Design for a Compact Neutron Interferometer

22.033 Neutron Interferometer Design Team
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Abstract

A new, compact neutron interferometer system has been designed based on the four-blade perfect crystal interferometer configuration. This design is motivated by the current interferometer facility at the National Institute of Standards and Technology (NIST). The unique components in our design include a four-blade interferometer crystal, magnesium windows for the beam break, a strained silicon monochromator for neutron energy selection, compact modular spin polarizers and flippers, a helium-3 cryogenic system, and a single vacuum shell. The four-blade crystal makes the design less sensitive to vibration-induced noise, allowing us to develop a compact vibration isolation system. This compact system (dimensions 1 m x 0.75 m x 0.5 m) can be placed closer to the neutron source, increasing our neutron count by a factor of 26. The increased signal and compact design broaden the range of neutron interferometry applications, from condensed matter physics to biological and material sciences.
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Chapter 1

Introduction

We present a new neutron interferometer design that is compact and insensitive to low frequency vibrations, has a high signal to noise ratio, and is capable of handling cold samples. This device will allow a wider range of condensed matter physics research and will increase the access of neutron interferometry for other research fields, such as biology and materials science. In order to appreciate the uniqueness of this new interferometer, a basic understanding of neutron interferometry is necessary. To build this understanding, we first look at the neutron and see how its properties allow interferometry.

1.1 Properties of Neutron

The neutron, a neutral particle, was discovered in 1932 by Chadwick. Four years later, Elsasser [9] suggested the possibilities of neutron diffraction in powder, which Halban and Preiswerk, and Mitchell and Powers [1] confirmed in their experiments the same year. Since then, all of properties listed in table 1 [1], have been studied by interference. These properties have allowed neutrons to be used in many applications, from sample characterization and imaging to the search for gravitational waves.

Neutrons interact with matter through electromagnetism, the strong force, the
weak force and gravity [1]. Neutrons exhibit wave-particle duality, and here it is more interesting to view the neutron as a wave. This allows neutrons to exhibit photon-like phenomenon such as diffraction, reflection, and interference.

### 1.2 Brief Physics of Neutron Interferometry

Figure 1.1 below shows the top view of a three-blade single crystal interferometer, which currently is the most common type of interferometer. Notice that the neutron interferometer consists of two physically separated paths after which the beams are recombined. One path experiences a phase shift relative to the other due to the phase flag inserted in that path.

The neutron wave-function is spread over the two paths due to Bragg scattering at the first crystal. We will therefore describe the wave function in a basic set of path 1, $|1>$, and path 2, $|2>$. Once the neutron is Bragg scattered at the first blade, the wave function is

$$\Psi = \frac{1}{\sqrt{2}}(|1> + |2>).$$  \hspace{1cm} (1.1)
Figure 1.1: Top View a three-blade single crystal interferometer, which contains two paths of neutron. Path 1 exhibits a phase shift relative to path two due to insertion of a phase lag in path one. The two paths interfere at the third blade. The intensities are measured at two locations.

Notice here that we assume the transmission and reflection coefficients are equal. The phase flag in path one writes an experimentally controllable phase, $e^{i\theta}$, into the wave function.

$$\Psi = \frac{1}{\sqrt{2}}(e^{i\theta}|1\rangle + |2\rangle). \quad (1.2)$$

The second blade also splits the neutron beam into a transmitted and reflected paths via Bragg scattering. Since the transmitted beam never reaches the detector, we will ignore it in this simplified description and treat the blade as a mirror. The wave function at the O-detector, $\Psi_O$, and the H-detector, $\Psi_H$, are

$$\Psi_O = \frac{1}{\sqrt{2}}(e^{i\theta}|1\rangle + |2\rangle) \quad (1.3)$$

$$\Psi_H = \frac{1}{\sqrt{2}}(e^{i\theta}|1\rangle - |2\rangle) \quad (1.4)$$

The intensities at the detector correspond to $|<\Psi|\Psi>|^2$, which for this idealized picture are [1]

$$I_O = |<\Psi_O|\Psi_O>|^2 \propto [1 + \cos(\theta)] \quad (1.5)$$
\[ I_H = | \langle \Psi_H | \Psi_H \rangle |^2 \propto [1 - \cos(\theta)] \] (1.6)

Notice that particle conservation requires \( I_O + I_H = \text{constant} \), and as the phase \( \theta \) varies, the intensity goes back and forth between the O-beam and the H-beam. The neutron interferometer contrast is the modulation depth as the phase flag is varied. As described here, the contrast is 1. When we include the effects of experimental imperfections, the contrast is reduced. The application of a neutron interferometer requires that the property of interest must influence the contrast and the initial measurement must have a high contrast. A neutron interferometer is the most precise means of studying neutron physics, including tests of the standard model. In order to get to this level of precision, much effort has been expended towards improving neutron interferometry and developing the neutron interferometer setup.

1.3 Development of Neutron Interferometers

Neutron diffraction phenomena were quickly studied after the discovery of neutrons. Shull [10] developed the neutron scattering techniques in the 1940s to study neutron diffraction, and he won a Nobel Prize for this work in 1994 while he was professor at MIT. In 1962, the first neutron interferometry experiment was attempted by Leibnitz and Springer [9], [11]. There are many types of neutron interferometers that have been developed. Leibnitz and Springer [11] used wavefront division and bi-prism deflection. The applications of this interferometer were limited due to the small beam separation of around 100\( \mu \text{m} \). In fact, only the first order of neutron interference was observed. Introduction of a sample into this interferometer completely disturbed the neutron coherence [9], [11]. The setup of this interferometer is shown in figure 2a. Another type of interferometer, shown in figure 2b, is based on spin-up and spin-down superposition, is available for atom interferometry studies. The spin echo systems by Mezei [12] and the zero-field spin-echo systems by Gahler et al. [13] are examples of such studies [1]. The advantage of these systems is that no lateral beam splitting is
Figure 1.2: Different types of interferometer. a) Wave front division interferometer based on single-slit diffraction. b) Spin-echo interferometer based on spin state superposition [1].

required. The perfect single crystal interferometer is of most interest since this allows the study of quantum mechanics at a macroscopic scale [1]. It has been used to study advance neutron optical events as well. Rauch et al. [1] made the first perfect silicon crystal interferometer in 1974, and this interferometer produced the first interference of coherent neutron beams, shown in figure 1.3.

One of the most advanced single crystal interferometers is located at the National Institute of Standards and Technology (NIST) [14]. This interferometer was completed in 1994, and in 1996 it became available for experimentation. While the NIST interferometer is best suited for study of fundamental questions in physics, it does not allow one to fully explore the capabilities of neutron interferometry. The desire to use neutron interferometry in a wider range of research fields, including condensed matter physics, motivates the new interferometer design.
Figure 1.3: First perfect-crystal interferometer and the first interference pattern obtained by rotating an aluminum plate [1].
Chapter 2

Design Goals

The overall goal of the project is to design a new interferometer facility that is compact, has high contrast, and is compatible with a wider range of applications.

The new interferometer should have an improved Signal to Noise (S/N) ratio by a factor of 20 in comparison to the existing NIST interferometer. This can be achieved by moving the perfect crystal closer to the beam break, thus allowing more neutrons to enter the crystal and be detected. A higher signal to noise ratio helps to improve the contrast to noise and will shorten the measurement time in an experiment. An increase in the number of neutrons can be achieved by a more intense and less divergent beam. This can be accomplished by using a different set of monochromators.

The new interferometer should be compatible with a wider range of physics research compared to that of the existing setup. For example, it should permit the handling and measuring of cold samples at temperatures as low as 300 mK. In addition, the setup should be compatible with a spin-polarized beam to allow an extra degree of freedom, neutron spin, to be measured and manipulated. The neutron spin is an exquisite probe of magnetic properties.

A simpler set up while preserving high contrast is one of the goals of the design since this allows a less expensive and more compact interferometer to be built. This is another step forward in making neutron interferometry widely available to other
fields of research beyond physics. A simpler setup also allows for the use of more sample types as well as various sample geometries. This goal can be attained by a using a different vibration isolation system setup as well as by changing the geometry of the perfect crystal. The new device should have a reduced sensitivity to external conditions, such as temperature fluctuations, mechanical, and acoustic vibrations.
Chapter 3

Existing Design at NIST

3.1 Introduction

In 1984, the National Bureau of Standards agreed to fund a Cold Neutron Source for the NIST nuclear facility. This source allowed a variety of experiments to develop, including the Neutron Interferometer. The Neutron Interferometer facility was constructed in 1994 and is an enormous, isolated structure, which is connected to the cold neutron beam guide. It was extraordinarily expensive and was built with the goal of producing a very precise measurement with high contrast. It has provided many interesting results and performed several tests of the standard model of physics and other experiments. The issue that it quickly ran into was that its size, and thus the distance from the interferometer to the beam guide, along with the expense of the machine, limited the rate with which it could perform experiments and the variety of those experiments. The Neutron Interferometer design can be divided into three sections: the reactor and beam guides, the section from the beam break to the second monochromator, and the internal mechanics of the vibration isolation room. A picture of the last two sections is shown in figure 3.1.
3.1.1 Reactor to Beam Break

The NIST fission reactor is a 20 MW D$_2$O moderated research reactor. It has several beam guides for experiments and a cold neutron source with neutron beam guides branching from the cold source. The cold neutron source is the section of interest for the interferometer. It consists of a 5L ellipsoidal shell of liquid hydrogen kept at 20K. The neutrons are moderated to an average temperature of 40K. After moderation, the neutrons enter the beam guides, which are 15cm tall and 6 cm wide. There are two primary beam guides that come off the cold source: NG-6 and NG-7. The interferometer is on NG-7; its neutron spectrum was unavailable but the similar spectrum of the NG-6 line is shown in figure 3.2. [15]

The NG-7 beam line ends with an experiment focusing on slowing down neutrons and analyzing their decays and properties. These experiments require extremely cold neutrons, while the interferometer does not, so the two experiments can be placed on the same beam line. Therefore, a monochromator was placed in the path of the beam
to deflect certain wavelengths of semi-cold neutrons. This monochromator is made
from pyrolytic graphite and is 7.5 cm wide by 5 cm tall. The wavelength that is nor-
mally selected by the monochromator is $2.7 \, $Å. For more information on the operation
of these devices, please see appendix B. The monochromator is positioned inside the
beam guide and has a small housing around it to protect the crystal from damage.
The beam guides become brittle after prolonged use and are prone to implode, which
can damage the crystal. The monochromator also is positioned on a rotation and
pitch stage, to select other neutron energies. The reflected beam from the monochro-
mator is directed away from the beam line at an angle selected by Bragg’s Law, and
the neutrons enter the second phase of the interferometer.

3.1.2 Monochromator to Isolation Room

After selection by the first monochromator, the new beam of neutrons is directed out
of the beam line. It has to pass through a corridor through thick beam line protective
material, lithiated parafin, before interacting with the rest of the experiment. In this
section, the shutter, fission detector, and collimator are also placed.

The shutter consists of a positionable table with a rotating drum with a slit in it.
The cylinder can be positioned in such a way that no neutrons can escape, or it can allow selected wavelengths of neutrons to escape. The linear positioning is required to adjust for the distance between the monochromator and shutter. A shutter is required to be able to work on the interferometer safely.

The fission detector is required by protocol to be certain that the beam is either on or off. It measures the presence of the neutron flux through the beam guide. The detector, however, does not greatly affect the flux to the interferometer.

The collimator is a strip of cadmium with a thin slit designed to minimize the stray and scattered neutrons passing through to the interferometer. Collimators are used throughout the design to control the spread of the neutron and to keep stray neutrons from reaching the detectors. They produce some gamma rays, the maximum energy of which is 263 keV, but the intensity is not high enough to pose major health risks. The slit width is adjustable to allow varying the spread of allowed neutron energies.

After the collimator, there is a flight tube. The flight tube is one of several designs that are discussed in detail in appendix A. The purposes of the flight tube are two-fold: to meet health regulations regarding free neutron paths and to minimize the number of neutrons lost before reaching the experiment. These beam guides are wrapped in flexible boronated rubber shielding to capture stray neutrons. Boron is used because of its lack of high energy gamma ray emission [16]. Different length flight tubes are constructed so that different geometries of the monochromators can be accommodated. The extremes of the positions of the flight tube is shown in the figure 3.3.

The final device before the vibration isolation chamber is the second monochromator. This is, once again, a pyrolytic graphite monochromator, but this time it consists of nine separate blades positioned in a vertical parabola. This vertical assembly focuses the beam of neutrons, which increases number of neutrons incident on the interferometer. The last monochromator is positioned outside the flight tube
and before the final isolation building.

Figure 3.3: Figure showing the extreme positions of the beam guides in the NIST interferometer set-up. Wavelengths of neutrons are shown along with the angle at which the neutrons exit the beam guide. (Taken from presentation by Dimitry Pushin)

3.1.3 The Isolation Room

Interferometers, particularly three-blade interferometers, are highly susceptible to vibrations, and this forced the construction of the extremely large, expensive, and complex vibration isolation chamber. This system is isolated from the foundation of the NIST facility and consists of a two stage vibration isolation system along with pressure and temperature isolation.

The vibration isolation consists of a 40,000 kg concrete table on pneumatic actuators. This table supports the acoustic and thermal isolation along with the second stage of vibration isolation. The second stage is a 3,000 kg optical bench supported by another set of pneumatic actuators. The position of these systems is monitored
Existing Design at NIST

by proximity sensors and inclinometers, and the actuators are controlled by a computer [2]. This system provides vibration isolation of approximately $10^{-7}g$ [17]; a rough schematic of the room is shown in figure 3.4.

![Figure 3.4: Figure showing the relative sizes of the vibration isolation systems with respect to the interferometer. [2]](image)

Inside the isolation chamber the configuration tends to be variable. The dimensions of the room are approximately 4m x 4m on the floor and it is over 2m high, so there is ample room for repositioning the experiment. Often a second flight tube is used inside the isolation chamber along with supermirrors, helium-3 polarizers, and spin flippers (see the appendices for more information). The interferometer itself is held inside a box, which can be equipped with Helmholtz coils to maintain the spin of the neutrons inside the interferometer. The interferometer crystal is mounted on several positioning systems, and its base is 15 inches from the base of the box. At the end of the system, there are always He-3 neutron detectors.

3.1.4 Final Comments

It is interesting to note that the current system at NIST was designed with so many degrees of freedom that the system became difficult to use. For instance, several of the
isolation systems in the vibration isolation room had to be deactivated because they provided minimal results and introduced extra resonances in the system. Similarly the monochromators have too many positioning systems to accomplish what they need to do. The system is extremely versatile, but overly complex.

The current interferometer does have very good contrast, which it accomplishes with the complex arrangement of vibration isolation. This arrangement forces the interferometer to be positioned a sizable distance from the beam break; this distance varies given the experiment, but the neutron flight path can be anywhere from 4 - 8 meters long. This distance along with the spread of the neutron beam severely lowers the flux at the neutron detectors and increases the time required for any experiment to be performed.

Despite these limitations, the system has been extraordinarily successful. Many experiments have been performed on this interferometer, and the experiments are ongoing. There have been tests of the Quantum Entanglement [18], thin film density measurements [19], and various other experiments [20]. Using this system for industrial and medical applications, however, has not been feasible because of the long testing times, expense, and difficulty of use. As stated previously in the design goals, the new system being proposed will address these problems while building on the success of the current design by adopting many of its current systems and methods of operation.
Chapter 4

Description of the New Design

4.1 Overview

Our goal is to make neutron interferometry more accessible to the broader scientific community. To that end, we have designed a system that not only is compact and relatively inexpensive compared to existing interferometry systems, but that also provides high contrast measurements with a high signal to noise ratio.

The components of our proposed neutron interferometer design consist of: energy selection from the neutron beam guide by a strained-Si monochromator; beam collimation and focusing using Cd collimators and a multi-crystal Pyrolytic Graphite monochromator; a four-blade, perfect Si-crystal interferometer; and finally a measurement system of He-3 detectors. To accommodate neutron spin experiments, spin polarizers and flippers can be inserted at various locations along the neutron path of flight. Furthermore, our design includes a He-3 cryogenics system that can cool the sample down to 300 mK. These aspects of the design will extend the applicability of the neutron interferometer to fields beyond fundamental physics and materials science.

In this chapter, we provide a description of our interferometer design, discuss the design criteria for each component, and justify our design choices.
4.2 Strained Silicon Monochromator

The neutron beam used for this interferometer will be separated from the NIST reactor’s main beam using a strained silicon double-crystal monochromator (manufactured to specification by Grenoble Modular Instruments). The monochromator is used to select a specific energy of neutrons 2 to 4 Å from the incident beam. The monochromator has a cross-section of 5 cm by 5 cm. These width of the monochromator was optimized using geometrical consideration of the beam divergence by the time it hits the first collimator. Figure 4.1 is an annotated CAD model showing the dimensions and orientation of the strained silicon monochromator in relation the the beam break.

![Annotated CAD model showing the dimensions and orientation of the strained silicon beam break monochromator. The monochromator is set at 25.5° to select 2.7 Å neutron.](image)

Figure 4.1: Annotated CAD model showing the dimensions and orientation of the strained silicon beam break monochromator. The monochromator is set at 25.5° to select 2.7 Å neutron.
4.3 Magnesium Housing Windows for the Monochromator

The beam break is enclosed by magnesium windows with a thickness of 1 mm, to protect the monochromator from damage in the case of a structural failure in the main beam line. This thickness of magnesium will be able to withstand the ambient vacuum pressures with a safety factor of 2.8. The safety factor is defined as the maximum stress that a particular material can withstand divided by the maximum stress that will be present in a given configuration during its use. A safety factor of 2.8 means that the magnesium windows will maintain their structural integrity when enclosing vacuums up to 2.8 times stronger than the 10E-5 Torr vacuum they were designed to enclose. These magnesium windows transmit 90% of neutrons with wavelengths between 1 and 9 Angstroms incident on them. All relevant calculations can be found in Appendix B and J. Figure 4.2 shows a plot of the attenuation caused by the magnesium windows.

![Figure 4.2: Neutron attenuation due to the first magnesium beam break window.](image)

The existing NIST interferometer uses a highly ordered pyrolytic graphite monochro-
mator. Strained silicon has a narrower momentum acceptance range than pyrolytic graphite. This narrower acceptance range leads to a sharper scattered neutron spectrum and less noise in comparison with the existing NIST interferometer. However, a strained silicon monochromator admits a broader momentum spread of neutrons than a comparably-sized silicon monochromator. Admitting more neutrons into the vacuum chamber allows more neutrons to interact with the sample, thereby increasing the system’s signal to noise ratio. Figure 4.3 compares the selection of neutrons from the beam line by the two crystals. As demonstrated in the figure, the wavelength spectrum of neutrons selected by the strained silicon monochromator has a much narrower spread than that of pyrolytic graphite monochromator. A more detailed description of the physics underlying the monochromator’s operation can be found in Appendix B.

### 4.4 Shutter and Collimator

After being selected by the monochromator and passing through the magnesium windows, the beam then passes through a cadmium shutter and a collimator. The shutter blocks the incident neutron beam when the interferometer is not in use. During normal operation, the shutter will be controlled by servo motors. The shutter will also be mounted to a flange that will allow it to be manually rotated from the outside of the chamber, in the event that the servo motors fail. A cadmium collimator is used to collimate the incoming neutrons.

### 4.5 Fission Detector

A fission detector (manufactured by LND, Inc; part number 30753) has been mounted on servo motors (manufactured by Rockwell Automation; part number MPL-B960C) so that it can be moved into the beam line to determine the flux of incident neutrons.
Figure 4.3: Plot of flux as a function of wavelength for the remaining neutrons in the beam line, after passing through the beam splitter and Mg windows. The geometry of the system has been set to select 2.7 Å neutrons. This graph demonstrates that the strained-Si monochromator accepts a narrow range of neutron energies under Bragg diffraction and that passage through the Mg windows and beam splitter has small absorption effects (<90%) on the rest of the neutrons. Furthermore, the figure serves as a comparison between the use of strained-Si and pyrolytic graphite (PG) crystals for the monochromator. The latter is shown to have a much broader distribution in the selected neutrons.

The detector can be moved out of the beam line using the servo motors on a rack and pinion assembly after a definite neutron flux has been established. This detector has sensitivity of one count per 1000 neutrons.

4.6 HOPG Monochromator

The beam then strikes a highly-ordered pyrolytic graphite monochromator (manufactured to specification by Grenoble Modular Instruments). This nine-blade parabolic
monochromator, which extends 6 cm in the vertical direction and has a focusing length of 20 cm (Figure 4.4), focuses the beam and further selects energies. All relevant equations can be found in Appendix B. This monochromator is mounted on the same vibration stand as the strained silicon monochromator. Beam spreading is minimized because the two monochromators oscillate at the same frequency.

![Figure 4.4](image.png)

Figure 4.4: Schematic drawing of the nine-blade parabolic HOPG monochromator.

The two monochromators will be oriented using an x-ray source and detector mounted in parallel to the neutron flight path. X-rays are easier to produce, shield, and detect than neutrons. It will be more convenient for researchers to orient the monochromators by measuring x-ray diffraction, rather than by measuring neutron diffraction.

### 4.7 Helium-3 Spin Polarizer

The incident beam is then polarized in a helium-3 polarizer. Polarization at this point is also necessary for experiments in which the spin state of the neutrons are to be measured after the neutrons pass through the sample. The first spin polarizer that neutrons will encounter will have dimensions of 15cm long by 20cm diameter. This 20 cm diameter is equal to the diameter of the first monochromator. The other two spin flippers incorporated into our design will have dimensions of 15cm long by 10cm diameter. Or helium-3 polarizers are optimized to operate at an internal
Figure 4.5: Annotated CAD model showing main beam line, magnesium windows, strained silicon and graphite monochromators, shutter, collimator, and fission detector. The magnesium windows and the housing surrounding the beam break have been removed from this figure to more clearly show the strained silicon monochromator.

Pressure of 750 Torr. As shown in Figure 4.6, a helium-3 polarizer with 40% helium-3 polarization is capable of polarizing 80% of neutrons with 10% transmission. It is necessary to repolarize the rubidium in the polarizer approximately every 100 hours. A more detailed description of the physics underlying this device’s operation can be found in Appendix D.

It is also possible to polarize a neutron beam using a supermirror. However, in order to achieve a polarization efficiency of close to 1, the supermirror has to be on the order of two meters long, which renders it unfeasible for our compact design. It is desirable to keep the system as small as possible. Please see Appendix E for a description of the physics of supermirror operation.
4.8 Spin Flipper

A spin flipper is mounted downstream of the helium-3 detector. This interferometer uses an adiabatic RF spin flipper, as does the NIST interferometer, because it minimizes power consumption and neutron loss. Our design explicitly seeks to minimize neutron loss. Lowering the number of neutrons interacting with the sample decreases the signal to noise ratio. The spin flipper used in our design requires dimensions of 3cm x 2cm x 2cm. It requires a 10 Gauss backing field, which is provided by Helmholtz coils mounted about and below the vacuum chamber. The probability of spin flipping using an adiabatic RF flipper is close to 100 percent. Appendix F describes in greater detail the different varieties of spin flippers and the physics underlying their operation.

4.9 Interferometer Crystal and Sample, and Associated Cryogenic Systems

The beam passes through a four-blade silicon crystal crystal interferometer. The interferometry physics are described in more detail in section 1.2 and appendix C. The interferometer crystal is mounted on a set of rotation, tilting, and translation stages, manufactured by Newport Corporation, to provide six degrees of rotational and translation freedom.

The neutrons in our interferometer must travel approximately 88 cm between the beam break and the interferometer crystal. In the existing NIST design, the neutrons travel between 400 and 800 cm between the beam break and the interferometer crystal. Neutron intensity decreases as the inverse square of the distance the neutrons travel, so reducing the distance the neutrons travel by a factor of 5 increases the neutron count at the interferometer crystal by a factor of 25. Increasing the count increases the signal to noise ratio by a factor of 5, the square root of the increase in the neutron
count.

The sample can be cooled to 300 mK using a helium-3 finger. Cooling the sample reduces the thermal vibrations in the sample, thereby increasing the sharpness in the resulting image. Figure 4.6 shows the layout of the interferometer crystal, sample, and associated cryogenic systems. Additional descriptions of the devices’ operation and relevant thermodynamics calculations can be found in Appendix G.

Figure 4.6: Interferometer crystal, sample, and sample cooling system.

As shown in figure 4.6, the cryogenic system interfaces only with the sample, not with the interferometer crystal. Directly cooling the interferometer crystal would be useful, because a colder crystal has fewer thermal vibrations. Therefore, it would be able to generate an image with less noise. However, repeatedly cooling the crystal could introduce significant thermal strain, and might cause cracking. Also, a cryogenic system that directly interfaced with the crystal would introduce new vibrations in the crystal that would be difficult, if not impossible, to properly damp. After evaluating these considerations, we decided to directly cool the sample, and not directly cool the interferometer crystal.

It is necessary to thermally sink the interferometer crystal to reduce variation
in the crystal’s structure caused by variations in the ambient temperature. The crystal will be connected to a block of copper, measuring 22cm x 15cm x 20cm by sapphire rods. The copper block will extend through the bottom of the vacuum chamber and will be connected to a temperature stabilizer that will maintain the block’s temperature, and consequentially the interferometer crystal’s temperature, at 23 Celsius. Cooling the crystal to a lower temperature would reduce the thermal vibrations in the crystal, thereby reducing the noise in the image. However, cooling the crystal has the potential to introduce significant vibrations into the system. In particular, high frequency vibrations introduced by boiling of the cryogenic fluid used to cool the interferometer would introduce a great deal of noise into the system.

4.10 Additional Spin Polarizers and Spin Flippers

Additional polarizers and spin flippers are included downstream of the interferometer, as shown in Figure 4.6. These spin polarizers and spin flippers allow researchers to modify the neutrons’ spin state at many different points along their flight path, thereby increasing the types of experiments they can be performed with this interferometry system.

4.11 Helium-3 Detectors

The neutrons exiting the interferometer are detected by Helium-3 proportional counters (Saint-Gobain Crystals, part number 50H3/760/50). A more detailed description of the physics underlying this device’s operation can be found in Appendix H.

4.12 Vacuum Chamber

The vacuum chamber, which is kept at room temperature, has dimensions of 1.5 meters by 1 meter by 0.5 meters. It is made of 25.4 mm thick boron-doped stainless
steel. This thickness is sufficient to stop both neutrons and the gamma radiation produced by the neutrons’ interaction with the boron. With these dimensions, the top and sides of our vacuum chamber will weigh approximately 300kg. The top of the vacuum chamber will be fitted with handles so that the cranes available at the NIST facility will be able to lift it.

There are many possible approaches to providing adequate radiation shielding; boron-doped stainless steel represents only one options. Figure 4.7 shows the neutron attenuation as a function of neutron energy in the walls of the vacuum chamber. More detailed calculations describing the neutron and gamma radiation attenuation in the walls of the vacuum chamber can be found in Appendix I.

![Figure 4.7: Neutron attenuation as a function of incident neutron energy in the walls of the boron-doped stainless steel vacuum chamber.](image)

The components within the vacuum chamber must be repositioned to perform experiments using different neutron energies. Figure 4.8 shows the components’ position for imaging experiments using 2.7A neutrons, and Figure 4.9 shows the components’ position when using 4A neutrons. In Figure 4.9, the first monochromator is rotated counterclockwise by 7 degrees and the second monochromator is moved to the right by 14 cm with respect to the components position in Figure 4.8.
Figure 4.8: Component layout within vacuum chamber for 2.7A neutrons.

Figure 4.9: Neutron attenuation as a function of incident neutron energy in the walls of the boron-doped stainless steel vacuum chamber.

4.13 Vacuum Pumping System

During operation, the pressure inside the vacuum chamber will be maintained at 10e-3 Torr. This vacuum is generated using a Varian vacuum pump system (part number VHS-400). Based on the available specification sheets and approximation of the component out-gassing, it will take approximately 1 hour to pump the vacuum chamber to a pressure of 10e-3 Torr. The vacuum pump will not be running while the interferometer is in use, so these is no need to vibrationally isolate the vacuum pumps from the vacuum chamber.
4.14 Vibration Table

Our system is significantly less sensitive to external vibrations than current interferometer systems, because our system uses a four-blade interferometer crystal rather than a three-blade crystal. Therefore, we are able to streamline our vibration damping systems.

The vacuum chamber is supported by TMC vibration stands (part number 14-42H-84). This vibration table provides a vertical and horizontal vibration isolation efficiency of 80-90% at 5 Hz, and 90-99% vertical and horizontal vibration isolation efficiency at 10 Hz.

The entire system is mounted on the same set of vibration damping stands, so the components will not oscillate out of phase with one another. Figure 4.10 shows the position of the vibration stands supporting the vacuum chamber.

![CAD model showing TMC vibration stands supporting vacuum chamber.](image)

Figure 4.10: CAD model showing TMC vibration stands supporting vacuum chamber.

4.15 Future Tasks

The aim of this project was to develop a design for a compact neutron interferometer. Though we have described this design in detail throughout this chapter, there remain steps to be taken before this design is realized.
Before this design can be built, manufacturers or suppliers for each component of the design must be identified. For the spin polarizers and spin flippers, there are many options as these products are made by several companies. The exact dimensions and specifications needed for the system will determine which model is chosen for these modular components. Both monochromators also must be commercially bought. The model of the strained silicon monochromator must be chosen carefully because this component, while not new to the field, is new to neutron interferometry. Most likely, vibration stands from TMC will be incorporated as this company is trusted in the making of such optical benches. Lastly, a manufacturer for the vacuum pumps must also be identified.

Additionally, machining and determining the detailed specifications for the design must be explored. We have not addressed these tasks because their nature is such that they will be mapped out as the interferometer is built. However, these tasks include the determination of the exact, physical interface between each of the components, the so called “nuts and bolts” of the design.

Most importantly, the ingot must be cut. A 4-blade silicon interferometer has never been cut before; therefore, the provider of this crystal must be chosen carefully and must fully understand the function and importance of this component to the design.

While these tasks remain to be done, we hope that the design that we have described will act as a more than sufficient guide to building a new, compact neutron interferometer.
Chapter 5

Sensitivity Analysis

5.1 Introduction

Single crystal interferometer is extremely sensitive to external noise such as vibration. Therefore, a vibration isolation system is required for any single crystal interferometer. In this section, the sensitivity analysis of the four-blade crystal will be discussed and then compared to that of the original three-blade crystal.

5.2 Analysis

Figure 5.1 shows the schematic diagram for the three-blade and four-blade crystal system. According to D. Pushin [3], when the neutron is reflected, one can approximate the neutron as a particle and an interferometer blade as a moving wall, in which the interaction is fully understood. There is no interaction when the neutron is transmitted. According to D. Pushin [3], vibration along the z-axis will not affect the contrast since the velocity of the neutron in the z-direction and the path lengths are independent of the vibration along the z-axis. Therefore, we are only focusing on the vibration along x, y and angular directions. D. Pushin [3] describes in his thesis the details of how one obtain a phase difference, $\Delta \Phi$, between the two neutron paths.
Figure 5.1: A: A schematic diagram of the 3-blade neutron interferometer. B: A schematic diagram of the proposed interferometer with 4 blades. In both cases, there are four types of vibration: in x direction, y direction, and z direction and rotational about the center of mass. The vibration in any direction is assumed to be sinusoidal with the form \( \theta(t) = \theta_0 \sin(\omega t + \phi) \). \( \theta_0 \) is the amplitude of vibration, \( \omega \) is the frequency and \( \phi \) is the random phase at which the particle interacts with the blade. Taken from D. Pushin’s thesis [3]

subjected to vibration. The intensity measured at the O-detectors has the form

\[
I_O(\phi) = 1 + \cos(\Delta \Phi(\varphi) + \phi). \tag{5.1}
\]

The average of the intensity, \( I_0(\phi) \), over the random phase, \( \varphi \), which varies from 0 to \( 2\pi \) is what necessary to calculate the contrast, \( C \), which is defined [3]

\[
C = \frac{\max[I(\phi)] - \min[I(\phi)]}{\max[I(\phi)] + \min[I(\phi)]} \tag{5.2}
\]

We are interested in the changes in contrast as the amplitude and/or the frequency of the vibrations in direction of interests vary. Using the results for \( \Delta \Phi \) derived by D. Pushin [3], the sensitivity of the 4-blade crystal due to vibration are obtained.
Figure 5.2 shows the contrast as a function of vibration frequency in y direction. Here we assume the distance between any two blades in the 3-blade system is 5 cm. The same is applied for the 4-blade system except the distance between the two middle blades is 10 cm. The amplitude of vibration is 0.1\( \mu m \). Here, one observes a drop in contrast at around 50 Hz vibration for the 3-blade system while that of the 4-blade system does not change much until the frequency gets up to around 250 Hz. The same trend is observed when one plot the contrast as a function of amplitude of vibration in y direction for a constant frequency, 15 Hz in this case, and this is shown in figure 5.3. The four-blade system, therefore, is much less sensitive to vibration in the y-axis in comparison with the three-blade one.

![Contrast as function of vibration frequency in Y direction](image)

Figure 5.2: Contrast as function of vibration frequency in y direction for the 3-blade and 4-blade interferometer. The neutron velocity is approximately 2000 m/s, the vibration amplitude is 0.1 \( \mu m \). Four-blade: blue curve. Three-Blade: red curve.

In the x-axis, the four-blade crystal does not perform as well as the three-blade crystal. However, the difference here is smaller compared to that in y direction. The results are shown in figure 5.4 and 5.5.

However, in rotational direction, the three-blade system is much worst than the four-blade one. As figure 5.6 shows, the contrast of three-blade system drops quickly.
Figure 5.3: Contrast as function of vibration amplitude in y direction for the 3-blade and 4-blade interferometer. The neutron velocity is approximately 2000 m/s, the vibration frequency is 15 Hz. Four-blade: blue curve. Three-Blade: red curve.

even for vibration frequency of 10 Hz while that of the four-blade system only drops 50% at 20 Hz. Over all, the four-blade system is a lot less sensitive to vibrations compared to that of the three-blade system.
Figure 5.4: Contrast as function of vibration frequency in x direction for the 3-blade and 4-blade interferometer. The neutron velocity is approximately 2000m/s, the vibration amplitude is 0.1 $\mu m$. Four-blade: blue curve. Three-Blade: red curve.

Figure 5.5: Contrast as function of vibration amplitude in x direction for the 3-blade and 4-blade interferometer. The neutron velocity is approximately 2000m/s, the vibration frequency is 20 Hz. Four-blade: blue curve. Three-Blade: red curve.
Figure 5.6: Contrast as function of vibration amplitude in rotational direction for the 3-blade and 4-blade interferometer. The neutron velocity is approximately 2000m/s, the vibration amplitude is 1 micro radian. Four-blade: blue curve. Three-Blade: red curve.
Chapter 6

Examples of Applications

The existing neutron interferometer at NIST is dedicated to studying fundamental, nuclear, and solid-state physics and has historically been used for the observation of various quantum phenomena with great precision and detail. Our design accommodates for a wider range of conditions for both the sample and the interferometer, which can hopefully expand the breadth of applications for the new interferometry facility and therefore make the interferometer accessible to a broader scientific and industrial community.

6.1 New Conditions

Our design deviates from the existing one mainly in its compactness, improved vibration control, and capacity for a cooled sample. The compact design allows us to place the interferometer in closer proximity (\( \sim 1 \) m) to the cold neutron source, which vastly increases the overall neutron incidence at the detectors. Having a high neutron count is especially advantageous for applications in neutron imaging, since it reduces the total measurement time and improves the overall contrast. Neutron imaging experiments that can be performed using the neutron interferometer include neutron phase contrast imaging, 3D neutron tomography, and single crystal diffractometry.
Additionally, the effects of thermal and motional vibrations on the measurement results are minimized with the vacuum environment and the uses of the state-of-the-art vibration stands and a four-blade interferometer. This can make possible accurate measurements of the nuclear scattering properties of materials and therefore allow the structural and bulk properties of materials to be investigated with greater precision than existing setups. Phase detections by the neutron interferometer rely on neutron wave interference and are therefore particularly sensitive to vibrations. As a result, the improved vibration control can allow for better phase measurements to be performed in fundamental physics experiments, such as those for the studies of the fermion spinor and gravitational quantum interference.

Finally, our design accommodates a wide range of sample temperatures (from 300 mK to higher than room temperature) and can consequently be used to study samples previously deemed inappropriate for interferometry studies due to their thermal vibrational properties at room temperatures.

6.2 Examples of neutron interferometry experiments

The NIST neutron interferometer is being used to develop neutron imaging techniques, which include neutron phase contrast imaging, three-dimensional neutron tomography, and single crystal diffractometry [21]. These imaging techniques will be used to study the structural and bulk properties of materials and will be applicable to other materials science research and industrial applications. However, there are two fundamental and current applications of neutron interferometry: (1) accurate measurement of nuclear scattering properties of isotopes and (2) measurement of the Fourier Transform of the neutron momentum distribution, which is used to precisely study the surface properties of materials. Here we show examples of the current applications as well as an example of the developing neutron phase contrast imaging application [21].
6.2.1 Measurement of Nuclear Scattering Properties

Word and Werner from the University of Missouri Physics Department have used neutron interferometry to measure the nuclear scattering properties of isotopically pure $^{149}$Sm [4]. Near a resonance energy, the neutron scattering amplitude is dependent on the energy of the incident neutron. Neutron transmission experiments allow measurement of a material’s neutron absorption cross section, which is proportional to the imaginary part of the forward scattering amplitude. However, this method of scattering amplitude determination involves averaging over many Bragg scattering angles in the crystallographic. This averaging may obscure the presence of any angle-dependent variation seen in the resonant scattering amplitude. Word and Werner have, therefore, used neutron interferometry to directly measure the coherent part of the forward-scattering amplitude. Placing the sample, in this case $^{149}$Sm, in the interferometer causes a neutron’s wave function to undergo a phase shift that is proportional to the real part of the forward scattering amplitude [4]. Word and Werner have, therefore, used neutron interferometry to directly measure the coherent part of the forward-scattering amplitude. Placing the sample, in this case $^{149}$Sm, in the interferometer causes a neutron’s wave function to undergo a phase shift that is proportional to the real part of the forward scattering amplitude. The phase shift, $\Delta \phi$, is given by $\Delta \phi = R(nft\lambda)$, where $n$ is the atomic density of the sample, $t$ is the sample’s thickness, and $\lambda$ is the wavelength of the neutron [4]. The magnitude of the wave function, then, falls off as

$$r = e^{\pi(nft\lambda)}$$  \hspace{1cm} (6.1)

, which is proportional to the imaginary part of the forward scattering amplitude [4].

The experimental setup, shown below, for this study used a pyrolytic graphite double crystal monochromator, a highly perfect silicon crystal for the interferometer blades, and a slab of Si or Al of thickness $T$ that was used as a freely-rotating phase flag. In addition, a $^3$He detector was used to measure the intensity from the interference of
the two coherent waves emerging from the interferometer.

Figure 6.1: Word and Werner’s experimental setup. Three perfect silicon blades comprise the interferometer. A phase flag of either Si or Al intercepts both beams and can be rotated, while the sample intercepts one beam and is fixed. An $^3$He detector is used to measure the outgoing intensity of one of the beams. [4]

Given the angle $\delta$ between the phase flag slab and the interferometer blades and the phase shift $\Delta \phi$, shown above, the phase difference introduced into the two paths is determined to be

$$\beta_{rot} = 2\lambda v_{rot} T \frac{\sin \delta \sin \theta_B}{\cos^2 \theta_B \sin^2 \delta}.$$  \hfill (6.2)

The measured intensity is, therefore, sinusoidal and dependent on the angle $\delta$. Two sinusoidal curves were obtained for various angles as the $^{149}$Sm sample was placed in and out of the neutron path in the interferometer. The difference in phase of the two sinusoids was a measurement of the phase shift $\Delta \phi$, which was introduced by the $^{149}$Sm sample and is proportional to the real part of the forward scattering angle. Investigating nuclear scattering properties in this way therefore allows the study of $^{148}$Sm, or any sample, with a single species of atomic constituent [4].

### 6.2.2 Fourier Transform of Nuclear Momentum Distribution

Kneller et al of the University of Pennsylvania used neutron interferometry to study the hydration state of yeast cytochrome c (YCC) monolayers on soft interfaces. Wa-
ter hydration is important to maintaining the function of soluble proteins, such as YCC, which is a membrane protein [22]. Therefore, neutron interferometry was used to determine how much water hydrates each YCC molecule such that protein function is maintained. The water distribution in the monolayer profile structure can be seen from neutron scattering density profiles from neutron reflectivity for a monolayer hydrated by D2O versus a monolayer hydrated by H2O. Various multilayer substrate profiles were used in this study, and the normalized neutron reflectivity for the composite structures can be expressed in the kinematic limit for $q_z > q_{\text{critical}}$ by the equation [22]

$$|\Phi_{\text{kin}}(q_z)|^2 = |\Phi_k(q_z)|^2 + |\Phi_u(q_z)|^2 + 2|\Phi_k(q_z)||\Phi_u(q_z)|\cos\{[\psi_k(q_z) - \psi_u(q_z)] + 2\pi q_z A_{ku}\}$$

(6.3)

where $|\Phi_{\text{kin}}(q_z)|^2$, $|\Phi_k|^2$, and $|\Phi_u|^2$ are the moduli squared of the Fourier Transforms of the gradient of the neutron scattering length density profile for the composite structure, the known multilayer substrate, and the unknown protein overlayers, respectively [22]. $q_z$ is the momentum transfer normal to the substrate surface divided by $2\pi$. The moduli squared of these Fourier Transforms can be determined experimentally from the neutron reflectivities of these three samples, and the moduli can be compared to recover the unknown phase information or momentum distribution of the detected neutrons [22].
Chapter 7

Summary

We initially identified three design goals that would make this design a worthwhile contribution to the field of interferometry: an increase in the signal to noise ratio, a simpler setup, and compatibility with a wider range of applications. Throughout the course of developing this design, we have seen our decision to use a 4-blade interferometer has allowed us to achieve each of these goals. The 4-blade system simplifies the vibration isolation setup needed to make accurate measurements with the interferometer. Because several layers of isolation for vibration damping are no longer needed, our design has become more compact. We can, therefore, move the entire setup closer to the beam break. This shortens the overall distance between the main beam line and the interferometer, which decreases the divergence of the beam between system components and lowers the neutron loss throughout the design. We, therefore, calculate a neutron intensity of 1399 n/s compared to the intensity of 54 n/s at the current NIST facility, which is a 26-fold increase in our signal-to-noise ratio, as shown by Figure 7.1.

Figure 7.1 also demonstrates our achievement of a simpler setup for the interferometer. As previously mentioned, the 4-blade system allows us to consolidate the vibration isolation system, which allows a more compact design and greater proximity to the beam break. The simpler setup allows for more physical workspace, as can be
Figure 7.1: Comparison of the Proposed Design to the NIST Design.

seen by the dimension comparison of the proposed design to the current NIST design in Figure 7.1. Additionally, the compact design improves the accessibility of the interferometer with respect to ease of changing the sample between the interferometer blades, modifying the placement of spin flippers and polarizers, and employing the cryogenic cold finger in various experimental setups. The degree to which this design is accessible in addition to the inclusion of the cold finger and the modular spin flippers and polarizers adds the quality of experimental flexibility to this design. Depending on the experiment being conducted, the components of our system can be modified and relocated to satisfy the differing setups that may be required to conduct various experiments. In addition, the increased signal-to-noise ratio that we were able to achieve improves the resolution of any measurement made with this system. This increased resolution not only decreases the amount of time necessary to make measurements, but also allows this system to be used for materials science and biological applications, which require accurate measurements in order for such measurements to be useful. The increased resolution as well as the flexibility of experimentation built
into our design, therefore, has allowed us to achieve our third design goal of increasing the compatibility of neutron interferometry to a wider range of applications. This report describes the compact and flexible design of a neutron interferometer that can be used for a broad range of applications. We have provided a detailed description of this design in Chapter 4 and in the Appendices, and we have demonstrated the features of our system through numerical simulations, which are shown in Chapter 5. While we realize that every aspect of our proposed design may not materialize, we do expect that a version of this design will be built at NIST. In addition, it is our hope that the constructed design will retain the unique components that we have incorporated: the magnesium windows, the strained silicon monochromator, the 4-blade interferometer crystal, the compact spin polarizers and flippers, the cryogenic cold finger, and the single vacuum shell and optical bench. These unique components resemble our contribution to the field of interferometry, and it has been our sincere desire to not only contribute to this field, but to contribute in a marked and useful way.
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Appendices
Appendix A

Neutron Beam Guides and Windows

A.1 Introduction

While selecting long wavelength neutrons, it is important to isolate their flight path from the surrounding environment. This is done via a beam guide or flight tube. The most important metric for a beam guide is minimizing the interactions of the neutron beam, so that the neutrons are delivered to the experiment with a small momentum spread and at high intensity. In addition to the scientific necessity for a beam guide, there are legal requirements that prohibit an open neutron path of longer than 20cm. This makes beam guides effectively unavoidable in neutron system designs.

Windows allow neutrons to enter a beam guide or other device while keeping unwanted particles out of the system. They have a small cross-section for neutrons at the desired energy but are still structurally sound. These windows are typically thin since any structurally sound material will be dense and this increases the macroscopic cross section to which the neutrons are exposed.
A.2 Beam Guides

Because of beam guide requirements, there are two commonly used designs: Helium filled and vacuum. Both have advantages and disadvantages and both have been used with the current interferometer on NIST. The designs have a number of similarities. Both designs are 4.5 inch diameter vacuum tubes that are made of Aluminum, because it has a low cross section for the neutrons of interest (2.7 Å corresponding to an energy of 0.012 eV) see Figure A.1. Aluminum scatters and absorbs neutrons, but its scattering cross section is much larger than its absorption cross section. Because of the mass difference between Aluminum and a neutron, the scattering does not change the neutron spectrum too much. Activation of Aluminum does occur; however, its relaxation is fast in comparison to most materials, Al-28 has a 2.2 minute half life, so the danger is minimal. Finally, the strength to weight ratio of Aluminum is high so the weight of the tubes can be small, and Aluminum is inexpensive to use. For all of these reasons Aluminum is often used in vacuum projects in all fields of science.

All flight tubes have to be surrounded by shielding to minimize personnel exposure to neutron radiation. In NIST, Boroflex is used, which is a rubber permeated with Boron. All flight tubes require neutron windows to allow neutrons to enter and exit more or less freely; windows will be discussed in greater detail in the next section.

A number of these flight tubes have been built, in order to allow the interferometer to operate at slightly different neutron energies. The current design requires different distances from the beam break to the interferometer for different energies. The Figure A.2 shows one of the current NIST guides. The black material is the Boroflex, and the device at the end of the tube is a fission detector to measure the flux.

A.2.1 Helium Filled

One of the options for flight tubes is to fill them with Helium-4. Helium-4 has a very low neutron cross section, see Figure A.3, because it is has a doubly magic nucleus.
Finally, the strength to weight ratio of Aluminum is high so the weight of the tubes can be small. All of these reasons are why Aluminum is often used in many vacuum projects in all fields of science.

**Figure A.1:** Total and scattering cross sections for Aluminum-27. [5]

This means that a guide filled with Helium will not interact significantly with the neutron beam. Assuming that Helium is an Ideal Gas at atmospheric pressure and $300^\circ K$, we can find a macroscopic cross section and mean free path for the neutrons of interest. The microscopic cross section is approximately 1 barn.

\[
PV = nRT \tag{A.1}
\]

\[
\frac{n}{V} = \frac{101000 \ Pa}{8.314 \ \frac{j}{molK} \times 300^\circ K} \tag{A.2}
\]

\[
\frac{n}{V} = 2.4376 \times 10^{25} m^{-3} \tag{A.3}
\]

\[
\lambda = \frac{1}{(n/V)\sigma} = 411 m \tag{A.4}
\]
where $P$ is the pressure, $V$ is the volume, $n$ is the number of atoms, $K$ is Boltzmann’s constant, $T$ is the temperature, and $\lambda$ is the mean free path.

This mean free path signifies that the effect from the Helium in the tube is minimal on the neutron intensity. Helium can also have a higher than atmospheric pressure in the tube, which guarantees an outflow of gas from the tube. This permits having thin windows on the ends of the beam. The window that was used in the existing tubes is a sheet of aluminum tightened by two O-rings. The disadvantage of a Helium filled tube is that Helium diffuses very well through metal so there is a need for a constant flow of Helium; the Helium bottle had to be replaced approximately once a week.
Appendix: Neutron Beam Guides and Windows

Figure 2: Picture of current NIST beam guide.

Figure 3: Total and scattering cross sections for Helium-4. Scattering cross section is not visible under total. [5]

A.2.2 Vacuum

A vacuum tube has almost no inherent scattering in the tube, and this is the design that is currently installed on the NIST interferometer. This design requires a vacuum pump, currently a roughing pump is used, that evacuates the guide and is then turned off during operation to minimize vibrations throughout the experiment. The disadvantage of a vacuum system is that the window has to be significantly thicker to account for the pressure differences in the surrounding atmosphere and the internal atmosphere. This increase in the window thickness has been found to counteract any gains by removing the helium from the tube.
A.3 Neutron Windows

Neutron windows or filters have to be used to isolate any device, such as a beam guide, from the surrounding environment. As an added bonus the window can serve as a shield for delicate system parts such as monochromators, since windows are easier to replace than the precisely ground crystal. Several materials have been used for low energy neutron windows including: Aluminum, Magnesium, Zirconium, Teflon, and Kapton. For small pressure differences on the two sides of the window, a very small thickness of material can be used. For larger pressure differences, a thicker window has to be used. For the current NIST design, the material used is Aluminum. The windows are a thin sheet of Aluminum for the Helium system and a few millimeters of Aluminum for the vacuum system. The vacuum window has to be welded to the beam guide. Aluminum and Magnesium windows are currently under consideration the new interferometer design. The two cross sections of the materials are over plotted in figure A.4. While Aluminum has the smaller total cross section, Magnesium has a higher tensile strength, so a Magnesium window can be made thinner than an Aluminum window leading to a lower neutron loss.

Sometimes filters have been used as well as windows, where the filters attenuate unwanted neutrons. It is known that the neutron flux at the beam break interacting with the filter provides significant heating to the material. This forces the use of a liquid nitrogen cooling system for the window. In the past, cooled Aluminum filters have had problems with liquid condensation on them, which is extremely disruptive to the neutron flux, since water has a high interaction rate with the neutron beam. The NIST reactor beam guide is evacuated and the intended design of the new neutron interferometer will be vacuum, so condensation should not be present.
Figure A.4: Total cross sections for Aluminum and Magnesium. Aluminum shown in red. [5]
Appendix B

Si and HOPG monochromators

A crystal monochromator is a device that is used to select a defined wavelength (or energy) of radiation in neutron and x-ray optics. The device operates through the diffraction process according to Bragg’s Law [23].

The principle of wave-particle duality can be applied to neutrons, thus each neutron interacts with lattice crystal of the monochromator as a wave. The neutron interactions with the crystal to produce waves, which interfere either constructively or destructively to produce a diffraction pattern. Constructive interference is observed with a phase shift of $2\pi$ [24].

Bragg’s Law can be stated as

$$n\lambda = 2dsin\theta,$$

where $n$ is an integer, $d$ is the distance between the lattice planes of the crystal, $\lambda$ is the wavelength of the emerging neutrons, and $\theta$ is the angle between the incident ray and scattering plane, as shown in the figure below [24].

The scattering plane is the plane of the crystal in the monochromator. Our neutron interferometer design consists of two different monochromators: the first uses a strained silicon crystal while the second uses a pyrolyic graphite crystal. The angle of the crystal (and thus the scattering plane) relative to the incoming beam can be adjusted so as to select a certain particle energy from the incoming beam.
If the material has a low enough absorption cross section, the energies in the beam that were not reflected, or selected, by the monochromator crystal will transmit through the crystal and continue on their paths, as shown in Figure B.1. The interferometer design that we present here employs a double crystal monochromator system, by which the beam incident on the first monochromator is parallel to the beam reflected off the second monochromator [25].

B.1 Strained silicon monochromator

The main purpose of the first monochromator is to accurately select neutrons of energy 2–4 Å out of the neutron beam that emerges from the reactor while allowing longer wavelength neutrons to transmit such that cold-neutron experiments further along the main beam line can be conducted. We chose the first monochromator to consist of a single crystal of strained silicon because this crystal lattice allows the energy selection we aim for while keeping the neutron momentum distribution broad compared to perfect silicon and narrow compared to pyrolytic graphite. A perfect silicon crystal would mandate very precise geometry in order to reflect neutrons into our interferometer setup; it is therefore not as practical as the broader geometric
acceptance of strained silicon. Figure 4.3 demonstrates the rationale for choosing strained silicon over pyrolytic graphite. Though the first monochromator for many existing designs uses a pyrolytic graphite monochromator, strained silicon more accurately selects the energy of neutrons that are specified by the geometric arrangement. This broad distribution ensures that a substantial number of neutrons are entering the interferometer setup so as to yield a significant signal.

In addition, the distance between the first monochromator and the following Cd collimator had to be optimized in order to minimize neutrons loss. The beam of neutrons that is reflected from the first monochromator into our interferometry setup moves towards a Cd collimator while diverging. The collimator was placed to minimize the neutron loss given this beam divergence.

B.2 Pyrolytic graphite monochromator

The second monochromator is placed such that the reflected beam will be parallel to the beam incident on the first monochromator and will run toward the interferometer blades, as shown by Figure B.2. In existing designs that employ pyrolytic graphite for both the first and second monochromators, the same momentum spread of the beam is observed before the first monochromator and after the second [23].

The main purpose of the second monochromator is to vertically focus the beam while simultaneously providing a second round of energy selection. Pyrolytic graphite is used because its low neutron absorption cross section keeps as many neutrons as possible in the reflected beam, thus improving the system’s signal and increasing the effective neutron flux at the sample position. In addition, this material is relatively inexpensive. Unlike the first monochromator, the second device consists of nine crystals lined up vertically. Each of the nine crystals can be rotated or translated, but all nine crystals are positioned to resemble a slight parabola. This shape allows vertical focusing of the beam incident on the pyrolytic graphite crystals. Though both crystals
Figure B.2: Double Crystal Parallel Geometry of the Monochromators.

are made of different materials, the strained silicon allows an increase in the number of neutrons in the system, which improves the system’s signal. Ideally, the beam emerging from the second monochromator is focused, has a reasonably high neutron count, and has a known momentum distribution that corresponds to energies between 2 and 4 Å. The combination of the strained silicon and pyrolytic graphite monochromators allows us to achieve these three goals in the double crystal monochromator system.
Appendix C

The Four-Blade Interferometer

C.1 Design choice

Our design utilizes a four-blade interferometer [3]. This interferometer differs from the three-blade configuration in that the third blade is moved further away from the second blade such that the two Bragg-reflected beam paths emerging from the third blade refocus at the fourth blade. A diagram of the interferometer is shown in Figure C.1.

The four-blade interferometer is advantageous for obtaining measurements with high contrast because the low-frequency vibrations of the system are removed in the reference frame of the neutron beam [3], by having the divergent beams undergo a reflection operation before refocusing takes place. Its physics is almost equivalent to the three-blade interferometer and hence experiments that are currently studied using the existing set-up can be run without significant modifications. The only caveat of this configuration is the additional loss of transmitted neutrons at the third crystal blade, but this loss can be compensated by moving the device closer to the beam guide so that neutron losses due to beam divergence are reduced.
C.2 Physics

An unpolarized beam of neutrons is in a mixed state of randomly oriented spins. This is best expressed by a density matrix \( \rho = \frac{1}{2}(\lvert \uparrow \rangle \langle \uparrow \rvert + \lvert \downarrow \rangle \langle \downarrow \rvert) \). This unpolarized neutron beam passes through a spin filter that only allows spins of a certain orientation to pass through. For instance, if the He-3 atoms inside the filter are polarized in the +\( \hat{z} \) direction, the basis for the spin state of any neutron in the beam is \( \sigma_z \) (all neutron spins are aligned along \( \hat{z} \)) and only the spin-up (\( \lvert \uparrow \rangle_z \)) neutrons will be transmitted.

The resulting system can be approximated by a pure state. That is the spin state of the neutron beam can be described by a single wavefunction, \( \lvert \psi \rangle_{\text{spin}} \). \( \lvert \psi \rangle_{\text{spin}} \), after the passage of the neutron through a spin filter (at point I), is \( \lvert \uparrow \rangle_z \). So far we have had no need to define the spatial state of the neutron beam, since all neutrons are moving along the same beam before arriving at the interferometer, and the spatial wavefunction of the neutrons can be arbitrarily denoted as \( \lvert 1 \rangle \). Upon arriving at the first blade of the interferometer, however, the beam splits into two paths, by the processes of Bragg reflection and transmission. We assume that the reflection and transmission coefficients are both \( \frac{1}{2} \). The state of the beam as it emerges from the

\[ \text{Figure C.1: Schematic setup of the four-blade interferometer (as viewed from the top).} \]
first blade can be represented by the spatial wavefunction $|\psi_{\text{path}}\rangle$:

$$|\psi_{\text{path}}\rangle_{\text{II}} = \frac{1}{\sqrt{2}}(|1\rangle + |2\rangle),$$

(C.1)

where $|1\rangle$ and $|2\rangle$ respectively refer to the transmitted and reflected path along which the neutron is traveling, as illustrated in Figure C.1. The full description of the state of the neutron beam, $|\psi\rangle$, can be formed from putting the two wavefunctions together:

$$|\psi\rangle_{\text{II}} = |\psi_{\text{path}}\rangle \otimes |\psi_{\text{spin}}\rangle,$$

(C.2)

$$= \frac{1}{\sqrt{2}}(|1\rangle + |2\rangle) \otimes |\uparrow\rangle_z,$$

(C.3)

$$= \frac{1}{\sqrt{2}}(|1, \uparrow\rangle + |2, \uparrow\rangle).$$

(C.4)

We then subject only the neutrons in path $|2\rangle$ to a spin flipper, which rotates the spin of the neutron by $\theta$ along $\hat{x}$. A rotation operator can be used to represent the action of this device. The operator is

$$\hat{R}_x(\theta) = e^{-i\sigma_x \theta \sigma_z} = \begin{pmatrix} \cos(\theta/2) & -i\sin(\theta/2) \\ -i\sin(\theta/2) & \cos(\theta/2) \end{pmatrix}.$$  

(C.5)

$|\uparrow\rangle_z$ and $|\downarrow\rangle_z$ are respectively represented by $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$ and $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$, so the action of $\hat{R}_x(\theta)$ on $|\uparrow\rangle_z$ and $|\downarrow\rangle_z$ yields

$$\hat{R}_x(\theta)|\uparrow\rangle_z = \begin{pmatrix} \cos(\theta/2) & -i\sin(\theta/2) \\ -i\sin(\theta/2) & \cos(\theta/2) \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} \cos(\theta/2) \\ -i\sin(\theta/2) \end{pmatrix}.$$  

(C.6)

$$= \cos(\theta/2)|\uparrow\rangle_z - i\sin(\theta/2)|\downarrow\rangle_z$$

and

$$\hat{R}_x(\theta)|\downarrow\rangle_z = \begin{pmatrix} \cos(\theta/2) & -i\sin(\theta/2) \\ -i\sin(\theta/2) & \cos(\theta/2) \end{pmatrix} \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} -i\sin(\theta/2) \\ \cos(\theta/2) \end{pmatrix}.$$  

(C.7)

$$= -i\sin(\theta/2)|\uparrow\rangle_z + \cos(\theta/2)|\downarrow\rangle_z.$$  

(C.8)

The resulting $|\psi\rangle_{\text{III}}$ from the rotating operation of the spin flipper is

$$|\psi\rangle_{\text{III}} = \frac{1}{\sqrt{2}}(|1, \uparrow\rangle + \cos(\theta/2)|2, \uparrow\rangle - i\sin(\theta/2)|2, \downarrow\rangle).$$  

(C.10)
Appendix: The Four-Blade Interferometer

The beams subsequently pass through the second blade, whose action can be represented by a mirror, since both of the split beams are reflected [3]. The wavefunction becomes

$$|\psi\rangle_{IV} = \frac{1}{\sqrt{2}}(|2, \uparrow\rangle + \cos(\frac{\theta}{2})|1, \uparrow\rangle - i \sin(\frac{\theta}{2})|1, \downarrow\rangle).$$  \hfill (C.11)

After this we add a phase shift of $\phi$ along path 1, so that

$$|\psi\rangle_{V} = \frac{1}{\sqrt{2}}(e^{-i\phi}|2, \uparrow\rangle + \cos(\frac{\theta}{2})|1, \uparrow\rangle - i \sin(\frac{\theta}{2})|1, \downarrow\rangle).$$  \hfill (C.12)

The split beams pass through yet another blade and are again reflected:

$$|\psi\rangle_{VI} = \frac{1}{\sqrt{2}}(e^{-i\phi}|1, \uparrow\rangle + \cos(\frac{\theta}{2})|2, \uparrow\rangle - i \sin(\frac{\theta}{2})|2, \downarrow\rangle).$$  \hfill (C.13)

Finally the beams converge at the fourth blade and Eqn. (C.14) represents the wavefunction of the neutron beam (split again into two paths) as it emerges from the interferometer.

$$|\psi\rangle_{VII} = \frac{1}{\sqrt{2}}(e^{-i\phi}(|1\rangle + |2\rangle)\uparrow\rangle + (|1\rangle + |2\rangle)(\cos(\frac{\theta}{2})\uparrow\rangle - i \sin(\frac{\theta}{2})\downarrow\rangle).$$  \hfill (C.14)

Since the transmitted and reflected paths are divergent, one needs to observe the two paths separately; these are referred to as the O-beam and H-beam. The O-beam can be represented by

$$|\psi\rangle_{O} = \frac{1}{\sqrt{2}}(e^{-i\phi}|2\rangle\uparrow\rangle + |2\rangle(\cos(\frac{\theta}{2})\uparrow\rangle - i \sin(\frac{\theta}{2})\downarrow\rangle).$$  \hfill (C.15)

In experiments where it is necessary to select specific spin states for observation, the beams can be passed through a $\frac{\pi}{2}$ spin flipper and then a spin filter before detection. The effect of a $\frac{\pi}{2}$-rotation of the spin state of the neutron can be computed from applying $\hat{R}_x(\frac{\pi}{2})$ to $|\psi\rangle$. The actions of $\hat{R}_x(\frac{\pi}{2})$, according to Eqns. (C.7) and (C.9), are as follows:

$$\hat{R}_x(\frac{\pi}{2})|\uparrow\rangle_z = \frac{1}{\sqrt{2}}(|\uparrow\rangle - i|\downarrow\rangle)$$  \hfill (C.16)

$$\hat{R}_x(\frac{\pi}{2})|\downarrow\rangle_z = \frac{1}{\sqrt{2}}(-i|\uparrow\rangle + |\downarrow\rangle).$$  \hfill (C.17)
One can then show that the application of $\hat{R}_x(\frac{\pi}{2})$ on $|\psi\rangle_O$ gives

$$\hat{R}_x(\frac{\pi}{2})|\psi\rangle_O = \frac{1}{2}|2\rangle[(e^{-i\phi} + \cos(\frac{\theta}{2}) (|\uparrow\rangle - i|\downarrow\rangle)) - i\sin(\frac{\theta}{2})(-i|\uparrow\rangle + |\downarrow\rangle)]. \quad (C.18)$$

The beam subsequently goes through a filter that selects spins along positive $\hat{z}$ and the resulting wavefunction as the beam reaches the detector is

$$|\psi\rangle_{\text{VII}} = \hat{R}_x(\frac{\pi}{2})|\psi\rangle_O = \frac{1}{2}|2, \uparrow\rangle(e^{-i\phi} + \cos(\frac{\theta}{2}) - \sin(\frac{\theta}{2})). \quad (C.19)$$

The intensity observed at the detector is now a function of both $\phi$ and $\theta$:

$$I = |\langle \psi |\psi \rangle_{\text{VII}}| \quad (C.20)$$

$$= \frac{1}{4}(e^{-i\phi} + \cos(\frac{\theta}{2}) - \sin(\frac{\theta}{2}))(e^{i\phi} + \cos(\frac{\theta}{2}) + \sin(\frac{\theta}{2})) \quad (C.21)$$

$$= \frac{1}{2}(1 - \frac{1}{2}\sin(\theta) + \cos(\phi)(\cos(\frac{\theta}{2}) - \sin(\frac{\theta}{2}))) \quad (C.22)$$

Most interferometry experiments involve only a $\pi$ flip ($\theta = \pi$), so the interference pattern becomes

$$I = \frac{1}{2}(1 + \cos(\phi)). \quad (C.23)$$

### C.3 Significance

For a non-divergent, monochromatic neutron beam, the contrast measured at the O-beam detectors is 1. The interference pattern in Eqn. (C.23) can be observed by rotating the phase flag from 0 to 180°. In cross-section measurements, a sample is often placed along the path of one of the beams. The insertion of the sample has the effect of introducing an overall phase shift to the interference pattern, from which one can infer the scattering properties of the sample.

With the addition of spin polarizing and flipping components, the interferometer is capable of handling experiments involving spin-measurements or the observation of magnetic and spin effects. Examples include EPR interferometry [26] and spinor experiments [27].
Appendix D

He-3 Spin Polarizers

D.1 Introduction

Neutron scattering experiments usually require spin-polarized neutrons to study more physical phenomena. Up until recently, Heusler alloy monochromators and supper-mirrors have been used as the neutron polarizer [2]. However, $^3\text{He}$ polarizer has been of great interest for the last several years due to its ability to polarize neutrons of a wide range of energy up to epithermal energy [3]. In this section, the basic operation of a $^3\text{He}$ polarizer will be discussed as well as the selection for the $^3\text{He}$ polarizer for the interferometer will be presented.

D.2 Operation of $^3\text{He}$ polarizer

A $^3\text{He}$ polarizer operates based on the capture reaction in equation D.1

$$^3\text{He} + n \rightarrow ^4\text{He}^* \rightarrow \text{Tritium} + p + 740\text{KeV} \quad \text{(D.1)}$$

The cross section for such reaction depends on the orientation of a neutron’s spin relative to the polarization of the $^3\text{He}$ molecule. An anti-parallel spin neutron has a large capture cross section ($\sigma \uparrow\downarrow \approx 6000 * \lambda[\text{Å}]$) while that of parallel spin neutron is
Appendix: He-3 Polarizers

only $\approx 5$ barns [1]. Therefore, polarization of a neutron can be selected by orienting the polarization of the $^3$He molecule.

Transmission ratio and polarizer efficiency are the two important quantities of any $^3$He polarizer. The transmission ratio determines the fraction of neutrons of a specific spin to pass through the $^3$He polarizer while the efficiency determines the fraction of neutrons that get polarized going through the polarizer. The transmission ratio for parallel (denoted as $+$) and anti-parallel (denoted as $-$) neutrons is given in equation D.2 [4]

$$T_{\pm} = \frac{1}{2} e^{-\left(\sigma_o \mp \sigma_p P_3 n L\right)}$$

where $P_3$ is the polarization of the $^3$He molecule and $L$ and $n$ are the length and the number density, respectively. $\sigma_o$ and $\sigma_p$ are the respective cross section for the unpolarized and polarized $^3$He, which are defined as

$$\sigma_o = \frac{1}{2}(\sigma_+ + \sigma_-)$$

$$\sigma_p = \frac{1}{2}(\sigma_+ - \sigma_-)$$

where $\sigma_\pm$ are the cross section for the parallel and anti-parallel spin, respectively. The polarization of a neutron beam ($P_n$) and its transmission efficiency ($T_n$) after passing through a polarized $^3$He sample are defined as [4]

$$P_n = \frac{T_+ - T_-}{T_+ + T_-} = -\tanh(\sigma_p P_3 n L)$$

$$T_n = T_+ + T_- = T_o \cosh(\sigma_p P_3 n L),$$

where $T_o = e^{-\sigma_o n L}$. For $^3$He, it is common to approximate $\sigma_p = \sigma_o$ [4], which turns equation D.5 and D.6 into the following [1]:

$$P_n = \frac{T_+ - T_-}{T_+ + T_-} = \tanh(\sigma P_3)$$

$$T_n = T_+ + T_- = e^{-\sigma \cosh(\sigma P_3)},$$
Appendix: He-3 Polarizers

where \( O = nL\sigma_o = 0.0732O' \), and \( O' = p_{He}[\text{bar}] \cdot \lambda[\AA] \cdot L[\text{cm}] \) [1] for convenience of calculation. The quality factor (Q) determines how well a \(^3\text{He}\) polarizer functions, and is defined as

\[
Q = T_n \cdot P_n^2. \tag{D.9}
\]

A high value of Q is desirable.

D.2.1 \(^3\text{He}\) Polarization

In order for a \(^3\text{He}\) polarizer to function, the \(^3\text{He}\) needs to be polarized periodically. Currently there are two available methods: Metastability Exchanged Optical Pumping (MEOP) and indirect polarization using Rb vapor [2].

Metastability Exchanged Optical Pumping (MEOP)

Using a gas discharge, a \(^3\text{He}\) atom at ground state can be excited into the metastable state \( 1s2sS_1 \), which then can be polarized by optical pumping using a light laser of 1083 nm. The pressure of \(^3\text{He}\) is 1 mbar. This requires compression by a factor of 10,000 in order to be used as neutron polarizer, which operates at atmospheric pressure. Furthermore, the polarization of \(^3\text{He}\) decays with time, which means that the changes in neutron beam polarization and intensity need to be taken into account during an experiment. Currently, the time constant of the \(^3\text{He}\) polarization is about 82 hours [2]. According to Cussen et al. [2], for a 20 cm cell with a gas pressure of about 3 bar, 50 percent polarization \(^3\text{He}\) can be achieved. This allows 84 percent polarization of the neutron with a transmission efficiency of around 12 percent for a 4.5 Å wavelength neutron.

D.2.2 Polarization of \(^3\text{He}\) using Rb vapor

In this method, the \(^3\text{He}\) is polarized indirectly. Usually, the polarizer cell has both \(^3\text{He}\) and Rb vapor. The Rb vapor is first polarized by shining circularly polarized
light of wavelength 793 nm. The polarization is then transferred to the $^3He$ upon collision of the Rb vapor and the $^3He$ molecule via hyperfine interactions. The physics behind the spin exchange process is explained by Pomeroy [28] in his thesis. However, maintaining the uniform polarization and increasing in magnitude of the polarization has raised some difficulties as the pressure of the $^3He$ gas increases. However, at the $^3He$ gas in the cell increases, maintaining the uniform polarization while increasing the polarization in magnitude has met some difficulties. Sakai et al. [5] has developed a $^3He$ polarizer of this type which allows 40 percent polarization of $^3He$ at 3 atm. The $T_1$ relaxation time of the $^3He$ measured by NRM was $38.5 \pm 7.2$ hours. Pomeroy [28] also reported the fabrication of 64% polarized cells with polarization decay lifetimes of 180 hours.

### D.3 Selection of $^3He$ Polarizer for the Interferometer

Our interferometer system will consist of three $^3He$ polarizers. The first $^3He$ polarizer, located immediately after the pyrolytic graphite monochromator, will be 15 cm long with a 20 cm diameter, which corresponds to the size of the monochromator. The other two $^3He$ polarizers will of the same length but will have a 10 cm. They will be located before the O-beam and H-beam detectors, respectively. The cells are made of cesium-coated quartz with since this allows for a $T_1$ relaxation time on the order of 100 hours [1]. The cells will consist of $^3He$ gas, a small amount Rb and small amount of $N_2$ gas, similar to the design of Sakai et al. [5]. The $^3He$ will be re-polarized through spin exchange with Rb, and the process will not be done in situ with the experiment. The instruments, that are required for $^3He$ repolarization, such as lasers, oven, Helmholtz coil for magnetic field, are too cumbersome to be included within the vacuum shell. It is easier to remove the $^3He$ polarizer cell from the shell and allow the $^3He$ to be repolarized outside. One of the main concerns is that this will break
the vacuum in the shell while replacing the $^3He$ polarizers and that this breakage will require extra time to pump the shell back to vacuum pressure again.

Using the expressions in equation D.7 and D.8, we calculate the transmission coefficient of the polarizers specified here and their transmission efficiency as a function of the polarization of $^3He$ in the cell. The results are shown in figure D.1. Here, the operating pressure of the $^3He$ polarizer is 1 bar and the wavelength of the neutron is 2.7 Å. As the figure shows, even with only 40 percent $^3He$ polarization, an 80 percent

![Figure D.1: Transmission Coefficient, Polarization Efficiency and Quality Factor as a function of $^3He$'s polarization](image)

neutron polarization can be achieved with 10 percent transmission. Also, a higher $^3He$ polarization gives higher values for $T_n$ and $P_n$, which may be needed for some experiments.
Appendix E

Supermirrors

E.1 Introduction

In neutron optics, the quality of the beam and the neutron’s polarization are very important. Neutrons in a beam are usually selected using a monochromator. The selected neutrons are then guided to an experiment by a neutron optical mirror system. Polarizers are used to select neutron polarization. Supermirrors, devices consisting of a multilayer mirror with alternating layers of two materials, have been one of the most useful devices for neutron experiments, especially for slow neutron studies [29]. Supermirrors can be used as advanced neutron guides, polarizers and analyzers. We will cover some basic neutron optics to explain how a supermirror functions as a guide and/or a polarizer. Examples of many supermirrors that have been developed will be discussed here to illustrate the many applications of supermirrors in neutron research. Finally, a discussion of the possible uses of supermirrors in the new neutron interferometer will be presented.
E.2 Review of Neutron optics

E.2.1 Neutron reflection and refraction

Neutrons exhibit properties of both waves and particles. Therefore, neutrons can reflect and refract in many ways similar to light. Analysis of neutron behavior within neutron optics, therefore, is not significantly different compared to such an analysis with light. However, the standard notations of neutron optics are slightly different from light optics, and hence create some different mathematical results. The basic physics, however, is the same. Figure E.1 represents the notation in neutron optics.

![Neutron Optics Notation](image)

Figure E.1: Neutron Optics Notation [6].

In the figure, $\theta_i$, $\theta_R$ and $\theta_T$ are the angle of incident (glancing angle), angle of reflection and transmitted angle, respectively. $n_1$ and $n_2$ are the indexes of refraction of materials above and below the interaction surface, respectively. The angles here are exaggerated for the sake of a better description. According to Snell’s law,
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equation (E.1),

\[ n_2 \cos(\theta_T) = n_1 \cos(\theta_i) \quad (E.1) \]

at the boundary of the two media with different indices. A neutron beam can be completely reflected if the glancing angle is below the critical angle, \( \theta_c \), where \( \theta_c = \frac{n_2}{n_1} \).

The value of \( \theta_c \) is usually of order minute of an arc per Å. Notice that the incident angle and the reflective angle (when there is reflection) are equal. This critical angle is the basis for the neutron guide and selection based on neutron polarization.

E.2.2 Supermirror as neutron guide and polarizer

As previously stated, a supermirror is a multilayer mirror consisting of alternating materials of varying thickness. With the correct choice of materials having a large difference in the index of refraction, one can increase the value \( \theta_c \), which enables more neutrons to be totally reflected at the boundary. The refractive index of neutrons depends not only on the material but also on the velocity of the neutron according to equation E.2 [30] [31]

\[ n = \left(1 - \frac{\sqrt{V}}{E}\right)^{\frac{1}{2}} = 1 - \frac{N\lambda^2 b}{2\pi} \quad (E.2) \]

where \( N \) is the nuclei density, \( \lambda \) is the neutron wavelength, \( b \) is the coherent scattering length. For a single neutron mirror, Nickel gives the highest critical angle and this is known as \( m = 1 \) mirror. Usually, the quality of a supermirror is determined by its \( m \) value. Another way to increase the \( \theta_c \) is to have more bilayers. The general structure of a supermirror is shown in figure E.2.

In the figure, layer P and layer S are the protective layer and the substrate layer, respectively. The substrate layer is assumed to be infinitely thick relative to the other layers. The bilayer with thickness \( d_{k1} \) and \( d_{k2} \) in the \( k^{th} \) level has refractive index \( n_1 \)
Figure E.2: Geometry of a general multi-bilayer structure composed of 2N alternating thin films of generic materials 1 and 2 with refractive indices $n_1$ and $n_2$, respectively. The thickness $dk_1, dk_3,...$ corresponds to layer 1, 2..., respectively. Substrate S has refractive index $n_S$ and the mirror is immersed in air with refractive index $n_A$ [7].

and $n_2$ respectively. For a bilayer, the incident angle $\theta$ satisfies Bragg diffraction when $d_\mu = \frac{\lambda}{4\psi_\mu}$ where $\mu$ is material specific [32]and $\psi_\mu$ is the angle with respect to the horizontal plane of material $\mu$. The critical angle for the multilayer depends on the thickness of the bilayers. The thickness dependency is described by Hayter and Mook [7]. Essentially, with the different materials, the thickness has to be chosen such that the Bragg condition is met, subsequently gives total reflection. Supermirrors are usually built up layer by layer. The desired reflectivity determines the supermirror structure. For the use of a supermirror as a polarizer, one of the two materials in the bilayer must be ferromagnetic, which provides a constant magnetic field in the device.
Appendix: Supermirrors

The neutron optical phenomena are the same except that the index of refraction in equation E.2 has an extra magnetic potential term. This is shown in equation (3) [32], where (+) denotes parallel, (-) denotes anti-parallel to the magnetic field, and μ is the magnetic moment of neutron. B is magnetic induction and H is the magnetic field strength of the material.

\[ n_{\pm} = (1 - \frac{V_{\text{new}}}{E})^{\frac{1}{2}} = 1 - \lambda^2 \left( \frac{Nb}{2\pi} \mp m_n(B - H) \frac{\mu}{\hbar^2} \right) \]  

(E.3)

Using Snell’s law, one can again solve for the critical angle, which now depends on both wavelength and spin of the neutron, given in equation (E.4) [32].

\[ (\theta_c)_{\pm} = \lambda \left[ \frac{Nb}{\pi} \mp m_n(B - H) \frac{\mu}{\hbar^2} \right]^{1/2} \]  

(E.4)

For the appropriate choice of material, where \( \frac{Nb}{\pi} = m_n(B - H) \frac{\mu}{\hbar^2} \)\(^{1/2} \), only one spin state can be reflected since the \( \theta_c \) of the other spin is 0. This is how supermirror can act as a polarizer.

E.3 Developments and Applications

The idea of a graded multilayer was first proposed in 1966, but the first working supermirror for neutrons was not made until the 1970s [33]. However, since then, many supermirrors have been developed, tested and used for many applications. Currently, supermirrors with m value of 4 (the performance is four times better than the Nickel mirror) can be obtained commercially.

In 1980, Hayter et al. measured critical reflection angle for copper on glass surface and two organic materials. They found that the reflectivity profile depends greatly on the surface and the thickness of the inter-facial layer.

Ten years later, Mezei [34] described the operation of a polarizing cavity, which he claimed more advantageous compared to conventional supermirror. In 1999, using Monte-Carlo code, Soyama [35] reported a guide tube system made of supermirrors that can provided 5.6 times that of natural Nickel guide tube. A year later, Hino et
al. [29] showed the feasibility of producing a broad-band supermirror polarizers using sputtering techniques for magnetic field of 1mT. In 2001, Semadeni et al. [36] showed three main advantages of benders, which includes: spin flippers are not needed, spin selection independent of neutron wavelength and no cross-talk between neutron spin flippers. Then, Stahn and Clemens [37] produced a Fe/Si supermirror that gave 96 to 98% polarization efficiency by adding reactive gases O$_2$ and N$_2$ to the Silicon. Funahashi et al. showed coherence of neutron reflected of supermirror, which confirm the possible use in cold neutron interferometry.

### E.4 Supermirror in Neutron Interferometry

Supermirrors have been well developed and have been shown to be feasible in Neutron Interferometry as both neutron guide and polarizers. Since we are only interested in using supermirror as polarizer in the current design of the neutron interferometer, we will consider the disadvantage and advantages of supermirror for such system.

The major advantages of supermirror included its well characterized properties and its variety. Current supermirror can polarize neutron of higher than 95% efficiency. Coherent neutron beam, a necessity for neutron interferometry, were measured from supermirror reflections. In addition, a wide range of neutron wavelengths can be selectedly reflected or transmitted using supermirror. The 2-3 Å range will present no problem for current existing supermirror.

The major disadvantage of supermirror is its length due to its small critical cut-off angle. Usually, supermirrors are of order 1m long, which is inconvenient for the new system’s design, where we want the interferometer facility as compact as possible. Mezei [34] reported a supermirror polarizer of size 215cm by 3cm, and another device at Budapest with dimension of 150cm by 2.5cm. Finally, production of supermirrors is always subjected to design error due to the intricacy of the supermirror. Any surface’s roughness in any bilayer can affect the supermirror’s function.
E.5 Conclusion

Supermirrors have been well developed and tested. While a supermirror can be used as polarizer in cold neutron interferometry, its length of at least 1m long can present an inconvenient for a new interferometer design. We want the crystal to be as close to the source as possible and the whole system need to be in vacuum. The large size of supermirror can greatly affect those two factors. Unless a new polarizing system with supermirror’s efficiency but with smaller dimensions can be found, supermirror will be an ideal polarizer for the new interferometer.
Appendix F

Spin Flippers

F.1 Description of a spin flipper

In a fully polarized beam, a neutron’s spin will be aligned with the applied field. The purpose of a spin flipper device is to either flip the direction of the spin by $\pi$ (so that $|\uparrow\rangle \rightarrow |\downarrow\rangle$ or vice versa), or turn $\mathbf{P}$ along a different axis than $\mathbf{B}_0$. Here we review various methods of changing spin orientation.

By the adiabatic principle, during a slow evolution of the guide field ($\omega_{\text{evolution}} \ll \omega_L$), the polarization vector $\mathbf{P}$ will continue to precess around $\mathbf{B}_0$. It is then possible to change the orientation of the polarization vector by slowly rotating the guide field to the desired direction.

On the other hand, the spin direction is not changed during a sudden reversal of the magnetic field ($\omega_{\text{evolution}} \gg \omega_L$). This result can be shown using the “sudden approximation” method in quantum mechanics [38]. By this principle, neutrons can be passed through a current sheet in which the current direction is orthogonal to the spin polarization, such that the guide field direction is flipped from one side of the current sheet to another. The spin of the neutron emerging from the current sheet is therefore reversed with regards to the guide field. This process is shown in Figure F.1.

Generally speaking, in cases where the change of guide field direction is neither
Appendix: Spin Flippers

Figure F.1: The passage of a spin-polarized neutron through a folded current sheet. Picture taken from [8].

slow nor sudden, spin flip requires that the neutrons be temporarily subject to a new field. This can be done by passing the neutron beam through another field of $B_1$. The strength and direction of $B_1$, along with the time it takes for the neutron beam to pass through the field, can be varied to achieve different flip angles. Fig.(F.2) demonstrates the process of spin flipping.

In the case illustrated on the left, $P$ is turned from $B_0$ by $\frac{\pi}{2}$. $B_1$ is pointed along $\hat{y}$ and has the same strength as $B_0$. Within the new field, the spin precesses around the vector sum of $B_0$ and $B_1$, which is rotated $\frac{\pi}{4}$ from $\hat{z}$. Since the neutron velocity is known for an incident neutron beam, the distance over which $B_1$ is applied can be adjusted to yield the precession angle $\pi$. With this precession angle, the polarization of the neutron beam as it emerges from field $B_1$ will be along $\hat{y}$. The polarization transformation, to follow the convention introduced previously, is $(P_x, P_y, P_z) \rightarrow (-P_x, P_z, P_y)$.

On the right, the spin entering the new field is subject only to $B \hat{y}$ and is allowed to precess for $\pi$ before exiting the field. When it emerges, its polarization changes to $(-P_x, P_y, -P_z)$. 
Figure F.2: *Left:* Demonstration of $\frac{\pi}{2}$ turn from $B_0$. The spin is initially along $\hat{z}$ but passes through a field of $B_{\text{HELM}}\hat{z} + B_{\text{FLIP}}\hat{y}$ during which it precesses by $\pi$. As it emerges from the field, it is aligned along $\hat{y}$. *Right:* $\pi$ flip. A spin initially aligned along $\hat{z}$ is subject to a field of $B\,\hat{y}$ and flipped by $\pi$ to $-\hat{z}$ by the time it emerges from the field. In the formulas, $v$ indicates neutron velocity and $x$ the distance traveled by the neutron in the field. Picture taken from NIST website: http://www.ncnr.nist.gov/instruments/nse/NSE technique/sld007.htm

F.2 Types of spin flippers

**RF flippers**

A radiofrequency (RF) flipper turns spins by subjecting them to an oscillatory field with frequency tuned to a resonant condition with regards to the Larmor frequency. Within a RF flipper, the magnetic field $B_{\text{rf}}$ can be expressed by

$$B_{\text{rf}} = B_0 \, \hat{z} + B_x \cos(\omega_s t + \chi) \, \hat{x},$$  \hspace{1cm} (F.1)

where $B_x$ is the magnitude an oscillating RF field, with $\omega_s$ as the oscillating frequency and $\chi$ as the phase shift [39]. The resonance condition for the flip is $\frac{\omega_s}{2} = \omega_z$ and the flip angle is defined as

$$\theta_{\text{flip}} = \frac{|\mu_N|}{\hbar} \frac{B_x \, d}{v},$$  \hspace{1cm} (F.2)

where $d$ and $v$ respectively represent the distance traveled by the spin in $B_{\text{rf}}$ and the velocity of the spin [39].
RF flippers remain commonly used in diffraction and interferometry studies with fairly high degrees of efficiency and accuracy. In recent years, an adiabatic component has been added to the RF flipper design to accommodate for a wider range of neutron velocities in the beam. The adiabatic design will be described in section 3.0.4.

Mezei coils

Mezei introduced a flat-coil spin flipper device in his study of spin-echo. DC current is run through the coil and a field $B_c$ is induced. Inside the coil, the field is the sum of $B_0$ and $B_c$. As the neutron enters the coil, it begins to precess around $B_0 + B_c$. The spin rotation experienced by the neutron during its passage through the coil can be adjusted by manipulating the magnitudes of $B_0$ and $B_c$, and the size of the coil [40, 41].

The advantages that the Mezei coil has over an rf flipper include [40]:

1. higher flipping efficiency

2. flipping occurs within well-defined beam volume

3. adjustable neutron path within flipper (allows for possibility of including a chopper)

However, correction coils are often needed to adjust for field inhomogeneities in the Mezei coil, which increase the size of the device. Furthermore, given the low flux condition of the cold neutron interferometer, passing the neutron beam through the coil can lead to absorption and hence even lower incidence at the detectors.

Leningrad flipper

The Leningrad flipper device consists of two coils with opposing fields and flips the beam passing axially through them by the changing sign of the solenoidal guide field. For more details, please see reference [42].
Adiabatic RF flipper

In a conventional RF flipper, the probability of spin reversal is a function of neutron velocity. Therefore, resonance spin flipping is only efficient for monoenergetic neutron beams. To accommodate for neutron beams with a wider range of energy, an adiabatic RF flipper can be used. This flipper is similar to the conventional RF flipper except that the static field now contains a gradient. The resonance condition is met at precisely the middle point of the flipper. In order to achieve spin flipping for the polychromatic neutron beam, the adiabatic condition has to be satisfied for the fastest neutrons in the beam. That is,

\[ \frac{dB}{dt} \ll 2\pi\gamma B_1. \]  

(F.3)

It has been reported that the probability of spin flipping using this method is close to 1 [43].

F.3 Dimension of the flipper

The spin flipper used for our design will be an adiabatic RF flipper. We opted for this type of flipper since it can be compact in size, does not require a large backing field (\(B_0 \sim 10\text{ G}\) will suffice), and will likely not cause additional losses in neutron counts since the beam is not being passed through a current sheet or coils (as in some DC flippers and the traditional RF flipper). Furthermore, adiabatic flippers can act on neutron beams with a wide range of energies with high flipping efficiency.

F.3.1 Rough calculations for the dimensions of the RF flipper

We use the relations

\[ B_1L = \frac{\pi\hbar v_0}{8\mu}, \text{ where} \]

\[ v_0 = \frac{2|\mu|B_0a}{\hbar}. \]  

(F.4)  

(F.5)
$a$ is the wavelength corresponding to the resonant frequency of the spin in the backing field and is different for each neutron energy. Using $B_0 = 10 \text{ G}$, $B_1 = 0.5 \text{ G}$, and a neutron wavelength of $\lambda = 2.7 \text{Å}$, we get $L \sim 0.01 \text{ m}$. Including the space needed for the hardware, the RF flippers will have dimensions on the order of 2-3 cm in length.
Appendix G

Sample Cryogenics

A cryostat is a device used to maintain a system at cryogenic temperatures. Helium-3 cryostats are used for cooling systems to the 0.3 K to 1 K temperature range. Often, helium-3 cooling systems have adjacent helium-4 and nitrogen cooling systems, to help maintain low temperatures by buffering the system from the lab’s ambient temperature. The helium-3 is liquefied by bringing it into contact with a helium-4 bath.

At equilibrium, the helium-3 has equal rates of condensation and vaporization. If some of the vapor phase is removed, some of the liquid phase then evaporates to restore equilibrium. This vaporization requires energy, because the gas phase has a higher entropy and therefore a higher Gibbs free energy than the liquid phase. Therefore, this vaporization lowers the kinetic energy of the remaining liquid. Alternatively, the thermodynamics of the system can be considered as follows: there is some variation in the temperature of the liquid helium-3 molecules. The molecules with the higher temperature (and therefore, by definition, the higher kinetic energy) are more likely to enter the vapor phase, leaving behind the colder molecules. In a helium-3 cryostat, the vapor phase is removed by being absorbed by charcoal.
Helium-3 has a higher rate of vaporization than helium-4, and therefore a faster rate of vaporization and a faster cooling rate for a given rate of vapor removal. Helium-3 has a significantly lower superfluid transition temperature than helium-4 (2.6 mK versus 2.17 K). Below this transition point, the liquid has zero viscosity and infinite thermal conductivity.
Appendix H

He-3 Detectors

There are several different configurations of proportional counters. This application uses a cylindrical counter containing a fixed amount of helium, with no helium flow-through. This configuration creates a voltage gradient when the anode and cathode are connected to a voltage source. It is impossible, however, to use this voltage gradient to detect the neutrons directly, because they are electrically neutral. Therefore, the detector must instead measure the charged particles produced when the helium-3 captures an incoming neutron. This capture reaction is written as follows:

\[ \text{neutron} + ^3\text{He} \rightarrow \text{proton} + \text{tritium} + 764\text{keV} \quad (\text{H.1}) \]

The 764 keV released corresponds to the mass difference of the reaction. This capture reaction causes prompt release of a proton. The proton is accelerated to the cathode, and this acceleration causes a cascade of ionization in the gas. These ions are further accelerated and measured when they hit the cathode. Looking at the energetics of the capture reaction, an ideal output energy curve would have one sharp peak at 764 keV. However, proportional counters exhibit wall effects, which occur when some of the reaction products enter the wall of the detector. A particle that is quickly absorbed by the detector wall does not cause an ionization cascade, so it is not detected. Because of momentum conservation, which says that the two produced...
particles must travel in opposite direction, we know that only one of the produced particles is absorbed by the wall in a given capture reaction. The other, non-absorbed particle is able to create an ion cascade and is therefore detected. This partial detection causes steps on the lower-energy side of the main peak. The shoulders of these peaks are at 191 keV and 574 keV, which are the energies of a tritium nucleus and a proton, respectively.

It is desirable to minimize wall effects so that the peak can be seen more clearly. This can be accomplished in several ways. Increasing the diameter of the detector decreases the probability of a given reaction occurring near the detector wall. Increasing the pressure of the helium-3 gas reduces the range of the charged particle reaction products. He-3 has a low atomic mass, so reaction products tend to have a long range, thereby increasing the prevalence of wall effects. It is also possible to add a small amount of heavier gas to the proportional counter to increase the stopping power.

There are also proportional counters filled with Boron-10 or Li-6, but He-3 has a significantly higher capture cross section than either of those two gases. Helium-3 proportional counters can also be operated at a higher pressure, so that they have a higher detector efficiency.
Appendix I

Shielding Calculations

I.1 Preliminary Comments

The shielding method that is described here is only one of the several possible methods. We use a thick stainless steel chamber in order to minimize risk and provide a reinforced outer shell. The disadvantages of this method are the weight, welding difficulty, and expense that comes with a thick outer chamber. Another possible method of shielding would be to place boronated sheets of plastic at locations where the neutron beam is likely to strike. The Cadmium filters can be surrounded in boxes of lead or stainless steel to shield from gamma rays. While this method is less expensive and lighter than the previous one, it leaves the possibility of human error exposing a laboratory worker to unnecessary amounts of radiation, so the method presented here is the safest and most robust one.

I.2 Properties

We assume that our chamber is made of stainless steel with 1.1% doped Boron. This material was selected because of its use in reactor control rods and our need to shield from neutrons and gamma rays. In order to simplify the calculations, the composition
of stainless steel is assumed to be 74% Iron-56, 18% Chromium-52, and 8% Nickel-58. The Boron doping is assumed to be 20% Boron-10 and 80% Boron-11. Our chamber walls will be 1 inch thick in most locations, and the two primary energies of radiation present will be 2.7 Å neutrons and 263 keV gamma rays. The gamma ray emission is from neutron capture in the Cadmium collimators.

I.2.1 Initial calculation

Using our previous assumptions, we find the atom density of the chamber walls. First, we find the average molar weight.

\[
Weight = (0.18 \times 58 + 0.08 \times 52 + 0.74 \times 56) \times 0.989 + (0.2 \times 10 + 0.8 \times 11) \times 0.011 = 54.9 \text{ (I.1)}
\]

Assuming a stainless steel density of 8.03 g/cm\(^3\), we find atom density.

\[
Atom \, Density = \frac{(8.03 \text{ g/cm}^3)}{(54.9 \text{ g/mol})} \times 6.022 \times 10^{23} \text{ atoms/mol} = 8.79 \times 10^{22} \text{ atoms/cm}^3 \text{ (I.2)}
\]

I.2.2 Neutron Shielding

Taking the neutron cross sections from the MCNP database [44], we can plot transmission through our chamber for a variety of energies. A graph of this transmission as a function of energy is shown in figure I.1. For the most probably wavelength neutron, 2.7 Å, the transmission is 2.5%. Since the highest energies that are expected to be isolated from the beam correspond to 2 Å, which also has a transmission of approximately 2.5%, none of the other wavelengths, which are longer, that are reflected from the monochrometer are expected to pose a major health risk.

Given that the transmission is below 5%, and that large fluxes at the chamber are not expected, this should shield the surrounding areas sufficiently for a safe working environment.
Appendix: Shielding Calculations

Figure I.1: Neutron transmission as a function of Energy.

I.2.3 Gamma Shielding

As stated earlier, the gamma rays in the system primarily occur from absorption by Cadmium. The most energetic emission seen from Cadmium is 263 keV. Finding the mass attenuation coefficients from the NIST website [45], and repeating the previous assumptions about the composition of the chamber walls, we can calculate the transmission of these gamma rays through the walls.

\[
\frac{I}{I_0} = e^{-\mu \times 1 \text{ in} \times 2.54 \text{ cm}} = 0.012
\]  \hspace{1cm} (I.3)

All other gamma rays are lower energy, and the mass attenuation is higher for them, so they pose less of a health risk than the 263 keV gamma emission.
Appendix J

Neutron Count Calculations

Here we calculate the neutron incidence at the He-3 detectors as well as the momentum acceptance spectra of the Si and PG crystals in the interferometer setup. A central wavelength of 2.7Å is chosen as the basis for our calculations.

J.1 First monochromator

The beam-splitter that is used to select neutrons of our desired wavelength (2-4 Å) is a piece of strained-Si crystal (111). Si-crystals have a lattice spacing of $d_{Si} = 3.1355\text{Å}$, corresponding to a Bragg diffraction angle of 25.5° for 2.7 Å neutrons. The range of energies (or equivalently the range of wavelengths $\Delta \lambda$) allowed under the Bragg condition at any given diffraction angle $\theta_0$ can be determined by considering the angular divergence $\Delta \theta$ of the crystal. The angular divergence is the angular offset in the alignment between the crystal lattice planes. The energy acceptance is represented by
\[ \Delta \lambda = \lambda (\theta_0 + \Delta \theta) - \lambda (\theta_0) = 2d \sin(\theta_0 + \Delta \theta) - 2d \sin \theta_0 \]
\[ = 2d(1 - \cos \Delta \theta) + 2d \cos \theta_0 \sin \Delta \theta \]
\[ \frac{\Delta \lambda}{2d \sin \theta_0} = \cot \theta_0 \sin \Delta \theta + (1 - \cos \Delta \theta), \]

where \( d \) refers to the lattice spacing and \( \lambda(\theta_B) \) is the Bragg wavelength evaluated at \( \theta_B \). For a small angular divergence (which is true for monochromator crystals), the above relationship is simplified to

\[ \frac{\Delta \lambda}{\lambda} = \Delta \theta \cot \theta_0. \]  

(J.1)

The strained-Si crystal has an angular divergence of \( \sim 0.05^\circ \). This gives a \( \Delta \lambda \) of 0.0049 Å. The distribution of the neutron beam reflected off the crystal can be approximated as gaussian [23]. Based on the simulated neutron spectrum in the NG-6 beam line, and assuming that the strained-Si has a reflectivity of 0.9, the flux of the reflected neutron beam is \( 7.56 \times 10^5 \) n/cm²/s. This figure also takes into account the attenuation of neutrons by the first Mg window. The integrated neutron flux varies for different selected wavelengths, depending on both the Bragg angle and the initial neutron spectrum. Figures J.1 and J.2 illustrate the integrated neutron flux as a function of central wavelength for the range of selected wavelengths from 2 to 4 Å.

### J.2 Cd Collimators

The dimensions of the beam size are optimized such that neutron losses due to beam divergence are minimized during collimation. For this reason, we assume that collimation will not greatly affect the momentum distribution of the neutron beam but will introduce neutron losses purely by absorption. The beam divergence is assumed to be 0.5° in the neutron flight path. For our reference neutron count calculation, we use a size of 5.5 cm × 2 cm for the first collimator and 0.7 cm × 1.5 cm for the second collimator.
Appendix: Neutron Count Calculations

<table>
<thead>
<tr>
<th>Wavelength (Å)</th>
<th>Number of neutrons (n/s/cm²)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0105</td>
<td></td>
</tr>
<tr>
<td>0.011</td>
<td></td>
</tr>
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<td>0.0115</td>
<td></td>
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<td>0.0135</td>
<td></td>
</tr>
<tr>
<td>0.014</td>
<td></td>
</tr>
</tbody>
</table>

Number of selected neutrons by the strained-Si crystal as a function of neutron wavelength (theoretical)

Figure J.1: Relative neutron flux of the neutron beam reflected by the strained-Si monochromator as a function of the wavelength satisfying the Bragg condition. Calculation is done without consideration of the incoming neutron beam spectrum and based on an angular divergence of 0.05°, a reflectivity of 0.9 for strained-Si crystal, and a gaussian distribution for the reflected neutron beam.

### J.3 Second PG monochromator

Pyrolytic graphite monochromators have a Bragg angle of 33.3° for 2.7Å neutrons and an angular divergence of $\Delta \theta = 0.5^\circ$. This gives $\Delta \lambda = 0.043\text{Å}$. Since this acceptance range is much larger than that for the strained-Si crystal, all incident neutrons on the PG crystals should be either transmitted or reflected. For the chosen PG crystal, the reflectivity is 0.9.

### J.4 Interferometer crystal

Through the neutron losses by collimation and transmission, around 11% of the selected neutrons reach the interferometer. For perfect Si crystals, the angular divergence is $< 2.78 \times 10^{-4}$, corresponding to a $\Delta \lambda$ of $2.75 \times 10^{-5}\text{Å}$. This means that only
2.2 Neutron Count Calculations

Figure J.2: Neutron flux of the neutron beam reflected by the strained-Si monochromator as a function of the wavelength satisfying the Bragg condition. Calculation is based on the original neutron spectrum in beam line and also taking into account the attenuation by the first Mg window.

a small subset of the incident neutrons are accepted by the interferometer. The range of accepted neutrons in the spectrum of the incident beam (with a cross-section of 0.5 cm by 1.5 cm) is illustrated in Figure J.3.

The transmission and reflection coefficients of the crystal are approximated to be 0.5 and since the transmitted neutrons leave the interferometer at the second and third blades, the fraction of neutrons lost through the interferometer crystal is 0.25.

J.5 Detectors

The number of neutrons reaching the O-beam He-3 detectors per second is around 1399 n/s. For a spin polarized neutron beam, the beam undergoes further attenuation and the neutron count is reduced to 350 n/s. Table J.1 lists the neutron flux and
Figure J.3: Distribution of wavelengths of the incident neutron beam before it enters the interferometer and the range of accepted wavelengths for the perfect-Si crystal. The count at various locations along the flight of path of the neutron beam and compares the numbers for our design to those computed for the existing NIST setup. The results presented on the table are purely from numerical simulations. The dimensions used for the calculating the neutron statistics for the existing setup are shown in Figure J.4.

Figure J.4: Reference dimensions used for calculations related to the existing setup. Picture taken from [3].
Table J.1: A comparison between the neutron statistics for our design and the existing facility at NIST. Results are based purely from numerical simulations using the simulated neutron wavelength spectrum in the NG-6 beam line we obtained from NIST.

<table>
<thead>
<tr>
<th>Location</th>
<th>Our Design</th>
<th>Existing Setup</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Flux (n/cm²/s)</td>
<td>Count (n/s)</td>
</tr>
<tr>
<td>Strained-Si Monochromator</td>
<td>7.56 ×10⁵</td>
<td>1.89 ×10⁷</td>
</tr>
<tr>
<td>Collimator 1</td>
<td>5.86 ×10⁵</td>
<td>6.44 ×10⁶</td>
</tr>
<tr>
<td>Pyrolytic Graphite Monochromator</td>
<td>5.37 ×10⁵</td>
<td>5.80 ×10⁶</td>
</tr>
<tr>
<td>Collimator 2</td>
<td>1.92 ×10⁶</td>
<td>2.01 ×10⁶</td>
</tr>
<tr>
<td>Interferometer Crystal</td>
<td>1.07 ×10⁴</td>
<td>1.12 ×10⁴</td>
</tr>
<tr>
<td>O-beam Detector</td>
<td>–</td>
<td>1399</td>
</tr>
</tbody>
</table>

**J.6 Significance**

According to calculations, the new design has a 26-fold improvement in neutron incidence (1399 n/s for a 0.9 cm by 1.5 cm beam) at the detectors over the existing setup (26 n/s for a 0.8 cm by 1.3 cm beam), suggesting that measurement results can be obtained within a shorter period of time and with higher resolution. This increase in neutron counts is mainly attributed to the compactness of the new interferometer, which allows the entire setup to be moved significantly closer to the beam line, thus
drastically reducing the neutron losses through beam divergence. Furthermore, it should be noted that with the selection of strained-Si monochromator crystal over PG, the fraction of neutron loss as the beam enters the perfect crystal interferometer is reduced.
Bibliography


