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Probabilistic updating of building models using incomplete modal data

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ABSTRACT

This paper investigates a new probabilistic strategy for Bayesian model updating using incomplete modal data. Direct mode matching between the measured and the predicted modal quantities is not required in the updating process, which is realized through model reduction. A Markov chain Monte Carlo technique with adaptive random-walk steps is proposed to draw the samples for model parameter uncertainty quantification. The iterated improved reduced system technique is employed to update the prediction error as well as to calculate the likelihood function in the sampling process. Since modal quantities are used in the model updating, modal identification is first carried out to extract the natural frequencies and mode shapes through the acceleration measurements of the structural system. The proposed algorithm is finally validated by both numerical and experimental examples: a 10-storey building with synthetic data and a 8-storey building with shaking table test data. Results illustrate that the proposed algorithm is effective and robust for parameter uncertainty quantification in probabilistic model updating of buildings.

1. Introduction

Identifying and updating the system parameters of a structural model, conditional on observed data, is a key component in structural health monitoring (SHM), since it is related to assessing the health condition, evaluating the integrity, and estimating the capacity to carry loads and risk of a structure. This topic has gained much attention recently (refer to, e.g., [1–14], among others).

In general, model updating seeks to determine a set of the most plausible parameters that best describe the structure given the measured system responses and, possibly, the external excitation. In the process of model updating, the parameters can be expressed as either specific values (deterministic) or probability distributions (probabilistic) [12]. Existing deterministic model updating strategies, such as the least squares-based methods [15,16], the heuristic algorithms [17–22], the filtering techniques [23–25], and sensitivity-based updating approaches [26–28], have been well studied and applied in SHM. Nevertheless, those strategies only find a single plausible model and have limitations in resolving issues related to model uncertainties.

Bayesian model updating techniques make possible to identify a set of plausible models with probabilistic distributions and to characterize the modeling uncertainties of a structural system. Recently, a number of Bayesian model updating

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approaches have been proposed for a reliable assessment of structural condition and a robust prediction of future structural responses. For example, Beck and Katafygiotis [29] first presented a comprehensive statistical framework for Bayesian model updating, which was then extended by their colleagues and applied to update various types of structural models using sampling techniques such as the Markov chain Monte Carlo (MCMC) simulations [30–32]. Mares et al. [33] studied stochastic model updating using a Monte-Carlo inverse procedure. Nichols et al. [34] applied the MCMC to sample the posterior parameter distributions of nonlinear structural systems and extended this approach to damage detection of composites. Beck [35] presented a rigorous framework to quantify modeling uncertainty and perform system identification using probability logic with Bayesian updating. Boulkaibet et al. [36] proposed a shadow hybrid MCMC approach for uncertainty quantification in finite element model updating. Green [37] presented a Data Annealing-based MCMC algorithm for Bayesian identification of a nonlinear dynamical system. Yan et al. [38] investigated a reverse jump MCMC method for Bayesian updating of flaw parameters.

It is quite popular to use the identified modal characteristics to update a model within the framework of Bayesian inference [39,40]. However, direct mode matching is typically required for the majority of existing Bayesian updating approaches using modal data. In practice, when incomplete measurements of mode shapes are only available, direct mode matching is not an easy task. In addition, when some of the measured modes are missing or the mode orders are unclear, direct mode matching becomes more difficult. Mode switching due to structural damage even makes the case worse [41]. Recently, Bayesian methods without requiring direct mode matching have been proposed for model updating [41–45]. This is realized through introducing the concept of system mode shapes. In the updating process, the system mode shapes become extra parameters to be updated as well. Since this method introduces additional unknown parameters into the updating process, the computational cost will increase, especially when Monte Carlo techniques are used to draw samples for the updating parameters. To alleviate this issue and to avoid direct mode matching, we propose a new strategy for Bayesian model updating using incomplete modal data. This is accomplished by employing a model reduction technique considering the available sensor locations. A MCMC simulation with adaptive random-walk steps is used to sample the posterior distributions of the model parameters.

The organization of this paper is given as follows. Section 2 presents the probabilistic model updating framework based on Bayesian inference using incomplete modal data, in which direct mode matching is not required. Modal identification as well as model reduction is also introduced. Section 3 describes the sampling technique using MCMC with adaptive random-walk steps. Sections 4 and 5 discuss numerical and experimental examples to validate the proposed model updating technique. Finally, Section 6 provides the concluding remarks of this work.

2. Probabilistic model updating without direct mode matching

The essential promise of structural model updating conditional on modal data is to modify a set of model parameters (e.g., denoted with $\theta \in \mathbb{R}^{N_\theta \times 1}$, where N_θ is the number of parameters), that minimize the discrepancy between the predicted and the measured modal quantities. The objective is to obtain an updated model which has the most probable consistency with the real structural system. Most of existing model updating strategies minimize the objective function defined as [41]

$$J(\boldsymbol{\theta}) = \sum_{i}^{N_s} \sum_{j}^{N_m} \left\{ \alpha_j \left[\omega_{i,j}^2(\boldsymbol{\theta}) - \tilde{\omega}_{i,j}^2 \right]^2 + \beta_j \left\| \boldsymbol{\phi}_{i,j}(\boldsymbol{\theta}) - \tilde{\boldsymbol{\phi}}_{i,j} \right\|_2^2 \right\} \tag{1}$$

where $\tilde{\omega}_{i,j}$ and $\tilde{\boldsymbol{\phi}}_{i,j}$ are the jth measured frequency and mode shapes of the ith data set, while $\omega_{i,j}$ and $\boldsymbol{\phi}_{i,j}$ are the corresponding predicted frequency and mode shapes from the model; α_j and β_j are the weighting coefficients; N_m is the total number of observed modes; N_s is the number of measured data sets used for model updating; and $\|\cdot\|_2$ denotes the L_2 norm of a vector. Noteworthy, there exist two major issues associated with the model updating techniques based on Eq. (1): (i) it can be seen from Eq. (1) that direct mode matching is required so as to compute $J(\boldsymbol{\theta})$; and (ii) the weighting coefficients α_j and β_j are typically empirically defined by the user, which may significantly affect the model updating result a lot. To alleviate those two issues, we propose a probabilistic strategy based on Bayesian inference for model updating using incomplete modal data, in which direct mode matching is not required.

2.1. Bayesian inference for model updating without direct mode matching

We herein consider a linear structure model with n degrees-of-freedoms (DOFs). The mass matrix $\mathbf{M} \in \mathbb{R}^{n \times n}$ is assumed to be known and the stiffness matrix $\mathbf{K} \in \mathbb{R}^{n \times n}$ is parameterized by $\boldsymbol{\theta}$, namely, $\mathbf{K} = \mathbf{K}(\boldsymbol{\theta})$. In Bayesian model updating, the posterior probability density function (PDF) of the model parameters ($\boldsymbol{\theta}$), given a specified model class, can be obtained based on the Bayes' theorem [8,30]:

$$p(\boldsymbol{\theta}|\mathcal{D}) = c^{-1} p(\mathcal{D}|\boldsymbol{\theta}) p(\boldsymbol{\theta}) \tag{2}$$

with c being the normalizing factor (the evidence given by data $\mathcal{D}$) which can be written as:

$$c = p(\mathcal{D}) = \int_{\Theta} p(\mathcal{D}|\boldsymbol{\theta}) p(\boldsymbol{\theta})\, \mathrm{d}\boldsymbol{\theta} \tag{3}$$

where Θ denotes the domain of integration; $p(\boldsymbol{\theta}|\mathcal{D})$ denotes the posterior PDF of $\boldsymbol{\theta}$ conditional on the measured data $\mathcal{D}$ consisting of the extracted modal data from measured system responses, namely,

$$\mathcal{D} = \mathcal{D}_1 \cup \mathcal{D}_2 \cup \ldots \mathcal{D}_i \cup \ldots \mathcal{D}_{N_s}$$
$$\mathcal{D}_i = \left\{\tilde{\omega}_{i,1}, \tilde{\omega}_{i,2}, \ldots, \tilde{\omega}_{i,N_m}, \tilde{\boldsymbol{\phi}}_{i,1}, \tilde{\boldsymbol{\phi}}_{i,2}, \ldots, \tilde{\boldsymbol{\phi}}_{i,N_m}\right\} \quad (4)$$

Here, $p(\boldsymbol{\theta})$ is the prior PDF of $\boldsymbol{\theta}$; and $p(\mathcal{D}|\boldsymbol{\theta})$ is the likelihood function which gives a measure of the agreement between the measured and the predicted data. Therefore, the problem of Bayesian model updating can be stated as follows: given the specified model class, the measured data $\mathcal{D}$, and the parameter prior PDF $p(\boldsymbol{\theta})$, one's objective is to determine the posterior PDF $p(\boldsymbol{\theta}|\mathcal{D})$.

2.1.1. Likelihood function

The likelihood function can be formulated by considering the prediction error $\boldsymbol{\epsilon}_{i,j} \in \mathbb{R}^{N_o \times 1}$ representing the discrepancy between the measured and the predicted modal data, where the subscripts i and j denote the $i$the data set and the jth mode, respectively. Note that N_o denotes the number of observed DOFs ($N_o \le$ the total number of DOFs), $i = 1, 2, \ldots, N_s$ and $j = 1, 2, \ldots, N_m$. In this work, $\boldsymbol{\epsilon}_{i,j}$ is expressed as the eigenvalue equation error only taking into account the sensing DOFs (e.g., the sensor locations), given by [46]

$$\boldsymbol{\epsilon}_{i,j} = \left[\mathbf{K}_R(\boldsymbol{\theta}) - \tilde{\omega}_{i,j}^2 \mathbf{M}_R\right] \tilde{\boldsymbol{\phi}}_{i,j} \quad (5)$$

where $\mathbf{M}_R$ and $\mathbf{K}_R$ are the reduced mass and stiffness matrices defined according to the sensing DOFs, which can be obtained using an iterated improved reduced system (IIRS) technique presented in Section 2.2. To describe the uncertainty of the prediction error, the characterization of $\boldsymbol{\epsilon}$ can be realized by a probability model that produces the maximum uncertainty based on the Principle of Maximum (Information) Entropy [47]. To wit, $\boldsymbol{\epsilon}_{i,j}$ can be modeled as a discrete zero-mean Gaussian process [44], namely, $\boldsymbol{\epsilon}_{i,j} \sim \mathbb{N}(\mathbf{0}, \Sigma_\epsilon) = \mathbb{N}(\mathbf{0}, \sigma_j^2 \mathbf{I})$ where $\mathbf{I} \in \mathbb{R}^{N_o \times N_o}$ is an identity matrix; σ_j^2 denotes the variance of the prediction error of the jth mode, which is an additional unknown variable and needs to be identified in the updating process. To this end, the likelihood function can be expressed below, following the multivariate Gaussian distribution:

$$p(\mathcal{D}|\boldsymbol{\theta}) = \frac{1}{\left[(2\pi)^{N_m} \prod_{j=1}^{N_m} \sigma_j^2\right]^{N_o N_s/2}} \exp\left[-\sum_{i=1}^{N_s} \sum_{j=1}^{N_m} \frac{1}{2\sigma_j^2} \left\| \left[\mathbf{K}_R(\boldsymbol{\theta}) - \tilde{\omega}_{i,j}^2 \mathbf{M}_R\right] \tilde{\boldsymbol{\phi}}_{i,j} \right\|_2^2\right] \quad (6)$$

To evaluate the model updating process, we define as *goodness-of-fit* function as the sum of least squares in the likelihood function, given by

$$Q(\boldsymbol{\theta}, \mathcal{D}) = \sum_{i=1}^{N_s} \sum_{j=1}^{N_m} \left\| \left[\mathbf{K}_R(\boldsymbol{\theta}) - \tilde{\omega}_{i,j}^2 \mathbf{M}_R\right] \tilde{\boldsymbol{\phi}}_{i,j} \right\|_2^2 \quad (7)$$

It is noteworthy that since σ_j^2 is an unknown parameter in Eq. (6), we slightly revise the likelihood function by adding σ_j^2, namely, $p(\mathcal{D}|\boldsymbol{\theta}, \boldsymbol{\sigma}^2) = p(\mathcal{D}|\boldsymbol{\theta})$, where $\boldsymbol{\sigma}^2 = \{\sigma_1^2, \ldots, \sigma_{N_m}^2\}^T$. Therefore, we obtain the augmented posterior PDF through hierarchical Bayesian inference [8], viz.,

$$p(\boldsymbol{\theta}, \boldsymbol{\sigma}^2|\mathcal{D}) \propto p(\mathcal{D}|\boldsymbol{\theta}, \boldsymbol{\sigma}^2) p(\boldsymbol{\theta}) p(\boldsymbol{\sigma}^2) \quad (8)$$

where $p(\boldsymbol{\theta}, \boldsymbol{\sigma}^2|\mathcal{D})$ is the augmented posterior PDF and $p(\boldsymbol{\sigma}^2)$ is the prior PDF of $\boldsymbol{\sigma}^2$.

2.1.2. Prior distributions

In Bayesian inference, we need to define the prior PDF of the structural parameters characterizing the model as shown in Eq. (8). Let us assume that the prior system parameter vector $\boldsymbol{\theta}$ follows a multivariate Gaussian distribution with mean $\overline{\boldsymbol{\theta}} \in \mathbb{R}^{N_\theta \times 1}$ and covariance matrix $\Sigma_\theta \in \mathbb{R}^{N_\theta \times N_\theta}$ [8], expressed as

$$p(\boldsymbol{\theta}) = \frac{1}{(2\pi)^{N_\theta/2} |\Sigma_\theta|^{1/2}} \exp\left[-\frac{1}{2}\left(\boldsymbol{\theta} - \overline{\boldsymbol{\theta}}\right) \Sigma_\theta^{-1} (\boldsymbol{\theta} - \overline{\boldsymbol{\theta}})^T\right] \quad (9)$$

where $\overline{\boldsymbol{\theta}}$ and Σ_θ needs to be determined by the user before implementing the Bayesian model updating. It is noteworthy that if the prior parameters are assumed to be independent from each other, $p(\boldsymbol{\theta}) = \prod_{k=1}^{N_\theta} p(\theta_k)$ holds. In this case, Σ_θ is a diagonal matrix with its diagonal elements being the parameters' standard deviations [12].

In addition, since $\boldsymbol{\sigma}^2$ is always positive, its prior distribution can be modeled by an inverse Gamma distribution, namely, $p(\sigma_j^2) \sim IG(\alpha, \beta)$, where α ($\alpha > 0$) and β ($\beta > 0$) are the constant "hyperparameters". We assume that, in a generic case, σ_j^2 is statistically independent with each other. Thus $p(\boldsymbol{\sigma}^2)$ can be written as

$$p(\boldsymbol{\sigma}^2) = \prod_{j=1}^{N_m} p\left(\sigma_j^2\right) = \prod_{j=1}^{N_m} \frac{\beta^\alpha}{\Gamma(\alpha)} \sigma_j^{-2(\alpha+1)} \mathrm{e}^{-\beta/\sigma_j^2} \quad (10)$$

where $\Gamma(\cdot)$ is the Gamma function. Note that, when the a priori information of the hyperparameters is missing, the hyperparameters can be chosen as $\alpha = 1$ and $\beta = 1 \times 10^{-16}$, leading to a non-informative process.

2.1.3. Final form of the posterior distribution

The substitution of Eqs. (6), (9) and (10) into Eq. (8) leads to the final form of the posterior PDF of the unknown parameters:

$$p(\boldsymbol{\theta},\boldsymbol{\sigma}^2|\mathcal{D}) \propto \frac{1}{\left[\prod_{j=1}^{N_m}\sigma_j^2\right]^{N_oN_s/2+\alpha+1}}\exp\left\{-\sum_j^{N_m}\frac{1}{2\sigma_j^2}\left[\sum_i^{N_s}\left\|\left[\mathbf{K}_R(\boldsymbol{\theta})-\tilde{\omega}_{i,j}^2\mathbf{M}_R\right]\tilde{\boldsymbol{\phi}}_{i,j}\right\|_2^2+2\beta\right]-\frac{1}{2}\left(\boldsymbol{\theta}-\overline{\boldsymbol{\theta}}\right)\Sigma_\theta^{-1}(\boldsymbol{\theta}-\overline{\boldsymbol{\theta}})^T\right\} \tag{11}$$

The total number of unknown parameters in Eq. (11) is $N_\theta+N_m$. If one assumes $\sigma^2=\sigma_j^2$, the number of unknown parameters reduces to be $N_\theta+1$. The conditional posterior PDFs of $p(\boldsymbol{\theta},\boldsymbol{\sigma}^2|\mathcal{D})$ in Eq. (11) can be sampled using the MCMC with adaptive random-walk steps described in Section 3. Noteworthy, the analytical solution of the conditional distribution for σ_j^2 is written as

$$p(\sigma_j^2|\boldsymbol{\theta},\mathcal{D}) = IG\left(\alpha+\frac{N_oN_s}{2},\beta+\frac{1}{2}\sum_i^{N_s}\left\|\left[\mathbf{K}_R(\boldsymbol{\theta})-\tilde{\omega}_{i,j}^2\mathbf{M}_R\right]\tilde{\boldsymbol{\phi}}_{i,j}\right\|_2^2\right) \tag{12}$$

It can be seen from Eq. (12) that $p(\sigma_j^2|\boldsymbol{\theta},\mathcal{D})$ can be analytically determined given the estimate of $\boldsymbol{\theta}$.

2.2. Model reduction

Let us write the generalized eigenvalue problem of a n-DOF linear system containing the first m modes, with the partitioned mass and stiffness matrices and mode shapes governed by the master and slave DOFs, as follows [48]

$$\begin{bmatrix}\mathbf{K}_{mm} & \mathbf{K}_{ms}\\ \mathbf{K}_{ms}^T & \mathbf{K}_{ss}\end{bmatrix}\begin{Bmatrix}\boldsymbol{\Phi}_{mm}\\ \boldsymbol{\Phi}_{sm}\end{Bmatrix} = \begin{bmatrix}\mathbf{M}_{mm} & \mathbf{M}_{ms}\\ \mathbf{M}_{ms}^T & \mathbf{M}_{ss}\end{bmatrix}\begin{Bmatrix}\boldsymbol{\Phi}_{mm}\\ \boldsymbol{\Phi}_{sm}\end{Bmatrix}\boldsymbol{\Lambda}_{mm} \tag{13}$$

where $\mathbf{M}$ and $\mathbf{K}\equiv\mathbf{K}(\boldsymbol{\theta})$ are the mass and stiffness matrices, respectively; $\boldsymbol{\Phi}\in\mathbb{R}^{n\times m}$ is the mass-normalized mode shape matrix; $\boldsymbol{\Lambda}\in\mathbb{R}^{m\times m}$ is the diagonal eigenvalue matrix consisting of the eigenvalues λ_i $(i=1,2,\ldots,m)$; m and s denote the number of master and slave DOFs, respectively, satisfying $m+s=n$. Let us denote $\boldsymbol{\Phi}_{sm}=\mathbf{t}\boldsymbol{\Phi}_{mm}$, where $\mathbf{t}\in\mathbb{R}^{s\times m}$ is a transformation matrix, and substitute it into the second set of Eq. (13) to obtain

$$\mathbf{t} = -\mathbf{K}_{ss}^{-1}\mathbf{K}_{ms}^T+\mathbf{K}_{ss}^{-1}\left[\mathbf{M}_{ms}^T+\mathbf{M}_{ss}\mathbf{t}\right]\boldsymbol{\Phi}_{mm}\boldsymbol{\Lambda}_{mm}\boldsymbol{\Phi}_{mm}^{-1} \tag{14}$$

The substitution of $\boldsymbol{\Phi}=[\boldsymbol{\Phi}_{mm}\ \boldsymbol{\Phi}_{sm}]^T=\mathbf{T}\boldsymbol{\Phi}_{mm}$ into Eq. (13) pre-multiplied by $\mathbf{T}^T$ yields

$$\mathbf{M}_R^{-1}\mathbf{K}_R = \boldsymbol{\Phi}_{mm}\boldsymbol{\Lambda}_{mm}\boldsymbol{\Phi}_{mm}^{-1} \tag{15}$$

where $\mathbf{T}=[\mathbf{I}\ \mathbf{t}]^T$ and $\mathbf{I}\in\mathbb{R}^{m\times m}$; $\mathbf{M}_R$ and $\mathbf{K}_R$ are the mass and stiffness matrices of the reduced order model, namely,

$$\mathbf{M}_R=\mathbf{T}^T\mathbf{M}\mathbf{T} \quad \text{and} \quad \mathbf{K}_R=\mathbf{T}^T\mathbf{K}\mathbf{T} \tag{16}$$

The substitution of Eq. (15) into (14) yields

$$\mathbf{t} = -\mathbf{K}_{ss}^{-1}\mathbf{K}_{ms}^T+\mathbf{K}_{ss}^{-1}\left[\mathbf{M}_{ms}^T+\mathbf{M}_{ss}\mathbf{t}\right]\mathbf{M}_R^{-1}\mathbf{K}_R \tag{17}$$

It is noted that Eq. (17) forms an implicit function with $\mathbf{t}$ as the unknown parameter which can be solved through an iterative process. Friswell et al. [49] proposed an IIRS technique to solve for Eqs. (16) and (17) iteratively to obtain the reduced mass and stiffness matrices (e.g., in terms of the model parameters $\boldsymbol{\theta}$), namely, $\mathbf{K}_R(\boldsymbol{\theta})$.

2.3. Modal identification

Since we employ the modal data for structural model updating, the modal quantities have to be first identified from the measured time histories. The choice of modal identification depends on the availability of the measurements, e.g., input–output or output-only.

In this study, we employ the Eigensystem Realization Algorithm (ERA) together with an Observer Kalman Filter Identification (OKID) algorithm, viz., ERA-OKID [50,51], for input–output modal identification. The fundamental principle of ERA-OKID is that system's Markov parameters are first identified using a refined OKID and then used in ERA for the identification of the modal characteristics such as frequencies and mode shapes. Note that a refinement of the identified state space models by OKID can be conducted using the procedure proposed by Luş et al. [50]. The input–output scenario could be earthquake-induced vibration, in which the time history of the ground motion is measured along with the structural responses.

For the case of output-only measurements, we apply a blind source separation (BSS) [52–54] to extract the modal parameters. BSS attempts to find latent components contained in measured data which is a linear mixture. Herein, a second-order blind identification (SOBI) [55] approach is used to identify the modal responses and mode shapes. Frequencies can be obtained through analyzing the modal responses in the frequency domain. The output-only modal identification can be carried out when, for example, the measurements are recorded for structural ambient vibrations.

3. Realization of Bayesian inference by the Monte Carlo technique

In general, since the posterior distributions in Bayesian inference are usually complicated in a normalized form, it is difficult to directly draw independent samples, based on the joint posterior PDF as shown in Eq. (2), using classic Monte Carlo (MC) methods. Nevertheless, the Markov chain makes possible to draw a dependent sequence of samples representing the posterior samples. The corresponding MC simulation process is called MCMC. Typically, MCMC simulation is treated as an alternative choice for effective sampling. It is noted that the MCMC is able to produce a stationary distribution which is as close to the target distribution as possible. Therefore, in this study, we apply MCMC to quantify the conditional probability distributions of the system parameters of a structural model.

One of the most famous MCMC approaches is called Metropolis–Hastings (M–H) algorithm [56,57], which provides a simple implementation procedure for sampling. The basic idea of the M–H algorithm is to draw samples with acceptance and rejection governed by a probability. The general principle of the M–H algorithm can be described as follows. Let us assume that random samples are generated from a target distribution denoted with $\pi(\boldsymbol{\theta})$. Herein, $\pi(\boldsymbol{\theta})$ can be the posterior PDF $p(\boldsymbol{\theta}, \boldsymbol{\sigma}^2|\mathcal{D})$ as illustrated in Eq. (11). The M–H algorithm generates a sequence of samples $\boldsymbol{\theta}^{(p)}$ from the target distribution through a rejection sampling procedure. At a generic pth stage (iteration), a candidate solution $\boldsymbol{\theta}^*$ is generated based on the current value $\boldsymbol{\theta}^{(p-1)}$, which can be sampled from a chosen proposal or a transition distribution function $g\left(\boldsymbol{\theta}^*|\boldsymbol{\theta}^{(p-1)}\right)$. A Bernoulli trial is then performed with a success probability defined as

$$\gamma = \min\left\{\frac{\pi(\boldsymbol{\theta}^*)g\left(\boldsymbol{\theta}^{(p-1)}|\boldsymbol{\theta}^*\right)}{\pi\left(\boldsymbol{\theta}^{(p-1)}\right)g\left(\boldsymbol{\theta}^*|\boldsymbol{\theta}^{(p-1)}\right)}, \quad 1\right\} \tag{18}$$

Note that if the result of the trial is successful (e.g., $r_0 \le \gamma$, where r_0 is a uniform random number sampled from $[0, 1]$), $\boldsymbol{\theta}^{(p)}$ is replaced by $\boldsymbol{\theta}^*$; otherwise (e.g., $r_0 > \gamma$), $\boldsymbol{\theta}^{(p)}$ is kept as $\boldsymbol{\theta}^{(p-1)}$. The corresponding description of the successful/unsuccessful state is called moving/staying. Noteworthy, the rejection sampling process is repeated for a sufficient number of iterations, until the resulting Markov chain becomes stationary, which can be used to represent the target distribution [38]. Nevertheless, there is a non-stationary period of iterations before the chain gets stationarily converged, when one starts the algorithm from an arbitrary state. The corresponding process is called the "burn-in" period. Only the rest of iterations (e.g., called the "retained" period) are considered in the final representation of the posterior distribution.

Remark 1. The transition distribution $g\left(\boldsymbol{\theta}^*|\boldsymbol{\theta}^{(p-1)}\right)$ is typically a symmetric proposal function [34]. In this study, we apply a normal distribution to describe the transition proposal, e.g.,

$$g\left(\boldsymbol{\theta}^*|\boldsymbol{\theta}^{(p-1)}\right) = \prod_{k=1}^{N_\theta} \frac{1}{\sqrt{2\pi}\eta_k^{(p-1)}\theta_k^{(p-1)}} \exp\left[-\frac{\left(\theta_k^* - \theta_k^{(p-1)}\right)^2}{2\left(\eta_k^{(p-1)}\theta_k^{(p-1)}\right)^2}\right] \tag{19}$$

Here, θ_k^* can be sampled from

$$\theta_k^* \sim \mathbb{N}\left(\theta_k^{(p-1)}, \left(\eta_k^{(p-1)}\theta_k^{(p-1)}\right)^2\right) \tag{20}$$

where $\mathbb{N}$ denotes a normal distribution; θ_k^* and $\theta_k^{(p-1)}$ represent the kth ($k = 1, 2, \ldots, N_\theta$) parameter in θ^* and $\theta^{(p-1)}$; $\eta_k^{(p-1)}$ is the random-walk coefficient of variance (c.o.v.) of the normal distribution for θ_k. Here, $\eta_k^{(p-1)}$ changes adaptively along with the iterations. For example, $\eta_k^{(p)} = \kappa_k \eta_k^{(p-1)}$, where κ_k is the adaptivity coefficient which is updated randomly in the updating process. In the burn-in period, if the sampling trial is successful, κ_k is sampled uniformly from the range [1, 1.05], e.g., $\kappa_k \sim U(1, 1.05)$; otherwise (sample is rejected), κ_k is sampled uniformly from [0.95, 1], e.g., $\kappa_k \sim U(0.95, 1)$. The initial value $\eta_k^{(0)}$ is set to be 0.05. The use of an adaptive random-walk step leads to a more efficient sampling process, meanwhile, keeping the tuning capability of the algorithm.

The MCMC algorithm with adaptive random-walk steps is illustrated in Algorithm 1. Note that the MCMC sampler is implemented sequentially: the model parameters $\boldsymbol{\theta}$ are first sampled, and then used for the sampling of $\boldsymbol{\sigma}^2$.

Table 1
Statistical results for the most probable stiffness parameters of the 10-storey building.

Normalized k_i	Identified value	c.o.v. (%)	Normalized k_i	Identified value	c.o.v. (%)
k_1	0.9186	0.56	k_6	1.0397	1.58
k_2	0.9757	1.43	k_7	1.1058	1.04
k_3	0.9838	0.68	k_8	0.9786	0.85
k_4	1.1415	1.56	k_9	0.9649	0.57
k_5	0.9693	1.43	k_{10}	1.1017	0.81

Note that c.o.v. represents the coefficient of variance.

Algorithm 1. MCMC sampler with adaptive random-walk steps for probabilistic model updating.

Input the prescribed chain length N_{mc}, the burn-in period length N_b, the initial random-walk step $L_k^{(0)}$, the initial guess for the model parameters $\boldsymbol{\theta}^{(0)}$, and the mass information of the structure $\mathbf{M}$;
Initialize the chain index $p \leftarrow 0$;
while $j < N_m$ **do**
  Compute $\mathbf{M}_R$ and $\mathbf{K}_R\left(\boldsymbol{\theta}^{(0)}\right)$ using IIRS presented in Section 2.2;
  Sample the prediction error variance parameter $\sigma_j^{2(0)}$ using Eq. (12).
end while
while $p < N_{mc}$ **do**
  Update the chain index $p \leftarrow p+1$;
  while $k < N_\theta$ **do**
    set $\boldsymbol{\theta}^{(p-1)} \leftarrow \left\{\theta_1^{(p)}, \ldots, \theta_{k-1}^{(p)}, \theta_k^{(p-1)}, \theta_{k+1}^{(p-1)}, \ldots, \theta_{N_\theta}^{(p-1)}\right\}$;
    Sample θ_k^* using Eq. (20) and set $\boldsymbol{\theta}^* \leftarrow \left\{\theta_1^{(p)}, \ldots, \theta_{k-1}^{(p)}, \theta_k^*, \theta_{k+1}^{(p-1)}, \ldots, \theta_{N_\theta}^{(p-1)}\right\}$;
    Compute $\mathbf{M}_R$ and $\mathbf{K}_R(\boldsymbol{\theta}^*)$ using IIRS presented in Section 2.2;
    Compute the rejection probability γ using Eq. (18);
    Generate a uniform random number $r_0 \sim U(0,1)$;
if $r_0 \le \gamma$ **then**
      Set $\theta_k^{(p)} \leftarrow \theta_k^*$;
      Sample κ_k using $\kappa_k \sim U(1, 1.05)$;
      Set $\eta_k^{(p)} \leftarrow \kappa_k \eta_k^{(p-1)}$;
    else
      Set $\theta_k^{(p)} \leftarrow \theta_k^{(p-1)}$;
      Sample κ_k using $\kappa_k \sim U(0.95, 1)$;
      Set $\eta_k^{(p)} \leftarrow \kappa_k \eta_k^{(p-1)}$;
    end if
  end while
  while $j < N_m$ **do**
    Compute $\mathbf{M}_R$ and $\mathbf{K}_R\left(\boldsymbol{\theta}^{(p)}\right)$ using IIRS presented in Section 2.2;
    Sample the prediction error variance parameter $\sigma_j^{2(p)}$ using Eq. (12).
  end while
end while

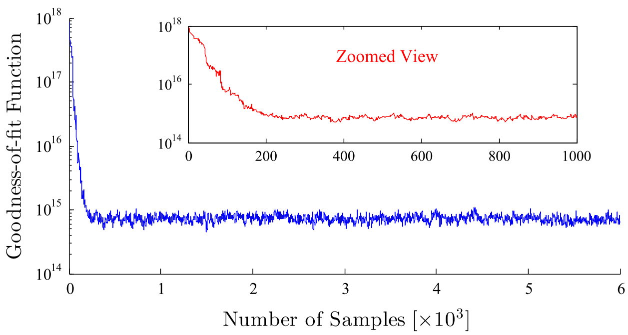

Fig. 1. Convergence of the goodness-of-fit function obtained by MCMC using 20% RMS noise measurement.

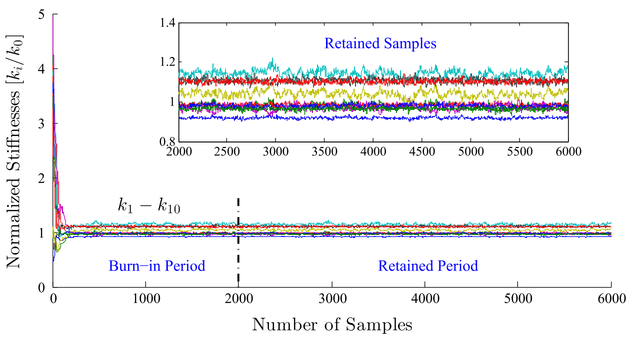

Fig. 2. Samples of 10 stiffness parameters of the 10-storey building obtained by MCMC using 20% RMS noise measurement.

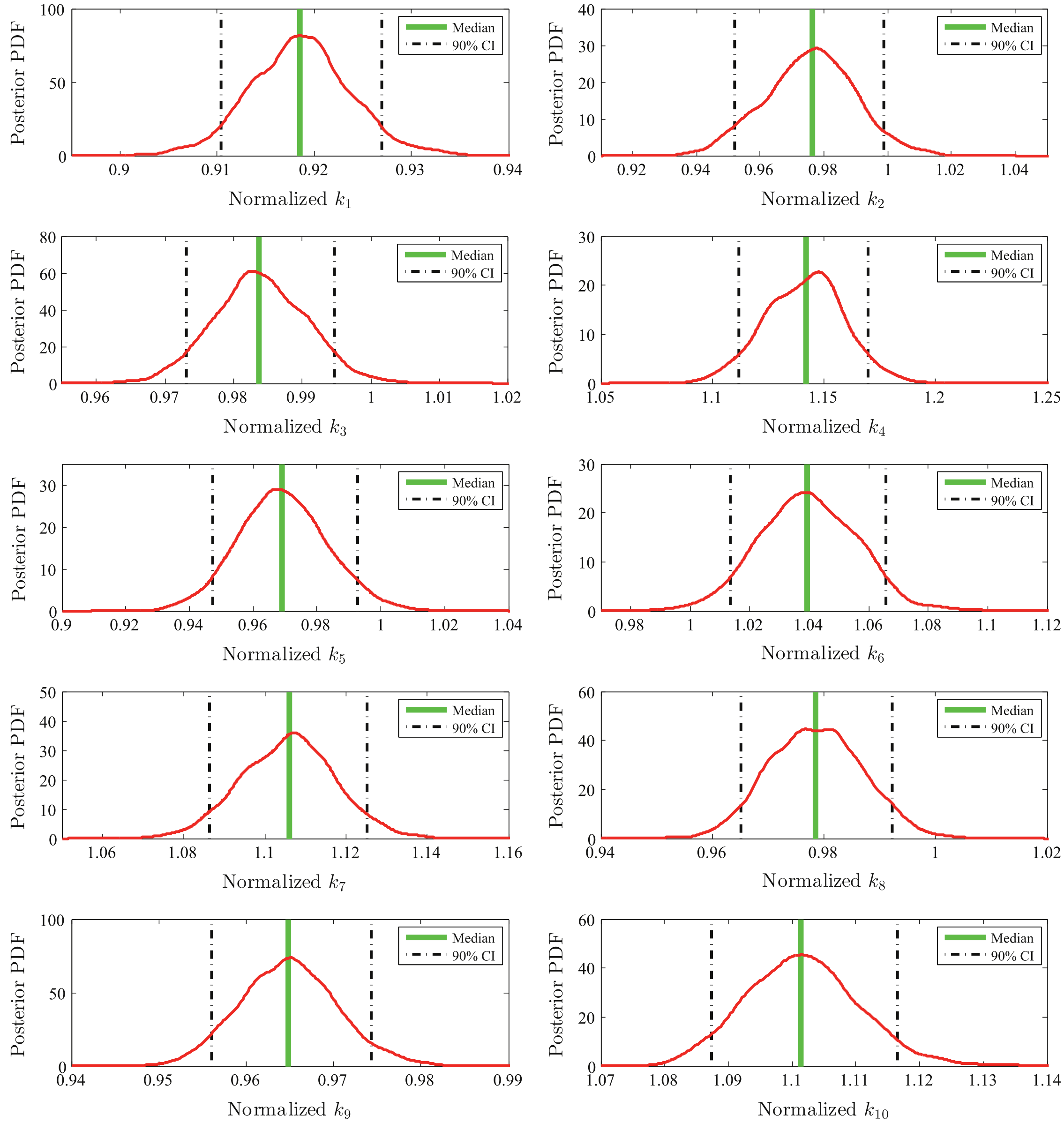

Fig. 3. The stiffness parameter posterior PDFs of the 10-storey building identified by MCMC using 20% RMS noise measurement. Note that CI represents the confidence interval.

4. Numerical example: a 10-storey shear-type building

In order to test the performance of the proposed algorithm for probabilistic model updating, a 10-storey shear-type building with synthetic measurements is studied here. The building has uniformly distributed mass and stiffness parameters, e.g., $\overline{m} = 100$ metric tons and $\overline{k} = 176.729$ MN/m, which was previously studied in [41]. The first five natural frequencies are 1.00, 2.98, 4.89, 6.69 and 8.34 Hz.

We first generate the synthetic response time histories simulated from ambient vibrations. We assume this building is subjected to ambient ground motions modeled by Gaussian white noise sequences. The classic modal damping is employed to describe the damping mechanism of the structure (e.g., the damping ratio of each mode is chosen 3% in the simulation). The ambient acceleration responses are acquired at the 1st, 3rd, 6th, 8th and 10th floors. A 100-s long signal with the sampling frequency of 100 Hz is recorded. To test the effect of measurement noise on parameter updating, noise pollution has been considered. The noisy output measurements are generated by adding a zero mean Gaussian white noise sequence, whose root-mean-square (RMS) is a certain percentage of the RMS of the clean signal, to the noise free signal. Six data sets are synthesized with 20% RMS noise and used for modal identification (e.g., N_s=6). Note that since the ambient ground

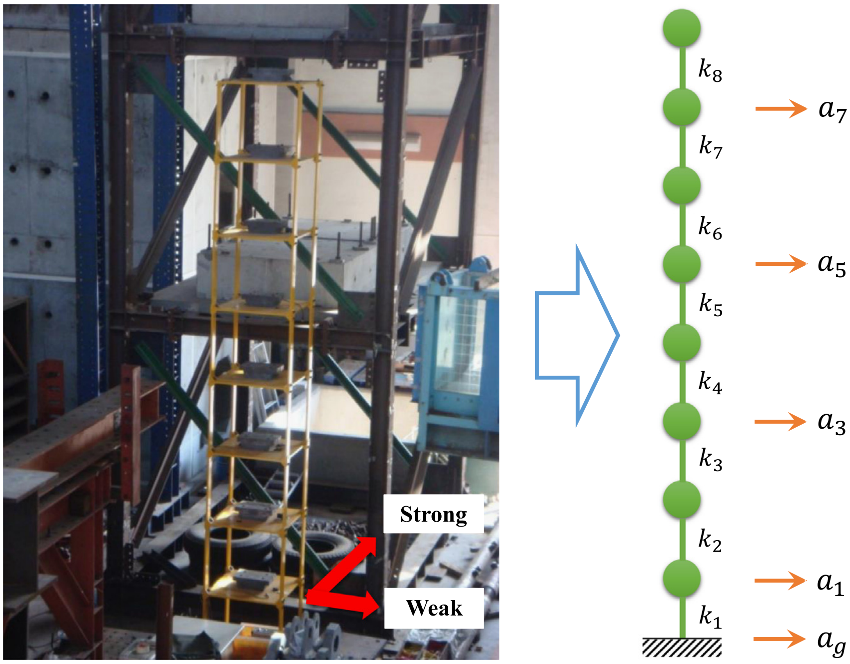

Fig. 4. Shaking table test setup of the 8-storey building. Note that measurements were recorded along the weak direction. The building is modeled by a 8DOF linear system.

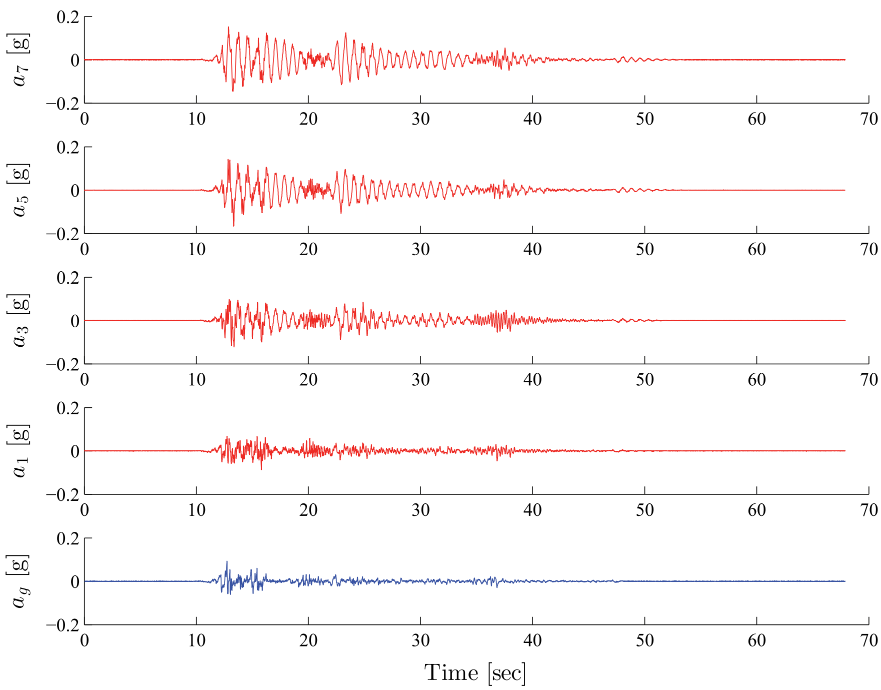

Fig. 5. A typical set of measured acceleration time histories of the 8-storey building.

motion input is assumed to be unmeasured, it naturally forms an output-only identification problem. Therefore, BSS is applied in this example to identify the modal characteristics of this building. Though the first five modes can be identified, only four of them (e.g., 1st, 2nd, 3rd, and 5th) are used for model updating. Thus, we have $N_m=4$. In the model updating process, we assume that we do not have any knowledge on the missing mode (e.g., the 4th mode). The mean values of the identified modal frequencies for the selected four modes by BSS are 0.99, 2.97, 4.88 and 8.37 Hz. The identified mode shapes are not presented herein.

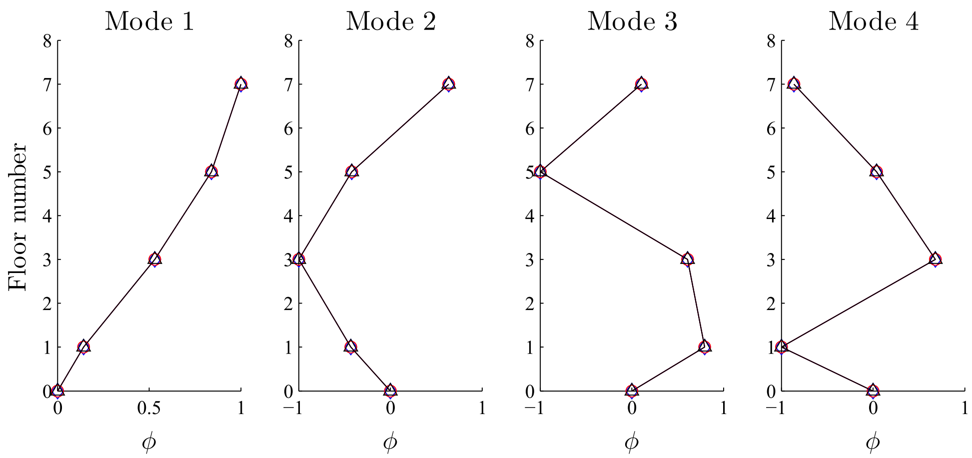

Fig. 6. Mode shapes of the 8-storey building identified by ERA-OKID using the shaking table test measurement. Note that the markers (○, △, and ⋄) represent three data sets used herein.

Table 2
Statistical results for the most probable stiffness parameters of the 8-storey building.

k_i	Identified value (kN)	c.o.v. (%)	k_i	Identified value (kN)	c.o.v. (%)
k_1	221.914	1.02	k_5	217.557	2.47
k_2	164.809	2.34	k_6	185.929	1.99
k_3	159.425	1.84	k_7	171.558	1.47
k_4	142.759	1.62	k_8	148.994	0.64

Note that c.o.v. represents the coefficient of variance.

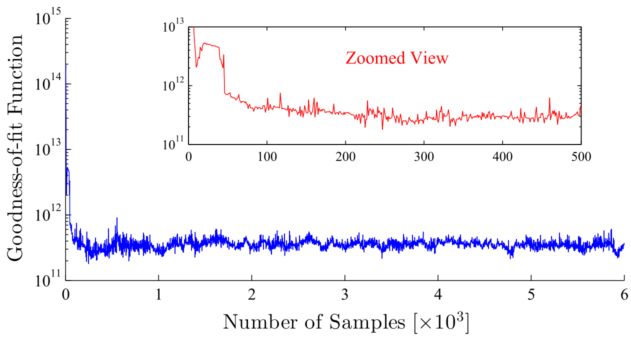

Fig. 7. Convergence of the goodness-of-fit function obtained by MCMC using the shaking table test measurement.

The chain length used in MCMC is $N_{mc} = 6 \times 10^3$ and the burn-in period is $N_b = 2 \times 10^3$. The lower and upper bounds for the parameters are zero and four times the true values. Since the masses are assumed to be known, the updating parameters become $k_1 \sim k_{10}$ and $\sigma_1^2 \sim \sigma_4^2$ (e.g., $N_\theta = 14$). The prior stiffness parameters follow the normal distribution with the mean value of 200 MN/m and the c.o.v. of 30%. The numerical analyses are programmed in MATLAB (The MathWorks, Inc., MA, USA) on a standard Intel (R) Core (TM) i7-4930K 3.40 GHz PC with 32G RAM.

Table 1 summarizes the statistical identification results for the most probable stiffness parameters of the 10-storey building. It can be seen that the updated normalized stiffness values are in general good. The estimated c.o.v. values are quite small, representing small uncertainties of the model parameters. Nonetheless, the largest error of the most probable stiffness value is less than 20% (e.g., about 14%). This error is acceptable because the measurements are corrupted with a high level of noise (e.g., 20% RMS) and a very limited number of modes (e.g., 4 incomplete modes) are used in model updating. In fact, the most probable stiffness error is comparable to the 20% RMS noise level.

Fig. 1 shows the convergence of the goodness-of-fit function as presented in Eq. (7). It is seen that the algorithm converges quite fast, e.g., the goodness-of-fit function becomes stationary after about 300 iterations. Fig. 2 depicts the samples of 10 stiffness parameters in the MCMC updating process. Similar to the goodness-of-fit function, the samples become stationary after 300 iterations. It is noteworthy that the burn-in period of 4000 iterations is sufficient and the retained

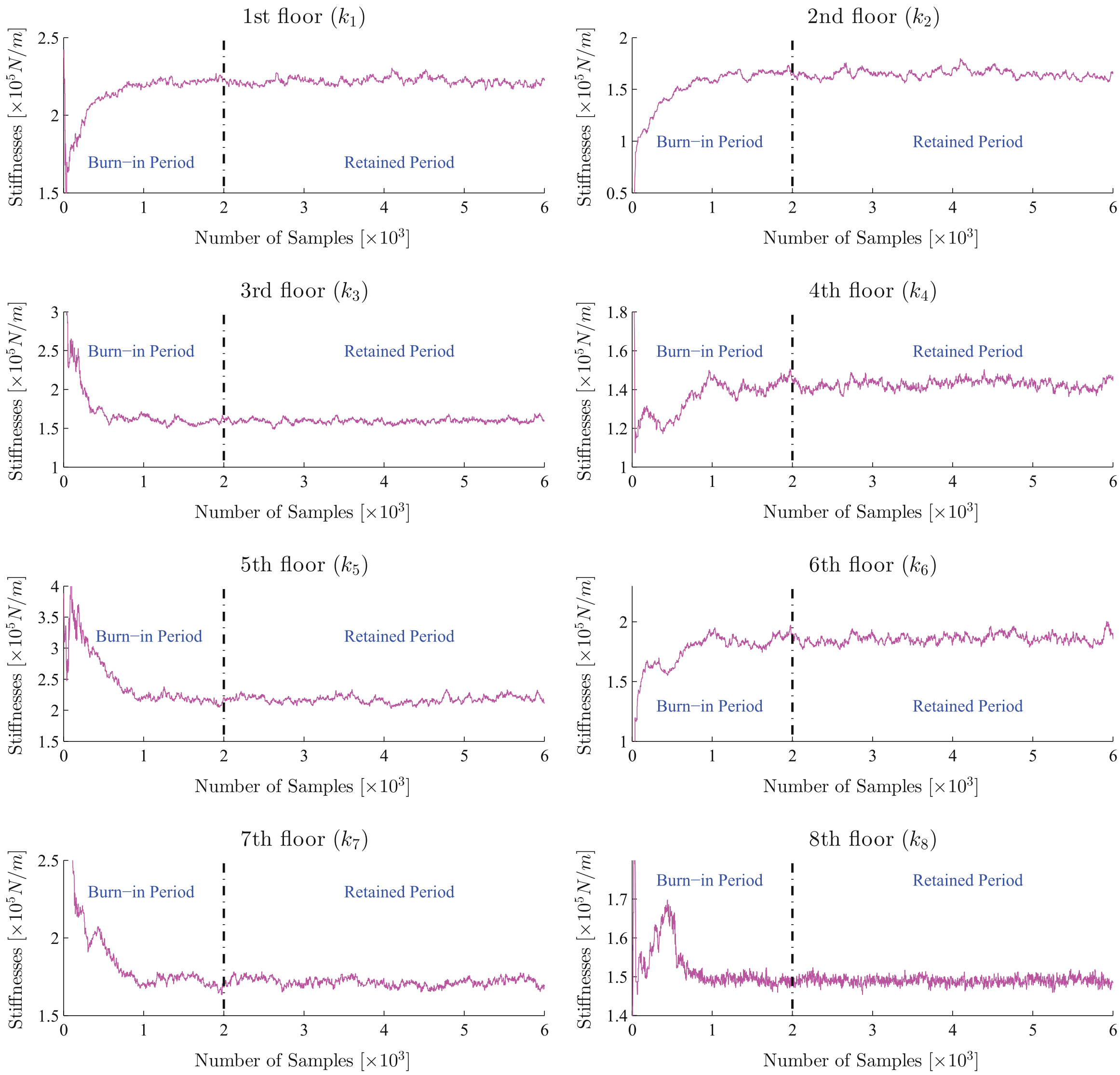

Fig. 8. Samples of eight stiffness parameters of the 8-storey building obtained by MCMC using the shaking table test measurement.

samples can be used for the representation of the posterior PDFs of the stiffness parameters. Fig. 3 shows the quantified posterior PDFs of the 10 stiffness parameters by MCMC. The 90% confidence intervals are also listed in Fig. 3. It takes about 3.9 min CPU time to complete the model updating process in this example. In general, the proposed algorithm performs quite well for probabilistic updating of the model stiffness parameters.

5. Experimental example: an 8-storey building shaking table test

We further validate the proposed probabilistic mode updating strategy using measurements from a shaking table test. The experimental structure utilized in this example is shown in Fig. 4, which was built by the National Center for Research on Earthquake Engineering (NCREE) in Taiwan [58]. It is a model of a 8-storey shear-type building. The frame consists of four steel columns and one steel plate for each floor, which are connected with each other by bolts. The plate has a dimension of 43×45 cm along the weak and strong direction, respectively, as shown in Fig. 4. The inter-storey height is 33 cm. An additive mass (50 kg) is placed in the center of each floor plate. The structure is mounted on a hydraulic uniaxial shaking table of NCREE. The ground excitation is applied along the weak direction only. Accelerators are installed at all the floors, including the base floor, to measure the responses along the structural weak direction. Though we have complete measurements, we only consider incomplete measurements from the ground, 1st, 3rd, 5th and 7th floors in the updating process as illustrated in Fig. 4. The structure is modeled by a linear shear-type system consisting of eight mass parameters ($m_1 \sim m_8$)

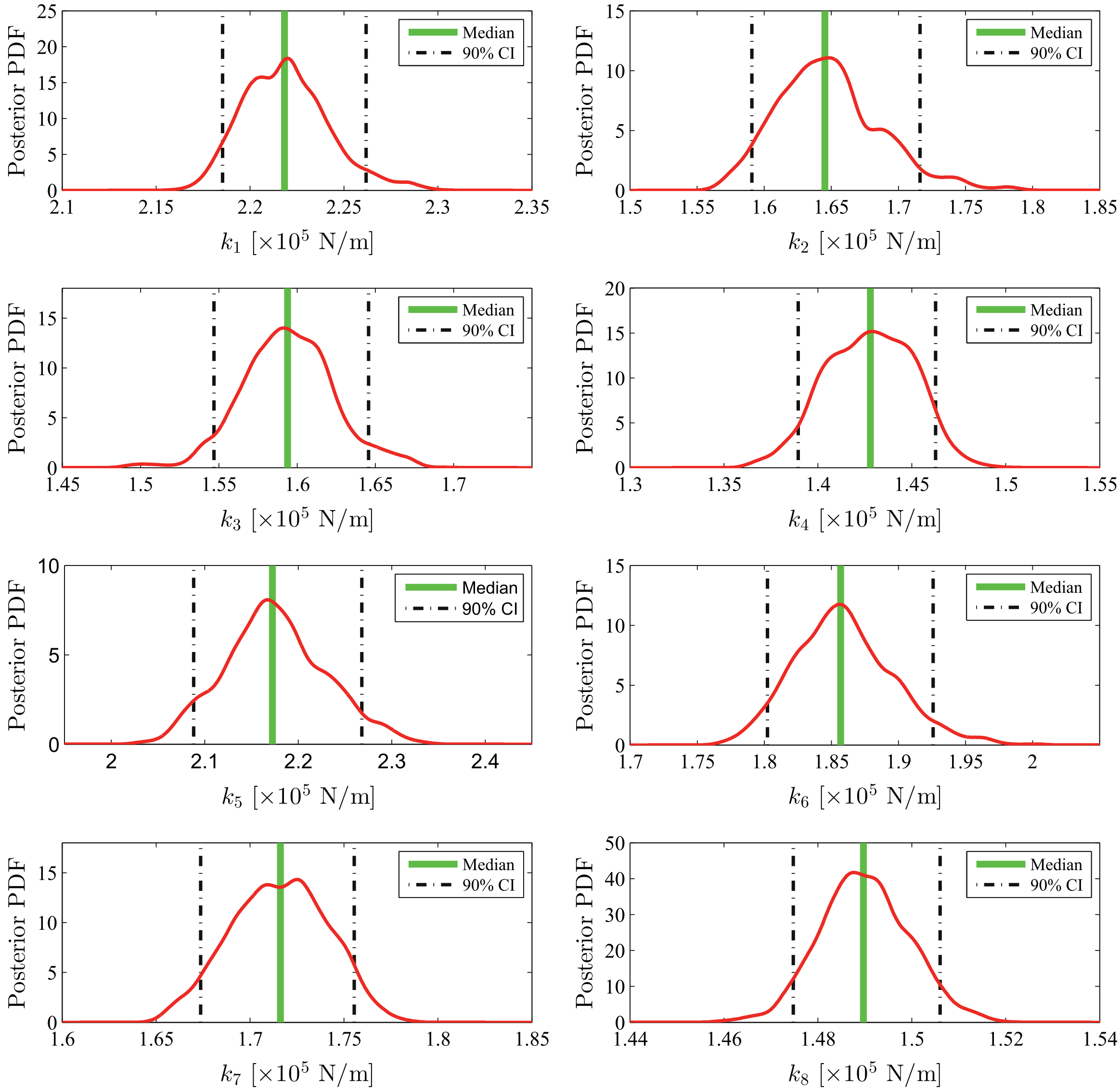

Fig. 9. The stiffness parameter posterior PDFs of the 8-storey building identified by MCMC using the shaking table test measurement. Note that CI represents the confidence interval.

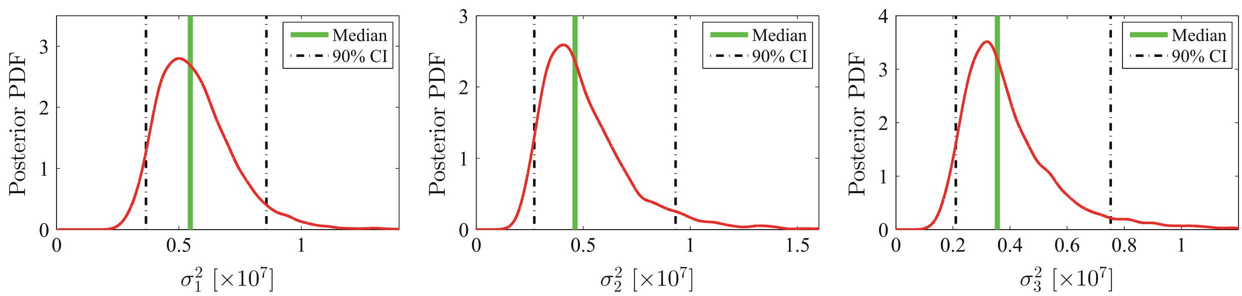

Fig. 10. The posterior PDFs of σ_1^2, σ_2^2 and σ_3^2 sampled by MCMC using the shaking table test measurement. Note that CI represents the confidence interval.

and eight stiffness parameters ($k_1 \sim k_8$). The estimation of the mass values are $m_1 = 80$ kg and $m_2 \sim m_8 = 75$ kg, which are regarded as known parameters.

Three sets of scaled ground motions (El Centro earthquake) are used to excite the building (e.g., $N_s = 3$), with the scaled peak ground acceleration (PGA) being 0.05g, 0.07g and 0.09g, respectively. The sampling frequency for the test is 200 Hz and

13660 data points are recorded for each data set. Fig. 5 shows a typical set of measured acceleration time histories of the test building (e.g., PGA=0.09g).

We employ the ERA-OKID algorithm to identify the modal quantities of the test building.The first four modes are successfully identified; nevertheless, only the 1st, 2nd and 4th modes used for model updating (N_m=4). The 3rd mode is manually missing in the updating process. The mean values of the identified modal frequencies for the selected three modes are 1.28, 4.31, 10.05 Hz. The corresponding identified mode shapes are presented in Fig. 6. In this example, the chain length used in MCMC is $N_{mc} = 6 \times 10^3$ and the burn-in period is $N_b = 2 \times 10^3$. The lower and upper bounds for the parameters are zero and 1×10^3 kN, respectively. In this example, the updating parameters are $k_1 \sim k_8$ and $\sigma_1^2 \sim \sigma_3^2$ (e.g., $N_\theta = 11$). The prior PDFs for stiffness parameters are selected to be independent distributions, namely, k_i $(i = 1, \ldots, 8)$ follow the normal distribution with means equal to $\overline{k}_i = 200$ kN/m and the corresponding c.o.v. of 30%.

Table 2 gives the statistical identification results for the most probable stiffness parameters of the test structure. In general, the most probable values of the stiffness parameters are presented in the range of 142 ~ 222 kN. It is observed from Table 1 that the c.o.v. values are small, which indicates a robust parameter estimation.

Fig. 7 shows the convergence of the goodness-of-fit function in the MCMC updating process. Fig. 8 illustrates the samples of the eight stiffness parameters. It is seen that the Markov chain converges and becomes stationary reasonably fast (e.g., after about 1000 iterations), indicating an efficient updating process. Stationary samples can be observed from Fig. 8 in the retained period. Fig. 9 depicts the quantified posterior PDFs as well as the 90% confidence intervals of the stiffness parameters for the 8-storey test building. In consistency with the results listed in Table 2, it can be observed that the identified stiffness parameter deviations are small, representing a robust identification. The overall performance of the proposed algorithm for probabilistic model updating is satisfactory. Fig. 10 shows the posterior PDFs of the prediction error variance σ_1^2, σ_2^2 and σ_3^2 with small deviations. Noteworthy, the CPU time for this example is only about 2.9 min.

6. Conclusions

We propose a new probabilistic strategy based on Bayesian inference for model updating in SHM. Incomplete modal data is first identified using a modal identification approach and then used as the measurement in the updating process. A model reduction technique (e.g., IIRS) is employed in the process of prediction error calculation such that direct mode matching is avoided. A MCMC algorithm with adaptive random-walk steps is proposed to draw samples for the representation of model parameter distributions and uncertainties. A sequential sampling is carried out to draw samples for the model parameters and the prediction error variances successively. Both numerical and experimental examples are used to illustrate the effectiveness and efficiency of the proposed method for probabilistic model updating. The proposed algorithm gives a possible uncertainty analysis of the model parameters and shows its potential to be used as a tool for probabilistic assessment of structural health conditions. Nevertheless, the examples for validation of the proposed algorithm are less complex structural systems. Future studies will be verifying the applicability of the proposed algorithm to more complex structural systems with field measurements.

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