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Identification of traffic-induced nodal excitations of truss bridges through heterogeneous data fusion

Hao Sun and Oral Büyüköztürk

Department of Civil & Environmental Engineering, MIT, Cambridge, MA 02139, USA
E-mail: haosun@mit.edu and obuyuk@mit.edu

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Abstract
We propose a time domain Bayesian inference-based regularization approach for the identification of traffic-induced nodal excitations of truss bridges through heterogeneous data fusion. The measurements (e.g., accelerations, strains and displacements) are fused via a state space realization and rescaled for force identification. The unknown excitation time histories are inverted by solving an ill-posed least squares problem using the proposed Bayesian regularization approach. A smoothing operator is used in the regularization process for the purpose of de-noising. Uncertainties due to measurement noise are considered in the process of force identification. Finally, the proposed algorithm is numerically illustrated by a 27 bar truss bridge. Results demonstrate the robustness and effectiveness of the proposed algorithm for traffic-induced excitation identification with high accuracy.

Keywords: force identification, heterogeneous data fusion, bayesian inference, regularization, traffic-induced excitation, truss bridge

(Some figures may appear in colour only in the online journal)

1. Introduction

Estimation of traffic-induced excitations has been a significant component in structural health monitoring (SHM) of bridge structures. Awareness of the real loadings induced by traffic brings benefits for bridge response prediction, condition assessment, maintenance operation, traffic flow management and control, as well as the improvement of future design. Traditional methods for external excitation estimation use force transducers (loading cells), installed at the locations where the forces are applied, to directly measure the external forces. Despite the fact that various types of force transducers have been developed and widely applied in the industry, these methods have obvious practical application limitations in the estimation of traffic-induced excitations, e.g., due to the accessibility issues and complicated loading conditions.

Recent advancements in sensor and computer technologies now make it possible to ‘indirectly measure’ the excitation time histories through solving an identification problem. Thus, indirect methods are becoming an alternative and popular strategy for external excitation estimation. In general, the indirect method formulates the force evaluation as an inverse problem based on the observed structural responses and solves the inverse problem so as to obtain the time histories and/or the locations of the external excitations (refer to, for example, [1–7], among others).

It is noted that to reconstruct the force time histories, a regularization method is usually applied [8]. Given the structural model (e.g., the updated finite element model based on the measurements), one alternative to relate the unknown input time history to the measured response is done through a state space model. In this manner, a lower block triangular Toeplitz matrix consisting of the system Markov parameters can be built, and a classic linear least squares regression problem is formulated [3] (e.g., $Ax = b$), where $A$ denotes the Toeplitz matrix, $x$ is the unknown input vector and $b$ represents the measurement vector. Inversion algorithms such as the Tikhonov regularization have been applied to solve the ill-

1 Author to whom any correspondence should be addressed.

Among others, it is noteworthy to list some recent works on applying Tikhonov regularization techniques to moving force identification. For example, Law et al [2] studied a regularization method to identify forces moving on a beam from measured acceleration responses through solving an ill-conditioned inverse problem. Zhu and Law [13] proposed a method based on modal superposition and regularization to identify moving loads on an elastically supported multi-span continuous bridge deck. Wu and Law [14] presented a new stochastic technique for moving load identification in which statistics of the moving force time histories are identified from samples of the structural responses.

Note that the conventional Tikhonov regularization requires the process of singular value decomposition of the Toeplitz matrix A for regularization parameter selection, as a result leading to a significant computational burden when the size of A is large. To alleviate this issue, another alternative to solve the ill-posed linear regression problem is Bayesian inference, which provides a statistical and efficient mechanism for the regularization parameter selection through hierarchical modeling. For instance, Jin and Zhou [15] presented a Bayesian inference approach to solve the regularized Cauchy problem in steady-state heat conduction. Yuen and Mu [16] proposed a novel probabilistic method for robust parametric identification and outlier detection in linear regression problems. It then appears natural to apply the Bayesian inference to solve the force identification problem.

Nevertheless, the majority of the work on unknown load identification in the literature employs a single type of measurement (e.g., acceleration response). Since heterogeneous sensing has been widely used in SHM, there is a need to develop heterogeneous data fusion algorithms for unknown external excitation identification. For example, displacement transducers (such as camera vision-based ‘sensors’), strain sensors (such as optic fibers and long strain gauges) and accelerometers are often used to monitor the bridge responses. Naturally, the recorded acceleration, strain and displacement time histories can then be simultaneously used to identify the traffic-induced excitations.

The contributions in this paper are listed as follows: we propose a Bayesian inference-based regularization approach for the identification of traffic-induced nodal excitations of truss bridges through heterogeneous data fusion. The heterogeneous measurements are fused and rescaled using a state space model. The excitation time history as well as the regularization parameters are iteratively updated via a proposed Bayesian learning scheme. In the updating process, a smoothing operator is applied to de-noise the identified time histories so as to obtain a more accurate result.

This paper is organized as follows. Section 2 presents the heterogeneous data fusion via a state space realization. Section 3 describes a Bayesian inference regularization approach for traffic-induced excitation identification via solving an ill-posed inverse problem. In section 4, a numerical example of a 27 bar truss bridge, installed with heterogeneous sensors, is illustrated. Finally, concluding remarks are presented in section 5.

2. Heterogeneous data fusion via a state space realization

2.1. Equations of motion

Consider a generic truss bridge with n degrees-of-freedom (DOFs) and m elements. The system equation of motion subjected to the traffic-induced loading can be written as

$$\mathbf{M}\mathbf{x}(t) + \mathbf{C}\dot{\mathbf{x}}(t) + \mathbf{K}\ddot{\mathbf{x}}(t) = \mathbf{L}\mathbf{f}(t)$$

(1)

where \( \mathbf{x}(t) \in \mathbb{R}^{n \times 1} \), \( \dot{\mathbf{x}}(t) \in \mathbb{R}^{n \times 1} \) and \( \ddot{\mathbf{x}}(t) \in \mathbb{R}^{n \times 1} \) are the system’s response components representing displacement, velocity, and acceleration vectors, which are partially measured by the sensors; \( \mathbf{M} \in \mathbb{R}^{n \times n} \), \( \mathbf{C} \in \mathbb{R}^{n \times n} \) and \( \mathbf{K} \in \mathbb{R}^{n \times n} \) are the mass, damping, and stiffness matrices, which are obtained from finite element assembly; \( \mathbf{f}(t) \in \mathbb{R}^{m \times 1} \) is the traffic-induced load vector acting on the truss nodes (\( n_f \) is the number of input forces), which needs to be identified; and \( \mathbf{L} \in \mathbb{R}^{m \times n_f} \) is the input location matrix.

To relate the input forces to the strain measures, the system responses (\( \mathbf{x}, \dot{\mathbf{x}}, \ddot{\mathbf{x}} \)) in the displacement field can be mapped into the strain field by pre-multiplying a transformation matrix \( \mathbf{\psi} \in \mathbb{R}^{m \times n} \), namely,

$$\mathbf{e}(t) = \mathbf{\psi}\mathbf{x}(t), \quad \dot{\mathbf{e}}(t) = \mathbf{\psi}\dot{\mathbf{x}}(t) \quad \text{and} \quad \ddot{\mathbf{e}}(t) = \mathbf{\psi}\ddot{\mathbf{x}}(t)$$

(2)

where \( \mathbf{e}(t) \in \mathbb{R}^{m \times 1} \), \( \dot{\mathbf{e}}(t) \in \mathbb{R}^{m \times 1} \) and \( \ddot{\mathbf{e}}(t) \in \mathbb{R}^{m \times 1} \) are the strain, strain rate, and strain acceleration vectors. The substitution of equation (2) into equation (1) yields

$$\mathbf{M}\mathbf{\psi}\ddot{\mathbf{e}}(t) + \mathbf{C}\mathbf{\psi}\dot{\mathbf{e}}(t) + \mathbf{K}\mathbf{\psi}\mathbf{e}(t) = \mathbf{L}\mathbf{f}(t)$$

(3)

where \( \mathbf{\psi}^{\dagger} \) denotes the pseudo-inverse of \( \mathbf{\psi} \), namely, \( \mathbf{\psi}^{\dagger} \approx (\mathbf{\psi}^{\top}\mathbf{\psi})^{-1}\mathbf{\psi}^{\top} \). Pre-multiplying equation (3) by \( \mathbf{\psi} \) yields the equation of motion in the strain field, given by

$$\mathbf{\tilde{M}}\ddot{\mathbf{e}}(t) + \mathbf{\tilde{C}}\dot{\mathbf{e}}(t) + \mathbf{\tilde{K}}\mathbf{e}(t) = \mathbf{L}\mathbf{f}(t)$$

(4)

where \( \mathbf{\tilde{M}} = \mathbf{\psi}^{\dagger}\mathbf{M}\mathbf{\psi}^{\dagger} \in \mathbb{R}^{m \times m} \), \( \mathbf{\tilde{C}} = \mathbf{\psi}^{\dagger}\mathbf{C}\mathbf{\psi}^{\dagger} \in \mathbb{R}^{m \times m} \) and \( \mathbf{\tilde{K}} = \mathbf{\psi}^{\dagger}\mathbf{K}\mathbf{\psi}^{\dagger} \in \mathbb{R}^{m \times m} \) are the equivalent mass, damping and stiffness matrices in the strain field; and \( \mathbf{L} = \mathbf{\psi}\mathbf{L} \in \mathbb{R}^{m \times n_f} \) is the modified input location matrix.
In the forward problem, one can solve equations (1) and (4) for the system responses given the input forces and the system matrices. Nevertheless, in this work, we consider to solve an inverse problem (e.g., identify the input forces) given some measured heterogeneous system responses based on a finite element model.

2.2. State space representation

The representation of equations (1) and (4) in the continuous state space is written as:

$$\dot{z}(t) = A_c z(t) + B_c f(t)$$

(5)

where

$$z(t) = [x(t) \ e(t) \ \dot{x}(t) \ \dot{e}(t)]^T \in \mathbb{R}^{(n+m) \times 1}$$

represents the system responses in both the displacement and strain fields. The time invariant continuous state matrix \(A_c \in \mathbb{R}^{(n+m) \times (n+m)}\) and input matrix \(B_c \in \mathbb{R}^{(n+m) \times n_f}\) are given by

$$A_c = \begin{bmatrix}
    0_{n \times n} & 0_{n \times m} & I_{n \times n} & 0_{n \times m} \\
    0_{m \times n} & 0_{m \times m} & 0_{m \times n} & I_{m \times n} \\
    -M^{-1}K & 0_{n \times m} & -M^{-1}C & 0_{n \times m} \\
    0_{m \times n} & -M^{-1}K & 0_{m \times m} & -M^{-1}C
  \end{bmatrix}$$

$$B_c = \begin{bmatrix}
    0_{n \times n_f} \\
    0_{m \times n_f} \\
    M^{-1}L \\
    \tilde{M}^{-1}L
  \end{bmatrix}$$

The complete output vector \(y(t) = [x(t) \ e(t) \ \dot{x}(t) \ \dot{e}(t)]^T \in \mathbb{R}^{(2n+m) \times 1}\), including the displacement, acceleration and strain responses, can be written as

$$y(t) = C_c z(t) + D_c f(t)$$

(7)

where the time invariant continuous output matrix \(C_c \in \mathbb{R}^{(2n+m) \times (n+m)}\) and feedthrough matrix \(D_c \in \mathbb{R}^{(2n+m) \times n_f}\) are expressed as

$$C_c = \begin{bmatrix}
    I_{n \times n} & 0_{n \times n} & 0_{n \times n} & 0_{n \times m} \\
    0_{m \times n} & I_{m \times m} & 0_{m \times n} & 0_{m \times m} \\
    -M^{-1}K & 0_{n \times m} & -M^{-1}C & 0_{n \times m} \\
    0_{m \times n} & -M^{-1}K & 0_{m \times m} & -M^{-1}C
  \end{bmatrix}$$

$$D_c = \begin{bmatrix}
    0_{n \times n_f} \\
    0_{m \times n_f} \\
    \tilde{M}^{-1}L
  \end{bmatrix}$$

(8)

Since the responses are typically measured at discrete time intervals, equations (5) and (7) are expressed in the following discrete state space form:

$$z(k+1) = A_d z(k) + B_d f(k)$$

(9a)

$$y(k) = C_d z(k) + D_d f(k)$$

(9b)

where \(z(k), y(k)\) and \(f(k)\) are discrete vectors at time \(t_k = k \Delta t\) (\(k = 0, 1, 2, \ldots, N\)). \(N\) is the number of observed data points and \(\Delta t\) is the sampling time. Herein, \(A_d, B_d, C_d\) and \(D_d\) are the discrete versions of the continuous system state space matrices, i.e.,

$$A_d = \exp(A_c \Delta t)B_d = A_c^{-1}(A_d - I)B_d,$$

$$C_d = C_c \quad \text{and} \quad D_d = D_c$$

(10)

where \(I \in \mathbb{R}^{(n+m) \times (n+m)}\) is an identity matrix. It is noteworthy that the system in equations (9a) and (9b) is completely state controllable and observable.

2.3. Fusing heterogeneous data for force identification

The output vector of the measurement can be determined by the sensor type and locations. One can assemble the heterogeneous measured outputs as follows:

$$y_i(k) = R_i y(k) = R_i C_d z(k)$$

(11a)

$$y_0(k) = R_0 y(k) = R_0 C_d z(k)$$

(11b)

$$y_a(k) = R_a y(k) = R_a C_d z(k) + R_a D_d f(k)$$

(11c)

where \(y_i(k) \in \mathbb{R}^{n_i \times 1}\), \(y_0(k) \in \mathbb{R}^{n_0 \times 1}\) and \(y_a(k) \in \mathbb{R}^{n_a \times 1}\) denote the displacement, strain and acceleration measurements, respectively, at time \(t_k\); \(n_i, n_0\) and \(n_a\) represent the number of displacement sensors, strain gauges (or optic fibers) and accelerometers, respectively; and \(R_i \in \mathbb{R}^{n_i \times (2n+m)}, \quad R_0 \in \mathbb{R}^{n_0 \times (2n+m)}\) and \(R_a \in \mathbb{R}^{n_a \times (2n+m)}\) are the corresponding output influence matrices whose elements consist of 0 and 1.

The substitution of equation (9a) into equations (11a)–(11c) and applying zero initial conditions (e.g., \(z(0) = 0\)) yield

$$y_i(k) = R_i C_d \sum_{p=0}^{k-1} A_d^{-p-1} B_d f(p)$$

(12a)

$$y_0(k) = R_0 C_d \sum_{p=0}^{k-1} A_d^{-p-1} B_d f(p)$$

(12b)

$$y_a(k) = R_a C_d \sum_{p=0}^{k-1} A_d^{-p-1} B_d f(p) + R_a D_d f(k)$$

(12c)

where \(k \geq 1\). For the case of \(k = 0\), we have \(y_i(0) = 0, \quad y_0(0) = 0\) and \(y_a(0) = R_a D_d f(0)\).

Equations (12a)–(12c) can be written as the matrix form as follows:

$$Y_i = H_i F, \quad Y_0 = H_0 F \quad \text{and} \quad Y_a = H_a F$$

(13)

where \(Y_i \in \mathbb{R}^{n_i \times N}\) is the assembled displacement measurement, \(Y_0 \in \mathbb{R}^{n_0 \times N}\) is the assembled strain measurement, \(Y_a \in \mathbb{R}^{n_a \times N}\) is the assembled acceleration measurement, and \(F \in \mathbb{R}^{n \times N}\) is the assembled unknown force vector, expressed as

$$Y_i = \begin{bmatrix}
    y_i(1) & y_i(2) & \cdots & y_i(N-1) & y_i(N)
  \end{bmatrix}^T$$

(14a)

$$Y_0 = \begin{bmatrix}
    y_0(1) & y_0(2) & \cdots & y_0(N-1) & y_0(N)
  \end{bmatrix}^T$$

(14b)

$$Y_a = \begin{bmatrix}
    y_a(0) & y_a(1) & \cdots & y_a(N-2) & y_a(N-1)
  \end{bmatrix}^T$$

(14c)

$$F = [f(0) \ f(1) \ \cdots \ f(N-2) \ f(N-1)]^T$$

(14d)

Here, \(H_i, H_0, H_a \in \mathbb{R}^{n_i \times n_f}, \quad H_i, H_0, H_a \in \mathbb{R}^{n_a \times n_f}\) and \(H_a \in \mathbb{R}^{n_a \times n_f}\) are the convolution matrices consisting of the Markov
parameters, given by
\[
H_x = \begin{bmatrix}
R_c C_d B_d & 0_{n_x \times n_f} \\
R_c C_d A_d B_d & R_c C_d B_d \\
R_c C_d A_d^2 B_d & R_c C_d B_d \\
\vdots & \vdots \\
R_c C_d A_d^{N-1} B_d & R_c C_d A_d^{N-2} B_d \\
0_{n_x \times n_f} & \cdots & 0_{n_x \times n_f} \\
0_{n_x \times n_f} & \cdots & 0_{n_x \times n_f} \\
R_c C_d B_d & \cdots & 0_{n_x \times n_f} \\
\vdots & \vdots & \vdots \\
R_c C_d A_d B_d & R_c C_d B_d \\
\end{bmatrix}
\]

Since the magnitudes of heterogeneous measurements are significantly different, we scale the measured data by multiplying rescaling coefficients. Therefore, the rescaled linear systems in equation (13) are written as
\[
\tilde{Y}_f = \tilde{H}_f F, \quad \tilde{Y}_e = \tilde{H}_e F \quad \text{and} \quad \tilde{Y}_a = \tilde{H}_a F \tag{16}
\]
where
\[
\tilde{Y}_f = \alpha_f Y_f \quad \text{and} \quad \tilde{H}_f = \alpha_f H_f \\
\tilde{Y}_e = \alpha_e Y_e \quad \text{and} \quad \tilde{H}_e = \alpha_e H_e \\
\tilde{Y}_a = \alpha_a Y_a \quad \text{and} \quad \tilde{H}_a = \alpha_a H_a \tag{17}
\]
respectively, which are determined as follows:
\[
\alpha_f = \|Y_f\|^{-1}, \quad \alpha_e = \|Y_e\|^{-1}, \quad \text{and} \quad \alpha_a = \|Y_a\|^{-1} \tag{18}
\]
where \(\|\cdot\|\) denotes the \(L_2\) norm of a vector. The use of rescaling coefficients results in a robust regularization and yields a more accurate solution when one solves an ill-posed problem using heterogeneous data with different orders of magnitude (see section 4.1.2).

The assembly of the linear systems in equation (16) into a single matrix form yields
\[
\begin{align*}
Y &= H F \\
Y &= \tilde{Y}_f, \quad \text{and} \quad H = \begin{bmatrix} \tilde{H}_f \\ \tilde{H}_e \\ \tilde{H}_a \end{bmatrix} \tag{19}
\end{align*}
\]
where
\[
Y = \begin{bmatrix} \tilde{Y}_f \\ \tilde{Y}_e \\ \tilde{Y}_a \end{bmatrix} \quad \text{and} \quad H = \begin{bmatrix} \tilde{H}_f \\ \tilde{H}_e \\ \tilde{H}_a \end{bmatrix} \tag{20}
\]

To this end, the measured heterogeneous data (e.g., accelerations, displacements and strains) are fused by equations (19) and (20).

Given the heterogeneous measurement vector \(Y\) and the parameter matrix \(H\) (e.g., defined by the finite element model), the unknown traffic-induced excitation vector \(F\) can be determined by solving equation (19).

3. Bayesian regularization for traffic-induced excitation identification

It is noteworthy that to solve for \(F\), equation (19) is ill posed when noise is present in the measurements (e.g., \(H\) may be ill conditioned or singular). Therefore, ordinary least squares fail to give a bounded solution. Methods using regularization can produce well-posed solutions. For example, Tikhonov regularization has been successfully adopted for deterministic force identification in recent years \([7, 8]\), wherein the regularization parameter is determined by the L-curve method \([11]\) via the singular value decomposition of the coefficient matrix \(H\).

In this work, we propose a statistical approach to solve equation (19) through a statistical analysis which is undertaken within the context of Bayesian inference. The regularization is explicitly modeled by the prior distribution. The traffic-induced excitation is updated iteratively through Bayesian learning.

3.1. Bayesian inference regularization

A statistical hierarchical modeling is employed to obtain the posterior probability density function (PDF) of the traffic-induced excitation vector \(F\) using Bayesian inference. In details, \(F\) is encapsulated in the posterior PDF \(p(F, \sigma^2, \lambda^2 | Y)\) conditional on the heterogeneous measurements \(Y\), expressed as \([16, 17]\)
\[
p(F, \sigma^2, \lambda^2 | Y) \propto p(Y | F, \sigma^2)p(F | \lambda^2)p(\sigma^2)p(\lambda^2) \tag{21}
\]
where \( p(Y|F, \sigma^2) \) is the likelihood function and \( p(F|\lambda^2) \) is the prior PDF of \( F \), given by

\[
p(Y|F, \sigma^2) \propto \frac{1}{\sigma^{n_N}} \exp\left(-\frac{1}{2\sigma^2} \|HF - Y\|^2\right) \tag{22}
\]

\[
p(F|\lambda^2) \propto \frac{1}{\lambda^{n_N}} \exp\left(-\frac{1}{2\lambda^2} \|GF\|^2\right) \tag{23}
\]

where \( n_n \) is the total number of observations (e.g., \( n_n = n_t + n_s + n_n \)) \( \sigma \) is the standard deviation of data noise and modeling error; \( \lambda \) is a scale parameter representing the force variation; and \( G \) is the regularization smoothing operator which is used to de-noise and smooth the identified traffic-induced excitation time histories. A common choice of \( G \) is the identity matrix when smoothing is ignored (e.g., \( G = I \in \mathbb{R}^{n_N \times n_N} \)). Nevertheless, in this paper, we employ a discrete approximation of derivatives as the regularization operator which is used to de-noise and smooth the identified traffic-induced excitation time histories. A common choice of \( G \) is the identity matrix when smoothing is ignored (e.g., \( G = I \in \mathbb{R}^{n_N \times n_N} \)).

The substitution of equations (22), (23), (25a) and (25b) into equation (26) yields

\[
p(F, \sigma^2, \lambda^2|Y) \propto \frac{\lambda^{2(a_1+1)-n_sN} \sigma^{2(a_0+1)+n_nN}}{\sigma^{2(a_0+1)+n_nN}} \times \exp\left(-\frac{1}{2\sigma^2} \|HF - Y\|^2 - \frac{1}{2\lambda^2} \|GF\|^2 - \beta_0\sigma^2 - \beta_1\lambda^2\right) \tag{26}
\]

The Bayesian inference approach maximizes the posterior PDF \( p(F, \sigma^2, \lambda^2|Y) \) so as to obtain an \textit{a posteriori} estimate of the parameter set, e.g. \( \{\hat{F}, \hat{\sigma}^2, \hat{\lambda}^2\} \), namely,

\[
\{\hat{F}, \hat{\sigma}^2, \hat{\lambda}^2\} = \arg \max_{\{F, \sigma^2, \lambda^2\}} \left\{ p(F, \sigma^2, \lambda^2|Y) \right\} \tag{27}
\]

Calculating the derivatives of \( p(F, \sigma^2, \lambda^2|Y) \) with respect to the unknown variables \( F, \sigma^2 \) and \( \lambda^2 \), respectively, and setting the derivatives equal to zero, we obtain the optimality systems of equation (27) accordingly, given by

\[
\left[ n_n N + 2(\alpha_0 + 1) \right] \hat{\sigma}^2 - \|HF - Y\|^2 = -2\beta_0 = 0 \tag{28a}
\]

\[
\left[ n_n N + 2(\alpha_1 + 1) \right] \hat{\lambda}^2 - \|GF\|^2 = -2\beta_1 = 0 \tag{28b}
\]

The optimal solutions \( \{\hat{F}, \hat{\sigma}^2, \hat{\lambda}^2\} \) are obtained provided equations (28a)–(28c) are solved. A sequential Bayesian learning scheme is proposed to interpret the optimality systems, as illustrated in algorithm 1. The traffic-induced excitations are identified as the algorithm satisfies the convergence criterion. Of note, the hyperparameters of the conjugate priors \( \{\alpha_0, \beta_0, \alpha_1, \beta_1\} \) are as small as possible and coincide with the orders of magnitude of \( \sigma^2 \) and \( \lambda^2 \), which can be roughly estimated from the measured data.

**Algorithm 1.** The sequential Bayesian learning algorithm for excitation identification

Given the convergence tolerance \( \epsilon \), the Hankel matrix \( H \), the heterogeneous measurement vector \( Y \), and the hyperparameters of the conjugate priors \( \{\alpha_0, \beta_0, \alpha_1, \beta_1\} \):

- set \( \hat{F}_0 \) as a vector with all elements being 1 and set \( \delta_0 = 1 \times 10^{-4} \) and \( \delta_k = 10 \times 1 ; k = 0 \);
- while \( \|\hat{F}_{k+1} - \hat{F}_k\|/\|\hat{F}_k\| > \epsilon \) do
  - Update the identified force: \( \hat{F}_{k+1} = \left( H^2 + \frac{\delta_k^2}{\delta_k} G^T G \right)^{-1} H^T Y \);
  - Calculate \( \delta_k^2 = [n_n N + 2(\alpha_0 + 1)]^{-1} \|H\hat{F}_{k+1} + Y\|^2 - 2\beta_0 \);
  - Calculate \( \lambda_k^2 = [n_n N + 2(\alpha_1 + 1)]^{-1} \|G\hat{F}_{k+1}\|^2 - 2\beta_1 \);
  - \( k = k + 1 \);
- end while

3.2. Traffic-induced excitation identification procedure

The detailed procedure for the identification of traffic-induced nodal excitations of truss bridges is illustrated as follows.

- Step 1. Carry out the field measurement of the target truss bridge from heterogeneous sensing to obtain \( y_0, y_s \) and \( y_t \); rescale the heterogeneous data and assemble them into the vector form, e.g. \( Y \), as shown in equations (14a)–(14c) and (20).
- Step 2. Build a finite element model of the truss bridge and update the mass, stiffness and damping matrices; build the state space model for heterogeneous data fusion following equations (6), (8) and (10); obtain the Markov parameters, rescale them, and assemble the rescaled parameters into the Hankel matrix, e.g. \( H \), as shown in equations (15a)–(15c) and (20).
- Step 3. Select the convergence tolerance \( \epsilon \) and the hyperparameters of the conjugate priors \( \{\alpha_0, \beta_0, \alpha_1, \beta_1\} \); run algorithm 1 to solve for the traffic-induced excitation vector \( \hat{F} \) until the convergence criterion are met (e.g., \( \epsilon = 1 \times 10^{-6} \)).
- Step 4. Reassemble the identified force vector to obtain the traffic-induced excitation time histories applied at certain nodes, e.g. \( f(t_k), k = 0, 1, 2, \ldots, N - 1 \).
Remark. Since the proposed algorithm is a model-based approach, we assume that the finite element model used herein is well established or updated using existing model-updating strategies [17–19]. Of note, the proposed algorithm works for both small and large data sets which are related to the value of $N$. We also assume that a sufficient number of sensors are given and adequately placed on the structure to ensure the solution’s uniqueness of the inverse problem [20].

4. Numerical example: a 27 bar truss bridge

To demonstrate the effectiveness and applicability of the proposed algorithm to the identification of traffic-induced excitations using heterogeneous measurements, a truss bridge, as shown in figure 1, is studied. The truss bridge is a simple supported structure with 27 bars and 15 nodes (e.g., the numbering system is shown in figure 1). The excitations applied to the bridge are induced by moving vehicles. The traffic-induced loads are applied to the bridge deck and transferred to the truss nodes through the lateral beams as illustrated in figure 1. The road surface roughness of the bridge deck is considered. It is noted that to generate the synthetic measurements, Gaussian white noise sequences are used to approximate the moving load time histories induced by vehicles. Therefore, the proposed algorithm can be used to identify any generic traffic-induced loads without the need of knowing any vehicle-specific information such as the mass, damping, stiffness, etc. The objective herein is to test the proposed algorithm for generic traffic-induced force identification through solving an inverse problem. For the target structure considered here, we have six traffic-induced forces acting on the bottom nodes of the truss bridge, which need to be identified.

The truss bridge is instrumented with 9 heterogeneous sensors in total, including 1 displacement sensor (installed at node 9), 5 strain sensors (installed at elements 3, 9, 13, 17, 23) and 3 accelerometers (installed at nodes 4, 7, 11). The vertical displacement and acceleration responses and the axial dynamic strains are measured. It is noted that either the linear variable differential transformer (LVDT) or the vision-based sensor using cameras can be used to measure the displacement time histories, while optical fibers or long strain gauges can be employed to measure the dynamic strains. The geometry, dimensions, loading condition and sensor locations of a 15-node 27 bar truss bridge are given in figure 1.

In this example, numerical synthetic data are employed as the measurements with a sampling rate of 1 kHz. To test the effect of measurement noise on the identification accuracy, noise has been explicitly considered. The noisy output measurements are generated by adding a zero mean Gaussian white noise sequence, whose root mean square (RMS) is a certain percentage of the clean signal RMS, to the noise-free signal. Four noise levels are considered here, namely, 2, 5, 10 and 20%.

The material and geometric properties of the truss elements are Young’s modulus $E = 200$ GPa, mass density $\rho = 7860$ kg m$^{-3}$, cross-sectional area of 0.01 m$^2$ for the elements numbered $\{1, 2, 4, 6, 8, 10, 12, 14, 16, 18, 20, 22, 24, 26, 27\}$ and $0.005$ m$^2$ for the elements numbered $\{3, 5, 7, 9, 11, 13, 15, 17, 19, 21, 23, 25\}$. The natural periods of the first two modes become 0.127 sec and 0.051 sec, which are...
indicative of a very stiff structure. A Rayleigh damping model is used to describe the damping mechanism [21], with the two coefficients $a$ and $b$ chosen as $2.117 \times 10^{-4}$ and $3.471 \times 10^{-4}$, respectively, resulting in a 3% damping ratio for the first two modes.

The accuracy of the identification is evaluated by the identification error defined below:

$$e_i = \frac{\| f_i^{\text{id}} - f_i^{\text{true}} \|}{\| f_i^{\text{true}} \|} \times 100\%$$

(29)

where $e_i$ denotes the identification error of the $i$th force ($i = 1, 2, ..., n_f$); $f_i^{\text{id}}$ and $f_i^{\text{true}}$ are the $i$th identified and actual force time histories, respectively.

Herein, we consider two cases regarding the number of vehicles passing through the bridge: (i) a single vehicle and (ii) multiple vehicles. The corresponding traffic-induced nodal excitations acting on the bottom nodes are identified. The hyperparameters of the conjugate priors used in algorithm 1 are given as follows: $\alpha_0 = \beta_0 = 1 \times 10^{-6}$ and $\alpha_1 = \beta_1 = 0.1$ kN·s. The numerical studies were carried out using MATLAB® (The MathWorks, Inc., MA, USA) on an Intel (R) Core (TM) i7-4930 K 3.40 GHz PC with 32 G RAM.

<table>
<thead>
<tr>
<th>Traffic-induced excitation</th>
<th>2% noise</th>
<th>25% noise</th>
<th>210% noise</th>
<th>20% noise</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f_1(t)$</td>
<td>6.648</td>
<td>5.452</td>
<td>6.648</td>
<td>8.761</td>
</tr>
<tr>
<td>$f_3(t)$</td>
<td>2.301</td>
<td>1.564</td>
<td>2.301</td>
<td>2.477</td>
</tr>
<tr>
<td>$f_5(t)$</td>
<td>2.153</td>
<td>1.477</td>
<td>2.153</td>
<td>2.185</td>
</tr>
<tr>
<td>$f_6(t)$</td>
<td>7.661</td>
<td>5.010</td>
<td>7.661</td>
<td>7.265</td>
</tr>
</tbody>
</table>

$^a$ $\Gamma = 1$ represents non-smoothing and $\Gamma \neq 1$ means smoothing using equation (24).

The identification errors are presented in %. 

Figure 3. Identified traffic-induced nodal excitations by a single vehicle using the rescaled data with 20% RMS noise corruption. Note that the solid lines represent the reference values and the dashed lines denote the identified values.

Figure 4. Error distributions of the identified traffic-induced nodal excitations by a single vehicle using the rescaled data under different levels of noise corruption.

Figure 5. Identified traffic-induced nodal excitations by a single vehicle using the unscaled data with 20% RMS noise corruption. Note that the solid lines represent the reference values and the dashed lines denote the identified values.
4.1. Identification of traffic-induced nodal excitations by a single vehicle

In the first case, we consider a single vehicle going across the bridge from the left to the right side with a speed of 60 km h\(^{-1}\). The vehicle has two axles inducing two moving forces (e.g., front and rear) acting on the bridge deck. The moving load is the summation of the allocated force (weight) at the axle and the road surface roughness-induced load. The road surface roughness-induced load is simulated by a white noise sequence with the mean value identical to zero and the RMS equal to 1 kN. The front axle force is 10 kN and the rear axle force is 15 kN. The spacing between the two axles is 3 m.

Table 2. The vehicle parameters, the moving speeds, and the moving directions as well as the entering time.

<table>
<thead>
<tr>
<th>Vehicle no.</th>
<th>Front load (ton)</th>
<th>Rear load (ton)</th>
<th>Axle spacing (m)</th>
<th>Speed (km h(^{-1}))</th>
<th>Entering time (sec)</th>
<th>Direction (L/R)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.0</td>
<td>1.5</td>
<td>3.0</td>
<td>75</td>
<td>0</td>
<td>L to R</td>
</tr>
<tr>
<td>2</td>
<td>1.5</td>
<td>2.0</td>
<td>4.5</td>
<td>60</td>
<td>0.5</td>
<td>L to R</td>
</tr>
<tr>
<td>3</td>
<td>1.1</td>
<td>1.6</td>
<td>3.0</td>
<td>75</td>
<td>1.0</td>
<td>L to R</td>
</tr>
<tr>
<td>4</td>
<td>2.0</td>
<td>2.9</td>
<td>6.0</td>
<td>60</td>
<td>0.2</td>
<td>R to L</td>
</tr>
<tr>
<td>5</td>
<td>1.2</td>
<td>1.8</td>
<td>3.0</td>
<td>75</td>
<td>0.8</td>
<td>R to L</td>
</tr>
</tbody>
</table>

L/R represents left/right.

Figure 6. Traffic-induced nodal excitations by multiple vehicles (5 vehicles): the moving forces of the front and the rear axles of the vehicles considering the road surface roughness and the corresponding forces applied at the bottom nodes of the truss bridge.
4.1.1. The effect of the smoothing operator

The six nodal forces induced by the vehicle are identified by the proposed algorithm through fusing heterogeneous measurements with noise corruption. The effect of the smoothing operator on the accuracy of force identification has been tested herein. Figure 3 shows the identified single vehicle-induced excitations using the rescaled measured data with 20% RMS noise corruption: figure 3(a) shows the case with the smoothing operator and figure 3(b) shows the case without the smoothing operator. It is seen from figure 3(a) that the traffic-induced nodal excitations are well identified by the proposed algorithm with the smoothing operator; even the measured data are corrupted by 20% RMS noise. Though the general trends of the excitations are fairly identified when the smoothing operator is not used (see figure 3(b)), large oscillations and errors are present in the identified time histories. Comparing figures 3(a) and (b), we observe that the smoothing operator helps de-noise the signals and reduce the oscillations and errors significantly. The overall performance of the proposed algorithm is excellent. Of note, the identified traffic-induced nodal excitations for the case of lower-level noise corruptions are not reported here, since they show an even better performance of the proposed algorithm.

Table 1 presents the identification errors of single vehicle-induced excitations under different levels of noise corruption. The corresponding identification error distributions for different cases are shown in figure 4. Again, it is observed that, in general, the smoothing operator yields smaller identification errors as compared with the non-smoothing cases. Of note, it is interesting that in the case of 5% RMS noise, some errors with the smoothing operator ($\Gamma \neq I$) are slightly larger than those without the smoothing operator ($\Gamma = I$). This phenomenon might be caused due to the noise uncertainty in the measurements. Though we assume that the noise level is identical, the noise sequence might be different. It is possible that for two sets of noise sequences, the noise amount present is larger in one noise set compared with the other one. In addition, it is noted that the maximum error produced by the proposed algorithm with the smoothing operator is close to 10% for the case of 20% RMS noise corruption, which demonstrates that the proposed algorithm is robust against measurement noise. Moreover, it is seen from figure 4 that the identification errors of $f_1(t)$, $f_2(t)$, $f_3(t)$ and $f_4(t)$ are, in general, larger than those of $f_5(t)$ and $f_6(t)$. This phenomenon is related to the quality of sensor placement, which raises a new direction for future work on optimal heterogeneous sensor placement for load identification. Attempts on optimal placement of accelerometers for unknown load identification have been done by Wang et al [22]. It is also noteworthy that the average CPU time for each case is about 220 sec, illustrating an efficient identification.

4.1.2. The effect of data rescaling

We also test the effect of data rescaling, proposed in section 2.3, on the identification of traffic-induced nodal excitations. Figure 5 illustrates the identified single vehicle-induced excitations using the unscaled data with 20% RMS noise corruption (e.g., $\alpha_1 = \alpha_2 = \alpha_3 = 1$). It is observed that distinct drifts are present in the identified force time histories, leading to large identification errors. As a result, most of the traffic-induced nodal excitations are misidentified. As illustrated in the work by Sun et al [23], when a single type of measurement is
employed, data rescaling is unnecessary and the proposed Bayesian regularization approach is able to yield a precise identification of the unknown load time histories. It is then concluded that when heterogeneous data are measured and used for force identification, rescaling the data (by setting the magnitudes of the heterogeneous response time histories to be of a similar order) is able to produce an effective and accurate regularization process. The correct regularization then yields a correct solution of the inverse problem in equation (19) as well as a robust identification of the traffic-induced nodal excitations. The proposed data rescaling approach, as shown in equations (16)–(18), is able to successfully provide an

Figure 8. Identified traffic-induced nodal excitations by multiple vehicles (5 vehicles) using the rescaled data with 20% RMS noise corruption. Note that the solid lines represent the reference values and the dashed lines denote the identified values.

Figure 9. Predicted responses of the truss bridge of unmeasured nodes and elements for the case of 20% RMS noise corruption.
accurate identification result as shown in figure 3. It proves that the rescaling of heterogeneous data is indispensable in state space-based force identification using heterogeneous data fusion.

4.2. Identification of traffic-induced nodal excitations by multiple vehicles

The second case is to test the performance of the proposed algorithm on the identification of excitations induced by multiple vehicles. We simulate five vehicles in total going across the bridge bidirectionally, three of which move from the left to the right and the other two move from the right to the left. The vehicle parameters, the moving speeds, and the moving directions, as well as the entering time, are given in table 2. The road roughness is also considered in this case, which is simulated by white noise sequences as used in section 4.1. Figure 6 illustrates the traffic-induced nodal excitations by five vehicles, including the synthetic moving forces of the front and the rear axles acting on the bridge deck, considering the road surface roughness, and the corresponding nodal excitation time histories.

Table 3 summarizes the identification errors of multiple vehicle-induced excitations under different levels of noise corruption, and figure 7 shows the corresponding error distributions using the proposed algorithm with/without the smoothing operator. Similar to the case of single traffic-induced excitation identification, the performance of the proposed algorithm is proved to be satisfactory for the identification of excitations induced by multiple vehicles. The identification errors remain quite small. For example, the maximum error is around 8% even when a large amount of noise is present in the measurements (e.g., for the case of 20% RMS noise and \( I \neq 1 \)). Again, we observe that the identification errors decrease as the smoothing operator is employed. It is noted that the CPU time for the multiple traffic-induced excitation identification is around 465 section.

Figure 8 depicts the identified traffic-induced nodal excitations by five vehicles using the rescaled data with 20% RMS noise corruption. It is observed that the identified force time histories match the actual forces quite well, despite the fact that slight oscillations are present due to the measurement noise. Furthermore, the identified traffic-induced nodal excitations can be used to predict dynamic structural responses at locations where sensors are unavailable or difficult to install. As a demonstration, the identified excitation time histories for the case of 20% RMS noise are used to predict the displacement, strain and acceleration responses. Figure 9 shows the predicted responses of the truss bridge of unmeasured nodes and elements. It can be seen that the overall agreement between the predicted and the actual (reference) responses is quite good. It is noted that the discrepancies are caused by the measurement noise. This good agreement again demonstrates the good performance of the proposed algorithm for traffic-induced excitation identification.

5. Conclusions

We present a statistical regularization approach for the identification of traffic-induced nodal excitations of truss bridges using Bayesian inference. The measured data are obtained through heterogeneous sensing, which can be acceleration, displacement and strain time histories. The heterogeneous data are fused through a state space model and rescaled for force identification. An ill-posed least square problem is formulated to relate the heterogeneous measurements and the unknown traffic-induced nodal excitations. The inverse problem is solved by a proposed Bayesian inference regularization approach. The unknown force vector and the regularization parameters are iteratively updated through Bayesian learning. A smoothing operator is encapsulated into the regularization process for the de-noising purpose.

Finally, a truss bridge with 27 bars is used to numerically validate the effectiveness and the accuracy of the proposed algorithm for traffic-induced excitation identification. Noise effects are well studied, and an excellent performance of the proposed algorithm is observed. Of note, the proposed approach can be further applied to moving load identification, provided that some vehicle information, such as the axle spacing, is known. Nevertheless, a possible direction of the future work could be the experimental study and testing the applicability of the proposed algorithm to various types of structures instrumented with heterogeneous sensors.

References