Keynote Paper: Remarks on the development of finite element methods and software

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Abstract: A brief personal, historical account of some developments of finite element methods and software is given, and then some recent advances are surveyed. The paper focuses on the research and developments of the author and his colleagues/students, and the developments of the programs SAP IV, NONSAP and ADINA.

Key words: ADINA, finite element methods, finite element software, history of FE development, NONSAP, SAP IV.

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1 INTRODUCTION

The finite element method has truly gone through a remarkable development. Since the early inception of the method for engineering use in the 1950s with the seminal papers by Turner *et al.* [1], Argyris and Kelsey [2], and Clough [3] the method has evolved into a most powerful procedure for general engineering analysis. Finite element programs are used today in a routine manner for the analysis of complex structures and solids, fluids, magnetic field problems and so on.

Courses on finite element analysis are taught at practically every major university, and the research and development conducted in the field has resulted in numerous publications [4]. Indeed, the method has become an integral part of computer-aided-engineering [5].

The objective in this paper is briefly to give some historical remarks regarding the author's earlier research and development in the field of finite element analysis and then to give some thoughts regarding the current state of the field. The paper is focused on the experiences of the author and therefore gives in various ways a rather subjective (indeed personal) view.

2 DEVELOPMENTS IN THE 1970s

Having left Germany after the completion of his high school diploma, the author was in the 1960s going through his undergraduate education at the University of Cape Town, South Africa. The educational experience was excellent in all respects, but in particular the desire for further studies of and possible developments in the computer analysis of structural systems was awakened. A tremendous potential for the use of computers in engineering analysis was recognized, and an excitement to work in this area evolved [6].

After graduate studies for an MS degree in Calgary, Canada, the author continued his graduate studies towards a doctoral degree at the University of California, Berkeley, under the stewardship of Professor E.L. Wilson. The structural engineering faculty and graduate students at Berkeley comprised at that time a world-renowned centre of research on finite element methods.

In his dissertation, the author developed the subspace iteration method for the solution of frequencies of large finite element systems [7]. The demand for more accurate dynamic analysis of buildings, dams, bridges and other civil engineering structures, immediately created much interest in this development and at present the subspace iteration method is used in many finite element programs as a primary procedure for calculating eigenvalues.

A most significant contribution in the development of finite element methods and software was the creation of the SAP series of programs by E.L. Wilson [8]. These programs contained state-of-the-art finite element procedures for structural analysis, and this author contributed to the development of SAP III [9] and SAP IV [10].

In addition to SAP IV, in the period 1972 to 1974, the NONSAP program, a nonlinear finite element code, was developed [11].

The SAP IV and NONSAP programs have been distributed since their publication in 1974 for a nominal fee by the University of California, Berkeley, and have found very wide use in industry and academe. Some of the methods used in these programs constitute valuable and general basic procedures for finite element analysis.

While these developments in the early 1970s were most significant, the field of finite element analysis has advanced very significantly since that time. These advances have taken place in establishing much improved finite elements, more general and powerful linear and nonlinear analysis procedures, the effective use of finite elements in computational fluid dynamics (CFD) and fluid-structure interactions, and so on. Within the endeavours to advance the field of finite element technology, the author has directed the development of the ADINA program system [12].

We shall point to some of the important advances in finite element methods in Section 4.

However, before summarizing these developments, we want to focus on one aspect of utmost importance, which frequently is not given sufficient recognition – namely, the need for *reliable* finite element procedures. As has been pointed out by this author repeatedly, only reliable methods are truly valuable in engineering practice.

3 THE NEED FOR RELIABLE FE METHODS

A most important aspect of a finite element technique and its program implementation – if the method is to be used in an industrial environment – is that the method and implementation be reliable. For example, the finite element formulation must be mechanistically clear and well-founded, the discretization must be stable and convergent and must not depend on adjustable numerical factors. Indeed, the reliability of a solution is much more important than the efficiency. While of course an efficient solution is very desirable, it is however quite inappropriate to aim for results in 'acceptable' computer run times if these results are not reliable and therefore should in any case be regarded as useless.

The present pressing competitive issues of our society, unfortunately, frequently render 'fast' computational methods more attractive than reliable methods. Considering advanced analysis, this means that it frequently appears more important to obtain 'any' results in a short period of time than to spend a little more effort (and more resources) to obtain reliable results that are truly useful for engineering design.

Of course, the development of solution methods that are reliable and effective – and that significantly advance the state-of-the-art of solution techniques – is a very difficult task. In research, such developments should be the only aim, because indeed only reliable techniques make a permanent contribution to the field.

In the following section, we briefly summarize advances in finite element methods that have been accomplished by the author's research and development groups during the recent years. The reader will notice that particular emphasis is placed on the reliability of the solution techniques (see for example [18, 26]). Of course, the field of research on finite element methods is now huge and the reader should seek other references to obtain a more balanced view [4].

4 SOME RESEARCH AND DEVELOPMENTS

The research and developments briefly mentioned in this section are not given in order of importance, but merely in a natural order from rather general to more specific contributions.

Mathematical conditions on finite elements

In this aspect of our work, we have studied the stability and accuracy conditions for mixed finite element formulations. The importance of the ellipticity and inf-sup conditions has been researched and the shortcomings of rules too simple for an assessment of stability have been delineated [13–15].

It is once again emphasised that elements based on simple reduced integration with spurious zero energy modes should not be used in engineering practice.

Selection of models

The selection of an appropriate mathematical model for the solution of a physical problem is the key step of any analysis [16]. It is clearly a difficult task in nonlinear analysis, but may also present some challenging issues in linear analysis. We have studied in particular the effects of different boundary conditions in the analysis of plate structures governed by the Reissner-Mindlin plate theory [16, 17].

Plates and shells

Our emphasis in this research area was on the development of new and effective plate and shell elements. As mentioned above, the reliability of these elements for practical analysis is of primary concern. This research resulted in the development of the MITC plate and shell elements that have no spurious zero energy modes and high predictive capability [18–22].

Incompressible analysis

Many materials behave as almost incompressible, and for the analysis of such material conditions special finite element formulations are needed [15]. We have developed the u/p (displacement/pressure) formulation for large

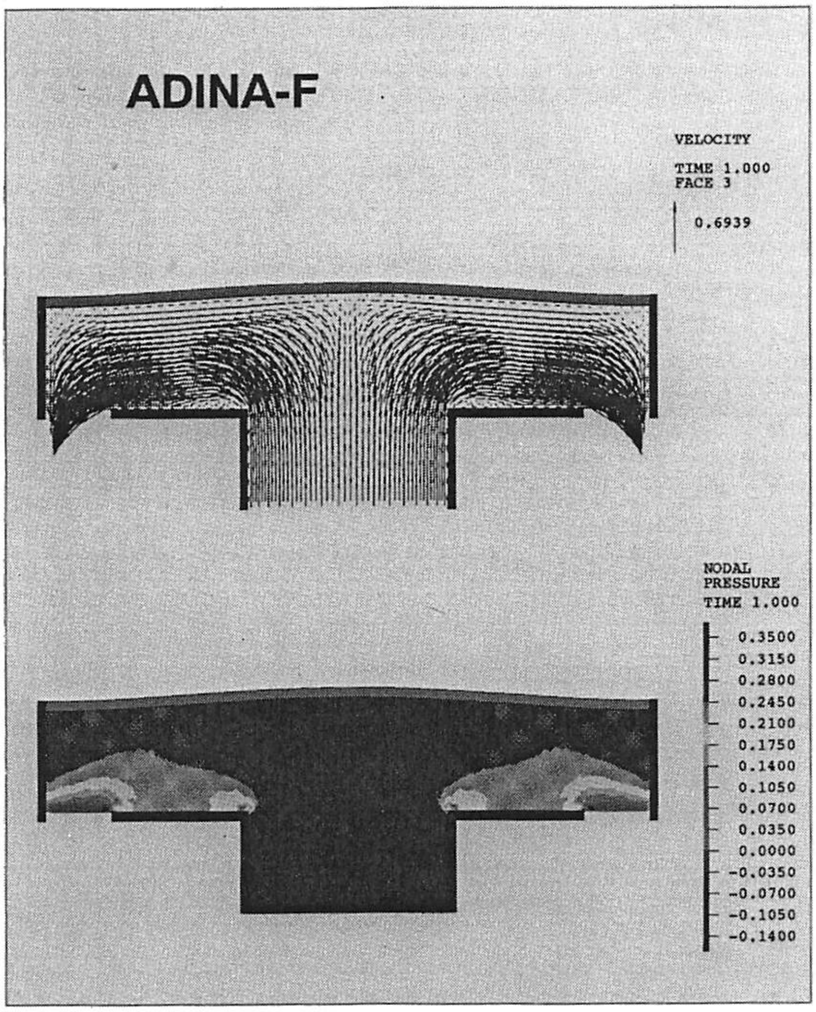

Figure 1(a) *Flow in a nozzle. Large deformations of initially horizontal cap. Re ~ 10^5.*

strains in elastic and inelastic response conditions [23, 24].

Elasto-plastic and creep analysis

The elasto-plastic and creep analysis of metals and geological materials continues to be a challenge, in particular for large strain conditions.

The solution approach we have developed consists of a reliable and effective procedure that is based on the total elastic strain computed from the total deformation gradient and an evolution of the plastic deformation gradient. The stresses are calculated using the effective stress function algorithm [25–27].

Modelling and analysis of concrete structures

The analysis of practical concrete structures requires a realistic but tractable concrete model as well as effective modelling features to include reinforcing steel, to possibly model the contact between steel tendons and the surrounding concrete, and so on. We have developed a concrete model that includes the effects of concrete cracking, crushing and strain-softening [28–29]. The model has found increasing use in engineering practice.

Analysis of contact problems

Many engineering analysis problems involve contact between bodies and we have developed a new algorithm for general contact analysis [30]. The solution procedure computes the actual area of contact and the conditions on the contact surfaces. The algorithm is based on constraint functions that when satisfied impose the contact conditions in the response of the finite element model.

Analysis of fluid flows and fluid-structure interactions

During the recent years, we have focused a considerable effort upon the development of solution techniques for fluid flows and fluid-structure interactions [31, 32]. The methods developed can now be used to solve very complicated fluid flows (modelled by the full Navier-Stokes equations) and fluid-structure interactions. Figure 1 shows the results of an analysis of a fluid flow structural interaction problem [33].

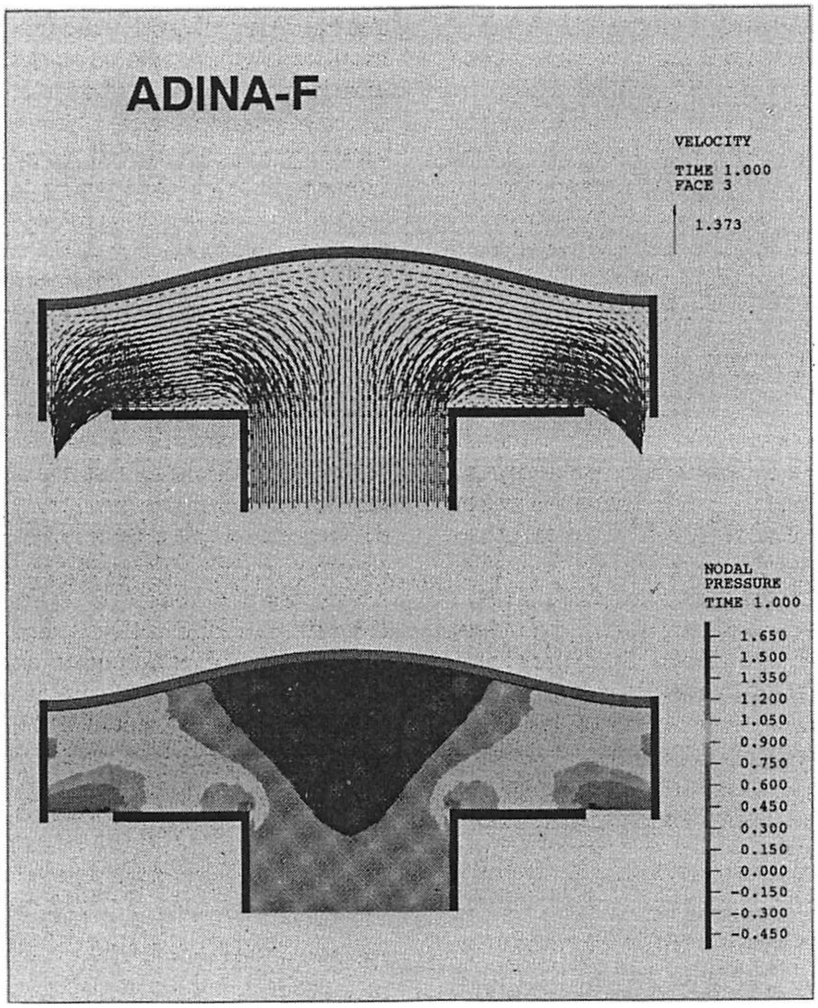

Figure 1(b) *Flow in a nozzle. Large deformations of initially horizontal cap. Re ~ 10^5.*

Error evaluation and mesh selection

The automatic evaluation of finite element discretization errors is a most important ingredient of a comprehensive analysis tool. Based on the observed discretization errors, the analyst can construct, with the available mesh generation procedures, new refined meshes and thus continue the analysis process until the required accuracy is achieved.

We have proposed and extensively used the iso-bands of stresses as an 'eyeball-norm' for an engineer to assess the accuracy of solution [34], and have studied elements [35] and developed error measures and procedures for adaptive refinement of finite element meshes [24, 36].

Solution of large systems

A most significant aspect is that effective iterative solvers are now available for the solution of large finite element systems on PCs and engineering workstations (EWS) [33]. The reduction of storage used, by factors of 10 and more, means that much larger systems than before can now be solved on EWS, and the reduction of solution time means a direct increase in productivity of the engineering analyst. Figures 2 and 3, and Tables 1 and 2, summarize some experiences with the solution of finite element systems, and in particular give the reduction in storage and solution times reached by the iterative solution (compared to the standard direct solution) [33, 37].

Table 1 *Static analysis of the dam of Figure 2.*

Number of equations: 22 103
Elements used: 3-D SOLID, 20-node elements, 8-node SHELL, MITC8 elements
Type of analysis: linear static
Number of iterations (ITESOL): 547
Computer: DEC Alpha AXP 3000 workstation

	Direct solver	*Iterative solver*	*Ratio*
Elapsed time	2179 s (~1/2 hr)	507 s (~8 min)	4.30
Storage Requirements (words)	30 322 866	2 614 260	11.6

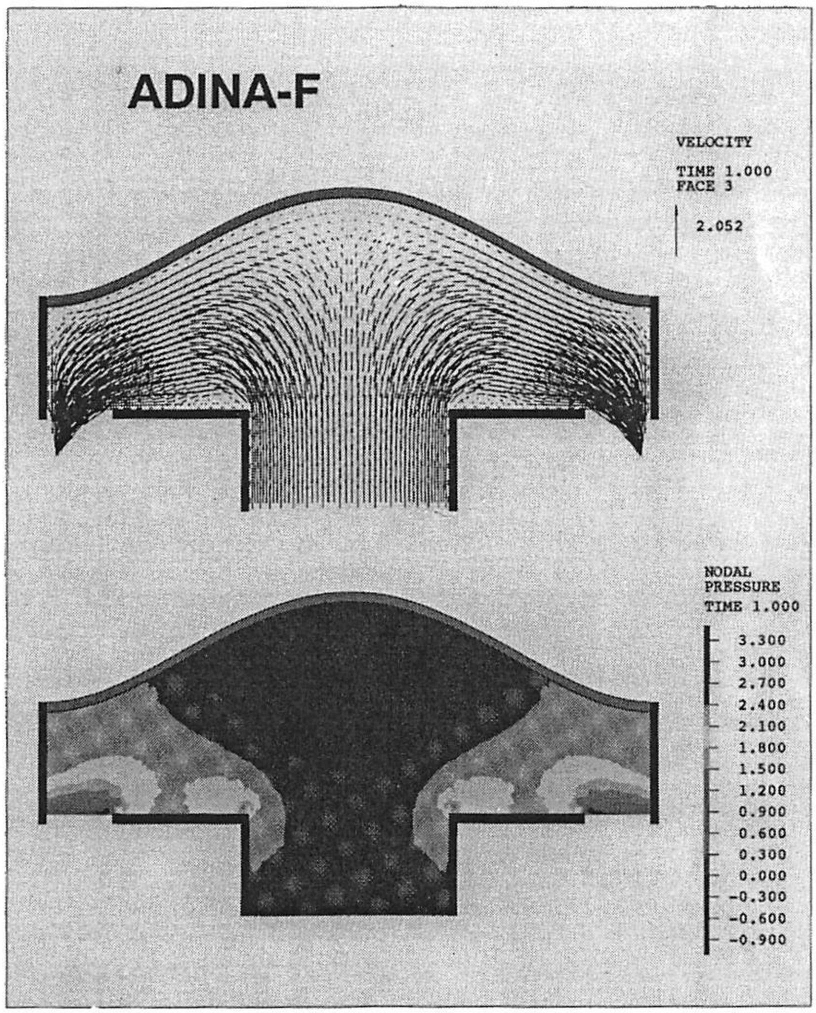

Figure 1(c) *Flow in a nozzle. Large deformations of initially horizontal cap. Re ~ 10^5.*

Table 2 *3-D contact analysis of the F16 jet fighter wheel in Figure 3.*

Number of equations: 51 159
Elements used: 3-D SOLID, 20-node elements
Type of analysis: elastic with 3-D contact
Number of Newton-Raphson iterations: 8
Mean number of iterations per equation solution (ITESOL): 759
Computer: DEC Alpha AXP 3000 workstation

	Direct solver	*Iterative solver*	*Ratio*
Elapsed time	49 987 s (~14 hrs)	15 131 s (~$4\frac{1}{4}$ hrs)	3.30
Storage Requirements (words)	119 210 734	9 523 056	12.5

5 CONCLUDING REMARKS

Research and development in, and teaching of, finite element methods has been very exciting. Intensive research and development in the area of finite element procedures is continuing, in many research centres, and significant further enhancements must be anticipated. For example, developments in the following areas will be very valuable:

- further improvements in finite element methods, in practically all fields of analysis, but in particular for fluid flows and coupled problems;
- enhancements in the automatic meshing and error control in finite element analysis;
- improved integration of the use of finite element methods in computer-aided-design;
- improvements in the process of interaction of the user with finite element software.

In all these areas significant advances must be expected, but as pointed out in this paper, particular emphasis should be directed towards the reliability of the methods proposed for use in engineering practice.

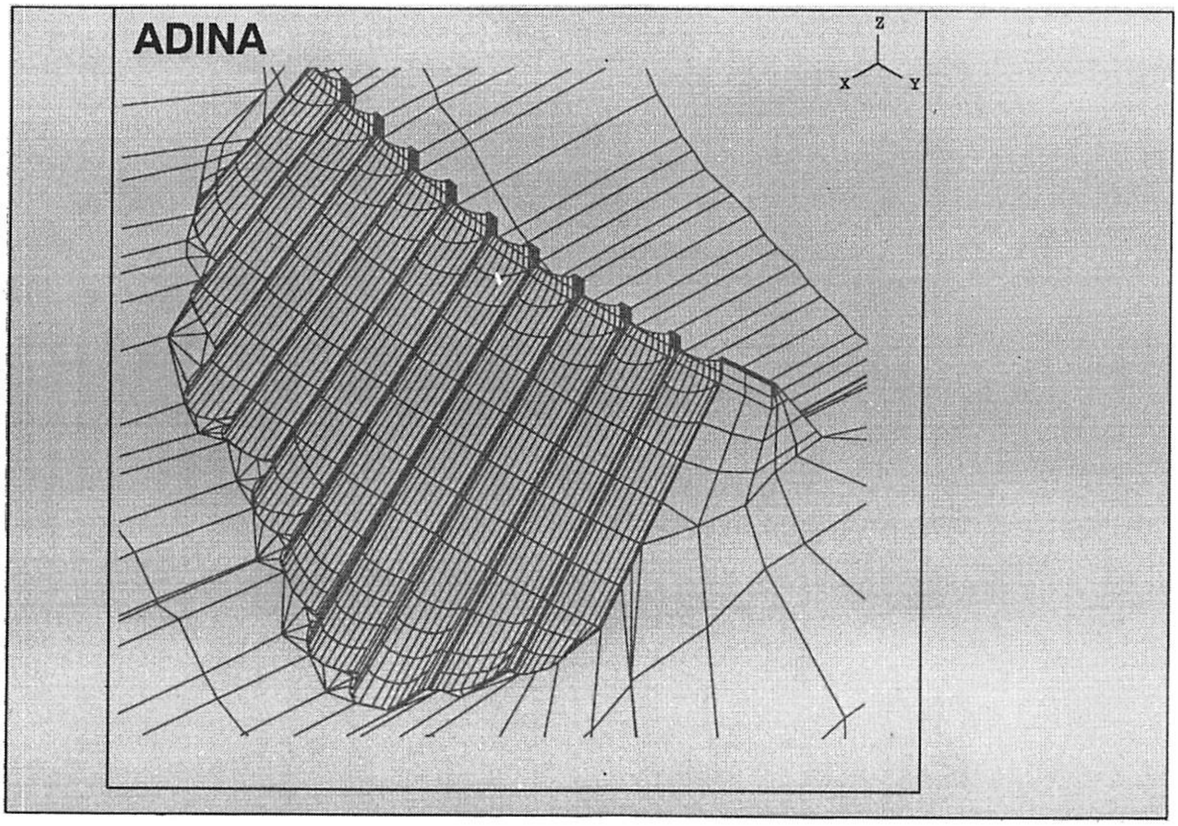

Figure 2 *View of model of dam.*

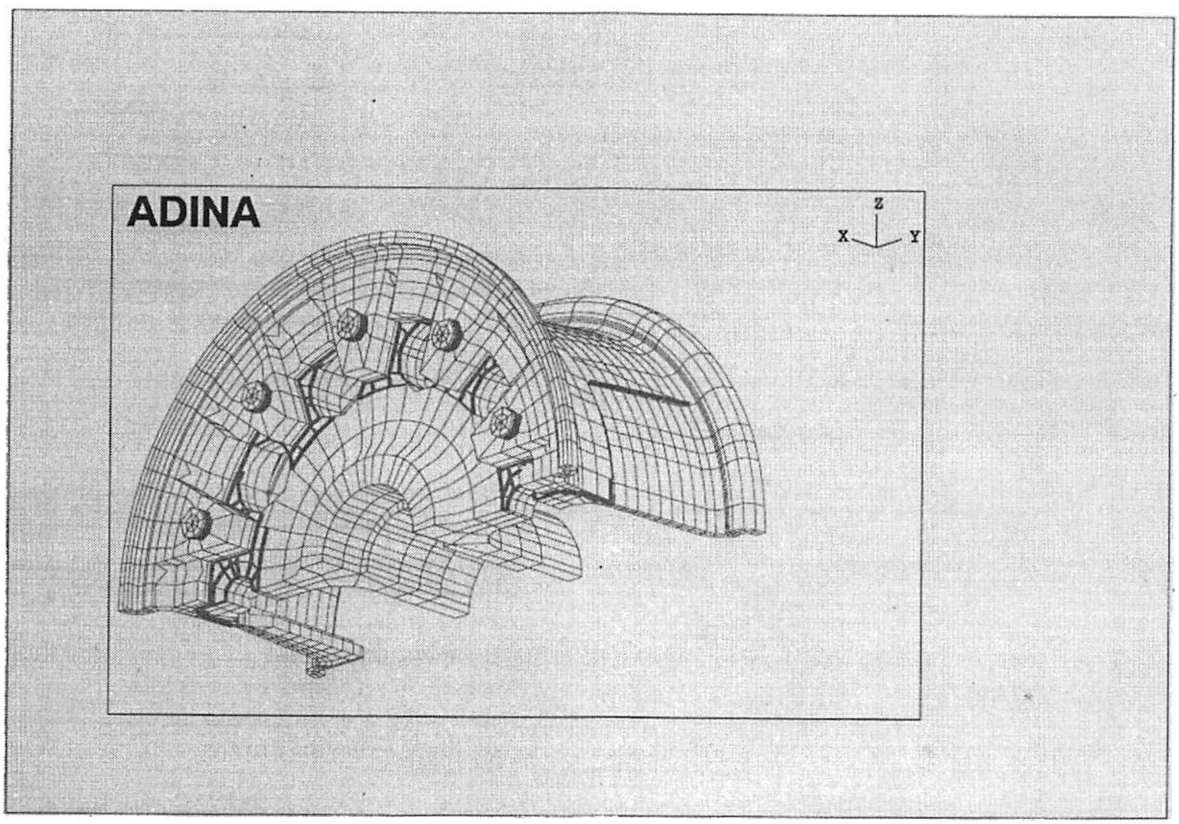

Figure 3 *View of model of F16 jet fighter wheel.*

REFERENCES

1 Turner, M.J., Clough, R.W., Martin, H.C. and Topp, L.J. (1956) Stiffness and deflection analysis of complex structures, *Journal of Aeronautical Science*, Vol. 23, pp. 805–823.
2 Argyris, J.H. and Kelsey, S. (1955) Energy theorems and structural analysis', *Aircraft Engineering*, Vols. 26 and 27.
3 Clough, R.W. (1960) 'The finite element in plane stress analysis', Proceedings, 2nd ASCE Conference on Electronic Computation, Pittsburgh, PA, September.
4 Noor, A.K. (1991) 'Bibliography of books and monographs on finite element technology', ASME, *Appl. Mech. Rev.*, Vol. 44, No. 6, June.
5 Bathe, K.J. *Finite Element Procedures*, Prentice-Hall, Inc., in press.
6 Bathe K.J. (1967) 'The use of the electronic computor in structural analysis', *Impact*, Journal of the University of Cape Town Engineering Society , pp. 57–61.
7 Bathe, K.J. (1971) 'Solution methods for large generalized eigenvalue problems in structural engineering', Report UC SESM 71–20, Department of Civil Engineering, University of California, Berkeley, November.
8 Wilson, E.L. (1971) 'Solid SAP – a static analysis program for three dimensional solid structures', Report UC SESM 71–19, Department of Civil Engineering, University of California, Berkeley.
9 Wilson, E.L., Bathe, K.J., Peterson, F.E. and Dovey, H.H. (1972) *Computer Program for Static and Dynamic Analysis of Linear Structural Systems*, Earthquake Engineering Research Centre, Report No. 72–10, November.
10 Bathe, K.J., Wilson, E. L. and Peterson, F.E. (1974) 'SAP IV – a structural analysis program for static and dynamic response of linear systems', Earthquake Engineering Research Center Report No. 73–11, College of Engineering, University of California, Berkeley, June 1973, revised April 1974.
11 Bathe, K.J. and Wilson E.L. (1973) 'NONSAP – a general finite element program for nonlinear dynamic analysis of complex structures', Paper No. M3-1, Proceedings of the Second Conference on Structural Mechanics in Reactor Technology, September.
12 Bathe, K.J. (ed.) (1991/93) 'Nonlinear finite element analysis and ADINA', *J. Computers & Structures*, Vol. 40, No. 2, 1991 and Vol. 47, No. 4/5, 1993.
13 Brezzi, F. and Bathe, K.J. (1986) 'Studies of finite element procedures — the Inf-Sup condition, equivalent forms and applications', in *Reliability of Methods for Engineering Analysis* (K.J. Bathe and D.R.J. Owen, eds.), Pineridge Press.
14 Brezzi, F. and Bathe, K.J. (1990) 'A Discourse on the stability conditions for mixed finite element formulations', *J. Computer Methods in Applied Mechanics and Engineering*, 82, pp. 27–57.
15 Chapelle, D. and Bathe, K.J. (1993) 'The Inf-Sup test', *J. Computers & Structures*, Vol. 47, No. 4/5.
16 Bathe K.J., Lee, N.S. and Bucalem, M.L. (1990) 'On the use of hierarchical models in engineering analysis ', *J. Computer Methods in Applied Mechanics and Engineering*, 82, pp. 5–26.
17 Häggblad, B. and Bathe, K.J. (1990) 'Specifications of boundary conditions for Reissner/Mindlin plate bending finite elements', *Int. J. Num. Methods in Engineering*, 30 pp. 981–1011.
18 Bathe, K.J. and Dvorkin, E.N. (1986) 'A formulation of general shell elements – the use of mixed interpolation of tensorial components', *Int. Journal for Numerical Methods in Engineering*, 22, pp. 697–722.
19 Bathe, K.J. and Brezzi, F. (1987) 'A simplified analysis of two plate bending elements – the MITC4 and MITC9 elements', Proceedings, Numerical Methods in Engineering: Theory and Applications, Univ. College Swansea, Wales, July.
20 Brezzi, F., Bathe, K.J. and Fortin, M. (1989) 'Mixed-interpolated elements for Reissner/Mindlin plates', *Int. J. Num. Methods in Engineering*, 28, pp. 1787–1801.
21 Bathe, K.J., Bucalem, M. and Brezzi, F. (1990) 'Displacement and stress convergence of our MITC plate bending elements', *J. Eng. Computations*, 7, no. 4, pp. 291–302.
22 Bucalem, M.L. and Bathe, K.J. (1993) 'Higher-order MITC general shell elements', *Int. J. for Numerical Methods in Engineering*, Vol. 36, pp. 3729–3754.
23 Sussman, T. and Bathe, K.J. (1987) 'A finite element formulation for nonlinear incompressible elastic and inelastic analysis', *J. Computers & Structures*, 26, No. 1/2, pp. 357–409.
24 Kato, K., Lee, N.S. and Bathe, K.J. (1993) 'Adaptive finite element analysis of large strain elastic response', *J. Computers & Structures*, Vol. 47, No. 4/5, pp. 829–855.
25 Kojić, M. and Bathe, K.J. (1987) 'The "effective stress-function" algorithm for thermo-elasto-plasticity and creep', *Int. J. for Numerical Methods in Engineering*, 24, pp. 1509–1532.
26 Eterovic, A.L. and Bathe, K.J. (1990) 'A hyperelastic-based large strain elasto-plastic constitutive formulation with combined isotropic-kinematic hardening using the logarithmic stress and strain measures'. *Int. J. Numerical Methods in Engineering*, 30, pp. 1099–1114.
27 Eterovic, A.L and Bathe, K.J. (1990) 'On large strain elasto-plastic analysis with frictional contact conditions', Proceedings, Conf. Numerical Methods in Applied Science and Industry, Politecnica di Torino, Torino, Italy, June.
28 Bathe, K.J., Walczak, J., Welch, A. and Mistry, N. (1989) 'Nonlinear analysis of concrete structures', *J. Computers & Structures*, 32, No. 3/4, pp. 563–590.
29 Bathe, K.J. and Walczak, J. (1990) 'On the nonlinear analysis of reinforced concrete structures', Proceedings, 2nd Int. Conference on Computer Aided Analysis and Design of Concrete Structures, Zell am See, Austria, April.
30 Eterovic, A. and Bathe, K.J. (1991) 'On the treatment of inequality constraints arising from contact conditions in finite element analysis', *J. Computers & Structures*, 40, No. 2, pp. 203–209.
31 Bathe, K.J. and Dong, J. (1989) 'Studies of finite element procedures – the use of ADINA-F in fluid flow analyses', *J. Computers & Structures*, 32, No. 3/4 pp. 499–516.
32 Nitikitpaiboon, C. and Bathe, K.J. (1993) 'An arbitrary Lagrangian-Eulerian velocity potential formulation for fluid-structure interaction', *J. Computers & Structures*, Vol. 47, No. 4/5, pp. 871–891.
33 Bathe, K.J., Walczak, J. and Zhang, H. (1993) 'Some recent advances for practical finite element analysis', *J. Computers & Structures*, Vol. 47, No. 4/5, pp. 511–521.
34 Sussman, T. and Bathe, K.J. (1986) 'Studies of finite element procedures – stress band plots and the evaluation of finite element meshes', *J. Engineering Computations*, 3, No. 3, pp. 178–191.
35 Lee, N.S. and Bathe, K.J. (1993) 'Effects of element distortions on the performance of isoparametric elements', *Int. J. for Numerical Methods in Engineering*, Vol. 36, pp. 3553–3576.
36 Lee, N.S. and Bathe, K.J. (1994) 'Error indicators and adaptive remeshing in large deformation finite element analysis', *Finite Elements in Analysis and Design*, 16, pp. 99–139.
37 Tan, L.-H. and Bathe, K.J. (1991) 'Studies of finite element procedures – the conjugate gradient and GMRES methods in ADINA and ADINA-F', *J. Computers & Structures*, 40, No. 2, pp. 441–449.