**The Conditional CAPM Does Not Explain Asset-
Pricing Anomalies**

Jonathan Lewellen[*]
Dartmouth College and NBER
jon.lewellen@dartmouth.edu

Stefan Nagel[+]
Stanford University and NBER
Nagel_Stefan@gsb.stanford.edu

January 2006
Forthcoming in *Journal of Financial Economics*

We are grateful to Joe Chen, Kent Daniel, Ken French, Ravi Jagannathan, Leonid Kogan, Martin Lettau, Sydney Ludvigson, Jun Pan, Bill Schwert, Jay Shanken, Ken Singleton, Tuomo Vuolteenaho, two referees, and workshop participants at numerous institutions for their helpful comments. We thank Ken French and Sydney Ludvigson for providing data.

[*] Tuck School of Business, 305 Tuck Hall, Hanover, NH 03755.
[+] Graduate School of Business, 518 Memorial Way, Stanford, CA 94305.

The Conditional CAPM Does Not Explain Asset-Pricing Anomalies

Abstract

Recent studies suggest that the conditional CAPM holds, period-by-period, and that time-variation in risk and expected returns can explain why the unconditional CAPM fails. We argue, however, that variation in betas and the equity premium would have to be implausibly large to explain important asset-pricing anomalies like momentum and the value premium. We also provide a simple new test of the conditional CAPM using direct estimates of conditional alphas and betas from short-window regressions, avoiding the need to specify conditioning information. The tests show that the conditional CAPM performs nearly as poorly as the unconditional CAPM, consistent with our analytical results.

1. Introduction

The unconditional CAPM does not describe the cross section of average stock returns. Most prominently, the CAPM does not explain why, over the last forty years, small stocks outperform large stocks, why firms with high book-to-market (B/M) ratios outperform those with low B/M ratios (the 'value premium'), or why stocks with high prior returns during the past year continue to outperform those with low prior returns ('momentum'). In this paper, our goal is to understand whether a conditional version of the CAPM might explain these patterns.

Theoretically, it is well known that the conditional CAPM could hold perfectly, period-by-period, even though stocks are mispriced by the unconditional CAPM (e.g., Jensen, 1968; Dybvig and Ross, 1985; Jagannathan and Wang, 1996). A stock's conditional alpha (or pricing error) might be zero, when its unconditional alpha is not, if its beta changes through time and is correlated with the equity premium or with market volatility, as we discuss further below. Put differently, the market portfolio might be conditionally mean-variance efficient in every period but, at the same time, not on the unconditional mean-variance efficient frontier (Hansen and Richard, 1987).

Several recent studies argue that time-varying betas do, in fact, help explain the size and B/M effects. Zhang (2005) develops a model in which high-B/M stocks are riskiest in recessions when the risk premium is high, leading to an unconditional value premium. Jagannathan and Wang (1996), Lettau and Ludvigson (2001), Santos and Veronesi (2005), and Lustig and Van Nieuwerburgh (2005) show that the betas of small, high-B/M stocks vary over the business cycle in a way that, according to the authors, largely explains why those stocks have positive unconditional alphas.[1]

In this paper, we question whether the conditional CAPM can really explain asset-pricing anomalies, either in principle or in practice. Our analysis has two components. We argue, first, that if the conditional CAPM truly holds, we should expect to find only small deviations from the unconditional

[1] These studies consider both the simple and consumption CAPMs, and we use 'beta' in this paragraph to refer to risk measured either way. Our paper focuses on the simple CAPM but, as we explain later, the arguments apply to the consumption CAPM as well. Other recent studies on the conditional CAPM include Wang (2003), Adrian and Franzoni (2005), Ang and Chen (2005), and Petkova and Zhang (2005).

CAPM – much smaller than those observed empirically. Second, we provide direct empirical evidence that the conditional CAPM does not explain the B/M and momentum effects.

The first point can be illustrated quite easily. Suppose, for illustration only, that market volatility is constant. If the conditional CAPM holds, we show that a stock's unconditional alpha depends primarily on the covariance between its beta and the market risk premium, $\alpha^u \approx \text{cov}(\beta_t, \gamma_t)$. This implied alpha will typically be quite small. For example, suppose that a stock's monthly beta has a standard deviation of 0.3, about our estimate for a long-short B/M strategy, and that the monthly risk premium has a standard deviation of 0.5%, large relative to its average (also around 0.5%). Then, if the conditional CAPM holds, the stock's unconditional alpha can be at most 0.15% monthly [$\text{cov}(\beta_t, \gamma_t) \le \sigma_\beta\ \sigma_\gamma$], an upper bound achieved only if β_t and γ_t are perfectly correlated. Empirically, the B/M strategy has an alpha of 0.59% monthly (std. error, 0.14%), and a momentum strategy has an alpha of 1.01% monthly (std. error, 0.28%), both substantially larger than our estimates for plausible alphas.[2] In short, we argue that observed pricing errors are simply too large to be explained by time variation in beta.

The second part of the paper provides a simple new test of the conditional CAPM. The test is based on direct estimates of conditional alphas and betas from short-window regressions. For example, we estimate CAPM regressions every month, quarter, half-year, or year using daily, weekly, or monthly returns. The literature has devoted much effort to developing tests of the conditional CAPM, but a problem common to all prior approaches is that they require the econometrician to know the 'right' state variables (e.g., Harvey, 1989; Shanken, 1990; Jagannathan and Wang, 1996; Lettau and Ludvigson, 2001). Cochrane (2001, p. 145) summarizes the issue this way: "Models such as the CAPM imply a conditional linear factor model with respect to *investors'* information sets. The best we can hope to do is test implications conditioned on variables that we observe. Thus, a conditional factor model is *not testable*!" (his emphasis). Our methodology gets around this problem since it does not require any

[2] The data are described later. Briefly, the portfolios consist of all NYSE and Amex stocks on CRSP and Compustat from 1964 to 2001. The B/M strategy invests in the top quintile and shorts the bottom quintile of firms ranked by B/M. The momentum strategy invests in the top decile and shorts the bottom decile when stocks are ranked by past 6-month returns.

conditioning information. As long as betas are relatively stable *within* a month or quarter, simple CAPM regressions estimated over a short window – using no conditioning variables – provide direct estimates of assets' conditional alphas and betas.

Using the short-window regressions, we estimate time series of conditional alphas and betas for size, B/M, and momentum portfolios from 1964 – 2001. The alpha estimates enable a direct test of the conditional CAPM: average conditional alphas should be zero if the CAPM holds, but instead we find they are large, statistically significant, and generally close to the portfolios' unconditional alphas. The average conditional alpha is around 0.50% for our long-short B/M strategy and around 1.00% for our long-short momentum strategy (we say 'around' because we estimate alphas in several ways; all methods reject the conditional CAPM but their point estimates differ somewhat.) The estimates are more than three standard errors from zero and close to the portfolios' unconditional alphas, 0.59% and 1.01%, respectively. We do not find a size effect in our data, with conditional and unconditional alphas both close to zero for the 'small minus big' strategy.

Our tests show that betas do vary considerably over time – just not enough to explain large unconditional pricing errors. A nice feature of the short-window regressions is that they allow us to back out the volatility of true conditional betas. Specifically, the variance of estimated betas should equal the variance of true betas plus the variance of sampling error, an estimate of which is provided by the short-window regressions (see also Fama and French, 1997). Using this relation, we estimate that beta has a standard deviation of roughly 0.30 for a 'small minus big' portfolio, 0.25 for a 'value minus growth' portfolio, and 0.60 for a 'winner minus loser' portfolio. The betas fluctuate over time with variables commonly used to measure business conditions, including past market returns, Tbill rates, aggregate dividend yield, and the term spread. However, we find no evidence that betas covary with the market risk premium in a way that might explain the portfolios' unconditional alphas (indeed, the covariances often have the wrong sign).

Overall, the evidence supports our analytical results. Betas vary significantly over time but not enough to explain observed asset-pricing anomalies. Although the short-horizon regressions allow betas to vary without restriction from quarter-to-quarter and year-to-year, the conditional CAPM performs nearly as poorly as the unconditional CAPM.

Our analysis focuses on the Sharpe-Lintner CAPM but the conclusions should apply to other models as well: as a rule, time-variation in risk should have a relatively small impact on cross-sectional asset-pricing tests. In intertemporal models, consumption betas and the consumption risk premium would need to vary enormously over time for a conditional model to significantly outperform an unconditional one. While our empirical tests cannot be applied directly to the consumption CAPM, because they require high-frequency data, preliminary results using the mimicking-portfolio approach of Breeden, Gibbons, and Litzenberger (1989) provide no evidence that time-varying consumption betas explain momentum or the value premium (these results are available on request).

Our conclusions counter those of Jagannathan and Wang (JW 1996), Lettau and Ludvigson (2001), Santos and Veronesi (2005), and Lustig and Van Nieuwerburgh (2005), who argue that conditioning is very important for asset-pricing tests. The difference is that they focus on cross-sectional regressions, not time-series intercept tests, and ignore key restrictions on the cross-sectional slopes (which are automatic in our tests). To illustrate, JW show that a one-factor conditional CAPM implies a two-factor unconditional model, $E[R_i] = \beta_i \gamma + \lambda_i$, where R_i is the stock's excess return, β_i is the stock's average beta, γ is the average risk premium, and λ_i measures how the stock's beta covaries though time with the risk premium. The four studies estimate this cross-sectional equation using various measures of β_i and λ_i (or transformations thereof). The key issue is that the cross-sectional slope on λ_i should be one if the conditional CAPM holds but that constraint isn't imposed. And our calculations suggest the actual estimates are much too large. In essence, by ignoring the cross-sectional restrictions, the papers don't provide a full test of the conditional CAPM.

The paper is organized as follows. Section 2 explores the impact of time-varying risk and expected returns on unconditional CAPM regressions. Section 3 introduces the data and describes our testing approach. Section 4 presents the main empirical results, and Section 5 discusses other papers which test the conditional CAPM. Section 6 concludes.

2. The CAPM with time-varying betas

Asset-pricing tests often assume that betas are constant over time. Such 'unconditional' tests may reject the CAPM even if it holds perfectly, period-by-period. In this section, we derive expressions for a stock's unconditional alpha and beta when expected returns, volatility, and covariances all change over time. Our goal is to understand whether the pricing errors induced by time-varying betas might be large enough to explain important asset-pricing anomalies.

2.1. Notation and assumptions

Let R_{it} be the excess return on asset i and R_{Mt} be the excess return on the market portfolio. The joint distribution of R_{it} and R_{Mt} can change over time without restriction, except that (i) it must have well-defined conditional and unconditional moments, and (ii) the conditional CAPM is assumed to hold. Conditional moments for period t given information at t-1 are labeled with a t subscript: the market's conditional risk premium and standard deviation are γ_t and σ_t, and the stock's conditional beta is β_t. The corresponding unconditional moments are denoted γ, σ_M, and β^u. The unconditional β^u will generally differ from the expected conditional beta, denoted $\beta \equiv E[\beta_t]$.

2.2. Unconditional alphas and betas

The conditional CAPM says that expected returns are proportional to conditional betas: $E_{t-1}[R_{it}] = \beta_t \gamma_t$. Taking unconditional expectations, this relation implies that $E[R_{it}] = \beta \gamma + \text{cov}(\beta_t, \gamma_t)$, as observed by Jagannathan and Wang (1996). The asset's unconditional alpha is defined as $\alpha^u \equiv E[R_{it}] - \beta^u \gamma$, and substituting for $E[R_{it}]$ yields:

$$\alpha^u = \gamma (\beta - \beta^u) + \text{cov}(\beta_t, \gamma_t). \tag{1}$$

Under some conditions, discussed below, a stock's unconditional and expected conditional betas will be similar, in which case α^u is approximately equal to the covariance between beta and the market risk premium. More generally, the Appendix shows that

$$\beta^u = \beta + \frac{\gamma}{\sigma_M^2}\operatorname{cov}(\beta_t, \gamma_t) + \frac{1}{\sigma_M^2}\operatorname{cov}[\beta_t, (\gamma_t - \gamma)^2] + \frac{1}{\sigma_M^2}\operatorname{cov}(\beta_t, \sigma_t^2)\,. \tag{2}$$

This expression says that β^u differs from the expected conditional beta if β_t covaries with the market risk premium (2nd term), if it covaries with $(\gamma_t - \gamma)^2$ (3rd term), or if it covaries with the conditional volatility of the market (last term). Roughly speaking, movement in beta that is positively correlated with the market risk premium or with market volatility, γ_t or σ_t^2, raises the unconditional covariance between R_i and R_M (the other term is generally quite small, as we explain in a moment). Substituting (2) into (1), the stock's unconditional alpha is

$$\alpha^u = \left[1 - \frac{\gamma^2}{\sigma_M^2}\right]\operatorname{cov}(\beta_t, \gamma_t) - \frac{\gamma}{\sigma_M^2}\operatorname{cov}[\beta_t, (\gamma_t - \gamma)^2] - \frac{\gamma}{\sigma_M^2}\operatorname{cov}(\beta_t, \sigma_t^2)\,. \tag{3}$$

Eq. (3) provides a very general formula for the unconditional pricing error. It says that, even if the conditional CAPM holds exactly, we should expect to find deviations from the unconditional CAPM if beta covaries with γ_t, $(\gamma_t - \gamma)^2$, or with conditional market volatility.

Fig. 1 illustrates these results for a scenario in which beta is positively correlated with the risk premium. For simplicity, the graph assumes that β_t and γ_t are bivariate normal and, conditional on these parameters, returns are normally distributed with constant volatility (the last terms in eqs. 2 and 3 drop out). The dark curve shows $E[R_i \mid R_M]$, the predicted return on the stock as a function of the realized market return, while the light line shows the unconditional linear regression of R_i on R_M.

The graph shows that, when beta and the risk premium move together, the relation between R_i and R_M becomes convex because the slope tends to be high when the market return is high. The true $E[R_i \mid R_M]$ goes through zero but a linear regression fitted to the data has a positive intercept – that is, the stock has a positive unconditional alpha. [This is true unless the average risk premium is huge, shifting the graph so far to the right that the point where the line drops below the curve occurs at a positive R_M; see eq. (3).] The effects all reverse in sign if beta and the risk premium are negatively correlated: the true

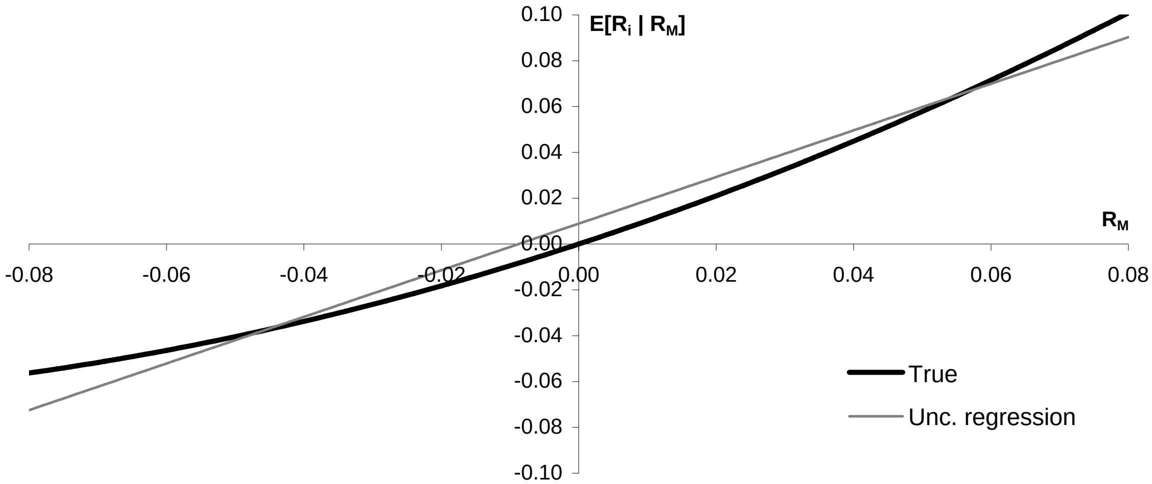


Figure 1. The unconditional relation between R_i and R_M.
The figure shows the excess return on stock i predicted as a function of the excess market return. The dark line shows the true $E[R_i \mid R_M]$ and the thin line shows the unconditional linear regression of R_i on R_M. Returns are conditionally normally distributed, with constant volatility, and the CAPM holds period-by-period. Beta and the expected risk premium are perfectly positively correlated.

relation is concave and the unconditional alpha is negative. The graph would also change if market volatility varied over time. For example, if volatility is positively correlated with beta, the slope of the curve is high in both tails, inducing a cubic-like relation. This effect would push up the stock's unconditional beta (eq. 2) and push down its unconditional alpha (eq. 3).

2.3. Magnitude

Our goal is to understand whether α^u in eq. (3) might be large enough to explain observed anomalies. We begin with a few observations to simplify the general formula. Notice, first, that the market's squared Sharpe ratio, γ^2 / σ_M^2 in the first term, is very small in monthly returns: for example, using the CRSP value-weighted index from 1964 – 2001, $\gamma = 0.47\%$ and $\sigma_M = 4.5\%$, so the squared Sharpe ratio is 0.011. Further, the quadratic $(\gamma_t - \gamma)^2$, in the second term, is also quite small for plausible parameter values: if γ equals 0.5% and γ_t varies between, say, 0.0% and 1.0%, the quadratic is at most $0.005^2 = 0.000025$. Plugging a variable this small into the second term would have a negligible effect on alpha. These observations suggest the following approximation for α^u:[3]

$$\alpha^u \approx \text{cov}(\beta_t, \gamma_t) - \frac{\gamma}{\sigma_M^2}\text{cov}(\beta_t, \sigma_t^2)\,. \tag{4}$$

Eq. (4) says that, when the conditional CAPM holds, a stock's unconditional alpha depends primarily on how β_t covaries with the market risk premium and with market volatility.

To explore the magnitude of eq. (4), it is useful to consider the simplest case when β_t covaries only with the market risk premium: $\alpha^u \approx \text{cov}(\beta_t, \gamma_t) = \rho\ \sigma_\beta\ \sigma_\gamma$, where σ denotes a standard deviation and ρ is the correlation between β_t and γ_t. Table 1 reports the α^u implied by various combinations of ρ, σ_β, and σ_γ. The parameters are chosen as follows:

- We consider three values for σ_β – 0.3, 0.5, and 0.7 – which probably span or, more likely exceed, standard deviations encountered in practice. Note, for example, that if $\beta = 1.0$ and $\sigma_\beta = 0.5$, a two-standard-deviation interval for beta extends all the way from 0.0 to 2.0. In comparison, Fama and French (1992) estimate unconditional betas for beta-sorted decile portfolios and find a minimum of 0.79 and a maximum of 1.73. Further, we estimate later that size, B/M, and momentum portfolios have σ_β's between 0.25 and 0.60, while Fama and French (1997) estimate that 48 industry portfolios have σ_β's between 0.12 and 0.42.

- We consider five values for σ_γ ranging from 0.1% to 0.5% monthly. The average risk premium from 1964 – 2001 is 0.47%, using the CRSP value-weighted index, so a standard deviation as high as 0.5% implies very large changes in the risk premium relative to its mean (a two-standard-deviation interval extends from –6% to 18% annualized). For additional perspective, a simple OLS regression of NYSE returns on log dividend yield suggests that $\sigma_\gamma = 0.3\%$ from 1946 – 2000 (Lewellen, 2004), while the calibrations of Campbell and Cochrane (1999) produce a standard deviation between 0.4% and 0.5% monthly (using statistics in their Tables 2 and 5).

[3] The approximation becomes perfect as the return interval shrinks because γ^2 and $(\gamma_t - \gamma)^2$ go to zero more quickly than the other terms in eq. (3). We thank John Campbell for this observation.

Table 1
Implied deviations from the unconditional CAPM
The table reports unconditional alphas (% monthly) implied by the conditional CAPM for various assumptions about time-variation in beta (β_t) and the market risk premium (γ_t). σ_β is the standard deviation of β_t, σ_γ is the standard deviation of γ_t, and ρ is the correlation between β_t and γ_t. Market volatility and β_t are assumed to be uncorrelated.

ρ = 0.6	σ_β			**ρ = 1.0**	σ_β		
	0.3	0.5	0.7		0.3	0.5	0.7
	Unconditional alpha (%)				Unconditional alpha (%)		
σ_γ = 0.1	0.02	0.03	0.04	σ_γ = 0.1	0.03	0.05	0.07
0.2	0.04	0.06	0.08	0.2	0.06	0.10	0.14
0.3	0.05	0.09	0.12	0.3	0.09	0.15	0.21
0.4	0.07	0.12	0.17	0.4	0.12	0.20	0.28
0.5	0.09	0.15	0.21	0.5	0.15	0.25	0.35

- Finally, we consider two values for ρ, 0.6 and 1.0. The first correlation is chosen arbitrarily; the second provides an upper bound for the pricing error.

The key result in Table 1 is that unconditional alphas are generally small relative to observed anomalies. The alphas are typically less than 0.20%, with a maximum of 0.35% for our most extreme combination of parameters (which we regard as quite generous). We estimate later that a long-short B/M strategy has σ_β = 0.25, so Table 1 suggests that time-variation in beta can explain an unconditional alpha of at most 0.15% monthly, small in comparison to our empirical estimate of 0.59% (std. error, 0.14%). The same is true of a momentum strategy, for which we estimate an unconditional alpha of 1.01% and a σ_β of 0.60. The bottom line is that, for reasonable parameters, the pricing error induced by time-variation in beta seems far too small to explain important asset-pricing anomalies.

Our analysis extends easily to cases in which beta covaries with market volatility as well as the risk premium. In fact, time-varying volatility might well strengthen our conclusions: eq. (4) shows that unconditional alphas are increasing in cov(β_t, γ_t) but decreasing in cov(β_t, σ_t^2). Thus, if the risk premium and volatility move together, the impact of time-varying volatility would tend to offset the impact of the risk premium, making implied α^us even smaller. The connection between γ_t and σ_t^2 is difficult to estimate, since returns are so noisy, but there is strong indirect evidence that the relation is positive

(French, Schwert, and Stambaugh, 1987; Campbell and Hentschel, 1992; Ghysels, Santa-Clara, and Valkanov, 2004). Many asset-pricing theories also predict that volatility and the equity premium move together over time, including Merton (1980) and Campbell and Cochrane (1999).[4] We skip a detailed calibration with time-varying volatility, in the interest of brevity, but our later empirical results show that changes in volatility have only a small impact on unconditional alphas. In short, with or without time-varying volatility, α^u seems too small to explain significant asset-pricing anomalies.

3. Testing the conditional CAPM

We believe the conclusions above are quite robust, but the calibration relies, in part, on our view of reasonable parameter values. In the remainder of the paper, we estimate some of the parameters and provide a simple direct test of the conditional CAPM.

3.1. Methodology

The basic framework for our tests is standard. We focus on time-series CAPM regressions for a handful of stock portfolios:

$$R_{it} = \alpha_i + \beta_i R_{Mt} + \varepsilon_{it}, \tag{5}$$

where R_{it} is the excess return on portfolio i and R_{Mt} is the excess return on the market. The CAPM predicts, of course, that α_i is zero. For unconditional tests, we estimate (5) using the full time series of returns for each portfolio, restricting α_i and β_i to be constant. For conditional tests, a common approach (e.g., Shanken, 1990; Ferson and Schadt, 1996; Lettau and Ludvigson, 2001) is to model betas as a function of observed macroeconomic variables. However, these tests, and alternatives suggested in the literature, are strictly valid only if the econometrician knows the full set of state variables available to investors (see Cochrane, 2001, for a review).

[4] Merton models the risk premium as $\gamma_t \approx \phi\, \sigma_t^2$, where ϕ is aggregate relative risk aversion. Given this relation, α^u in eq. (4) is very close to zero because the impact of time-varying volatility almost perfectly offsets the impact of a time-varying risk premium. In Campbell and Cochrane's model, γ_t and σ_t are both decreasing functions of the surplus consumption ratio but volatility moves less than the risk premium (see Lettau and Ludvigson, 2003); thus, the effects of time-varying γ_t and σ_t only partially offset.

We propose a simple way to get around this problem: we directly estimate conditional alphas and betas using short-window regressions. That is, rather than estimate (5) once using the full time series of returns, we estimate it separately every, say, quarter using daily or weekly returns. The result is a direct estimate of each quarter's conditional alpha and beta – without using any state variables or making any assumption about quarter-to-quarter variation in beta. We use the time series of alpha and beta estimates to test the conditional CAPM in two ways. Our main test simply asks whether average conditional alphas are zero. In addition, we test whether betas vary over time in a way that might explain stocks' unconditional alphas, via the mechanisms discussed in Section 2: do betas covary with the market risk premium or market volatility? For robustness, we estimate regressions over a variety of interval lengths – monthly, quarterly, semiannually, and yearly – and using daily, weekly, or monthly returns.

The key assumption underlying our tests is that beta is fairly stable during the month or quarter, so each short-window regression can treat it as constant. The idea is that, if beta is constant during the quarter, a simple OLS regression $R_{it} = \alpha_i + \beta_i R_{Mt} + \varepsilon_{it}$ should produce an unbiased estimate of the true conditional alpha and beta. And that's all our tests require. Each regression uses a small number of observations and produces a noisy estimate of the parameters, but our tests have reasonable power because they use a long time series of estimates.

The assumption that beta is stable within a month or quarter seems fairly mild. Empirical tests often assume beta is stable for five or more *years*, and studies that model beta as a function of macroeconomic variables typically use very persistent series, like Tbill rates and dividend yield, implying that betas also change quite slowly. Moreover, we doubt that high frequency changes in beta, if they do exist, would affect the results significantly. The impact of, say, daily changes in beta on quarterly regressions parallels the impact of time-varying betas on unconditional regressions, except that now only *intra*quarter variation (i.e., changes missed by the short-horizon regressions) is important. We argued in Section 2 that ignoring *all* variation in beta has little impact on asset-pricing tests. The point obviously has greater force once we account for a significant portion of time-varying betas via the short-window regressions: betas, market volatility, and the risk premium would have to show incredibly large variation within the quarter – and would have to covary strongly with each other – in order to explain the pricing errors from our short-window regressions. The Appendix explores these ideas more fully. Simulations in which risk and expected returns change daily or weekly suggest that our short-window regressions capture nearly all of the impact of time-varying betas (i.e., our short-window alpha estimates are close to zero, on average, if the conditional CAPM truly holds).

3.2. Microstructure issues

While most asset-pricing studies use monthly returns, we use daily or weekly returns since the regressions are estimated over such short intervals. Doing so raises two concerns. First, alphas and betas for different return horizons should differ slightly because of compounding (Levhari and Levy, 1977; Handa, Kothari, and Wasley, 1989). For example, if daily returns are IID, then expected N-day returns are $E[1+R_i]^N - 1$ and the N-day beta is

$$\beta_i(N) = \frac{E[(1+R_i)(1+R_M)]^N - E[1+R_i]^N E[1+R_M]^N}{E[(1+R_M)^2]^N - E[1+R_M]^{2N}} . \tag{6}$$

From (6), it can be shown that betas spread out as the horizon lengthens: $\beta_i(N)$ increases in N if $\beta_i(1) > 1$ but decreases in N if $\beta_i(1) < 1$. In addition, if the CAPM holds for daily returns, a stock with $\beta_i(1) > 1$ will have N-day alphas that are negative, while the opposite is true if $\beta_i(1) < 1$. Fortunately, these effects are tiny and can be ignored in the remainder of the paper. For example, if the market return has mean 0.5% and standard deviation 5% monthly, then a stock with a daily beta of 1.300 would have a monthly beta of 1.302 and a monthly alpha of –0.001%.

Second, and more important, nonsynchronous price movements can have a big impact on short-horizon betas. Lo and MacKinlay (1990) show that small stocks tend to react with a week or more delay to common news, so a daily or weekly beta will miss much of the small-stock covariance with market returns. To mitigate the problem, our tests focus on value-weighted portfolios and exclude NASDAQ stocks. Also, following Dimson (1979), we include both current and lagged market returns in the regressions, estimating beta as the sum of the slopes on all lags (alpha is still just the intercept). For daily returns, we include four lags of market returns, imposing the constraint that lags 2 – 4 have the same slope to reduce the number of parameters:

$$R_{i,t} = \alpha_i + \beta_{i0} R_{M,t} + \beta_{i1} R_{M,t-1} + \beta_{i2} [(R_{M,t-2} + R_{M,t-3} + R_{M,t-4})/3] + \varepsilon_{i,t}. \quad (7)$$

The daily beta is then $\beta_i = \beta_{i0} + \beta_{i1} + \beta_{i2}$. (Adding a few more lags doesn't affect the results.) For weekly returns, we include two lags of market returns:

$$R_{i,t} = \alpha_i + \beta_{i0} R_{M,t} + \beta_{i1} R_{M,t-1} + \beta_{i2} R_{M,t-2} + \varepsilon_{i,t}, \quad (8)$$

where the weekly beta is again $\beta_i = \beta_{i0} + \beta_{i1} + \beta_{i2}$. To increase precision, we estimate (8) using overlapping returns (i.e., consecutive observations overlap by four days). Finally, we estimate monthly betas including one lag of market returns:

$$R_{i,t} = \alpha_i + \beta_{i0} R_{M,t} + \beta_{i1} R_{M,t-1} + \varepsilon_{i,t}, \quad (9)$$

where the monthly beta is $\beta_i = \beta_{i0} + \beta_{i1}$. As discussed below, Dimson betas are not a perfect solution but our results do not seem to be driven by measurement problems. Indeed, unconditional alphas estimated by (7) – (9) are nearly identical for our test portfolios.

3.3. The data

The empirical tests focus on size, B/M, and momentum portfolios from July 1964 – June 2001. Prices and returns come from the CRSP daily stock file and book values come from Compustat. As we mentioned above, the portfolios are value-weighted and contain only NYSE and Amex common stocks. Our market proxy is the CRSP value-weighted index (all stocks), and we calculate excess returns on all portfolios net of the one-month T-bill rate.

The size and B/M portfolios are similar to those of Fama and French (1993). In June of every year, we form 25 size-B/M portfolios based on the intersection of five size and five B/M portfolios, with breakpoints given by NYSE quintiles. Size is the market value of equity at the end of June, while B/M is the ratio of book equity in the prior fiscal year (common equity plus balance sheet deferred taxes) to market equity at the end of December. Our tests are then based on six combinations of the 25 size-B/M portfolios: 'Small' is the average of the five portfolios in the lowest size quintile, 'Big' is the average of the five portfolios in the highest size quintile, and 'S-B' is their difference. Similarly, 'Growth' is the average of the five portfolios in the low-B/M quintile, 'Value' is the average of the five portfolios in the high-B/M quintile, and 'V-G' is their difference. Our 'S-B' and 'V-G' portfolios are much like Fama and French's SMB and HML factors except that we exclude NASDAQ stocks and start with 25 basis portfolios (rather than six).

The momentum portfolios are constructed separately using all stocks on CRSP with the required data (i.e., not restricted to Compustat firms). We sort stocks every month into deciles based on past 6-month returns and hold the stocks for overlapping 6-month periods, as in Jegadeesh and Titman (1993). This means, in effect, that one-sixth of the momentum portfolio is rebalanced every month. Again, the tests focus on a subset of the 10 portfolios: 'Losers' is the return on the bottom decile, 'Winners' is the return of the top decile, and 'W-L' is their difference.

The tests use daily, weekly, and monthly returns. 'Weekly' returns are calculated by compounding daily returns over five-day intervals rather than calendar weeks. We use five-day windows in part because they are easier to align with calendar quarters and in part because the changing number of trading days in a week (sometimes as few as three) would complicate some of the tests. Monthly returns are calculated in the standard way, compounding within calendar months. For long-short strategies, we compound each side of the strategy and then difference.

To set the stage, Table 2 reports summary statistics for the portfolios from 1964 – 2001. Panel A shows average daily, weekly, and monthly excess returns. The estimates are all expressed in percent monthly; the daily estimates are multiplied by 21 (trading days per month) and the weekly estimates are multiplied by 21/5. Excess returns exhibit the usual cross-sectional patterns: small stocks outperform large stocks (0.71% vs. 0.50% using monthly returns), high-B/M stocks outperform low-B/M stocks (0.88% vs. 0.41%), and winners outperform losers (0.91% vs. 0.01%). Estimates of average returns are always lowest using daily returns and highest using monthly returns. A very small portion of this pattern could be attributed to compounding, but it more likely reflects positive autocorrelation in daily returns.

Table 2
Summary statistics for size, B/M, and momentum portfolios, 1964 – 2001
The table reports average returns and unconditional CAPM regressions for size, B/M, and momentum portfolios. The regressions use daily, weekly, or monthly returns, correcting for nonsynchronous trading as described in the text. Average returns and alphas are expressed in percent monthly; the daily estimates are multiplied by 21 and the weekly estimates are multiplied by 21/5. The portfolios are formed from all NYSE and Amex stocks on CRSP / Compustat. We begin with 25 size-B/M portfolios (5×5 sort, breakpoints determined by NYSE quintiles) and 10 return-sorted portfolios, all value weighted. 'Small' is the average of the five low-market-cap portfolios, 'Big' is the average of the five high-market-cap portfolios, and 'S-B' is their difference. Similarly, 'Growth' is the average of the five low-B/M portfolios, 'Value' is the average of the five high-B/M portfolios, and 'V-G' is their difference. Return-sorted portfolios are formed based on past 6-month returns. 'Losers' is the bottom decile, 'Winners' is the top decile, and 'W-L' is their difference.

		Size			B/M			Momentum		
		Small	Big	S-B	Grwth	Value	V-G	Losers	Winrs	W-L
Panel A: Excess returns										
Avg.	Day	0.57	0.49	0.08	0.32	0.81	0.49	-0.10	0.87	0.97
	Week	0.63	0.50	0.13	0.37	0.84	0.47	-0.04	0.91	0.95
	Month	0.71	0.50	0.21	0.41	0.88	0.47	0.01	0.91	0.90
Std error	Day	0.28	0.20	0.19	0.27	0.23	0.13	0.33	0.28	0.26
	Week	0.26	0.18	0.18	0.26	0.22	0.12	0.30	0.26	0.25
	Month	0.34	0.19	0.23	0.30	0.26	0.16	0.35	0.28	0.27
Panel B: Unconditional alphas										
Est.	Day	0.09	0.10	-0.01	-0.21	0.39	0.60	-0.64	0.35	0.99
	Week	0.05	0.10	-0.05	-0.22	0.37	0.59	-0.66	0.37	1.03
	Month	0.07	0.11	-0.03	-0.20	0.39	0.59	-0.63	0.38	1.01
Std error	Day	0.15	0.06	0.17	0.10	0.12	0.12	0.18	0.13	0.26
	Week	0.14	0.06	0.16	0.09	0.11	0.11	0.17	0.12	0.25
	Month	0.18	0.07	0.20	0.11	0.13	0.14	0.19	0.13	0.28
Panel C: Unconditional betas										
Est.	Day	1.07	0.87	0.20	1.18	0.94	-0.25	1.22	1.17	-0.06
	Week	1.25	0.86	0.39	1.27	1.03	-0.24	1.33	1.16	-0.17
	Month	1.34	0.83	0.51	1.30	1.05	-0.25	1.36	1.14	-0.22
Std error	Day	0.03	0.01	0.03	0.02	0.03	0.02	0.03	0.02	0.05
	Week	0.03	0.01	0.04	0.02	0.03	0.03	0.04	0.03	0.06
	Month	0.05	0.02	0.06	0.03	0.04	0.04	0.06	0.04	0.08

Specifically, monthly expected returns are $\mu_{mon} = E[\prod_i(1+R_i)] - 1$. If daily returns are IID, the right-hand side becomes $(1 + \mu_{day})^{21} - 1$, essentially identical to $21 \times \mu_{day}$. But notice that the monthly expected return is higher if daily returns are positively autocorrelated since the expectation would have additional covariance terms. This observation is consistent with the fact that average daily and monthly returns are most different for small stocks.

Panel B shows unconditional alphas for the portfolios (% monthly). The estimates are remarkably similar for the three return horizons. Focusing on the long-short portfolios, S-B has a daily alpha of –0.01% and a monthly alpha of –0.03%, V-G has a daily alpha of 0.60% and a monthly alpha of 0.59%, and W-L has a daily alpha of 0.99% and a monthly alpha of 1.01%. Thus, after adjusting for risk, the size effect is absent in our data but the B/M and momentum effects are strong. Using monthly returns, the latter two are about 4 standard errors from zero.

The contrast between Panels A and B is interesting: excess returns increase with the return horizon but alphas do not. Panel C shows why: betas increase, roughly speaking, at the same rate as excess returns, so the net effect is that alphas ($\alpha_i = E[R_i] - \beta_i E[R_M]$) are nearly constant across horizons. As a result, nonsynchronous prices have important effects on excess returns and betas, especially for small stocks, but little impact on CAPM tests for any of our portfolios.

4. Empirical results

We now turn to the main empirical results. As discussed above, we provide both a direct test of the conditional CAPM – are conditional alphas zero? – and an indirect test based on the time-series properties of beta. The volatility, persistence, and cyclical behavior of betas should be of interest beyond their implications for the CAPM (see, e.g., Franzoni, 2002).

The main inputs for the empirical tests are the time series of conditional alpha and beta estimates from the short-window regressions (see Section 3.1). We have explored a variety of window lengths and return horizons and report results for short-window regressions estimated several ways: (i) quarterly using daily returns; (ii) semiannually using both daily (Semiannual 1) and weekly (Semiannual 2) returns; and (iii) annually using monthly returns. The estimates are corrected for nonsynchronous trading using the methodology described in Section 3.2.

4.1. Conditional alphas

The most basic test of the conditional CAPM is whether conditional alphas are zero. Unlike prior studies, we can test this hypothesis without using any state variables because each quarterly or semiannual regression produces a direct estimate of a portfolio's conditional alpha. Our tests focus on the average conditional alpha for each portfolio, using the time-series variability of the estimates to obtain standard errors (in the spirit of Fama and MacBeth, 1973).

The average conditional alphas, in Table 3, provide strong evidence against the conditional CAPM. Most important, B/M and momentum portfolios' alphas remain large, statistically significant, and close to the unconditional estimates. Depending on the estimation method, V-G's average conditional alpha is between 0.47% and 0.53% (t-statistics of 3.05 to 3.65), compared with an unconditional alpha around 0.59%. W-L's average alpha shows more dispersion, ranging from 0.77% to 1.37% for the different estimation methods (t-statistics of 2.66 to 5.12), but the estimates are in line with an unconditional alpha of about 1.00%. The size effect continues to be weak, as in unconditional tests, but small stocks show a hint of abnormal returns in quarterly regressions. Overall, the conditional CAPM performs about as poorly as the unconditional CAPM.

The close correspondence between conditional and unconditional alphas supports our analytical results in Section 2, i.e., that time-varying betas should have a small impact on asset-pricing tests. The short-window regressions allow betas to vary without restriction from quarter-to-quarter or year-to-year, and we show later that betas do, in fact, vary significantly over time. Yet compared with unconditional tests, the alpha for the long-short B/M strategy drops by only about 0.10%, from 0.60% to 0.50%, and the alpha for the momentum strategy stays close to 1.00%. Thus, time-variation in beta has only a small impact on measures of CAPM pricing errors.

A couple features of Table 3 deserve highlighting. First, recall that the standard errors aren't taken directly from the short-window regressions but are based instead on the sample variability of the conditional alphas (e.g., the standard deviation of the 148 quarterly estimates divided by the square root of 148). The tests are therefore robust to both heteroskedasticity, which doesn't affect the standard error of a sam-

Table 3
Average conditional alphas, 1964 – 2001
The table reports average conditional alphas for size, B/M, and momentum portfolios (% monthly). Alphas are estimated quarterly using daily returns, semiannually using daily and weekly returns, and annually using monthly returns. The portfolios are formed from all NYSE and Amex stocks on CRSP / Compustat. We begin with 25 size-B/M portfolios (5×5 sort, breakpoints determined by NYSE quintiles) and 10 return-sorted portfolios, all value weighted. 'Small' is the average of the five low-market-cap portfolios, 'Big' is the average of the five high-market-cap portfolios, and 'S-B' is their difference. Similarly, 'Growth' is the average of the five low-B/M portfolios, 'Value' is the average of the five high-B/M portfolios, and 'V-G' is their difference. Return-sorted portfolios are formed based on past 6-month returns. 'Losers' is the bottom decile, 'Winners' is the top decile, and 'W-L' is their difference. Bold denotes estimates greater than two standard errors from zero.

	Size			B/M			Momentum		
	Small	Big	S-B	Grwth	Value	V-G	Losers	Winrs	W-L
Average conditional alpha (%)									
Quarterly	**0.42**	0.00	0.42	-0.01	**0.49**	**0.50**	**-0.79**	**0.55**	**1.33**
Semiannual 1	0.26	0.00	0.26	-0.08	**0.40**	**0.47**	**-0.61**	**0.39**	**0.99**
Semiannual 2	0.16	0.01	0.15	-0.12	**0.36**	**0.48**	**-0.83**	**0.53**	**1.37**
Annual	-0.06	0.08	-0.14	-0.20	0.32	**0.53**	**-0.56**	0.21	**0.77**
Standard error									
Quarterly	0.20	0.06	0.22	0.12	0.14	0.14	0.20	0.13	0.26
Semiannual 1	0.21	0.06	0.23	0.12	0.14	0.15	0.19	0.14	0.25
Semiannual 2	0.21	0.06	0.23	0.14	0.15	0.16	0.20	0.15	0.27
Annual	0.26	0.07	0.29	0.16	0.17	0.14	0.21	0.17	0.29

Quarterly and Semiannual 1 alphas are estimated from daily returns, Semiannual 2 alphas are estimated from weekly returns, and Annual alphas are estimated from monthly returns.

ple average, and autocorrelation, which shouldn't exist (in alphas) if the conditional CAPM holds because every alpha estimate should have a conditional mean of zero.

Second, our short-window regressions ignore high frequency changes in beta, but we doubt that such changes affect the results significantly. As noted earlier, daily changes in beta are a concern only if they are very large and covary strongly with high frequency changes in the risk premium or volatility. Indeed, betas, volatility, and the risk premium would have to exhibit enormous variation within the quarter – much more than they show across quarters – in order to explain the pricing errors from our short-window regressions. The Appendix explores these issues more fully. Simulations in which betas change daily or weekly, and otherwise calibrated to the data, suggest that our short-window regressions capture nearly all of the impact of time-varying betas.

4.2. Conditional betas

Table 3 provides direct evidence against the conditional CAPM: conditional alphas are large and significant. An alternative approach is to ask whether betas vary over time in a way that might explain portfolios' *unconditional* alphas via the mechanisms discussed in Section 2: do betas covary strongly with the market risk premium or volatility? The fact that our estimates of conditional and unconditional alphas are similar tells us the answer must be no, but it's useful to look at time-variation in betas to get additional perspective for what's driving the results. These tests reinforce our conclusion that betas don't vary enough to redeem the conditional CAPM.

Table 4 reports summary statistics for conditional betas. Average betas, in panel A, are generally close to our earlier estimates of unconditional betas. Focusing on the semiannual estimates from weekly returns (Semiannual 2), S-B has an average conditional beta of 0.32 (vs. an unconditional weekly beta of 0.39), V-G has an average beta of –0.19 (vs. an unconditional beta of –0.24), and W-L has an average beta of –0.14 (vs. an unconditional beta of –0.17).

Panels C and D indicate that betas fluctuate significantly over time. In Panel C, the standard deviation of *estimated* betas is often greater than 0.30 and sometimes higher than 0.40 (for momentum portfolios). Part of the variability is due to sampling error, so we focus more on the implied variability of true betas. Specifically, we can think of the estimated betas as $b_t = \beta_t + e_t$, where β_t is the true conditional beta and e_t is sampling error. As long as beta is stable during the estimation window and the regression satisfies standard OLS assumptions, b_t is an unbiased estimate of β_t, implying that β_t and e_t are uncorrelated and that $\text{var}(b_t) = \text{var}(\beta_t) + \text{var}(e_t)$. We use this equation to back out the volatility of true betas, where $\text{var}(e_t)$ is the average sampling variance of $b_t - \beta_t$ from the short-window regressions (see Fama and French, 1997).

As shown in Panel D, the volatility of betas remains substantial even after removing sampling error. Focusing on the long-short strategies, S-B's beta has a standard deviation around 0.30, V-G's beta has a standard deviation around 0.25, and W-L's beta has a standard deviation around 0.60. The

Table 4
Time-variation in betas, 1964 – 2001

The table reports summary statistics for the conditional betas of size, B/M, and momentum portfolios. Betas are estimated quarterly using daily returns, semiannually using daily and weekly returns, and annually using monthly returns, correcting for nonsynchronous trading as described in the text. Panels A and C report the time-series mean and standard deviation of beta, Panel B reports the average standard error of beta from the short-window regressions, and Panel D reports the implied time-series standard deviation of true betas.

The portfolios are formed from all NYSE and Amex stocks on CRSP / Compustat. We begin with 25 size-B/M portfolios (5×5 sort, breakpoints determined by NYSE quintiles) and 10 return-sorted portfolios, all value weighted. 'Small' is the average of the five low-market-cap portfolios, 'Big' is the average of the five high-market-cap portfolios, and 'S-B' is their difference. Similarly, 'Growth' is the average of the five low-B/M portfolios, 'Value' is the average of the five high-B/M portfolios, and 'V-G' is their difference. The return-sorted portfolios are formed based on past 6-month returns. 'Losers' is the bottom decile, 'Winners' is the top decile, and 'W-L' is their difference.

	Size			B/M			Momentum		
	Small	Big	S-B	Grwth	Value	V-G	Losers	Winrs	W-L
Panel A: Average betas									
Quarterly [a]	1.03	0.93	0.10	1.17	0.98	-0.19	1.19	1.24	0.05
Semiannual 1	1.07	0.93	0.14	1.19	0.99	-0.20	1.20	1.24	0.05
Semiannual 2	1.23	0.91	0.32	1.25	1.06	-0.19	1.33	1.19	-0.14
Annual	1.49	0.83	0.66	1.36	1.17	-0.19	1.38	1.24	-0.14
Panel B: Average std error [b]									
Quarterly	0.13	0.06	0.17	0.10	0.10	0.12	0.17	0.14	0.24
Semiannual 1	0.09	0.04	0.12	0.07	0.07	0.08	0.12	0.09	0.17
Semiannual 2	0.16	0.07	0.20	0.11	0.12	0.14	0.20	0.15	0.29
Annual	0.36	0.13	0.42	0.22	0.24	0.30	0.40	0.28	0.57
Panel C: Std deviation of estimated betas									
Quarterly	0.35	0.15	0.38	0.22	0.30	0.28	0.41	0.37	0.68
Semiannual 1	0.31	0.13	0.32	0.19	0.29	0.25	0.33	0.32	0.58
Semiannual 2	0.35	0.13	0.38	0.20	0.33	0.33	0.44	0.36	0.71
Annual	0.54	0.14	0.56	0.27	0.46	0.41	0.52	0.44	0.83
Panel D: Implied std deviation of true betas [c]									
Quarterly	0.32	0.13	0.33	0.19	0.28	0.25	0.36	0.33	0.63
Semiannual 1	0.29	0.12	0.30	0.18	0.28	0.24	0.30	0.30	0.55
Semiannual 2	0.31	0.10	0.32	0.16	0.31	0.29	0.36	0.32	0.62
Annual	0.35	--	0.25	0.04	0.37	0.19	0.19	0.29	0.52

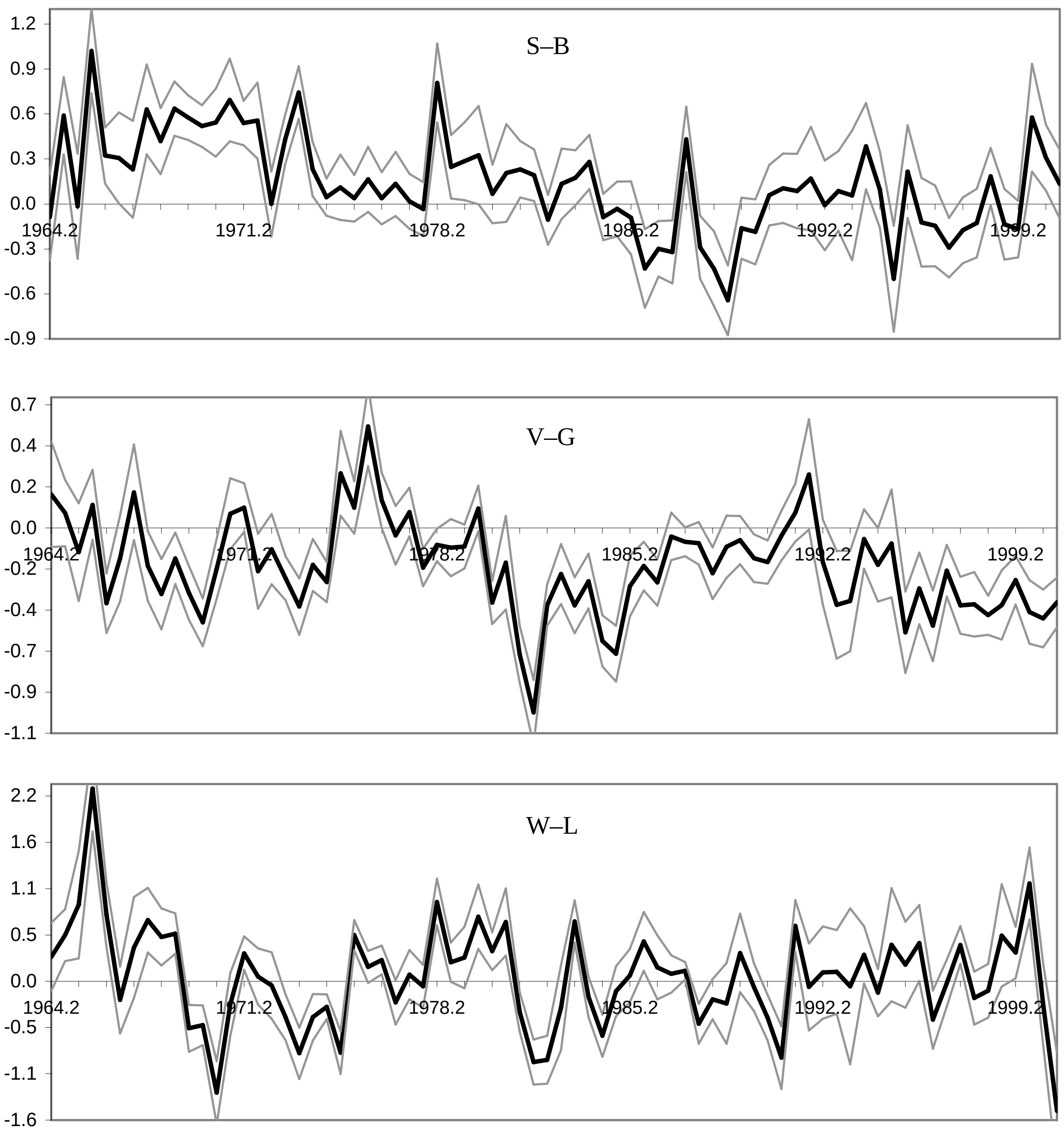
[a] Quarterly and Semiannual 1 betas are estimated from daily returns, Semiannual 2 betas are estimated from weekly returns, and Annual betas are estimated from monthly returns.

[b] Average standard error from the short-window regressions, not the standard error of the average.

[c] The implied variance of true betas equals $\mathrm{var}(b_t) - \overline{\mathrm{var}}\,(e_t)$, the difference between the variance of estimated betas and the average variance of the sampling error in b_t (from the regressions). The standard deviation is undefined for Big using annual windows / monthly returns because the implied variance is negative.

Figure 2
Conditional betas, 1964 – 2001

The figure plots conditional betas for size, B/M, and momentum portfolios. The dark line is the point estimate and the light lines indicate a two-standard-deviation confidence interval. Betas are estimated semiannually (non-overlapping windows) using daily returns. The portfolios are formed from all NYSE and Amex stocks on CRSP / Compustat. We begin with 25 size-B/M portfolios (5×5 sort, breakpoints determined by NYSE quintiles) and 10 return-sorted portfolios, all value weighted. 'S–B' is the average return on the five low-market-cap portfolios (Small) minus the average return on the five high-market-cap portfolios (Big). 'V–G' is the average return on the five high-B/M portfolios (Value) minus the average return on the five low-B/M portfolios (Growth). Return-sorted portfolios are formed based on past 6-month returns. 'W–L' is the return on the top decile (Winners) minus the return on the bottom decile (Losers).



magnitude of these standard deviations can be seen most easily in Figure 2. In particular, S-B's beta varies from a high of 1.02 (t-stat = 7.22) in 1966 to a low of –0.64 (t-stat = –5.51) in 1989. V-G's beta reaches a maximum of 0.54 (t-stat = 5.13) in 1976 before falling to a minimum of –0.99 (t-stat = –11.39) just six years later. The momentum strategy's beta is the most volatile, which is not surprising given that the strategy almost certainly has the highest turnover. In Figure 2, W-L's beta varies from a high of 2.25 (t-stat = 8.98) to a low of –1.51 (t-stat = –4.47).

Beta and the market risk premium

Section 2 showed that, if the conditional CAPM holds and beta covaries with the risk premium γ_t, a portfolio's unconditional alpha is approximately $\alpha^u \approx \text{cov}(\beta_t, \gamma_t) = \rho\ \sigma_\beta\ \sigma_\gamma$. At the time, we considered values of σ_β ranging from 0.3 to 0.7 to illustrate that implied alphas are relatively small for 'plausible' parameters (see Table 1). This range seems reasonable given the results in Table 4.

We can also estimate $\text{cov}(\beta_t, \gamma_t)$ directly from the data. As a first step, Table 5 explores the correlation between betas and several state variables that have been found to capture variation in the equity premium. The state variables are lagged relative to beta (i.e., known prior to the beta estimation window), so the correlations are predictive. $R_{M,-6}$ is the past 6-month return on the market portfolio; TBILL is the one-month Tbill rate; DY is the 12-month rolling dividend-to-price ratio on the value-weighted NYSE index; TERM is the yield spread between 10-year and 1-year Tbonds; and CAY is the consumption-to-wealth ratio of Lettau and Ludvigson (2001). The portfolios' lagged betas, denoted β_{t-1}, are also included to test for persistence. Table 5 focuses on betas estimated semi-annually using daily returns, the same as those used in Figure 2.

Panel A reports the correlation between betas and the state variables. The first row shows that betas are persistent but that autocorrelations are far from one, with estimates between 0.45 and 0.68 for

Table 5
Predicting conditional betas, 1964 – 2001

The table reports the correlation between various state variables and the conditional betas of size, B/M, and momentum portfolios. Betas are estimated semiannually using daily returns. The state variables are lagged relative to the beta estimates. β_{t-1} is the portfolio's lagged beta; $R_{M,-6}$ is the past 6-month market return; TBILL is the one-month Tbill rate; DY is the log dividend yield on the value-weighted NYSE index; TERM is the yield spread between 10-year and 1-year Tbonds; CAY is the consumption to wealth ratio of Lettau and Ludvigson (2001). Panel A reports simple correlations between estimated conditional betas and the state variables, and Panel B reports slope estimates when betas are regressed on all of the state variables together.

The portfolios are formed from all NYSE and Amex stocks on CRSP / Compustat. We begin with 25 size-B/M portfolios (5×5 sort, breakpoints determined by NYSE quintiles) and 10 return-sorted portfolios, all value weighted. 'Small' is the average of the five low-market-cap portfolios, 'Big' is the average of the five high-market-cap portfolios, and 'S-B' is their difference. Similarly, 'Growth' is the average of the five low-B/M portfolios, 'Value' is the average of the five high-B/M portfolios, and 'V-G' is their difference. The return-sorted portfolios are formed based on past 6-month returns. 'Losers' is the bottom decile, 'Winners' is the top decile, and 'W-L' is their difference.

	Size			B/M			Momentum		
	Small	Big	S-B	Grwth	Value	V-G	Losers	Winrs	W-L
Panel A: Correlation between betas and state variables									
β_{t-1}	0.55	0.68	0.43	0.58	0.67	0.51	0.30	0.45	0.37
$R_{M,-6}$	-0.05	-0.01	-0.05	-0.18	0.00	0.14	-0.53	0.47	0.56
TBILL	-0.04	0.11	-0.08	0.15	-0.12	-0.25	0.14	-0.25	-0.21
DY	0.22	0.64	-0.04	0.37	0.40	0.18	0.13	-0.12	-0.14
TERM	-0.20	0.19	-0.27	-0.12	0.01	0.10	-0.01	-0.08	-0.04
CAY	-0.12	0.50	-0.31	-0.01	0.17	0.20	0.09	-0.09	-0.10
Panel B: Betas regressed on the state variables [a]									
Slope estimate									
β_{t-1}	**0.12**	**0.05**	**0.11**	**0.10**	**0.12**	**0.08**	**0.10**	**0.15**	**0.22**
$R_{M,-6}$	0.05	-0.01	0.04	0.02	0.04	0.04	**-0.19**	**0.20**	**0.39**
TBILL	**-0.13**	-0.02	**-0.11**	-0.03	**-0.14**	**-0.13**	0.09	**-0.14**	**-0.24**
DY	**0.14**	**0.05**	0.09	**0.06**	**0.16**	**0.10**	-0.07	**0.11**	**0.19**
TERM	**-0.10**	0.00	**-0.10**	-0.02	**-0.08**	**-0.07**	0.07	**-0.11**	**-0.19**
CAY	-0.05	0.02	-0.08	-0.03	-0.01	0.03	0.00	-0.01	-0.01
t-statistic									
β_{t-1}	3.53	3.99	2.83	4.24	3.88	2.62	3.03	5.31	4.49
$R_{M,-6}$	1.52	-0.45	1.17	0.73	1.58	1.41	-5.63	7.25	7.63
TBILL	-2.56	-1.39	-2.09	-1.06	-3.19	-2.98	1.79	-3.41	-3.22
DY	2.82	3.05	1.74	2.10	3.64	2.65	-1.50	2.87	2.65
TERM	-2.40	-0.25	-2.21	-0.81	-2.40	-1.99	1.60	-3.07	-2.81
CAY	-1.32	1.86	-1.81	-1.34	-0.17	0.98	0.07	-0.22	-0.13
Adj R^2	0.37	0.60	0.26	0.34	0.52	0.32	0.35	0.56	0.53

[a] The state variables – including lagged beta – are scaled by their standard deviations. The slopes can be interpreted as the predicted change in beta associated with a one-standard-deviation change in the state variable. Bold denotes estimates greater than 1.96 standard errors from zero.

most of the raw portfolios and a bit lower, 0.37 to 0.51, for the long-short strategies.[5] Momentum betas are both the least persistent and the most highly correlated with past market returns. Winner betas increase (correlation of 0.47) and Loser betas decrease (correlation of –0.53) after the market does well. This pattern is intuitive: we expect the Winner portfolio to become weighted towards high-beta stocks when the market goes up since those stocks do best (Ball, Kothari, and Shanken, 1995; Grundy and Martin, 2001). Panel A also shows that Value betas are positively correlated with CAY. Hence, our short-window regressions capture the same variation in Value betas found by Lettau and Ludvigson (2001) (though our pricing conclusions differ substantially).

Panel B studies the joint explanatory power of the state variables. For this panel, the state variables are scaled by their standard deviations, so the regression slopes can be interpreted as the change in beta predicted by a one-standard-deviation change in the state variables. The slopes indicate that betas vary significantly with TBILL, DY, and TERM. Small, Value, and Winner stocks have high betas when TBILL and TERM are low (slopes of –0.08 to –0.14) and when DY is high (slopes of 0.11 to 0.14). The effect of $R_{M,-6}$ on momentum betas is also quite strong, with a slope of 0.39 for the Winner minus Loser portfolio. CAY, the consumption-to-wealth ratio of Lettau and Ludvigson (2001), shows little relation to betas once we control for the other variables. In sum, betas fluctuate with state variables that have been found to capture variation in the equity premium.

With that prelude, we estimate $\text{cov}(\beta_t, \gamma_t)$ in two ways. Our first estimate is simply $\text{cov}(b_t, R_{Mt})$, where we have replaced the true conditional beta with our estimate b_t and replaced the risk premium with the realized market return R_{Mt}. The logic here is that, under the assumptions of OLS, sampling error in beta should be uncorrelated with market returns, so the covariance between b_t and R_{Mt} provides an unbiased estimate of $\text{cov}(\beta_t, \gamma_t)$:

$$\text{cov}(b_t, R_{Mt}) = \text{cov}(\beta_t, R_{Mt}) = \text{cov}(\beta_t, \gamma_t), \tag{10}$$

[5] The standard error of the estimates is roughly $1/\sqrt{T} = 0.12$ under the null that the autocorrelations are zero. Also, true betas should be more highly autocorrelated than estimated betas: sampling error in beta, if serially uncorrelated, would attenuate the autocorrelations by $\text{var}(\beta_t) / \text{var}(b_t)$. The statistics in Panels C and D of Table 4 suggest that the attenuation bias is small for our data.

where the last equality uses the fact that unexpected market returns must be uncorrelated with β_t. Eq. (10) is necessarily true if returns are conditionally normal, but it may not hold for alternative distributions. Empirically, Ang and Chen (2002) show that stocks covary more strongly in down markets, suggesting that e_t and s_t might be correlated for some firms. Therefore, we report this first estimate primarily as a benchmark rather than as a perfect estimate of cov(β_t, γ_t).

Table 6, Panel A, shows the results. The numbers can be interpreted as the unconditional monthly alpha (in %) that we should observe if the conditional CAPM holds, i.e., $\alpha^u \approx$ cov(β_t, γ_t). Like our earlier tests, the results provide no evidence that time-varying betas salvage the CAPM: the implied alphas are either close to zero or have the wrong sign. The covariance estimates for S-B and W-L betas are generally negative (between –0.04% and –0.39% for quarterly and semiannual betas), while the covariance estimates for V-G are small and positive (between 0.04% and 0.11%). Thus, conditional betas do not seem to covary with the risk premium in a way that can explain the unconditional alphas observed for B/M and momentum portfolios.

Our second estimate uses the predictive regressions from Table 5. In particular, the estimator is given by cov(b_t^*, R_{Mt}), where b_t^* is the fitted value from the regression of b_t on the state variables and its own lag. Because the predictor variables are known at the beginning of the period, it must be the case that cov(b_t^*, R_{Mt}) = cov(b_t^*, γ_t). The estimator will equal cov(β_t, γ_t) if the error in b_t^* is uncorrelated with the market risk premium, i.e., if cov(γ_t, $\beta_t - b_t^*$) = 0. This requires that the state variables do a good job capturing either time-variation in the risk premium or time-variation in betas (one is necessary, not both). The variables do capture a significant fraction of movements in betas – the regression R^2s in Table 5 range from 0.26 to 0.60 – but there clearly remains a large component unexplained. Thus, we again interpret the results with caution, although we have no particular reason to believe that the unexplained component of beta is correlated one way or another with γ_t.

The estimates, in Panel B, typically have the same sign as those in Panel A but are closer to zero. S-B's and W-L's betas still covary negatively with market returns, but only the size strategy's covariance

Table 6
Conditional betas and the market risk premium, 1964 – 2001
The table reports the covariance between market returns and the conditional betas on size, B/M, and momentum portfolios. Betas are estimated quarterly using daily returns (Qtr), semiannually using daily and weekly returns (Semi 1 and Semi 2), and annually using monthly returns (Annual). The market portfolio is the CRSP value-weighted index. Excess returns on the index are measured over the same window as betas (e.g., quarterly betas covary with quarterly returns), but the numbers are all expressed in percent monthly (e.g., the quarterly covariance is divided by 3). Panel A reports the market's covariance with contemporaneously estimated betas and Panel B reports the market's covariance with predicted betas, taken from the regression of estimated betas on lagged state variables (see Table 4). The size, B/M, and momentum portfolios are formed using all NYSE and Amex stocks on CRSP/Compustat.

		Size			B/M			Momentum		
		Small	Big	S-B	Grwth	Value	V-G	Losers	Winrs	W-L
Panel A: Covariance between estimated betas and market returns (% monthly)										
Est.	Qtr	**-0.32**	**0.07**	**-0.39**	**-0.20**	-0.12	0.09	0.16	**-0.23**	**-0.38**
	Semi 1	**-0.17**	**0.07**	**-0.24**	**-0.14**	-0.03	0.11	-0.03	-0.07	-0.04
	Semi 2	-0.12	**0.07**	**-0.19**	**-0.10**	-0.03	0.07	0.15	**-0.18**	**-0.33**
	Annual	0.06	0.03	0.03	-0.03	0.01	0.04	-0.08	0.11	0.20
Std err.[a]	Qtr	0.08	0.03	0.08	0.05	0.07	0.06	0.09	0.08	0.16
	Semi 1	0.07	0.03	0.07	0.04	0.07	0.06	0.08	0.07	0.13
	Semi 2	0.08	0.03	0.08	0.04	0.08	0.07	0.10	0.08	0.15
	Annual	0.12	0.03	0.13	0.06	0.10	0.09	0.12	0.10	0.19
Panel B: Covariance between predicted betas and market returns (% monthly)										
Est.	Qtr	-0.06	0.04	**-0.09**	-0.01	-0.02	0.02	0.06	-0.05	-0.12
	Semi 1	-0.07	0.03	**-0.10**	-0.02	-0.02	0.01	0.05	-0.07	-0.12
	Semi 2	-0.04	0.02	-0.05	0.00	-0.01	0.00	0.07	-0.08	-0.14
	Annual	0.03	0.01	0.02	0.00	0.01	0.03	0.05	-0.03	-0.08
Std err.[a]	Qtr	0.04	0.02	0.04	0.03	0.05	0.04	0.05	0.06	0.10
	Semi 1	0.05	0.02	0.04	0.03	0.05	0.04	0.05	0.06	0.10
	Semi 2	0.04	0.02	0.03	0.02	0.05	0.04	0.06	0.05	0.10
	Annual	0.05	0.02	0.05	0.03	0.06	0.04	0.06	0.05	0.09

[a] Standard errors are obtained by regressing market returns on the estimated or predicted betas, scaling the independent variable so that the slope equals a simple covariance. These standard errors implicitly condition on the sample variance of the betas.

is now significant. V-G's betas continue to show little relation to market returns, with estimates between 0.00% and 0.03% (standard errors of 0.04%).

In short, covariation between beta and the risk premium does not explain the unconditional alphas observed for B/M and momentum portfolios. Using the estimates in Panel B, the conditional CAPM predicts that V-G should have an unconditional alpha of 0.00% to 0.03%, a tiny fraction of the actual alpha, 0.59% (see Table 2). W-L should have an alpha of –0.08% to –0.12%, small and opposite in sign to the actual alpha, 1.03%. The results are consistent with our direct evidence that *conditional* alphas are large and significant, contrary to the conditional CAPM.

Beta and market volatility

Section 2 showed that unconditional alphas depend not only on the covariance between beta and the risk premium, but also on the covariance between beta and market volatility:

$$\alpha^u \approx \mathrm{cov}(\beta_t, \gamma_t) - \frac{\gamma}{\sigma_M^2} \mathrm{cov}(\beta_t, \sigma_t^2), \tag{11}$$

where the last term captures the impact of time-varying volatility. In untabulated results, we find that the volatility effect is economically quite small. To estimate $\mathrm{cov}(\beta_t, \sigma_t^2)$, we calculate conditional market volatility much like we do betas, using daily, weekly or monthly returns over short windows. [We adjust for autocorrelation using the approach of French, Schwert, and Stambaugh (1987).] We then estimate the covariance between market volatility and both estimated and predicted betas, similar to Table 6. The estimates of $\mathrm{cov}(\beta_t, \sigma_t^2)$ are between -0.02% and 0.02% for every portfolio (std. errors of 0.01 – 0.02), and, multiplying by γ / σ_M^2 = (0.0047 / 0.045^2) = 2.32, the implied impact on unconditional alphas is at most +/–0.05% monthly. Thus, accounting for time-varying market volatility does little to improve the performance of the conditional CAPM.

5. Comparison with other studies

Our empirical results, and generally skeptical view of conditioning, are opposite to the conclusions of Jagannathan and Wang (1996), Lettau and Ludvigson (2001), Santos and Veronesi (2005), and Lustig and Van Nieuwerburgh (2005). They argue that conditioning dramatically improves the performance of both the simple and consumption CAPMs. The studies have been influential, so it seems worthwhile to offer a few observations on why their conclusions are different.

The four papers differ from ours in many ways, but a key distinction is that they focus on cross-sectional regressions, not time-series intercept tests, and ignore important restrictions on the cross-sectional slopes. As such, the papers test only the qualitative implications of the conditional CAPM, that the effects of time-varying betas are cross-sectionally correlated with expected returns. They do not provide a full, quantitative test of the conditional CAPM.

This point can be seen most easily in the context of the simple CAPM. A full test is whether expected returns are cross-sectionally linear in conditional betas, $E_{t-1}[R_{it}] = \beta_{it}\, \gamma_t$, with a slope equal to the equity premium. However, following Jagannathan and Wang (1996), the papers instead focus on the unconditional relation $E[R_{it}] = \beta_i\, \gamma + \text{cov}(\beta_{it}, \gamma_t)$, estimating this cross-sectional regression using various measures of β_i and $\text{cov}(\beta_{it}, \gamma_t)$. In this regression, the slope on β_i should equal γ and the slope on $\text{cov}(\beta_{it}, \gamma_t)$ should equal one but the papers treat the slopes as free parameters. We believe this explains why they find conditioning to be so important – in particular, the estimated slopes on $\text{cov}(\beta_t, \gamma_t)$ appear to be much too large (as we illustrate in a moment).

To be fair, the papers don't estimate the cross-sectional regression, $E[R_{it}] = \beta_i\, \gamma + \text{cov}(\beta_{it}, \gamma_t)$, directly but, rather, consider transformations of it that obscure the restrictions implied by the conditional CAPM. For example, Jagannathan and Wang (1996) show that, under some assumptions, the terms β_i and $\text{cov}(\beta_{it}, \gamma_t)$ can be replaced by stocks' unconditional betas and their so-called 'premium betas,' $\beta^{\gamma} \equiv \text{cov}(R_t, \gamma_t) / \text{var}(\gamma_t)$. The other papers use β_i and a second loading δ_i that is proportional, under their assumptions, to $\text{cov}(\beta_{it}, \gamma_t)$. These substitutions make it more difficult to see exactly how their estimates violate the restrictions implied by the conditional CAPM. As an illustration, we offer here a detailed example from Lettau and Ludvigson (LL 2001).

LL's main conclusions concern the performance of the consumption CAPM. The 'CCAPM' implies that $E_{t-1}[R_{it}] = \beta_{it}\, \gamma_t$, where, in an abuse of notation, β_{it} is now an asset's consumption beta and γ_t is the consumption-beta risk premium (in the standard model, $\gamma_t \approx \phi\, \sigma_c^2$, where ϕ is aggregate relative risk aversion and σ_c^2 is the variance of consumption growth). Taking unconditional expectations, $E[R_{it}] = \beta_i\, \gamma$

+ $\text{cov}(\beta_{it}, \gamma_t)$, just as in the simple CAPM. To implement this empirically, LL estimate how stocks' consumption betas fluctuate with the consumption-to-wealth ratio CAY: $\beta_{it} = \beta_i + \delta_i \, CAY_t$ [β_i and δ_i are estimated in the first-pass regression $R_{it} = \alpha_{i0} + \alpha_{i1} \, CAY_t + \beta_i \, \Delta c_t + \delta_i \, CAY_t \, \Delta c_t + e_t$, as LL explain on p. 1266]. Substituting β_{it} into the unconditional relation above gives

$$E[R_{it}] = \beta_i \, \gamma + \delta_i \, \text{cov}(CAY_t, \gamma_t). \tag{12}$$

Thus, in LL's context, the conditional CCAPM implies that the slope on β_i should be the average consumption-beta risk premium and the slope on δ_i should be $\text{cov}(CAY_t, \gamma_t)$. In principle, the second restriction could be tested and, we believe, would almost certainly be rejected. Here we simply note that the estimated slope seems huge. In LL's Table 3, the slope on δ_i is around 0.06% or 0.07% quarterly. Interpreting this slope as an estimate of $\text{cov}(CAY_t, \gamma_t)$ and using the fact the covariance must be less than $\sigma_\gamma \, \sigma_{cay}$, LL's estimate implies that $\sigma_\gamma > 3.2\%$ quarterly (that is, if the slope is less than $\sigma_\gamma \, \sigma_{cay}$, then $\sigma_\gamma >$ slope / σ_{cay} = 0.0006 / 0.019). In contrast, LL estimate that the average risk premium is close to zero, between –0.02% and 0.22% quarterly. Thus, if the conditional CCAPM truly explains their results, the risk premium must be close to zero on average yet have enormous volatility (and, since γ_t must be positive, it must also have enormous skewness). These facts are difficult to reconcile – quantitatively – with the consumption CAPM.[6]

On a related note, the cross-sectional R^2s reported by all four papers should be interpreted with caution. The papers find a dramatic increase in R^2 for their conditional models, nicely illustrated by their figures showing predicted returns plotted on actual returns. But these R^2s aren't very informative. First, as discussed above, the papers ignore key restrictions on the cross-sectional slopes; the R^2s would likely drop significantly if the restrictions were imposed. Second, the papers all use returns on size–B/M portfolios that have two key features: the returns can be traced to three common factors (Fama-French

[6] Campbell and Cochrane (1999) provide a convenient benchmark. In their model, $\gamma_t \approx \phi \, \sigma_c^2 \, (1 + \lambda_t)$, where λ_t is the 'sensitivity function' that defines how the surplus consumption ratio responds to consumption. Calibrations in their paper assume that $\phi = 2$, $\sigma_c = 0.75\%$ quarterly, and generate λ_t with a mean of 15 and a standard deviation of 7.5 (roughly). Substituting into γ_t, these parameters imply that $\sigma_\gamma \approx 0.10\%$ quarterly, more than an order of magnitude smaller than the estimate implied by LL's regressions.

time-series R^2s above 90%) and betas on the factors explain most of the cross-sectional variation in expected returns. In this setting, it can be easy to find a high sample R^2 even when the population R^2 is zero. For example, we have simulated the two-pass regressions of Lettau and Ludvigson (2001, Table 3) using historical returns and consumption but substituting a randomly generated, normal and IID, state variable in place of CAY. In 10,000 simulations, the median R^2 is 0.43 and the 5th and 95th percentiles are 0.12 and 0.72, respectively (compared with a reported 0.66). These results suggest that, despite its increasing use, the cross–sectional R^2 isn't very meaningful (Lewellen, Nagel, and Shanken, 2006, explore this point more fully; see, also, Roll and Ross, 1994; Kandel and Stambaugh, 1995).

6. Conclusion

The main point of the paper is easily summarized: the conditional CAPM does not explain asset-pricing anomalies like B/M or momentum. Analytically, if the conditional CAPM holds, deviations from the unconditional CAPM depend on the covariances among betas, the market risk premium, and market volatility. We argue that, for plausible parameters, the covariances are simply too small to explain large unconditional pricing errors.

The empirical tests support this view. We use short-window regressions to directly estimate conditional alphas and betas for size, B/M, and momentum portfolios from 1964 – 2001. This methodology gets around the problem, common to all prior tests, that the econometrician cannot observe investors' information sets. We find that betas vary considerably over time, with relatively high-frequency changes from year to year, but not enough to generate significant unconditional pricing errors. Indeed, there is little evidence that betas covary with the market risk premium in a way that might explain the alphas of B/M and momentum portfolios. Most important, conditional alphas are large and significant, in direct violation of the conditional CAPM.

Appendix A

This appendix derives eq. (2), the expression for a stock's unconditional beta. Let R_{it} be the excess return on asset i, R_{Mt} be the excess return on the market portfolio, and β_t be the stock's conditional beta for period t (given information at t–1). Also, let $\beta_t = \beta + \eta_t$, where $\beta = E[\beta_t]$ and η_t is the zero-mean, time-varying component. According to the conditional CAPM, $R_{it} = \beta_t R_{Mt} + \varepsilon_t$, so the unconditional covariance between R_{it} and R_{Mt} equals:

$$\begin{aligned} \mathrm{cov}(R_{it}, R_{Mt}) &= \mathrm{cov}[(\beta + \eta_t) R_{Mt}, R_{Mt}] \\ &= \beta \sigma_M^2 + E[\eta_t R_{Mt}^2] - E[\eta_t R_{Mt}] E[R_{Mt}]. \qquad \text{(A.1)} \end{aligned}$$

Recall that $E[\eta_t] = 0$, $E_{t-1}[R_{Mt}] = \gamma_t$, $E_{t-1}[R_{Mt}^2] = \gamma_t^2 + \sigma_t^2$, and $E[R_{Mt}] = \gamma$. Therefore, the second term equals $\mathrm{cov}(\eta_t, \gamma_t^2 + \sigma_t^2)$ and the last term equals $\gamma\,\mathrm{cov}(\eta_t, \gamma_t)$. Substituting into (A.1) yields

$$\mathrm{cov}(R_{it}, R_{Mt}) = \beta \sigma_M^2 + \mathrm{cov}(\eta_t, \sigma_t^2) + \mathrm{cov}(\eta_t, \gamma_t^2) - \gamma\,\mathrm{cov}(\eta_t, \gamma_t). \qquad \text{(A.2)}$$

Finally, write $\gamma_t = \gamma + (\gamma_t - \gamma)$ and substitute into the second-to-last term of (A.2). Simplifying yields:

$$\mathrm{cov}(R_{it}, R_{Mt}) = \beta \sigma_M^2 + \mathrm{cov}(\eta_t, \sigma_t^2) + \gamma\,\mathrm{cov}(\eta_t, \gamma_t) + \mathrm{cov}[\eta_t, (\gamma_t - \gamma)^2]. \qquad \text{(A.3)}$$

Since $\beta_t = \beta + \eta_t$, we can simply replace η_t with β_t throughout this expression. The unconditional beta can then found by dividing both sides by the market's unconditional variance. The result, identical to eq. (2) in the text, is a general formula for the unconditional beta when expected returns, variances, and covariances all change over time and the conditional CAPM holds.

Appendix B

This appendix explores how high frequency changes in beta affect our short-window regressions. In principle, we expect the impact of, say, daily changes in beta on quarterly regressions to be similar to the effects discussed in Section 2 except that now only *intra*quarter variation in betas and expected returns (around their quarterly means) should be important. We use simulations to explore the effects formally, focusing on the case of constant volatility.

The simulations match many properties of the data but the parameters are guided in part by theory. We assume that both beta and the market risk premium follow weekly AR(1) processes:

$$\gamma_t = \phi\, \gamma_{t-1} + \xi_t, \text{ where } \xi_t \sim N[0, \sigma_\xi^2], \tag{A.4}$$

$$\beta_t = \kappa\, \beta_{t-1} + v_t, \text{ where } \nu_t \sim N[0, \sigma_v^2]. \tag{A.5}$$

The (realized) return on the market portfolio is $R_{Mt} = \gamma_t + s_t$, with $s_t \sim N[0, \sigma_s^2]$, and the return on the stock is $R_t = \beta_t R_{Mt} + \varepsilon_t$, with $\varepsilon_t \sim N[0, \sigma_\varepsilon^2]$. In addition, the simulations capture two potentially important features of the data: (i) shocks to the risk premium are allowed to covary negatively with market returns (prices drop if the risk premium goes up); and (ii) shocks to betas are allowed to covary negatively with the stock's returns (prices drop if risk goes up).

Specifically, we simulate weekly returns under several assumptions: (i) the correlation between betas and the risk premium is either 0.0 or 0.8; (ii) the correlation between shocks to the risk premium and shocks to realized market returns is 0.0, –0.4, or –0.8; and (iii) the correlation between idiosyncratic shocks to beta (the component that isn't correlated with the risk premium) and idiosyncratic stock returns is either 0.0 or –0.5. These parameters are chosen to cover a wide range of empirically-plausible values. In addition, all simulations assume that β_t and γ_t both have monthly autocorrelations of 0.98, β_t has a mean of 1.0 and volatility of 0.5, γ_t has a mean of 0.5% and volatility of 1.5% monthly, the market's conditional volatility is 4.5% monthly, and the asset's idiosyncratic volatility is 5%. We simulate returns with extreme variation in betas and the risk premium in order to generate unconditional alphas that are in line with their empirical values.

The simulation results are reported in Table A.1 (based on 50,000 quarters of weekly returns). True conditional alphas are zero in the simulations, since the CAPM holds in weekly returns. The top panel shows simulations in which β_t and γ_t are uncorrelated, so unconditional alphas are also zero, while the bottom panel shows simulations in which $\text{cor}(\beta_t, \gamma_t) = 0.8$, so unconditional alphas are roughly $\alpha^u \approx \text{cov}(\beta_t, \gamma_t) = 0.60$. The simulations confirm that unconditional alphas estimated using either weekly or monthly returns produce alphas close to the theoretical value. The fact that weekly and monthly alphas

Table A.1
The impact of high frequency variation in beta on CAPM regressions
The table reports unconditional and average short-window regressions from simulations in which beta and the risk premium vary weekly. The return generating process is described in the text. The table shows results for 12 scenarios that differ in three dimensions: (i) the correlation between β_t and γ_t is either 0.0 or 0.8; (ii) the correlation between idiosyncratic shocks to β_t (the component that isn't correlated with γ_t) and idiosyncratic stock returns is either 0.0 or –0.5 (labelled cor(β,ε) in the table); and (iii) the correlation between shocks to γ_{t+1} and shocks to R_{Mt} is either 0.0, –0.4, or –0.8 (labelled cor(γ,R_M) in the table). The conditional CAPM holds in weekly returns but the table shows unconditional regressions using both weekly and monthly returns.

Parameters			Unconditional regressions				Short-window regressions		
cor(β,γ)	cor(β,ε)	cor(γ,R_M)	α^u_{weekly}	β^u_{weekly}	$\alpha^u_{monthly}$	$\beta^u_{monthly}$	α_{weekly}	β_{weekly}	cov(β,R_M)
0	0	0	**0.01**	0.97	**0.01**	0.97	**0.00**	0.97	0.01
0	0	-0.4	**0.01**	0.97	**0.01**	0.97	**-0.01**	0.97	0.02
0	0	-0.8	**0.01**	0.97	**0.01**	0.97	**-0.01**	0.97	0.02
0	-0.5	0	**0.02**	0.97	**0.02**	0.97	**0.01**	0.97	0.01
0	-0.5	-0.4	**0.02**	0.97	**0.03**	0.97	**0.01**	0.97	0.02
0	-0.5	-0.8	**0.03**	0.97	**0.02**	0.97	**0.00**	0.97	0.02
0.8	0	0	**0.62**	0.99	**0.61**	1.00	**0.01**	0.98	0.61
0.8	0	-0.4	**0.62**	0.99	**0.61**	1.00	**0.06**	0.98	0.56
0.8	0	-0.8	**0.61**	0.99	**0.61**	1.00	**0.11**	0.99	0.50
0.8	-0.5	0	**0.63**	0.99	**0.63**	1.00	**0.02**	0.98	0.61
0.8	-0.5	-0.4	**0.63**	0.99	**0.63**	1.00	**0.08**	0.98	0.56
0.8	-0.5	-0.8	**0.63**	0.99	**0.63**	1.00	**0.13**	0.99	0.50

Alphas, in bold, are % monthly

are nearly identical suggests that the horizon over which the CAPM is assumed to hold isn't very important – if it holds at one frequency, it should hold nearly perfectly at others that aren't too different, absent the microstructure issues discussed in Section 3.

More important, the simulations show that our short-window regressions (quarterly using weekly returns) produce conditional alphas that are close to zero, even though betas and the risk premium vary wildly over time. For the six scenarios in the top panel, in which beta and the risk premium are uncorrelated, the short-window alphas are almost exactly zero. For the six scenarios in the bottom panel, in which $\alpha^u \approx 0.60\%$ monthly, the short-window alphas are between 0.01% and 0.13% monthly – that is, our short-window regressions capture 80–99% of the pricing impact of time-varying betas. The short-window regressions work almost perfectly as long as market returns are not too highly correlated with shocks to the risk premium, but they deteriorate somewhat (though still work extremely well) as that correlation approaches –1.0. The small deterioration seems to arise because the market return is also negatively correlated with shocks to beta; quarters in which beta goes down are quarters in which the market return is high, pushing the short-window alphas slightly positive. In general, the simulations suggest that our short-window regressions do a very good job capturing the impact of time-varying betas even when betas vary at high frequency.

References

Adrian, Tobias and Francesco Franzoni, 2005. Learning about beta: Time-varying factor loadings, expected returns, and the conditional CAPM. Working paper (Federal Reserve Bank of New York and HEC School of Management, Paris).

Ang, Andrew, and Joseph Chen, 2005. CAPM over the long-run: 1926 – 2001. Forthcoming in the *Journal of Empirical Finance*.

Ball, Ray, S.P. Kothari, and Jay Shanken, 1995. Problems in measuring portfolio performance: An application to contrarian investment strategies. *Journal of Financial Economics* 38, 79-107.

Breeden, Douglas, Michael Gibbons, and Robert Litzenberger, 1989. Empirical tests of the consumption CAPM. *Journal of Finance* 44, 231-262.

Campbell, John and John Cochrane, 1999. By force of habit: A consumption-based explanation of aggregate stock market behavior. *Journal of Political Economy* 107, 105-251.

Campbell, John and Ludger Hentschel, 1992. No news is good news: An asymmetric model of changing volatility in stock returns. *Journal of Financial Economics* 31, 281-318.

Cochrane, John, 2001. *Asset Pricing*. Princeton University Press, Princeton NJ.

Dimson, Elroy, 1979. Risk measurement when shares are subject to infrequent trading. *Journal of Financial Economics* 7, 197-226.

Dybvig, Philip and Stephen Ross, 1985. Differential information and performance measurement using a security market line. *Journal of Finance* 40, 383-399.

Fama, Eugene and Kenneth French, 1989. Business conditions and expected returns on stocks and bonds. *Journal of Financial Economics* 25, 23-49.

Fama, Eugene and Kenneth French, 1993. Common risk factors in the returns on stocks and bonds. *Journal of Financial Economics* 33, 3-56.

Fama, Eugene and Kenneth French, 1997. Industry costs of equity. *Journal of Financial Economics* 43, 153-193.

Fama, Eugene and G. William Schwert, 1977. Asset returns and inflation. *Journal of Financial Economics* 5, 115-146.

Ferson, Wayne and Rudi Schadt, 1996. Measuring fund strategy and performance in changing economic conditions. *Journal of Finance* 51, 425-462.

Franzoni, Francesco, 2002. Where is beta going? The riskiness of value and small stocks. Working paper (HEC School of Management, Paris).

French, Kenneth, G. William Schwert, and Robert Stambaugh, 1987. Expected stock returns and volatility. *Journal of Financial Economics* 19, 3-29.

Ghysels, Eric, Pedro Santa-Clara, and Rossen Valkanov, 2004. There is a risk-return tradeoff after all. Forthcoming in *Journal of Financial Economics*.

Grundy, Bruce and J. Spencer Martin, 2001. Understanding the nature of the risks and the source of the rewards to momentum investing. *Review of Financial Studies* 14, 29-78.

Handa, Puneet, S.P. Kothari, and Charles Wasley, 1989. The relation between the return interval and betas: Implications for the size effect. *Journal of Financial Economics* 23, 79-100.

Hansen, Lars Peter and Scott Richard, 1987. The role of conditioning information in deducing testable restrictions implied by dynamic asset pricing models. *Econometrica* 55, 587-614.

Harvey, Campbell, 1989. Time-varying conditional covariances in tests of asset pricing models. *Journal of Financial Economics* 24, 289-317.

Jagannathan, Ravi and Zhenyu Wang, 1996. The conditional CAPM and the cross-section of stock returns. *Journal of Finance* 51, 3-53.

Jegadeesh, Narasimhan, and Sheridan Titman, 1993. Returns to buying winners and selling losers: Implications for stock market efficiency. *Journal of Finance*, 48, 65-91.

Jensen, Michael, 1968. The performance of mutual funds in the period 1954 – 1964. *Journal of Finance* 23, 389-416.

Kandel, Shmuel and Robert Stambaugh, 1995. Portfolio inefficiency and the cross-section of expected returns. *Journal of Finance* 50, 157-184.

Lettau, Martin and Sydney Ludvigson, 2001. Resurrecting the (C)CAPM: A cross-sectional test when risk premia are time-varying. *Journal of Political Economy* 109, 1238-1287.

Lettau, Martin and Sydney Ludvigson, 2003. Measuring and modeling variation in the risk-return tradeoff. In: Ait-Sahalia, Y. and L. Hansen (Eds.), *Handbook of Financial Econometrics*.

Levhari, David, and Haim Levy, 1977. The capital asset pricing model and the investment horizon. *Review of Economics and Statistics* 59, 92-104.

Lewellen, Jonathan, 2004. Predicting returns with financial ratios. *Journal of Financial Economics* 74, 209-235.

Lewellen, Jonathan, Stefan Nagel, and Jay Shanken, 2006. A skeptical appraisal of asset-pricing tests. Working paper (Dartmouth College, Stanford University, and Emory University).

Lo, Andrew and A. Craig MacKinlay, 1990. When are contrarian profits due to stock market overreaction?. *Review of Financial Studies* 3, 175-205.

Lustig, Hanno and Stijn Van Nieuwerburgh, 2005. Housing collateral, consumption insurance, and risk premia: An empirical perspective. *Journal of Finance* 60, 1167-1219.

Merton, Robert, 1980. On estimating the expected return on the market: An exploratory investigation. *Journal of Financial Economics* 8, 323-361.

Petkova, Ralitsa, and Lu Zhang, 2005. Is value riskier than growth?. *Journal of Financial Economics* 78, 187-202.

Roll, Richard and Stephen Ross, 1994. On the cross-sectional relation between expected returns and betas. *Journal of Finance* 49, 101-121.

Santos, Tano and Pietro Veronesi, 2005. Labor income and predictable stock returns. Forthcoming in *Review of Financial Studies*.

Shanken, Jay, 1990. Intertemporal asset pricing: An empirical investigation. *Journal of Econometrics* 45, 99-120.

Wang, Kevin, 2002. Asset pricing with conditioning information: A new test. *Journal of Finance* 58, 161-196.

Zhang, Lu, 2005. The value premium. *Journal of Finance* 60, p. 67-103.