Heat transfer in flat-plate boundary layers: a correlation for laminar, transitional, and turbulent flow

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We Can’t Neglect the Transition Region

Transition region and laminar region have similar length

\[
h(x) = \begin{cases} 
\text{laminar} & \sim x^{1/2}, \\
\text{transition} & \sim x^{-1}, \\
\text{turbulent} & \sim x^{-0.2}.
\end{cases}
\]

The two step, laminar-then-turbulent model is incorrect!

We can’t neglect the transition region because the transition region and laminar region have similar length.

Typical Results (more data & fluids in paper)

Data from Multiple Independent Experiments

Typical Results

- For gases only, the following equation has similar accuracy:
  \[
  \text{Nu} = 0.332 \text{Re}^{1/2} \text{Pr}^{1/3}
  \]
- Eq. (9), Re = 610000, c = 2.6
- Eq. (9), Re = 1400000, c = 2.2
- Eq. (9), Re = 2700000, c = 2.2
- Eq. (6), Nu \text{turb} = 0.0296 \text{Re}^{0.8} \text{Pr}^{0.6}

Summary of the Correlation

For smooth, sharp-edged, flat plates with zero pressure gradient and either uniform wall temperature (UWT) or uniform heat flux (UHF)

Combining formula

\[
\text{Nu} = \left[ \text{Nu}_{\text{lam}}^6 + (\text{Nu}_{\text{turb}}^{10} + \text{Nu}_{\text{turb}}^{15})^{-1/2} \right]^{1/5}, \quad \text{Eq. (9)}
\]

Laminar region

\[
\text{Nu}_{\text{lam}} = \frac{0.332 \text{Re}^{1/2} \text{Pr}^{1/3}}{\text{UWT}}
\]

With an unheated starting length of \(x_0\) (UHF or UWT), use

\[
\text{Nu}_{\text{lam}} = \frac{0.453 \text{Re}^{1/2} \text{Pr}^{1/3}}{\text{UHF}}
\]

Transition region

\[
\text{Nu}_{\text{turb}} = \text{Nu}_{\text{turb}}(\text{Re}, \text{Pr}) \cdot \left[ 1 - \left( \frac{x}{x_0} \right)^{2/3} \right]^{1/3}, \quad \text{Eq. (6)}
\]

Re is the Reynolds number at onset of transition, \(x_0\)

\[
c = 0.9992 \log_{10} \text{Re}_i - 3.013 \quad \text{for Re}_i < 5 \times 10^5
\]

Turbulent region (UHF and UWT)

\[
\text{Nu}_{\text{turb}}(\text{Re}, \text{Pr}) = \frac{\text{Re}_i \text{Pr}_i}{1 + 12.7(\text{Pr}^{2/3} - 1)\sqrt{\text{Pr}_i}}\]

\[
\text{Pr}_i = \left( \frac{\text{C}_f(\text{Re}_i)}{2} \right) \frac{\text{Re}_i}{\text{Pr}_i}
\]

\[
\text{C}_f(\text{Re}_i) = \frac{0.455}{\left[ \ln(0.06 \text{Re}_i) \right]^{1/2}}
\]

For gases only, the following equation has similar accuracy

\[
\text{Nu}_{\text{turb}}(\text{Re}, \text{Pr}) = 0.0296 \text{Re}^{0.8} \text{Pr}^{0.6}
\]

Data from Multiple Independent Experiments

\[
0.7 < \text{Pr} < 257 \quad 4,000 \leq \text{Re} \leq 4,300,000
\]

Fully turbulent air data fit to std. dev. of ±5.5%

- Seban & Doughty
- Reynolds et al., Run 1
- Reynolds et al., Runs 2–4
- Reynolds et al., Runs 5–7
- Reynolds et al., Run 8
- Junkhan & Serovy, high \(u_i/u_m\)
- Kestin et al., low \(u_i/u_m\)
- Junkhan & Serovy, low \(u_i/u_m\)
- Blair, \(u_i/u_m = 0.5\%
- Blair, \(u_i/u_m = 1.0\%
- Blair, \(u_i/u_m = 2.0\%
- Nu_{\text{turb}} \text{Eq. (6)}

Classical Colburn analogy (1933)

Not recommended: Colburn’s \( \text{St} = (C_f/2) \text{ Pr}^{-1/3} \) was based on b.l. data for air and does not support a wide range of Pr. Colburn’s suggestion to use it for laminar flow compared a UWT formula to misplotted UHF data.

Similarity solution for UHF laminar b.l.

This result (Fage & Falkner, 1931; Imai, 1958) is not widely known

\[
\text{Nu}_{\text{lam}} = 0.4587 \text{Re}^{1/2} \text{Pr}^{1/3}
\]

but close to integral-method (replace 0.4587 by 0.4535). Pre-1950, wall boundary conditions often overlooked (Colburn 1933; Jakob & Dow 1946)