A fracture-based model for FRP debonding in strengthened beams

Oguz Gunes, Oral Buyukozturk, Erdem Karaca

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1. Introduction

FRP composites are becoming a material of choice in an increasing number of rehabilitation and retrofitting projects around the world. Depending on the design objectives, these materials can be used to improve one or more of the structural member characteristics such as the load capacity, ductility, and durability. A multi-national effort is underway to develop proper codes and guidelines to set the standards for material selection, design, installation, inspection, maintenance, and repair of FRP applications. Design of structural strengthening applications using externally bonded FRP composites is usually based on conventional design approaches with improvements to account for the presence and characteristics of the FRP material. Nonconventional design issues that are specific to the type of application require special considerations for their proper inclusion in the design process. One such design issue is the debonding problems in externally bonded FRP strengthening applications, the incorporation of which in the design has been a research challenge since the initial development stages of the method [1–3]. Due to the typical premature and brittle nature of debonding failures, inadequately designed strengthening applications may not only become ineffective, but may also reduce the level of safety of the member by decreasing its ductility. Design procedures that properly consider debonding problems are needed to ensure the safety and reliability of flexural members strengthened using FRP composites. This paper presents an experimental and analytical research study aimed at the understanding of debonding failures in FRP strengthened beams, and proposes a fracture-based model for the prediction of such failures as a basis for design of these systems.

2. Failure modes of FRP strengthened beams

Failure of FRP strengthened beams may take place through several mechanisms depending on the beam and strengthening parameters. In the recent publications of the ACI 440 Committee on Fiber Reinforced Polymer Reinforcement, these
failure modes are classified as: (1) concrete crushing before reinforcing steel yielding, (2) steel yielding followed by FRP rupture, (3) steel yielding followed by concrete crushing, (4) cover delamination, (5) FRP debonding. Oehlers further classified the debonding failure modes according to the type of crack causing debonding. In addition to these, shear failure occurs if the shear capacity of the beam cannot accommodate the increase in the flexural capacity. An investigation of each of these failure modes is required in the design process to ensure that the strengthened beam will perform satisfactorily.

2.1. Debonding failure mechanisms

The term debonding failure is often associated with a significant decrease in member capacity due to initiation and propagation of debonding. Debonding initiation in beams strengthened with FRP composites generally take place in regions of high stress concentration at the FRP–concrete interface. These regions include the ends of the FRP reinforcement, and those around the shear and flexural cracks. Fig. 1 shows examples of debonding failures from laboratory tests. The cover debonding mechanism shown in Fig. 1a is usually associated with high interfacial stresses, low concrete strength, and/or with extensive cracking in the shear span. If the concrete strength and the shear capacity of the beam are sufficiently high, potential debonding failure is most likely to take place through FRP debonding, as shown in Fig. 1b and c. Depending on the beam parameters and the strengthening configuration, such failures may initiate at the areas of high stress concentration at laminate ends and propagate towards the center of the beam, or may initiate at flexure-shear cracks and propagate towards the ends of the beam. Depending on the material properties, debonding may occur within the FRP laminate, at the FRP–concrete interface, or a few millimeters within the concrete.

A noteworthy issue regarding the failure behavior of FRP strengthened beams is the interaction between shear and debonding failures, which may have a causal relationship and sequential occurrence. It is often the case that the debonding failures and debonding + shear failures are not properly differentiated and reported. This is partly justified considering that the member is considered as failed in both cases. However, a fundamentally important difference between debonding and shear failures is the ductility behavior. Debonding failures significantly reduce the beam flexural capacity, however, provided that the beam has adequate shear capacity, it can still display the ductile failure behavior of a regular reinforced concrete beam. This is not the case for shear failures where total beam failure takes place in a brittle fashion. Thus, it is important to make sure that the beam has a shear capacity that is sufficiently higher than its flexural capacity and debonding failure load.

3. Previous research on debonding problems

Characterization and modeling of debonding in structural members strengthened with externally bonded reinforcements has long been a popular area of interdisciplinary research due to critical importance of debonding failures in bonded joints. In the last decade, there has been a concentration of research efforts in this area with respect to FRP strengthened flexural members, and considerable progress has been achieved in understanding the causes and mechanisms of debonding failures through numerous experimental, analytical, and numerical investigations. Modeling research in this area can be classified in general terms by their approach to the problem as strength or fracture mechanics approaches. In addition to these, a number of researchers have proposed relatively simple semi-empirical and empirical models that avoid the complexities of stress and fracture analyses and can be relatively easily implemented in design calculations.

Strength approach involves prediction of debonding failures through calculation of the interfacial or bond stress distribution in FRP strengthened members based on elastic material properties. Calculated stresses are compared with those corresponding to the strength of the materials to predict the debonding failure load and mechanism. The fact that debonding is essentially a crack propagation promoted by local stress intensities has raised interest among some researchers to take a fracture mechanics approach to the problem and develop predictive models that utilize elastic and fracture material properties. Several recent studies have conducted detailed investigations of opening mode, shear mode and mixed mode fracture processes during debonding. Despite the demonstrated success of various fracture models for specific bond test configurations, there is a need for models that can satisfactorily predict the debonding failure loads for the general case of FRP strengthened beams in which multiple mechanisms of debonding processes are simultaneously at

![Fig. 1. Debonding failure modes.](image-url)
work due to multiple cracks at unknown locations along the beam. Gunes [22] and Achintha and Burgoyne [23] used the global energy balance in a strengthened beam, with fundamental differences in characterization of energy components and debonding fracture, to predict the debonding failure loads based on a fracture mechanics approach. The general objective of empirical models is to provide a simple methodology to predict debonding failures without going into complex stress or fracture analyses. Several such models were proposed for FRP strengthened beams based on certain parameters that influence their debonding behavior [7]. The reader is referred to [6–8,12,16,20,24] for a comprehensive review of debonding research and modeling.

Guidelines by the ACI Committee 440 enforce a limit on the strain level developed in the FRP reinforcement to prevent debonding failures [4,5]. This limit is given by the following expression:

\[ \varepsilon_{ef} = \varepsilon_{cu} \left( 1 - \frac{h}{c} \right) - \varepsilon_{bi} \leq \kappa_m \varepsilon_{fu} \]

(1)

where \( \varepsilon_{fu} \) and \( \varepsilon_{ef} \) are the ultimate strain and the maximum allowed effective strain in the FRP reinforcement, respectively. \( \varepsilon_{cu} \) is the ultimate strain of concrete, \( h \) is the beam height, \( c \) is the neutral axis depth, \( \varepsilon_{bi} \) is the concrete substrate strain at the time of the FRP installation, and the limiting strain coefficient \( \kappa_m \) is given by:

\[ \kappa_m = \begin{cases} \frac{1}{60} \left( 1 - \frac{nE_f t_f}{50,000} \right) \leq 0.90 & \text{for } nE_f t_f \leq 180,000 \\ \frac{1}{60} \left( \frac{90,000}{nE_f t_f} \right) \leq 0.90 & \text{for } nE_f t_f > 180,000 \end{cases} \]

(2)

where \( n \) is the number of FRP reinforcement layers, \( t_f \) is the thickness of each layer, and \( E_f \) is the elastic modulus of the FRP reinforcement. From Eq. (2), it is apparent that the limiting strain in the FRP reinforcement is defined by the geometric and material properties of the FRP reinforcement only.

4. Experimental study

The experimental study presented herein is part of a comprehensive experimental program carried out to investigate the monotonic and cyclic load performance of precracked reinforced concrete beams strengthened in flexure and/or shear using FRP composite plates and sheets [22]. The focus of the study was characterization and prevention of debonding failures as affected by the shear strengthening and anchorage conditions. In this paper, primary test results from this experimental program are presented as a basis for the model presented in Section 5.

Laboratory size reinforced concrete beams were strengthened using carbon FRP composite plates in shear and/or flexure with and without anchoring of the flexural FRP reinforcement, and were loaded in four-point bending until failure. Properties of the materials used in the experimental program are given in Table 1. All beams were precracked prior to strengthening. The geometry and reinforcement details of the control specimen (CM1) are shown in Fig. 2a and the strengthening configurations of the tested beams are shown in Fig. 2b. All specimens shown in Fig. 2b were strengthened in flexure using 1270 mm (50 in) long, 38.1 mm (1.5 in) wide, and 1.2 mm (0.047 in) thick unidirectional FRP plates. For shear strengthening, 40-mm wide straight (beams S3PS1M and S3PS2M) and L-shaped (beams S4PS1M and S4PS2M) unidirectional FRP plates were used, the latter of which also served as anchorage for the flexural FRP reinforcement. In order to compare the influence of external shear strengthening vs. higher internal shear capacity on the debonding behavior, the shear capacity of a beam was increased through use of larger internal shear reinforcement (see beam S2PF7M in Fig. 2b). Using section analysis, the calculated flexural load capacity was 118.6 kN for the control beam (CM1) and 158.6 kN for all strengthened beams, with all expected to fail through concrete crushing. Calculated shear capacities were 202 kN for the control beam (D4 shear reinforcement), 339 kN for S2PF7M (#3 rebar shear reinforcement), and 300 kN in the sections of beams with FRP shear reinforcement [22].

The load–deflection curves obtained from the tests are shown in Fig. 3a and the corresponding load vs. mid-span FRP strain curves are shown in Fig. 3b. Table 2 summarizes the experimental results and indicates the observed failure mode for each specimen. All beams shown in Fig. 2b failed through FRP debonding except for beam S1PF1M which failed through cover debonding followed by shear failure. Comparing the load–deflection curves for beam S1PF1M and S2PF7M, the influence of the shear capacity of a beam on its debonding failure behavior is immediately apparent. Both beams were strengthened in the same configuration and essentially both failed through debonding, however, the failure load of S2PF7M, which

<table>
<thead>
<tr>
<th>Material</th>
<th>Compressive strength (MPa)</th>
<th>Yield strength (MPa)</th>
<th>Tensile strength (MPa)</th>
<th>Elastic modulus (MPa)</th>
<th>Ult. tensile strain (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Concrete</td>
<td>41.4</td>
<td>–</td>
<td>–</td>
<td>25,000</td>
<td>–</td>
</tr>
<tr>
<td>#3 and #5 rebars</td>
<td>–</td>
<td>440</td>
<td>–</td>
<td>200,000</td>
<td>–</td>
</tr>
<tr>
<td>D4 deformed bars</td>
<td>–</td>
<td>620</td>
<td>–</td>
<td>200,000</td>
<td>–</td>
</tr>
<tr>
<td>CFRP plate</td>
<td>–</td>
<td>–</td>
<td>2800.0</td>
<td>165,000</td>
<td>1.69</td>
</tr>
<tr>
<td>Epoxy adhesive</td>
<td>–</td>
<td>–</td>
<td>24.8</td>
<td>4500</td>
<td>1.00</td>
</tr>
</tbody>
</table>
Fig. 2. Geometry and strengthening configurations of beam test specimens (in mm).

(a) control specimen
(b) beams strengthened in shear and/or flexure

Fig. 3. Experimental results.

Table 2
Summary of experimental results.

<table>
<thead>
<tr>
<th>Beam</th>
<th>$P_y$ (kN)</th>
<th>$\delta_y$ (cm)</th>
<th>$\varepsilon_{fm,y}$ (%)</th>
<th>$P_u$ (kN)</th>
<th>$\delta_u$ (cm)</th>
<th>$\varepsilon_{fm,u}$ (%)</th>
<th>Failure mode</th>
</tr>
</thead>
<tbody>
<tr>
<td>CM1</td>
<td>110.0</td>
<td>5.2</td>
<td>–</td>
<td>117.4</td>
<td>17.9</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td>S1PF1M</td>
<td>123.0</td>
<td>5.8</td>
<td>0.31</td>
<td>131.9</td>
<td>7.7</td>
<td>0.40</td>
<td>Cover debonding + shear failure</td>
</tr>
<tr>
<td>S2PF7M</td>
<td>127.5</td>
<td>5.5</td>
<td>0.32</td>
<td>148.3</td>
<td>9.3</td>
<td>0.59</td>
<td>FRP debonding</td>
</tr>
<tr>
<td>S3PS1M</td>
<td>124.0</td>
<td>5.4</td>
<td>0.31</td>
<td>143.1</td>
<td>9.2</td>
<td>0.55</td>
<td>FRP debonding</td>
</tr>
<tr>
<td>S3PS2M</td>
<td>129.1</td>
<td>5.6</td>
<td>0.36</td>
<td>153.8</td>
<td>11.5</td>
<td>0.68</td>
<td>FRP debonding</td>
</tr>
<tr>
<td>S4PS1M</td>
<td>128.2</td>
<td>5.4</td>
<td>0.33</td>
<td>168.2</td>
<td>14.7</td>
<td>0.87</td>
<td>FRP debonding</td>
</tr>
<tr>
<td>S4PS2M</td>
<td>127.9</td>
<td>4.9</td>
<td>0.31</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Debonding and associated fracture processes result in global energy transformations in FRP strengthened members. In the early stages of loading, these fracture processes may be gradual and stable, whereas upon reaching a critical energy state, a sudden brittle failure may take place. The global energy dissipation, \( d\varphi \), in the system can be described in terms of the changes in the amount of externally supplied work, \( \varphi_{\text{ext}} \), and the energy stored in the system as free energy, \( W \), by the following expression:

\[
d\varphi = d\varphi_{\text{ext}} - dW \geq 0
\]

Introducing the potential energy of the system, \( \Pi \), in the following form:

\[
\Pi = W - \Phi
\]

where \( \Phi \) is the external work done by prescribed surface forces, the expression for total dissipation in Eq. (3) can be rewritten for constant prescribed surface forces and displacements as follows:

\[
d\varphi = -d\Pi \geq 0
\]

Thus, the amount of energy dissipated in the system during debonding can be determined by calculating the change in the potential energy of the system.

5.1. Energy dissipation during debonding

The mechanisms of energy dissipation in FRP strengthened RC beams under loading include cracking and crushing of concrete, reinforcement yielding and pullout, and FRP debonding. These mechanisms are shown in Fig. 4. Debonding failure may take place before or after steel reinforcement yielding depending on the RC beam geometry and reinforcement, and FRP strengthening configuration. The potential energy difference in strengthened beams upon debonding failure is depicted in Fig. 5 for the cases before and after reinforcement yielding. The difference between Fig. 5a and b in terms of energy dissipation is that the latter involves an added plastic energy dissipation term due to reinforcement yielding. Thus, the energy dissipation, \( \Delta \varphi \), given by the change in potential energy during debonding failure, can be written in general terms as:

\[
\Delta \varphi \simeq d\varphi = \int \gamma d\Omega + \int \sigma \cdot d\varepsilon d\Omega + \int G_d dA_f \geq 0; \quad \varepsilon = \varepsilon_y - \varepsilon_y \geq 0
\]

where \( \int \sigma \cdot d\varepsilon d\Omega \) is the plastic energy dissipation due to steel yielding when the strain in the reinforcing steel, \( \varepsilon_y \), is greater than its yield strain, \( \varepsilon_y \), and is equal to zero otherwise. The term \( \int G_d dA_f \) represents dissipation due to debonding process evaluated over the crack surface defined by the energy per unit area necessary for the crack formation called the interface fracture energy, \( G_d \), and the interfacial bond area \( A_f \), and the term \( \int \gamma d\Omega \) represents the bulk energy dissipation within the
system due to remaining mechanisms shown in Fig. 4 which consist mainly of concrete cracking under bending and shear effects.

Examination of Eq. (6) shows that debonding failures, before or after steel yielding, are not pure fracture processes. Thus, formulation of a debonding failure criteria based on fracture mechanics requires quantification of different energy dissipation mechanisms that are of significance. Experimental evidence shows that the bulk energy dissipation $\int \gamma d\Omega$ included in Eq. (6) is less significant compared to the remaining dissipation terms since much of the concrete cracking takes place before debonding, and only limited cracking occurs during debonding due to constant curvature and small change in the location of the neutral axis. As a first approximation, the bulk energy dissipation during debonding failure can be assumed to be insignificant and that the dominant modes of energy dissipation are the debonding fracture process and the plastic energy dissipation at the rebar. Thus, the total energy dissipation can be approximated as:

$$\Delta D \approx \int \sigma \cdot d\varepsilon d\Omega + \int G_f dA_f \geq 0; \quad \varepsilon^p = \varepsilon_i - \varepsilon_y \geq 0$$

(7)

Eq. (7) assumes that debonding failure before reinforcement yielding is a pure debonding fracture process, and that the only additional dissipation term in case of debonding after reinforcement yielding is the plastic energy dissipation due to rebar yielding. Quantification of these two mechanisms is sufficient for debonding failure modeling.

5.2. Plastic energy dissipation due to reinforcement yielding

In order to define a debonding criterion, an essential step is to characterize the plastic energy dissipation term in Eq. (7). It is apparent from Fig. 5 that in a displacement-controlled experiment the beam deflection and thus the curvature essentially stays constant upon debonding. Fig. 6 shows the strain profile in the beam cross-section before and after debonding failure. From the definition of curvature, $\phi$, one can write:

$$\phi = \frac{\varepsilon_c}{c} = \frac{\varepsilon_c^*}{c^*}$$

(8)

where $\varepsilon_c$ and $\varepsilon_c^*$ are the maximum concrete strain, and $c$ and $c^*$ are the neutral axis depth before and after debonding, respectively. Strain at rebars before and after debonding can be expressed using strain compatibility as:

![Fig. 4. Energy dissipation mechanisms in FRP strengthened beams.](image)

![Fig. 5. Energy dissipation during debonding failure.](image)
Thus, the change in rebar strain upon debonding is given by:

\[ \Delta e_s = \varphi (d - c') \]

Using Eq. (10), the plastic energy dissipation at the rebars during debonding failure can be determined as:

\[ W_p = \int \sigma \cdot d\varepsilon \cdot d\Omega = \sigma_c \Delta \varepsilon_c A_{sl} = f_y \varepsilon_c \left( 1 - \frac{c}{c'} \right) A_l \]

where \( A_s \) and \( f_y \) are the total cross-sectional area and yield strength of the steel reinforcement, and \( l_c \) is the length of the constant moment region.

5.3. Fracture energy dissipation due to FRP debonding

The energy dissipated at the FRP–concrete interface region during debonding goes to creating new surfaces along the bond area. Depending on the fracture properties of the materials that form the strengthened system, debonding fracture may take place within or at the interfaces of the materials, taking the path that requires the least amount of energy. The interface fracture energy \( G_f \) in Eq. (7) can be expressed as:

\[ G_f = C(\theta) \]

where the toughness of the interface \( C(\theta) \) can be regarded as an effective surface energy that depends on the mode of loading given by the phase angle \( \theta \):

\[ \theta = \tan^{-1}\left( \frac{\sigma_{II}}{\sigma_{I}} \right) \]

which is a measure of Mode II to Mode I loading acting on the interface crack [25]. The case in which \( \theta = 0^\circ \) corresponds to pure Mode I fracture and \( \theta = 90^\circ \) corresponds to pure Mode II fracture. In the case of brittle, isotropic and homogenous materials, fracture propagation follows a trajectory for which \( \sigma_{II} = 0 \), i.e. cracks for which the Mode II stress intensity is nonzero deflect out of their plane until the crack propagation is in pure Mode I. Interface cracks, however, may deviate from this behavior due to the inhomogeneous fracture energy of the bimaterial system. If the interface fracture energy is relatively small, the crack can propagate along the interface even when \( \theta = 0^\circ \) [25,26]. Depending on the fracture properties of the materials and interfaces, kinking of interface cracks into materials take place according to the following expression:

\[ \frac{G}{G_{\text{max}}} < \frac{\Gamma(\theta)}{\Gamma_c} \]

where \( \Gamma(\theta) \) and \( \Gamma_c \) are the interface fracture energy and Mode I fracture toughness of the substrate material, \( G \) is the energy release rate for continued interface cracking, and \( G_{\text{max}} \) is the maximum energy release rate at the kinked crack tip.

Experimental observations during laboratory tests have shown that debonding at the FRP–concrete interface generally takes place within concrete, although limited cases of kinking into the adhesive and the FRP composite were encountered. For a system of bonded dissimilar isotropic and elastic materials, crack propagation within the substrate parallel to the interface means that the Mode II loading conditions vanish at a small depth below the surface of the substrate [27,28]. In the case of FRP–concrete interface that is predominantly under shear loading, a possible mechanism for debonding propagation in the concrete substrate parallel to the FRP–concrete interface is the initial formation of inclined tensile microcracks in concrete under shear and their subsequent coalescence through various energy dissipating micromechanisms such as crushing of the concrete struts between the microcracks, and asperity contact and plasticity[26,29,30,16]. Such mechanisms result in an apparent shear mode (Mode II) fracture energy that is not a material property but depends on other material
properties as well as the geometry and loading. This shear mode fracture energy typically has a magnitude that is considerably higher than the opening mode (Mode I) fracture energy [26,30].

In bonded layers of dissimilar materials, mode mixity is a natural result of elastic mismatch even for symmetric loading or geometry [25]. Hence, all interface debonding processes are essentially mixed-mode fracture processes. There is a wealth of literature on mixed-mode cracking in layered dissimilar materials mainly developed for peeling of thin films and delamination of composites [25,26,31,32]. In the case of FRP–concrete interfaces, fracture under shear loading (such as those in simple bond shear tests or at mid-span regions of FRP-bonded beams) is generally referred to as shear mode (Mode II) fracture [33,13,16,17] whereas fracture due to combined shear and tensile loading, such as that takes place under a flexure-shear crack (also called intermediate crack debonding) [9,10,18–20], or edge debonding [21] is described as mixed-mode fracture.

Extensive mixed-mode cracking around flexure-shear cracks can be detrimental to the integrity of FRP strengthened beams [10,19,20]. Experimental observations show that this is especially the case if extensive transverse shear cracking takes place in the strengthened beam, which result in intermediate crack (IC) debonding or even critical intermediate crack (CIC) debonding failure [6]. Vertical component of the flexure-shear crack mouth displacement, which results in debonding through Mode I fracture is of critical importance considering that the Mode I fracture energy is an order of magnitude less than the Mode II fracture energy. However, if the strengthened beam is sufficiently strong in shear, as is the case in this research, not only the flexure-shear crack mouth displacements will be limited, but also the mixed-mode nature of the debonding fracture will quickly merge to near Mode II conditions upon debonding propagation. The study by Wang [20] verifies this argument as his numerical case study showed that the phase angle \( \theta \) around a flexure-shear crack neared Mode II conditions within millimeters of debonding propagation while the typical length of debonding that causes failure is an order of magnitude longer. Additionally, Neubauer and Rostasy [34] investigated the effect of mixed-mode fracture on the carbon FRP–concrete bond strength in beams using a truss model with shear crack friction and concluded that the reduction in bond strength due to mixed-mode fracture around flexure-shear cracks, including the combined effects of multiple flexure-shear cracks, is within 10% in most cases. Hence, the effect of mixed-mode debonding on the performance of FRP strengthened beams with sufficient capacity in shear can be neglected without significant error in the analysis.

Based on the above discussion, it is assumed in this research that all debonding at the FRP–concrete interface takes place sufficiently close to Mode II conditions, i.e.

\[ \theta = \tan^{-1}(K_{II}/K_I) \approx 90^\circ \]  

(15)

Furthermore, examination of the debonding surfaces during laboratory tests have revealed that the debonding at FRP–concrete interface generally takes place within the concrete substrate, while debonding of the bond anchorage takes place at the FRP composite interfaces. Thus, considering the assumption in Eq. (15), the associated fracture energies can be taken as:

\[
G_f = \begin{cases} 
G_{II}(\theta) \approx G_{II}(\text{FRP–concrete interface}) \\
\Gamma_{II}(\theta) \approx \Gamma_{II}(\text{FRP–FRP interface} - \text{transverse anchorage}) 
\end{cases}
\]  

(16)

where \( G_{II}(\theta) \) and \( \Gamma_{II} \) are the mixed mode and Mode II fracture energies of concrete, and \( G_{II}(\theta) \) and \( \Gamma_{II} \) are the mixed mode and Mode II fracture energy of the FRP–FRP bond anchorage interface. Now, the debonding energy dissipation term in Eq. (7) can be rewritten as:

\[
\int G_f dA_f \approx \int G_{II} dA_{fb} + \int \Gamma_{II} dA_{fa}
\]  

(17)

where \( A_{fb} = L b_f \) is the bond area at the FRP–concrete interface and \( A_{fa} = L b_a \) is the bond area between the FRP reinforcement and the bond anchorage reinforcement in the transverse direction.

5.4. Change in potential energy during debonding failure

The total change in the potential energy of the system after debonding failure is the difference between the recoverable energy stored in the beam before and after debonding. Eq. (5) gives the total dissipation as the negative change in the potential energy of the system. This change in potential energy can be calculated by means of the load–deflection curves at load points as shown in Fig. 5 based on the idealizations that the load–deflection relationship is bilinear, and that the unloading stiñnesses are equal to the pre-yield loading stiffnesses. Considering that the strain energy density is equal to the complementary strain energy density \( \psi = \psi' \) by the linearity assumption, the change in potential energy is equal to the change in global free energy or strain energy, \( W \) of the system, shown by the shaded areas in Fig. 5. Based on the homogeneous and linearly elastic materials assumption, the global free energy in beam elements is given by:

\[
W = \int \left\{ \frac{M^2}{2EI} + \frac{V^2}{2EA} \right\} dx
\]  

(18)

where \( L \) is the beam length, \( M \) is moment, \( V \) is shear, \( E \) and \( G \) are the elastic and shear moduli, and \( A \) and \( I \) are the area and moment of inertia of the beam cross-section, respectively. Neglecting the shear component and assuming that the FRP
The load values in Fig. 5 for before and after debonding can only be associated through the displacement; hence, the load–deflection (or moment–curvature) curve for the loading points must be constructed. This can be performed either through an iterative approach to construct an accurate nonlinear curve, or a bilinear curve defined by the calculated yield point and the point of ultimate failure. The former approach was used in this research [22], but the latter approach is more suitable for practical design since it significantly simplifies the problem for debonding after reinforcement yielding as shown in Fig. 5. Since the load capacity of an unstrengthened beam stays constant after reinforcement yielding, so does the strain energy of the strengthened beam after debonding, i.e. $W_2 = W_y = \text{const.}$ where $W_y$ is the strain energy of the system at steel yielding. From elasticity, the deflection at loading points is given by the following expression:

$$\delta_l = \frac{P}{24} \left[ 3L^2 - 4L^2 \right]$$

(20)

where the curvature, $\varphi$ is given by:

$$\varphi = \frac{c}{E_l} - \frac{P_l}{2EI_c}$$

(21)

Once the load–deflection curves are constructed, for the deflection at which debonding takes place, $\delta_{ld}$, the total dissipation in the system is given by:

$$\Delta \mathcal{W} = -\Delta \mathcal{H} = \frac{P_{ld}^2}{2K_2} - \frac{P_{yd}^2}{2K_1}$$

(22)

where $P_{ld} = P(\delta_l = \delta_{ld})$ and $P_{yd} = P(\delta_l = \delta_{yd})$ are the load values before and after debonding that takes place at deflection $\delta_{ld}$ under the application points. From Fig. 5 and Eqs. (19) and (22), the stiffness values for the strengthened and unstrengthened beams, $K_2$ and $K_1$ respectively, are given by:

$$K_2 = 2EI_2 \left[ \frac{L^2}{2} - \frac{L^2}{3} \right], \quad K_1 = 2EI_1 \left[ \frac{L^2}{2} - \frac{L^2}{3} \right]$$

(23)

where $I_1$ and $I_2$ are the moments of inertia of the transformed beam sections in cracked condition. With the total potential energy difference at hand, use of Eq. (7) now allows development of a debonding failure criterion.

### 5.5. Debonding failure criterion

Using Eqs. (7), (11), (17), and (22) a global debonding criterion can be developed based on the assumption that debonding takes place along the entire bond surface along the FRP reinforcement with concrete and if present with the transverse anchorage reinforcement:

$$\Delta \mathcal{W} = \frac{P_{ld}^2}{2K_2} - \frac{P_{yd}^2}{2K_1} = (G_{fr} I_2 b_f + \Gamma_{fr} I_2 a_2) + W_{pl}^f \geq 0$$

(24)

Eq. (24) indicates that for increasing beam curvature/deflection under loading, the portion of the energy stored in the strengthened beam in excess of that stored in the unstrengthened beam reaches a critical value that causes debonding failure and its dissipation through reinforcement yielding and debonding fracture. Using Eq. (24), the debonding failure load can be determined through iteration, trial and error, or through explicit solutions based on bilinear assumption for the beam load–deflection curves. If debonding takes place after reinforcement yielding, a simplification in Eq. (24) can be made by assuming $P_{yd} \approx P_y$ as illustrated in Fig. 5, in which case the expression becomes:

$$\Delta \mathcal{W} = \frac{P_{ld}^2}{2K_2} - \frac{P_{yd}^2}{2K_1} = (G_{fr} I_2 b_f + \Gamma_{fr} I_2 a_2) + W_{pl}^f \geq 0 (P_{yd} > P_y)$$

(25)

By Eq. (25) the strain energy of the unstrengthened beam becomes a constant after yielding, which greatly simplifies the analysis and design problem by enabling an explicit solution. A critical issue in use of Eq. (25) is the estimation of the fracture
energies \( GF_{II} \) and \( \Gamma_{II} \) which are not well known. A possible approximation for \( GF_{II} \) is the simple relation developed by Neubauer and Rostasy [35,36] based on the study by Holzenkämper [37] who developed a bond strength model for steel plates bonded to concrete using nonlinear fracture mechanics [38], in which the shear mode fracture energy is given by the following expression:

\[
GF = GF_{II} = \frac{1}{\pi} \int_0^\theta \tau(s) ds \approx c_f k_b f_{ctm}
\]  \hfill (26)

where \( \tau \) is the shear stress, \( s \) is the shear slip, \( f_{ctm} \) is the pull-off tensile strength of concrete measured according to DIN-1048 [39], \( k_b \) is a geometric factor that considers the influence of the plate width, \( b_p \), relative to the width of the concrete member, \( b_c \), according to the following expression:

\[
k_b = \sqrt{1.125 \frac{2 - b_p/b_c}{1 + b_p/400}} \text{ (SI)}
\]  \hfill (27)

and \( c_f \) is an experimentally determined constant that contains all secondary effects. Neubauer and Rostasy [35,36] determined this constant for carbon fiber reinforced polymer (CFRP) bonded concrete as \( c_f = 0.202 \) from 70 double shear bond tests and concluded that shear fracture in CFRP–concrete bond can be modeled using a triangular shear-slip model [38]. It is important to note that the model by Neubauer and Rostasy [35,36] is valid for normal strength concrete only since CFRP delamination failures were observed for high strength concrete with a compressive strength of 55 MPa. Lu et al. [12] tested the performance of this model along with 11 other bond strength models using 253 test results compiled from the literature and found that the model by Neubauer and Rostasy [35,36] is one of the better performing models with a high correlation coefficient (0.885) and a low coefficient of variation (0.168). Noting that the bond strength is directly proportional to the square root of the interface fracture energy \( \sqrt{GF} \) regardless of the shape of the bond–slip curve, performance of the model in estimating the bond strength also shows its performance in estimating the interface fracture energy [11]. Simplicity of the model by Neubauer and Rostasy [35,36] combined with its demonstrated accuracy justifies its use as a basis for estimating the shear fracture energy of the FRP–concrete bond in this research.

The expression proposed by Neubauer and Rostasy [35,36] gives a fracture energy that is at least an order of magnitude higher than the Mode I fracture toughness of concrete that can be calculated using the CEB–FIP Model Code expression [40,41]:

\[
GF = GF_{II} = z_f \left( \frac{f_{ctm}}{10} \right)^{0.7} \text{ (SI)}
\]  \hfill (28)

\[
z_f = (0.0469d_a^2 - 0.5d_a + 26) \times 10^{-3}
\]

where \( GF \) is in N/mm and \( z_f \), a function of the maximum aggregate size \( d_a \) (in mm), was calculated as 0.026 for the concrete used in this research.

Additional discussions and experimental studies [30,42–47] suggest that the Mode II fracture toughness of concrete may range from 10 to 25 times its Mode I fracture toughness, i.e. \( GF_{II} \approx (10–25)GF_{I} \) [22,30,16]. Although not a material property, Mode II fracture of concrete involves several effects such as friction, asperity contact and plasticity which considerably increase its fracture resistance [26]. The expression for the fracture energy given by Eq. (26) was developed from double shear bond tests where the bonded FRP reinforcement was subjected to uniaxial tension [36]. It is conceptually clear and experimentally evident [48] that the Mode II fracture energy of the FRP–concrete interface under flexural loading is likely to be higher for interface crack (IC) debonding than that of the bond test configuration due to the curvature effect that exerts additional compression on the interface. Considering the results obtained from laboratory tests and the reported range of Mode II fracture energy \( GF_{II} \) in relation to the Mode I fracture energy [22,30,16], the experimental constant in Eq. (26) was modified as \( c_f = 0.23 \) mm in this research, which produced a Mode II fracture energy for the FRP–concrete interface that is approximately 20 times the Mode I fracture energy of concrete calculated using the CEB–FIP Model Code [40] expression. Also, in order to make the expression simpler and more generally applicable, the pull–off tensile strength in Eq. (26) was approximated as:

\[
f_{ctm} \approx f_{ct} = 0.53 \sqrt{f_c} \text{ (SI)}
\]  \hfill (29)

where \( f_{ct} \) is the split cylinder tensile strength. With these modifications, Eq. (26) becomes:

\[
GF_{II} = 0.122k_b^\frac{1}{2} \sqrt{f_c} \text{ (SI)}
\]  \hfill (30)

Knowledge of interface fracture energy, \( \Gamma_{II} \), in Mode II between the longitudinal FRP reinforcement and the transverse anchorage reinforcement is very limited at this time. However, experimental observations indicate that the interface or interlaminar fracture energy of FRP, \( \Gamma_{II} \), is higher than \( GF_{II} \) but is in the same order since limited kinking of crack propagation into the composite was observed during the laboratory tests. Therefore, based on existing experimental observations and
data, $\Gamma_{\text{II}}$ was assumed to be twice the fracture energy of that between FRP–concrete interface, $G_{\text{FII}}$, for the purposes of this research [22].

6. Model implementation to experimental data

Implementation of the developed model was performed for the beam tests presented in Section 4 to compare the model prediction with the experimental results. Fig. 7a shows the experimental results obtained from representative beam tests, excluding the cover debonding failure case. This figure also shows the nonlinear load–deflection curves for the control and strengthened beams. These curves, shown with the dashed lines in the figure, were iteratively constructed using Hognestad’s nonlinear concrete model [49], and were used as a basis for determining the debonding failure loads according to Eq. (25) to increase the accuracy of the model prediction. The reader is referred to Gunes [22] for the details of constructing these curves. For practical design, a simpler approach can be taken by using bilinear load–deflection curves as shown in Fig. 5.

A comparison of the model predictions with the experimental results is shown in Fig. 7b. As can be seen from the figure, the developed fracture model yields a satisfactory prediction of the FRP debonding failure loads and performs better than the ACI 440 [4,5] provision given by Eqs. (1) and (2) which yield a constant debonding load for all three cases shown in Fig. 7b.

In order to perform further validation, the developed model was tested on a number of FRP debonding failure data sets produced by various independent research studies [29,50,51]. Fig. 8 compares the experimental results of these studies including this research with the model predictions. The overall success of the model in predicting FRP debonding failure loads for various sizes of beams and strengthening configurations shows the potential of fracture mechanics modeling approach for design against debonding failures.

7. Design of beams against FRP debonding failures

The developed FRP debonding failure model can be easily integrated into design of FRP strengthened beams to achieve safety against FRP debonding failures. The design approach is described in the following steps starting from the design of FRP strengthened beams for flexure and shear effects:

1. Perform the strengthened beam design using conventional ultimate strength analysis for design flexural loads [4,5]. The outcome of this step is the cross-sectional area of the bonded FRP reinforcement $A_f = b_f t_f$ needed for strengthening.
2. Perform design for shear strengthening of the beam using side bonded or wrapped FRP composites if the design shear load exceeds the beam shear capacity.
3. In order to ensure ductile behavior and failure of the beam, equate the debonding failure design load to the flexural capacity of the beam calculated in step 1, and calculate the total bond fracture resistance needed to resist the debonding failure load using Eq. (25):

$$\delta_d = (G_{\text{FII}} l_f b_f + \alpha_{\text{FII}} k_f b_f) = \frac{p_f^2}{2K_2} - \frac{p_y^2}{2K_1} - W_p^1 \geq 0$$  (31)
4. Once the required total debonding energy is determined, one has to make sure that the total fracture energy of the FRP–concrete bond and possible anchorage is sufficient to meet this energy demand. Since the required FRP reinforcement area $A_f$ is known from step 1, the first try would be to check if the bond area of the FRP (plate) reinforcement designed in step 1 is sufficient (or to arrange the width and thickness of the FRP (sheet) reinforcement to provide sufficient bond area) without any anchorage:

$$b_f = \frac{\sigma_d \cdot t_f}{C_{\text{fil}}} \leq b, \quad \frac{t_f}{b_f} = \frac{A_f}{b_f}$$

Special attention must be paid not to design the FRP reinforcement too thin to avoid FRP rupture due to stress concentrations at crack locations. If the bond area without any anchorage is not enough to meet the energy demand, then anchorage requirement needs to be calculated to provide additional fracture energy:

$$l_a b_a = \frac{\sigma_d - C_{\text{fil}} b_f}{T_{\text{fil}}}$$

so that the integrity of the bond is ensured under the design load. The calculated anchorage reinforcement should be placed close to the FRP reinforcement end regions.

5. It should be noted that the developed FRP debonding model does not address cover debonding failures since this failure type appears to be mainly influenced by the shear behavior and capacity of the beam. Until an accurate model is developed to address cover debonding failures, specification of a minimum bond anchorage in the FRP reinforcement end regions with a length approximately equal to the beam height is recommended as a safety assurance.

8. Summary and conclusions

Through experimental research and analytical modeling studies, a global fracture model was developed to predict FRP debonding failures in strengthened beams. The model includes the beam geometry, strengthening configuration, and additional bond anchorage effects considering energy balance in the system and energy dissipation through steel reinforcement yielding and FRP debonding. Implementation of the model to several sets of independently reported experimental data shows that the model can satisfactorily predict the FRP debonding failure loads for various sizes of beams strengthened in various configurations, with or without bond anchorage. The model can be further improved through better characterization of its components such as bulk energy dissipation in the concrete beam during debonding, and mixed-mode fracture energy values at the FRP–concrete and FRP–FRP interfaces.

The developed model can easily be integrated into the design of FRP strengthened beams to ensure that the debonding failure load is higher than the flexural capacity of the beam. A possible design approach is outlined to determine the bond area or additional bond anchorage area required to prevent brittle debonding failures. With further improvements and validation, the model may be used as a code provision for preventing FRP debonding failures.

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