lambda_h\|_{-1/2,\Gamma} \|\mathbf{v}_h\|_1} \geq \beta > 0 \tag{11}$$

The inf–sup condition is satisfied if the constant β is independent of the element size. The stability of the new contact algorithm has been assessed numerically, and it was found that with linear elements it is best to use a linear continuous pressure interpolation, whereas with quadratic elements the quadratic continuous pressure interpolation is optimal [3].

As mentioned above, the patch test is also passed by the algorithm [3].

5. Conclusions

A new segment-to-segment contact algorithm was developed which accurately evaluates the contact constraints between the contacting bodies. The algorithm provides optimal performance by satisfying both the stability and the contact patch conditions, using linear or quadratic element displacement interpolations. While the theory given here is directly applicable to 3D contact problems, the actual detailed solution algorithm needs still to be developed.

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