Asset Allocation with Conventional and Indexed Bonds

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1. Introduction

In January 1997 the U.S. Treasury began issuing 10-year inflation-protected bonds with principal and interest payments linked to the consumer price index of all urban consumers. We refer to these as indexed or inflation-linked bonds. In this paper we examine whether and how the availability of indexed bonds might affect investors' asset allocation decisions. We also study whether the availability of indexed bonds enables investors to construct superior mean-variance efficient portfolios.[1] The latter question is important. A change in an investor's asset allocation, without an accompanying increase in the risk-return trade-off, may not improve the investor's welfare. However, an improvement in the expected return for a given level of risk would certainly be attractive to investors.

Indexed bonds have long been a topic of interest to the investment community and government policymakers (see Campbell and Shiller, 1996). For example, advocates of indexed bonds have argued the benefits of such bonds to retirees and other investors who are vulnerable to inflation risk. Other benefits, like lowering financing costs to the Treasury, have also been suggested. Somewhat surprisingly, there has been limited work on the impact of the availability of indexed bonds on investors' asset allocation decision. Should investors hold a different mix of stocks and bonds in the presence of indexed bonds than otherwise? How does this change the risk-return trade-off facing investors? We offer guidance on this important question.

To examine the effect of indexed bonds on asset allocation, we compare the properties of a portfolio of stocks and indexed bonds with a portfolio of stocks and non-indexed or conventional bonds. Mean, variance, and covariance measures for stock and bond returns are examined. Since investors should, in principle, be concerned with the real purchasing power of

[1] A mean-variance efficient portfolio offers the maximum expected rate of return for a given level of risk, as measured by the standard deviation of portfolio return.

their nominal wealth, we emphasize results based on real returns, but present results using nominal returns as well.

While historical time series of stock and conventional bond prices are available, the same is not true for indexed bonds in the U.S. The U.S. Treasury only began issuing inflation-protected bonds in 1997. Therefore, we construct a time series of hypothetical zero-coupon indexed bond prices as if the bonds were available since the 1950s.[2] The indexed bond prices are calculated each month by discounting the payoff to a hypothetical zero-coupon bond by an estimated real rate of interest (see sections 2 and 3 for details). Returns are then computed from the hypothetical indexed bond prices. Optimal asset allocations are estimated using historical stock returns, together with the hypothetical indexed bond returns or historical returns on conventional bonds.

The results show that there can be considerable diversification benefits for investors using indexed bonds, rather than conventional bonds. Using data from 1953 to 1998, we find that the real returns we estimate for hypothetical indexed bonds are virtually uncorrelated with stock returns, whereas the non-indexed bonds exhibit strong positive correlation near 0.4. This is not surprising since numerous studies (e.g., Fama and Schwert, 1977, and Fama, 1981) show that stock prices, as well as conventional bond prices, react negatively to news of increased inflation. In addition, we find that real indexed bond returns are less variable than real non-indexed bond returns, thus favorably contributing to the risk-return tradeoff as well.

Naturally the risk-reduction benefits of indexed bonds must be weighed against the possibility of a reduction in expected return. Interest rates on conventional bonds can be viewed as compensation for expected inflation plus an expected real rate of return. This expected real rate, in turn, equals the real riskless rate on an indexed bond of the same maturity plus a premium

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[2] We implicitly assume that the covariance matrix of stock and conventional bond returns would not have been materially affected by the existence of indexed bonds. In principle, some endogenous adjustment might be expected, but consideration of such issues is beyond the scope of this research.

for inflation risk. If the inflation risk premium is positive, as might be expected, then indexed bonds will have lower expected returns than conventional bonds. However, as Campbell and Shiller note, in theory the "premium" could even be negative.

The pricing of indexed bonds currently traded in the market suggests that this may indeed be the case. The yield on conventional ten-year Treasury bonds at the end of 1999 was 6.44%, while the yield on ten-year indexed bonds was 4.37%. Therefore, the sum of expected inflation and the inflation risk premium was only a little more than 2% per year. Since expected inflation was apparently at least 2.5% at this time (see Monetary Trends, 2000, p. 8), the implied inflation risk premium is negative. Our base computation of indexed bond returns assumes the inflation risk premium is zero, but we also consider alternative scenarios.

Our current study is preliminary and could be expanded along several lines. First, we consider a one-year investment horizon and restrict ourselves to mean-variance analysis. In future work, we will extend the analysis to a multi-year horizon. Second, estimates of expected inflation play a critical role in estimating the prices of hypothetical zero-coupon indexed bonds. Future work will consider the effect of instability in the coefficients of the inflation-forecasting model. In addition, expectations based on survey data will be explored. Third, we examine the effect of the availability of five-year zero-coupon indexed bonds on asset allocation. However, indexed coupon bonds with maturities of up to 30 years are available. Variations in coupon and maturity will be considered in the future. We will also look at a broader asset allocation decision in which investment in both indexed and conventional bonds is considered, along with investment in stocks.

Fourth, we examine asset allocation based on our estimates of return variances and correlations. Future work will consider the sampling properties of these estimates, which are based on overlapping monthly stock- and bond-return data, through the use of simulations and bootstrapping. Finally, we work with the unconditional covariance matrix of returns here. It is likely that the covariance matrix changes over time, however, and it may be possible to model some of these changes in terms of observable variables that can be incorporated into the asset allocation decision.

Section 2 describes the research design at an intuitive, non-technical level. We present the inflation forecasting models, calculations of the real interest rates and prices of hypothetical indexed bonds, and descriptive statistics for inflation forecasting models and bond returns in section 3. An analysis of diversification across stocks and indexed or conventional bonds is given in section 4. The main results on asset allocation with indexed or conventional bonds, using nominal and real return data, appear in section 5. We summarize and conclude in section 6.

2. Research design

Our research design for assessing the potential effect of the availability of indexed bonds on investors' asset allocation decisions is intuitive and straightforward. We estimate the time series of annual returns on hypothetical indexed bonds from June 1953 to December 1998. Using these hypothetical bond returns and the time series of returns on the value-weighted portfolio of stocks, we compute the stock-bond allocation for the mean-variance optimal portfolio. We report the Sharpe ratio and M^2 value for the optimal portfolio and for portfolios with various stock-bond weights.

The Sharpe ratio is the ratio of mean excess return to standard deviation and is a commonly used portfolio reward-to-risk measure. The M^2 value is the mean excess return on a portfolio, levered up or down at the riskless rate, so as to have the same standard deviation as the stock market index; M^2 equals the Sharpe ratio of the portfolio times the standard deviation of the stock index return. M^2 provides the same ranking of portfolios as the Sharpe ratio, but is perhaps easier to interpret in that portfolios are compared on the basis of a return metric with risk held constant. Examining statistics for a range of portfolios provides some indication of the sensitivity of these performance measures to deviations from the estimated optimal weights. We also compare the optimal portfolio's stock-bond allocation with that of an optimal portfolio using conventional bonds and stocks.

Once a time series of returns on our hypothetical bonds is estimated, optimal portfolio construction employs standard optimization techniques in finance. The construction of a return series on hypothetical indexed bonds, however, is largely uncharted territory. We begin by assuming that the indexed bonds are five-year zero-coupon bonds. Corresponding non-indexed Treasury bond returns can be computed for the post-1953 period. Annual real return on an indexed bond equals the change in the discounted present value of the real principal, as a result of any changes in real riskless rates from the beginning to the end of the year, divided by the initial value.

To calculate this annual real return, we estimate a five-year indexed bond's real price at the beginning and end of the first year. Nominal return on the same bond is obtained by subtracting one from (1+ Real return)*(1 + Realized inflation). Price at the beginning of the first year, i.e., at the time of issuing the bond, is the present value of the principal calculated using the five-year real interest rate. The same bond's price at the end of the year is computed using the four-year real interest rate. However, since real interest rates are not observed, they must be estimated.

The difference between the nominal interest rate and the real interest rate is due to the market's expected inflation and an inflation risk premium. Neither expected inflation nor the inflation risk premium is observable. In estimating the real interest rate to be used in calculating the initial price of a five-year indexed bond, we subtract forecasted inflation for five years from the nominal yield on a conventional five-year bond. The price of the five-year bond at the end of the first year is calculated similarly, using the real interest rate on a four-year bond.[3] Thus, the inflation risk premium is ignored at this stage. The effect of a positive risk premium is considered later in our asset allocation analysis.

3. Estimation

In this section, we describe the estimation procedures that generate our time series of annual returns on hypothetical zero-coupon five-year indexed bonds. The empirical analysis begins with the estimation of expected inflation out to five years. We then back out real spot rates of interest. Finally, we estimate the time series of annual returns on hypothetical five-year zero-coupon indexed bonds. We present descriptive statistics at each stage of the estimation. After estimating expected inflation, we separately report the entire analysis using nominal and real (i.e., deflated by realized inflation for the year) returns.

All analysis is based on monthly time series of overlapping annual observations. Reported standard errors correct for biases due to serial dependence that arises from the use of overlapping data. The Newey and West, 1987, standard errors correct for heteroskedasticity as well as autocorrelation. All returns and inflation rates used in estimating expected inflation and calculating prices of hypothetical indexed bonds are continuously compounded in this section unless otherwise noted. Working with logs simplifies the analysis in that the usual discounted cash flow procedure becomes additive rather than multiplicative. However, the mean-variance portfolio analysis in section 4 uses buy-and-hold returns, not continuously compounded returns.

3.1 Estimating expected inflation

Since we analyze zero-coupon bonds with five-year maturity, we need forecasts of inflation up to five years out, denoted years t+1 to t+5. Unlike other studies, we forecast changes

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[3] More precisely, we work with log real prices and log (one plus) real returns following Barr and Campbell, 1997. We ignore the 3-month indexation lag. Prices are then backed out from the log prices and used to compute the actual return series. Further details are provided in section 3.2.

in adjacent-year inflation rates beyond the first year, rather than the levels. Insofar as expected inflation is well approximated by a random walk in univariate time series analysis (e.g., Nelson and Schwert, 1977, Fama and Gibbons, 1984), forecasted changes should be identically zero. In this case, working with inflation *levels* would require forecasts over an indefinitely long horizon. If there is some degree of mean-reversion in expected inflation, forecasted changes won't equal zero near-term, but will be approximately zero looking sufficiently far out. Our regression analysis suggests that there is no ability to forecast *changes* in inflation beyond the third year. Therefore, expected inflation for the fourth and fifth years is assumed to equal forecasted inflation for the third year. The forecasted changes from the first to the second and from the second to the third years are added to the forecasted level of the first year's inflation to give forecasts of two- and three-year-ahead inflation rates.

Inflation-forecasting model. We now describe the inflation-forecasting models for one-year-ahead inflation level and two- and three-year-ahead changes in inflation. Our regression approach implicitly assumes that investors have rational expectations that can be estimated from the *ex post* relation between realized inflation and the forecasting variables.

We forecast one-year-ahead inflation by regressing realized inflation for year t+1, Inf_{t+1}, on the nominal spot interest rate, Int_{t+1}, the spread between the nominal yield on a five-year zero-coupon bond and the spot interest rate, $Yld_{t,t+5} - Int_{t+1}$, lagged one-year inflation, Inf_t, and the sum of realized real returns on one-month bills over the past 12 months, $Realbill_t$.

Inclusion of the spot rate follows the early work of Fama, 1975, and is based on the simple observation that interest rates should impound expectations of future inflation. Insofar as expected real rates vary over time, however, the relation between spot rates and expected inflation is not perfect. Thus, forecasting power may be enhanced by the inclusion of other variables. The addition of lagged inflation is motivated by the work of Nelson and Schwert, 1977, who find that information in past inflation rates has incremental explanatory power beyond the spot rate for forecasting near-term inflation. The yield spread is not standard in this context, and is included as a general proxy for business conditions (Fama and French, 1989), which may be correlated with expected real rates. Similarly, the sum of real bill returns serves as a direct, albeit noisy, measure of expected real rates, as in the work of Fama and Gibbons, 1984.[4]

Formally, we have

$$Inf_{t+1} = b_0 + b_1\, Int_{t+1} + b_2\, (Yld_{t,t+5} - Int_{t+1}) + b_3\, Inf_t + b_4\, Realbill_t + e_{t+1}. \qquad (1)$$

The forecasting model for the change in inflation, from year t+1 to t+2, is

$$Inf_{t+2} - Inf_{t+1} = b_0 + b_1\, (Fint_{t+2} - Int_{t+1}) + b_2\, Inf_t + b_3\, Realbill_t + e_{t+2}, \qquad (2)$$

where $Fint_{t+2}$ is the time t forward interest rate for year t+2. This equation is motivated by the expectations hypothesis, in which the forward rate equals the expected spot rate for year t+2 which, as mentioned above, should incorporate information about expected inflation for that year. We then use the forward - spot spread to capture differences in expected inflation for years t+2 and t+1. Forecasted inflation for year t+2, $E_t(Inf_{t+2})$, equals $E_t(Inf_{t+1})$ plus the forecasted change in inflation from equation (2). The forecasting model for the change in inflation from year t+2 to t+3 is similar to equation (2), except that the difference between the forward rates for years t+3 and t+2 is used:

$$Inf_{t+3} - Inf_{t+2} = b_0 + b_1\, (Fint_{t+3} - FInt_{t+2}) + b_2\, Inf_t + b_3\, Realbill_t + e_{t+3} \qquad (3)$$

To estimate models (1) to (3), we use the following data. Annual inflation is calculated from the Consumer Price Index, CPI, as reported on the Wharton WRDS data base. We obtain continuously compounded annual inflation rates by taking the natural logarithm of the ratio of the CPI numbers one year apart. We use the Fama and Bliss, 1987, continuously compounded

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[4]We have also experimented with inflation forecasts from integrated moving average models of order one for monthly inflation, but find little difference between these results and those based on lagged inflation for one year. This is consistent with the fact that the moving average parameter is around –0.8, so that the forecast is highly correlated with an average of past monthly inflation rates. A similar model for expected real returns on one-month bills has also been examined.

zero-coupon bond yields from WRDS. The one-year yield serves as the one-year spot interest rate. We construct the forward rate for year t+k+1, $Fint_{t+k+1}$, by subtracting the total yield on the k-period zero-coupon bond from t to t+k from the total yield on the k+1-period bond from t to t+k+1. Spreads are then computed from the yields and forward rates.

The results of estimating models (1) through (3) appear in table 1. As expected, the spot rate is significantly positively related to one-year-ahead inflation. The real bill return variable is significantly negatively related to next year's inflation; holding the spot rate constant, a higher expected real rate implies lower expected inflation. Overall, the model explains over 70% of the variation in next-year's inflation rate and exhibits a statistically significant fit (i.e., a significant F-statistic).

[Table 1]

The forward spreads are significantly positively related to the changes in inflation two and three years ahead. That is, forward spreads contain reliable information about future inflation. The coefficient on lagged inflation is significantly negative for model (2), reflecting a degree of mean-reversion in inflation, with high inflation last year forecasting a negative change in inflation. This indication of mean-reversion implies a rejection of the random walk model for expected inflation. Like the one-year-ahead model, the two- and three-year-ahead inflation-forecasting models also exhibit a statistically significant fit.

3.2 Real interest rates

Background. The (continuously compounded) nominal interest rate in a period is the sum of the expected real interest rate and expected inflation for the period. The expected real interest rate is, in turn, the sum of the real risk-free rate and the inflation risk premium. In estimating real interest rates, we assume that the inflation risk premium is zero. Later, in our asset allocation analysis, we assume that the inflation risk premium for a given maturity bond is constant over time, although it may vary with bond maturity. In this case, the expected real return on an indexed zero-coupon bond equals the expected real return on a nominal bond of the same maturity minus a constant risk premium. Thus, if the risk premium in the nominal term structure is positive, ignoring it overstates the expected and realized returns on the indexed bond, but has no impact on the computation of second moments. Campbell and Shiller (1996) make a similar point.

While the assumption of a constant inflation risk premium is somewhat restrictive, our approach does not require that term premia in the real term structure are constant over time. In this regard, we extend the analysis in Campbell and Shiller (1996). They derive hypothetical indexed bond returns by assuming that the expected real returns on bonds of all maturities equal the expected real return on a three-month bill. This requires that real term premia, as well as the inflation risk premium, are constant over time. In contrast, we back out the real term premia from a broader set of bond prices at each point in time, thereby permitting the real term premia to change over time.

Following Barr and Campbell (1997), the log real price of a zero-coupon indexed bond at t equals the expected log real payoff minus the time t expected real return on the indexed bond from t to t+k. The time t expected real return from t to t+k on a *nominal* zero-coupon bond maturing in k years equals k times the (continuously compounded) k-period spot rate minus the sum of expected inflation rates for years t+1 to t+k. Thus, to compute the expected real return from t to t+k, it suffices to estimate the time t expected inflation rates and subtract the sum from k times the nominal rate. Without loss of generality, we assume that the bond's face value is one, so that the log real payoff equals zero. In this case, the log real price is the negative of the expected real return from t to t+k.. We use this relation to compute hypothetical indexed bond prices.

Estimation. For our five-year indexed zero-coupon bond with unit payoff, the log real price of the bond at time t is –1 times the total 5-year nominal yield minus the sum of the inflation forecasts for five years, t+1 to t+5. This procedure is repeated every month to obtain a time series of log real prices for a bond with five years to maturity. We similarly construct a price series for a bond with a 4-year maturity. These two price series are used to calculate a time series of annual buy-and-hold real returns on hypothetical five-year zero-coupon indexed bonds from June 1953 to December 1998. We obtain an overlapping time series of annual returns because prices and annual returns are calculated every month. Nominal returns for the same bonds are calculated by subtracting 1 from (1 + real return)*(1 + realized inflation), where realized inflation is the one-year inflation rate (not continuously compounded) calculated using the CPI index.

Nominal returns on non-indexed bonds are calculated directly using nominal prices for 4- and 5-year bonds from the WRDS database. We calculate real returns by subtracting 1 from (1 + nominal return)/(1 + realized inflation).

Finally, we estimate the one-year real riskless rate, which is used to calculate excess real returns on stocks and bonds for the purpose of mean-variance portfolio optimization. This return is not continuously compounded. If the inflation risk premium is zero, the one-year real riskless rate equals the expected real return on a conventional one-year bond. The expected real return is obtained by subtracting one from the product of one plus the nominal one-year spot rate and the expected value of 1/(1 + inflation). To evaluate this expected value, we assume the log of one plus inflation is normally distributed with mean equal to expected continuously compounded inflation, μ, the fitted value from the inflation forecasting model (1), and variance, σ^2, equal to the model's residual variance. The log of 1/(1 + inflation) is then normally distributed with mean $-\mu$ and variance σ^2. It follows that the required expected value of 1/(1 + inflation) is $\exp[-\mu + \sigma^2/2]$.

3.3 Descriptive statistics

Bond returns. Table 2 reports descriptive statistics for nominal and real annual returns on non-indexed and hypothetical indexed five-year zero-coupon bonds. Figure 1 portrays the time series of annual bond returns. There are 547 overlapping annual return observations from June 1953 to December 1998. The mean nominal and real returns on the hypothetical indexed bonds are slightly lower than that for the corresponding non-indexed bonds.

[Table 2]

[Figure 1]

Table 2 reveals two attractive properties of the indexed bond returns. First, they are less variable than their conventional bond counterparts. For example, the standard deviation of the indexed bond's real returns is 6.27% compared to 7.65% for the non-indexed bonds. Second, the correlation between indexed and non-indexed bond returns is only 0.52 using nominal returns and 0.57 using real returns. The relatively low correlation between the two series reflects the differential impact of unanticipated inflation on the indexed and non-indexed bond returns.

Non-indexed bond prices are, naturally, negatively impacted by unanticipated inflation. Indexed bond prices are only affected insofar as the unanticipated inflation is correlated with changes in real riskless rates, however, since the indexed cash flows are hedged against inflation. The correlation between annual changes in our estimates of expected inflation and the one-year real riskless rate is actually negative, about –0.4. Fama and Gibbons, 1982, likewise find a negative relation.

This is good news from a portfolio diversification standpoint for an investor considering a mix of stocks and bonds, given that previous research shows that stocks react negatively to unanticipated inflation (e.g., Fama and Schwert, 1977, and Fama, 1981). Thus, non-indexed bonds and stocks are expected to correlate positively because of their similar sensitivity to inflation, whereas indexed bond returns and stock returns might comove less strongly, as indexed bond returns are relatively unaffected by inflation.[5] We present empirical evidence on the correlation between stock returns and indexed and non-indexed bond returns in the next section.

We also note, in Table 2, that the standard deviation of real returns is lower than the standard deviation of nominal returns for indexed bonds, but higher for conventional bonds. To understand this, recall that real returns are roughly equal to nominal returns minus the inflation rate. The variance of this difference equals the sum of the variances of each component minus twice the covariance between them. Since increases in inflation lower non-indexed bond prices, this covariance term is negative and subtracting it increases the overall variability of real returns on conventional bonds. Now consider the nominal returns on indexed bonds, i.e., the sum of real returns and inflation. The real prices of indexed bonds increase when real interest rates fall. Insofar as movements in real rates are negatively related to expected inflation, as noted earlier, the covariance between the real returns on indexed bonds and inflation is positive. This effect tends to increase the volatility of nominal returns on indexed bonds relative to their real variability.

Real interest rate. Table 3 contains descriptive statistics for our estimates of the one-year real interest rate. The real interest rate averages 2.1% per year from 1953 to 1998 and it exhibits considerable variation (standard deviation = 2.2%). The maximum estimated real return, 9.3%, occurs in 1982 when the spot rate was over 14%. Tthe minimum is –2.6% in 1974. Previous research also reports negative real interest rates and large variation in estimated real returns. For example, Bodie (1988, p. 7) concludes "In the 1970s the real rate on bills was substantially negative, averaging –1% per year for the 10 years from January 1970 to December 1979. In the 1980s, it has averaged 4% per year." Estimates of *ex ante* real rates based on the Livingston survey of professional economists are also negative in the mid-70s.

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[5]A detailed examination of the separate roles inflation and real rates play in stock and bond returns will be conducted in future work.

[Table 3]

4. Diversification Analysis

We begin this section by examining the correlation between stock returns and indexed and non-indexed real and nominal returns. We then analyze the risk of portfolios of stocks and bonds with weights on each ranging from zero to 100%. This gives an intuitive feel for the benefits of diversification from using indexed bonds instead of conventional bonds with stocks.

Stock-bond correlations. Table 4 reports Pearson product-moment correlations between annual excess returns on the CRSP value-weighted stock index and excess returns on 5-year zero-coupon indexed and non-indexed bonds. The correlations are estimated using both real and nominal returns. Nominal indexed bond returns' correlation with stock returns is –0.09, compared to 0.25 using the non-indexed bond returns. Corresponding numbers using real returns are 0.07 and 0.40. Since both non-indexed bonds and stock returns are negatively impacted by unanticipated inflation, positive correlation between the two is not surprising. Similar logic suggests that indexed bond returns would be less strongly correlated with stock returns.

[Table 4]

The minimal correlation between indexed bond returns and stock returns is somewhat surprising. Higher correlation might have been expected because of the common effects of shocks to real interest rates on bonds and stocks. While indexed bonds would be negatively affected by shocks to real returns, the same does not appear to be true of stock returns, perhaps due to a positive relation between real rates and expected economic growth (Fama and Gibbons, 1982). This positive relation is plausible given our earlier observation that real rates and expected inflation are negatively correlated, as high inflation has also been negatively correlated with future real activity since the early 1950's (Fama, 1981).

The lack of correlation between indexed bond returns and stock returns has desirable diversification implications. Table 5 provides an indication of these benefits for portfolios of five-year zero-coupon bonds and the CRSP value-weighted stock index. The standard deviation of excess portfolio returns is presented, with excess returns computed by subtracting the one-year riskless rate, either nominal or real.[6] For given weights, the percentage reduction in risk achieved by investing in indexed bonds, rather than conventional bonds, is then computed from these numbers. For the real returns, the reduction in portfolio standard deviation is about 12% for an equal-weighted portfolio and it increases to over 25% when the portfolio weight on bonds is 80-90%. The reductions for nominal returns are smaller, as might be expected given the discussion of real and nominal bond risk in section 3.3.

[Table 5]

Recall that our estimation of the risk characteristics of indexed bonds proceeds under the assumption of a constant inflation risk premium. It seems likely that this assumption will bias upward our estimate of the correlation between real stock returns and indexed bond returns. Therefore, the actual diversification benefits of investing in indexed bonds may exceed those measured above. To see this, assume that inflation uncertainty increases with the level of inflation, and that the risk premium increases as well (Buraschi and Jiltsov, 1999). Also, suppose that stock prices tend to fall with increases in inflation (Fama and Schwert, 1977 and Fama, 1981). Under these assumptions, holding the real term structure constant, a rise in inflation should have no effect on the price of the indexed bond, as its real payoff is fixed. However, given the procedure outlined above, the increase in the inflation risk premium in nominal rates will carry over to the *imputed* real rate when expected inflation is subtracted. Thus, our hypothetical indexed bond price will be lowered through a discount rate effect, moving

[6] The real riskless rate is estimated, as discussed earlier. Working with excess returns can be viewed as conditioning on the riskless rate at each point in time.

in the same direction as stock prices. In future work, we will consider more general methods that accomodate changes in the inflation risk premium.

5. Asset Allocation

Numerous studies of portfolio optimization demonstrate that the portfolio weights obtained can be quite sensitive to relatively small changes in the expected return inputs. Moreover, it is well known that historical estimates of variances and covariances are much more reliable than sample means in terms of future prediction. In addition, some would argue that, as a result of the evolution of financial markets and changes in the world economy in recent years, investment in stocks is less risky today than in the past. Therefore, the required risk premium for stocks should perhaps be lower than the historical mean excess return, which was about 7.5% per year (nominal) for our 1953-98 sample (the mean excess return for five-year conventional bonds was less than 1%). Given these considerations, in our analysis of stock/bond asset allocation, we will specify the mean return inputs, rather than simply using ex post sample means. Our base case uses an expected excess return (nominal or real) of 6% for stocks and 1% for five-year bonds (indexed or non-indexed). We explore sensitivity to alternative inputs, including the possibility of a positive inflation risk premium in the pricing of conventional bonds.[7]

A potential criticism of our analysis is that the changes in financial markets just alluded to imply that the process governing inflation and interest rates has also changed over time. While this is probably true, it is important to appreciate that our forecasts of inflation, the key input to our simulated indexed bond prices, rely heavily on *current* market interest rates and recent inflation experience. The prevailing rates in the market at a given point in time reflect the perceptions of investors at that time, including recognized changes in the relation between inflation and economic growth or monetary policy. In this sense, such changes can be implicitly

[7] Grinold (1996) suggests that reasonable expected excess returns are 5-7% for stocks and 0.75-1.75% for bonds.

captured in our model. Nonetheless, variation in the coefficients over time cannot be ruled out and will be considered in future work.

Table 6 provides asset allocation results based on real returns analysis with the excess return inputs just discussed, i.e., 6% for stocks, and 1% for bonds. Panel A considers stocks and conventional (non-indexed) bonds. Given our inputs, the optimal asset allocation entails an investment of 10-20% of one's portfolio in conventional bonds and the rest in stock. Naturally, investment in bonds lowers both the expected return and risk of the overall portfolio. Surprisingly, though, the opportunity for diversification across stocks and bonds barely improves the Sharpe ratio (or M^2 value) over what would be achieved with full investment in stocks. This means that the stock index is close to the "tangency portfolio" in this case.

[Table 6]

In contrast, when indexed bonds are considered in panel B, the optimal allocation to bonds is between 60% and 70% of the portfolio. This assumes there is no inflation risk premium. With a risk premium of 50 basis points, the allocation to indexed bonds is still 50% of the portfolio, reflecting the diversification benefits of indexed bonds considered earlier. Without the risk premium, the optimal M^2 value is 6.9%, an increase of about 15% over the 6% value for stocks alone. Given the high investment in bonds, an investor wishing to bear the amount of risk implicit in the stock market (16.8%) would need to "lever up" the tangency portfolio.[8] With the inflation risk premium, the expected return on the indexed bond is cut in half, but the M^2 measure of 6.3% still exceeds that for stocks alone.

If we hold all other inputs constant and change the expected excess return on stocks to 7.5%, the improvement in M^2, when comparing indexed and non-indexed bonds, drops from 15% to 10% without the inflation risk premium (not shown). The allocation to indexed bonds

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[8]What this amounts to in practice is less clear for the real returns analysis than the nominal returns analysis. Thus, the calculations should perhaps be viewed as a general indication of opportunities for improving the risk-reward tradeoff in such cases.

drops only slightly, and is still around 60%. Including the risk premium reduces the benefits, as before, with the optimal allocation to indexed bonds now 40-50%. On the other hand, increasing the expected excess return on bonds to 1.5% increases the performance advantage of indexed bonds to about 25%.

Table 7 provides an identical analysis in terms of nominal returns. The results are quite similar to those in Table 6, with the relative advantage of indexed bonds over conventional bonds declining only slightly. Recall from section 3.3 that the differential impact of inflation on indexed versus non-indexed bond prices results in a risk advantage for indexed bonds when measured in real terms.

[Table 7]

6. Summary, conclusions, and recommendations

We have created a series of prices and computed the corresponding returns for a hypothetical five-year zero-coupon bond, as if it had existed back to 1953. To create the series, we develop models for expected inflation and use these models to back out real rates of interest from the historical series of nominal interest rates. Several important properties of the time series of indexed bond returns are observed.

First, the real (inflation-adjusted) returns on indexed bonds are less volatile than the returns on otherwise similar conventional bonds. Moreover, unlike conventional bonds, the real returns on indexed bonds are less volatile than the nominal indexed bond returns. Second, as a result of the inflation protection, the correlation between aggregate real stock returns and indexed bond returns is close to zero, whereas the correlation between real conventional bond returns and stocks is 0.40. Thus, indexed bonds provide better opportunities for diversification in a portfolio of stocks and bonds. As a result of the lower volatility and correlation, the standard deviation of an equal-weighted portfolio of stocks and bonds is about 12% lower using indexed bonds, as compared to conventional bonds. The risk-reduction more than doubles for conservative portfolios more heavily invested in bonds.

Of course, the asset allocation decision depends on expected returns as well as risk. If conventional bond yields contain a positive premium for inflation risk, then the expected returns on indexed bonds will be lower than those for non-indexed bonds. On the other hand, the indexed bond market is less liquid than the market for conventional bonds. Other things equal, this will tend to increase the yields on indexed bonds and make them more attractive to longer-term investors. Whatever the reason, there does not appear to be a positive premium in the current yields of conventional bonds.

We examine asset allocation between stocks, five-year bonds, and a riskless asset assuming expected excess returns of 6% per year for stocks and 1% for both indexed and non-indexed bonds. With these inputs, the optimal allocation to conventional bonds is less than 20% of the portfolio, while that for indexed bonds exceeds 60%. More importantly, the measures of portfolio performance, M^2 and Sharpe ratio, increase by about 15% with indexed bonds, as compared to conventional bonds. The relative advantage of indexed bonds increases to about 25% with expected excess bond returns of 1.5%. Assuming an inflation risk premium of 50 basis points, or an expected excess return of 7.5% on stocks, reduces the advantage of indexed bonds, but we continue to see a substantial allocation to indexed bonds in the optimal portfolio.

These observations suggest an important role for indexed bonds in a diversified investment portfolio. The risk-reduction benefits of indexed bonds reflect fundamental economic relations and are likely to persist in the future, though the magnitude of these benefits will vary with the inflationary environment. It is more difficult to speculate about the future yield spread for indexed vs. non-indexed bonds. However, if one thinks that the inflation risk premium is, in some sense, “too low” in the current marketplace, the anticipation of a future correction should enhance the attractiveness of indexed bonds in today’s asset allocation decision.

References

Barr, D., Campbell, J., 1997, Inflation, real interest rates, and the bond market: A study of UK nominal and index-linked government bond prices, Journal of Monetary Economics 39, 361-383.

Bodie, Z., 1988, Inflation, index-linked bonds, and asset allocation, working paper No. 2793, National Bureau of Economic Research.

Buraschi, A., Jiltsov, A., 1999, How large is the information risk premium in the U.S. nominal term structure? Working paper, London business School.

Campbell, J., Shiller, R., 1996, A scorecard for indexed government debt, In: B.S. Bernanke, J. Rotemberg (Eds.), NBER Macroeconomics Annual, MIT Press, Cambridge, MA.

Fama, E., 1975, Short-term interest rates as predictors of inflation, American Economic Review 65, 269-282.

Fama, E., 1981, Stock returns, real activity, inflation, and money, American Economic Review 71, 545-564.

Fama, E., 1990, Term-structure forecasts of interest rates, inflation, and real returns, Journal of Monetary Economics 25, 59-76.

Fama, E., Bliss, R., 1987, The information in long-maturity forward rates, American Economic Review 77, 680-692.

Fama, E., French, K., 1989, Business conditions and expected returns on stocks and bonds, Journal of Financial Economics 25, 23-49.

Fama, E., Gibbons, M., 1982, Inflation, real returns, and capital investment, Journal of Monetary Economics 9, 297-323.

Fama, E., Gibbons, M., 1984, A comparison of inflation forecasts, Journal of Monetary Economics 13, 327-348.

Fama, E., Schwert, G., 1977, Asset returns and inflation, Journal of Financial Economics 2, 115-146.

Grinold, R., 1996, Domestic grapes from imported wine, *Journal of Portfolio Management*, special issue, 29-49.

Nelson, C., Schwert, G., 1977, Short-term interest rates as predictors of inflation: On testing the hypothesis that the real rate of interest is constant, American Economic Review 67, 478-486.

Newey, W., West, K., 1987, A simple, positive semi-definite heteroscedasticity and autocorrelation consistent covariance matrix, Econometrica 55, 703-708.

Table 1
Inflation forecasting regressions: June 1953 to December 1998

Variable	Coefficient estimate	Newey-West		Hansen-Hodrick	
		Std error	t-stat	Std error	t-stat
Forecast of one-year-ahead inflation $Inf_{t+1} = b_0 + b_1\, Int_{t+1} + b_2\,(Yld_{t,t+5} - Int_{t+1}) + b_3\, Inf_t + b_4\, Realbill_t + e_{t+1}$					
Intercept	0.0092	0.0052	1.77	0.0064	1.43
Int_{t+1}	0.56	0.17	3.23	0.20	2.81
$Yld_{t,t+5} - Int_{t+1}$	-0.23	0.29	-0.80	0.35	-0.66
Inf_t	0.17	0.18	0.95	0.20	0.82
$Realbill_t$	-0.69	0.20	-3.42	0.23	-2.94
Adjusted R^2	70.6%				
Forecast of change in two-year-ahead inflation over one-year-ahead inflation $Inf_{t+2} - Inf_{t+1} = b_0 + b_1\,(Fint_{t+2} - Int_{t+1}) + b_2\, Inf_t + b_3\, Realbill_t + e_{t+2}$					
Intercept	0.0086	0.0043	1.99	0.0054	1.60
$Fint_{t+2} - Int_{t+1}$	0.84	0.26	3.23	0.31	2.69
Inf_t	-0.25	0.07	-3.32	0.09	-2.67
Realbill	-0.16	0.11	-1.43	0.13	-1.16
Adjusted R^2	25.8%				
Forecast of change in three-year-ahead inflation over two-year-ahead inflation $Inf_{t+3} - Inf_{t+2} = b_0 + b_1\,(Fint_{t+3} - Fint_{t+2}) + b_2\, Inf_t + b_3\, Realbill_t + e_{t+3}$					
Intercept	0.0006	0.0049	0.13	0.0061	0.10
$Fint_{t+3} - Fint_{t+2}$	0.87	0.3046	2.87	0.346	2.52
Inf_t	-0.07	0.08	-0.82	0.1048	-0.66
Realbill	0.02	0.12	0.13	0.147	0.10
Adjusted R^2	9.0%				

Inf_{t+1} = realized inflation for year t+1
Int_{t+1} = spot nominal interest rate for year t+1
$Yld_{t,t+5} - Int_{t+1}$ = the spread between the nominal yield on a five-year zero-coupon bond and the spot interest rate
Inf_t = lagged inflation
Realbill = sum of past 12 months real returns on one-month bills
$Fint_{t+k}$ = time t forward interest rate for year t+k.

Table 2

Descriptive statistics for annual overlapping bond returns from June 1953 to December 1998 (N = 547)

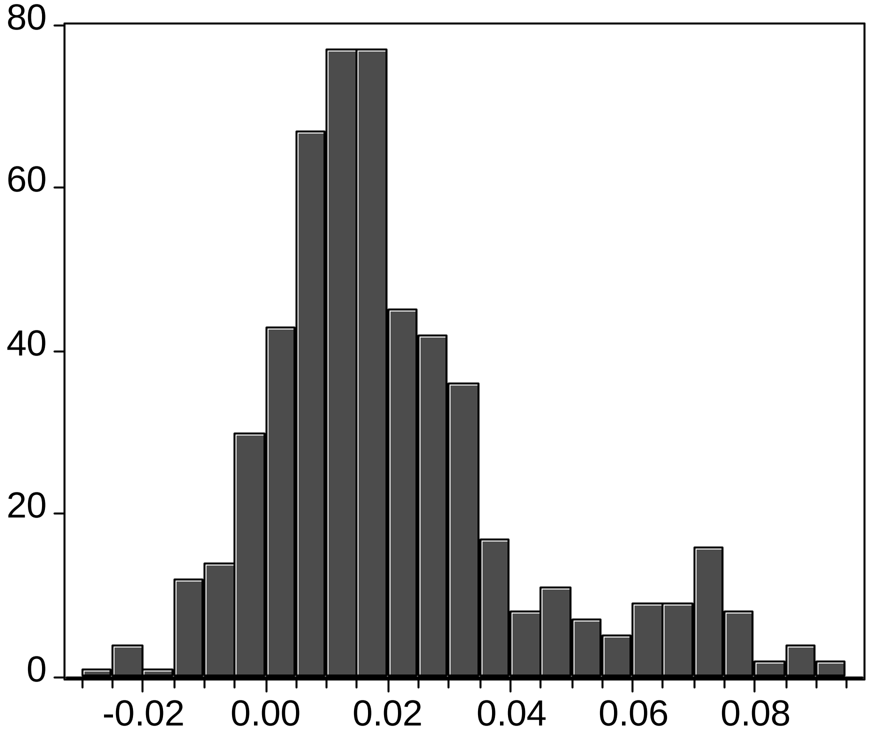
Statistic	Indexed bonds		Non-Indexed bonds	
	Nominal return, %	Real return, %	Nominal return, %	Real return, %
Mean	6.76	2.57	6.92	2.81
Median	5.66	1.93	5.30	2.05
Maximum	29.81	24.94	35.64	29.59
Minimum	-12.52	-20.15	-7.90	-17.11
Standard deviation	7.01	6.27	7.22	7.65
Skewness	0.311	0.242	1.265	0.819
Kurtosis	3.01	4.31	5.23	4.59

Correlation Matrix

	Indexed Nominal Bond Returns	Indexed Real Bond Returns	Non-Indexed Nominal Bond returns	Non-Indexed Real Bond Returns
Indexed Nominal	1	0.90	0.52	0.34
Indexed Real		1	0.59	0.57
Non-Indexed Nominal			1	0.93
Non-Indexed Real				1

Table 3

Descriptive statistics for one-year real interest rate



Series: EXPREALRET1YR
Sample 1953:06 1998:12
Observations 547

Mean	0.021125
Median	0.016364
Maximum	0.092985
Minimum	-0.026466
Std. Dev.	0.021822
Skewness	1.087871
Kurtosis	4.070089
Jarque-Bera	133.9909
Probability	0.000000

One-year real interest rate

Table 4

Correlations between real and nominal excess stock returns and indexed or non-indexed one-year returns on five-year zero-coupon bonds

Indexed bonds		Non-indexed bonds	
Nominal	Real	Nominal	Real
-0.09	0.07	0.25	0.40

Estimates based on monthly series of one-year historical excess returns (non-indexed bond/stock) or simulated excess returns (indexed bond) from June 1953 to December 1998.

Table 5

Standard Deviation of overapping annual returns on portfolios consisting of stocks and bonds: Data from 1953 to 1998

Panel A: Nominal Returns											
% Bond	0	10	20	30	40	50	60	70	80	90	100
Non-indexed	16.7	15.2	13.7	12.3	10.9	9.6	8.4	7.4	6.6	6.1	6.0
Indexed	16.7	15.0	13.3	11.7	10.1	8.6	7.3	6.3	5.7	5.7	6.2
% Reduction	0.0	1.4	3.0	4.9	7.1	9.7	12.1	13.6	12.2	6.5	-2.7
Panel B: Real Returns											
% Bond	0	10	20	30	40	50	60	70	80	90	100
Non-indexed	16.8	15.4	14.0	12.6	11.3	10.1	9.0	8.0	7.2	6.7	6.5
Indexed	16.8	15.1	13.5	11.8	10.2	8.7	7.3	6.1	5.2	4.8	5.0
% Reduction	0.0	1.5	3.3	5.6	8.5	12.0	16.3	21.2	25.4	26.8	23.3

Variances and covariances based on monthly series of one-year historical excess returns (non-indexed bond/stock) or simulated excess returns (indexed bond) from June 1953 to December 1998.

Table 6

Standard deviation, Sharpe ratio, and M^2 of stock-bond asset allocation portfolios using real returns: Data from 1953 to 1998

Panel A: Non-Indexed Bond											
% Bond	0	10	20	30	40	50	60	70	80	90	100
% Excess Return	6.0	5.5	5.0	4.5	4.0	3.5	3.0	2.5	2.0	1.5	1.0
% Std Deviation	16.8	15.4	14.0	12.6	11.3	10.1	9.0	8.0	7.2	6.7	6.5
Sharpe Ratio	0.36	0.36	0.36	0.36	0.35	0.35	0.33	0.31	0.28	0.22	0.15
% M^2	6.0	6.0	6.0	6.0	5.9	5.8	5.6	5.3	4.7	3.8	2.6
Panel B: Indexed Bonds with No Inflation Risk Premium											
% Bond	0	10	20	30	40	50	60	70	80	90	100
% Excess Return	6.0	5.5	5.0	4.5	4.0	3.5	3.0	2.5	2.0	1.5	1.0
% Std Deviation	16.8	15.1	13.5	11.8	10.2	8.7	7.3	6.1	5.2	4.8	5.0
Sharpe Ratio	0.36	0.36	0.37	0.38	0.39	0.40	0.41	0.41	0.39	0.31	0.20
% M^2	6.0	6.1	6.2	6.4	6.6	6.8	6.9	6.9	6.5	5.3	3.3
Panel C: Indexed Bonds with Inflation Risk Premium of 50bp											
% Bond	0	10	20	30	40	50	60	70	80	90	100
% Excess Return	6.0	5.5	4.9	4.4	3.8	3.3	2.7	2.2	1.6	1.1	0.5
% Std Deviation	16.8	15.1	13.5	11.8	10.2	8.7	7.3	6.1	5.2	4.8	5.0
Sharpe Ratio	0.36	0.36	0.36	0.37	0.37	0.37	0.37	0.35	0.31	0.22	0.10
% M^2	6.0	6.1	6.1	6.2	6.2	6.3	6.2	5.9	5.2	3.7	1.7

Variances and covariances are based on monthly series of one-year historical excess returns (non-indexed bond/stock) or simulated excess returns (indexed bond) from June 1953 to December 1998. Real riskless rate is simulated.

Table 7

Standard deviation, Sharpe ratio, and M^2 of stock-bond asset allocation portfolios using nominal returns: Data from 1953 to 1998

Panel A: Non-Indexed Bond

% Bond	0	10	20	30	40	50	60	70	80	90	100
% Excess Return	6.0	5.5	5.0	4.5	4.0	3.5	3.0	2.5	2.0	1.5	1.0
% Std Deviation	16.7	15.2	13.7	12.3	10.9	9.6	8.4	7.4	6.6	6.1	6.0
Sharpe Ratio	0.36	0.36	0.36	0.37	0.37	0.36	0.36	0.34	0.30	0.25	0.17
% M^2	6.0	6.0	6.1	6.1	6.1	6.1	6.0	5.7	5.1	4.1	2.8

Panel B: Indexed Bonds with No Inflation Risk Premium

% Bond	0	10	20	30	40	50	60	70	80	90	100
% Excess Return	6.0	5.5	5.0	4.5	4.0	3.5	3.0	2.5	2.0	1.5	1.0
% Std Deviation	16.7	15.0	13.3	11.7	10.1	8.6	7.3	6.3	5.7	5.7	6.2
Sharpe Ratio	0.36	0.37	0.38	0.39	0.40	0.41	0.41	0.40	0.35	0.26	0.16
% M^2	6.0	6.1	6.3	6.4	6.6	6.8	6.8	6.6	5.9	4.4	2.7

Panel C: Indexed Bonds with Inflation Risk Premium of 50bp

% Bond	0	10	20	30	40	50	60	70	80	90	100
% Excess Return	6.0	5.5	4.9	4.4	3.8	3.3	2.7	2.2	1.6	1.1	0.5
% Std Deviation	16.7	15.0	13.3	11.7	10.1	8.6	7.3	6.3	5.7	5.7	6.2
Sharpe Ratio	0.36	0.36	0.37	0.37	0.38	0.38	0.37	0.34	0.28	0.19	0.08
% M^2	6.0	6.1	6.2	6.2	6.3	6.3	6.2	5.7	4.7	3.1	1.3

Variances and covariances are based on monthly series of one-year historical excess returns (non-indexed bond/stock) or simulated excess returns (indexed bond) from June 1953 to December 1998. Riskless rate equals historical one-year riskless return.

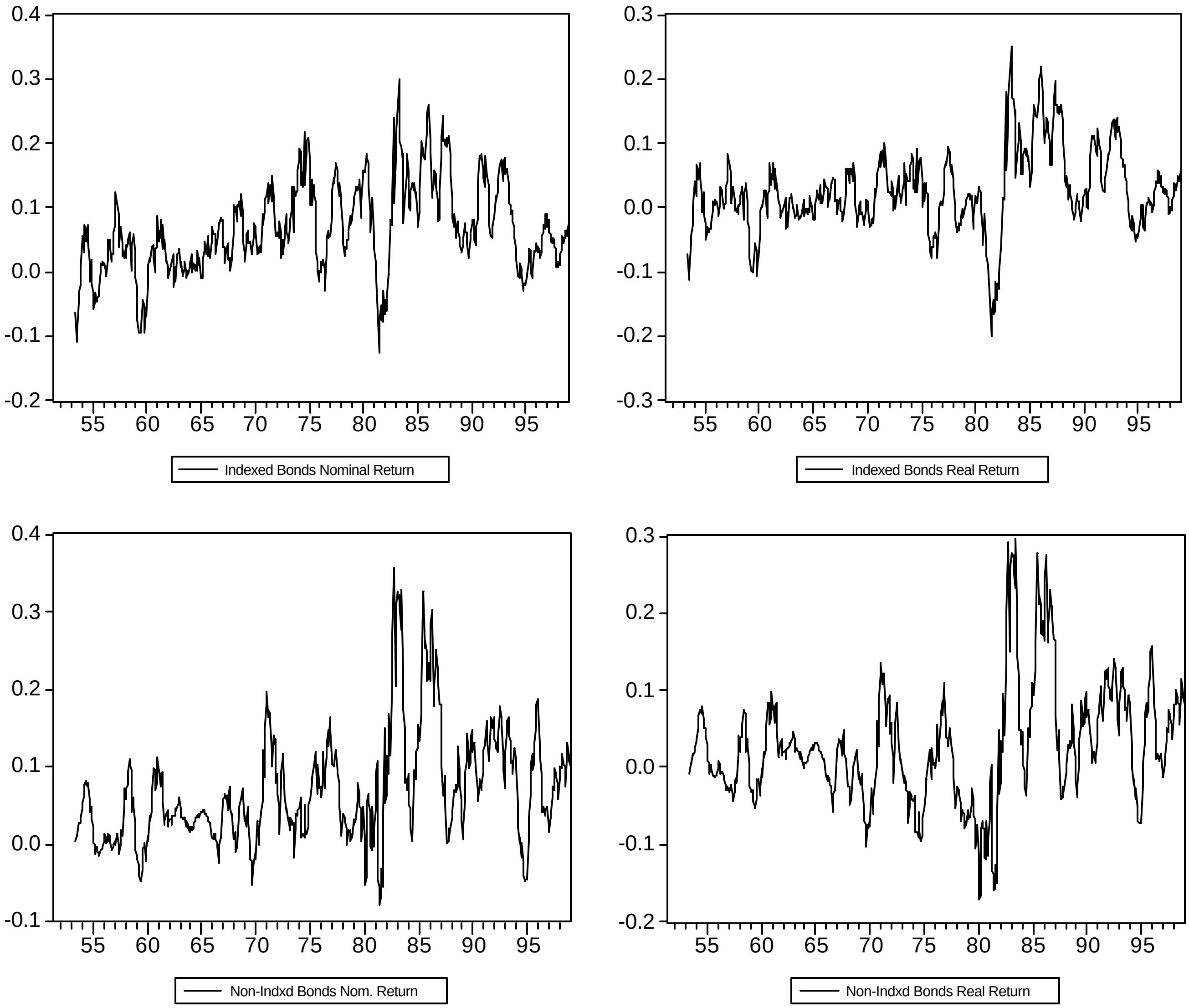


Figure 1a: Overlapping Annual Return Series for Indexed and Non-Indexed Five Year Zero Coupon Bonds

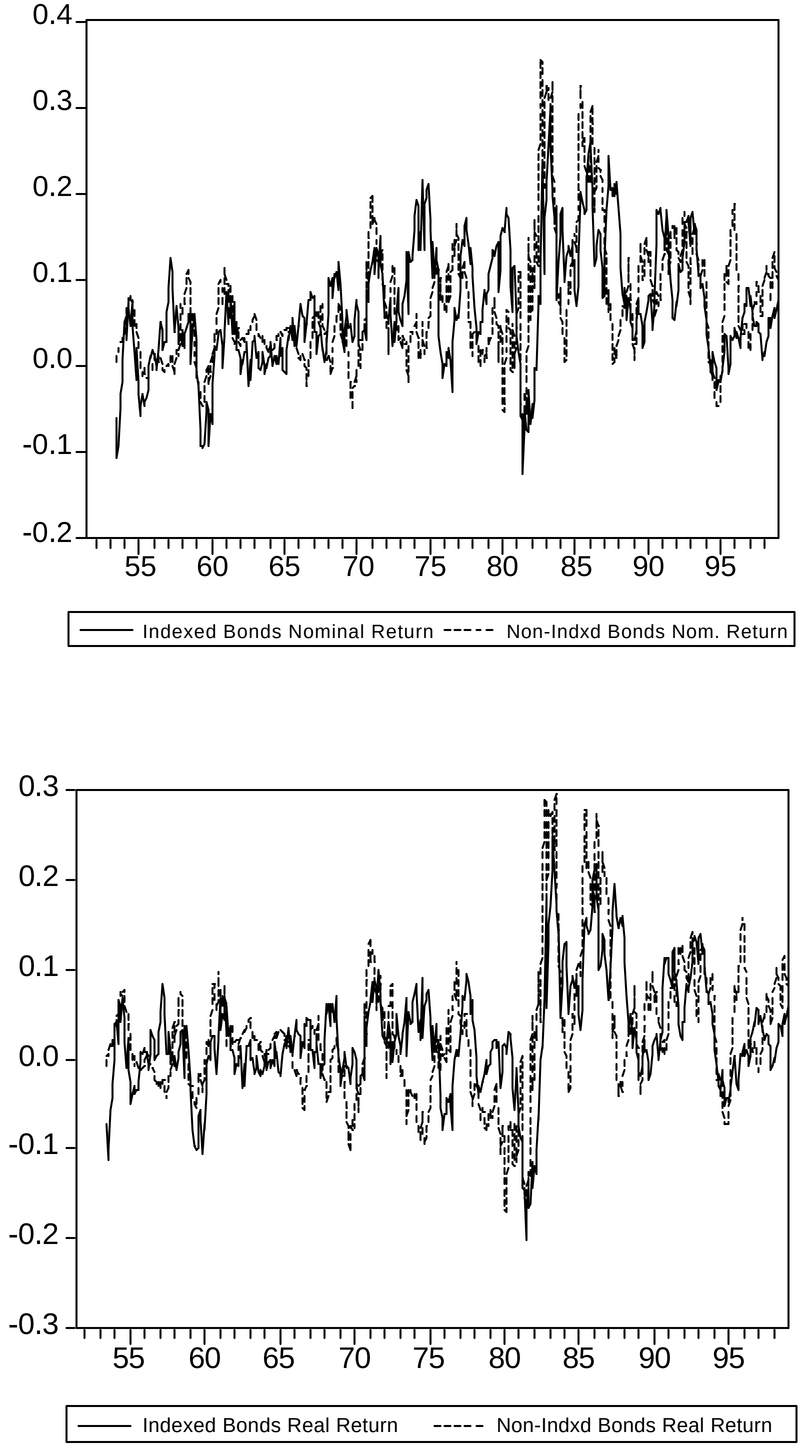


Figure 1b: Superimposed Overlapping Annual Return Series for Indexed and Non-Indexed Five Year Zero Coupon Bonds