Chapter 14
Modal Strain Energy Based Damage Detection Using Multi-Objective Optimization

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Abstract Modal strain energy has been reported by researchers as one of the sensitive physical measures that can be used as a damage index in structural health monitoring. Inverse problem-solving based approaches using single-objective optimization algorithms are also one of the promising damage identification methods. However, integration of these potential methods is currently limited with partial success in the detection of structural damages due to errors and noises. In this study, a novel damage detection approach using hybrid multi-objective optimization algorithms is proposed to detect multiple damages in a 3-dimensional steel structure. This study developed an approach to overcome the shortcomings of the single-objective genetic algorithm based approaches using multi-objective formulations for minimizing errors of damage indices between actual damaged structures and simulated damages. The performance of the proposed hybrid multi-objective genetic algorithm is compared to that of traditional single-objective optimizations based approaches. This study accurately detects the location and extent of induced multiple minor damages of the laboratory 3-dimensional steel structure.

Keywords Damage detection • Modal strain energy • Inverse problem solving • Multi-objective • Genetic algorithm • Structural health monitoring

14.1 Introduction

Modal strain energy has been considered in research as a damage index because it is sensitive to damages in structures and used in assessing the extent and location of the structural damages in 2-dimmensional structures [1–3]. Petro et al. [1] showed that modal strain energy based damage detection may be more sensitive than other mode shaped based approaches. The strain energy method was applied to a space truss model to detect multiple damages [2]. The method detected severe damages, while some of the less severe damages went undetected. A damage quantification method using changes in the modal strain energy (MSE) was applied to a simple 2-story plain structure [3]. The damage quantification method proved partially successful in quantification of the structural damage in a multiple damage case with noises. An efficient algebraic algorithm of element MSE sensitivity was proposed to detect location and severity of damages [4]. However, all these studies had limitations in localizing and identifying the extent of multiple damages with low accuracy and high noise, and some of the predicted damages were localized in undamaged structural elements.

In order to detect structural damages, inverse problem solving methods have also been used. As one of the inverse problem solving methods, genetic algorithms (GA) have been widely used to detect damages as a concept of model updating. GA was used to detect damages in a simple cantilever beam using modal assurance criterion of frequencies and mode shapes [5]. A two level micro GA was applied to detect multiple damages in a simply supported single span beam and a continuous three-span beam [6]. An implicit redundant representation (IRR) GA was also applied to detect damages using a frequency response function-based damage index for simple 2-dimensinal (2-D) structures. The performances of the IRR GA and those of the simple GA were compared to detect damages using the same damage index [7]. This study showed that the performance of the IRR GA based approach was superior to those of other simple genetic algorithm based approaches. Correlation-based methods were proposed to facilitate damage determination for 3-D truss bridge using multi-layer GA [8].

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From the first GA layer, damage-suspicious elements were divided into several groups and the groups were combined to larger groups and optimization process started over at the normalized point of the result of the first layer. This multi-layer GA based approach assumed that user knew approximate locations of the damage in the first-layer. This means that if the user chooses an incorrect location of the damage-suspicious elements, the damages may be assigned the wrong locations. Most studies cited above used single-objective GAs to detect damages in simple cantilever beams or idealized 2-D structures. Moreover, these simple GA based damage detection methods showed limitations to finding multiple damages in cases of 3-D structures. In addition, many trials were necessary to find proper parameters of the genetic algorithms in order to get satisfying results. The parameters include population size, crossover and mutation rates, and type of selection method.

In an effort to overcome these limitations, the present study proposes a multi-objective formulation based damage detection method by integrating modified modal strain energy as a damage index and multi-objective genetic algorithms as an inverse problem solving method to explore damages in 3-D structures. In this study, two objective functions are formulated using modified modal strain energy for the multi-objective formulation. To demonstrate the effectiveness of the proposed approach, a 3-D structure has been developed. Minor damages such as a 5 % reduction of stiffness of the structural elements have little effect to change of the modal properties of the structure. Consequently, these minor damages with multiple locations in a structure are difficult to detect using traditional damage detection methods based on modal properties. Thus, multiple minor damage scenarios are created to evaluate the proposed damage detection method. The results of the proposed method are compared to those of the methods based on traditional single-objective genetic algorithms.

### 14.2 Damage Detection Method Based on Inverse Problem Solving

In order to detect structural damages, inverse solving concept is adopted using heuristic genetic algorithms. The overall procedures for the damage detection using proposed multi-objective GA and traditional single-objective GA are described in a flowchart shown in Fig. 14.1. A baseline structural model is developed using finite element method (FEM) as a first step. A damage-induced structural model is also developed from the baseline structure by reducing Young’s modulus of the structural members. The damage-induced model is that damages are artificially created in the baseline structure as possible damage scenarios. Mass and stiffness matrices of the both damage-induced and damage-simulated models are obtained. From the obtained matrices, mode shapes of the damage-induced and the damage-simulated structures from genetic algorithms

![Flowchart for Damage Detection Method](image)

**Fig. 14.1** Overall procedure for the damage detection using multi-objective NS2-IRR GA and single-objective GA
are calculated. Modified modal strain energy is calculated using those calculated mode shapes. The modified modal strain energy uses absolute values of mode shapes. Thus, it is more convenient to compare the modal strain energy between damage-induced model and damage-simulated model because the global direction of the mode shapes is highly dependent on location of damages. In this study, objective functions are defined as the discrepancy of the modified modal strain energy between the induced damages and the simulated damages using GAs. If the discrepancy of the two modified modal strain energy is zero, then the simulated damages are the same as the induced damages. The simulated damage scenarios are the detected damages of the structure. Thus, in order to minimize the objective functions, the stiffness matrix of the damage simulated structure is dynamically changed based on fitness of the objective functions using the two types of genetic algorithms. The optimization process is stopped when the objective functions converge to zero value, otherwise it is continued to the next generation until the objective functions are converged to zero values or required criteria are met, such as the predefined maximum number of generations.

### 14.2.1 Strain Energy Based Damage Index

The modal strain energy is more effective for detecting damage than using modal properties as a damage index [3, 8, 9]. Thus, modified modal strain energy as a damage index is proposed to detect multiple minor damages for 3-D structures. The modal strain energy (MSE) is calculated as the product of the elemental stiffness matrix and the second power of the mode shape component. For the \( j \)th element and the \( i \)th mode, the MSE before and after occurrence of damage is given as

\[
MSE_s^{ij} = \Phi_i^T K_j \Phi_i^s
\]

(14.1)

\[
MSE_d^{ij} = \Phi_i^T K_j \Phi_i^d
\]

(14.2)

where \( MSE_s^{ij} \) is the MSE of the \( j \)th element for the \( i \)th mode shape of the damage simulated and \( MSE_d^{ij} \) is those of the damage-induced structural element, respectively, and \( \Phi_i^s \) and \( \Phi_i^d \) are \( i \)th mode shapes of the damage-simulated and damage-induced structures, respectively. In order to approximate \( MSE_d^{ij} \), the undamaged elemental stiffness matrix \( K_j \), for which the global coordinate is used for each elemental stiffness matrix, is used instead of the damaged one since the damage elements are not known. In order to calculate the change in the MSE, modified MSE is suggested as,

\[
mMSE_s^{ij} = \left| \Phi_i^s \right| K_j \left| \Phi_i^s \right|
\]

(14.3)

\[
mMSE_d^{ij} = \left| \Phi_i^d \right| K_j \left| \Phi_i^d \right|
\]

(14.4)

Using the modified MSE, the change of modal strain energy (MSEC) of the \( i \)th element for the \( j \)th mode is obtained as

\[
MSEC_{ij} = \left| \Phi_i^d \right| K_j \left| \Phi_i^d \right| - \left| \Phi_i^s \right| K_j \left| \Phi_i^s \right|
\]

(14.5)

where \( j \) and \( i \) denote the element number and mode number, respectively. Thus, the total energy difference of the modal strain energy between induced damaged structure and simulated structure is expressed as

\[
\text{Total change of MSE} = \sum_{i=1}^{ms} \sum_{j=1}^{el} \left| \Phi_i^d \right| K_j \left| \Phi_i^d \right| - \left| \Phi_i^s \right| K_j \left| \Phi_i^s \right|
\]

(14.6)

where \( ms \) and \( el \) denote the total number of elements of a structure and total number of the mode shapes considered, respectively. Structural damages are eventually expressed as reduction of the structural stiffness. Thus, in order to simulate damages to any extent and at any element of the structure, Young’s modulus of each structural element is reduced from the original value using the following formulation

\[
E_j^s = (1 - \alpha_j) E_o
\]

(14.7)
where $E_o$ is the intact Young’s modulus of elasticity, $E_j$ is the damage-simulated Young’s modulus of elasticity of $j$th element, and $\alpha_j$ is a percentage of the reduction of the Young’s modulus at $j$th element of the structure. Consequently, reduction in the Young’s modulus in the entire structure is expressed as

$$\alpha = (\alpha_1, \alpha_2, \ldots, \alpha_{el})$$ (14.8)

Therefore, any extent and any location of the damages in structures can be generated based on these randomly simulated $\alpha$ by this proposed multi-objective or single-objective genetic algorithms.

### 14.2.2 Single-Objective Genetic Algorithms

Implicit redundant representation genetic algorithms (IRR GA) as a single-objective GA has been shown to perform well for detection of damages in steel structures [7]. For this reason we choose IRR GA as single-objective GA. The IRR GA was first proposed by Raich and Ghaboussi [10]. The IRR GA uses gene locator indicating starting point of a gene instance containing encoded Young’s modulus information (i.e., $\alpha$), and redundant segments containing currently non-encoded segments that can become part of the encoded gene instances in later generations. These features of the IRR GA make it dynamic changes of the optimization variables, in which the dynamic changes can prevent early converges to local optimal solutions. In order to compare the performance of the proposed hybrid multi-objective genetic algorithms with traditional single-objective GA, the objective function of the single-objective GA process is also defined as a composite type of objective function:

$$\min \text{(Objective)}$$ (14.9)

$$\text{Objective} = \sum_{i=1}^{m} \sum_{j=1}^{el} \left| \Phi_i^j \right| K_j \left| \Phi_i^j \right| - \left| \Phi_i^j \right| K_j \left| \Phi_i \right|$$ (14.10)

### 14.2.3 Hybrid Multi-Objective Genetic Algorithms

Detection of structural damages is a complex nonlinear problem. In order to localize and quantify structural damages, hybrid multi-objective NS2-IRR GA which is the integration of best features of the implicit redundant representation (IRR) GA as encoding policy and non-dominated sorting genetic algorithm-II (NSGA-II) as a selection method, is proposed. This multi-objective NS2-IRR GA was proposed to find optimal layouts of the control devices and sensors for 3-D high-rise buildings [11–13]. The IRR encoding allows the percentage reduction of the Young’s modulus to dynamically change by the actions of crossover and mutation among individuals in the same population and in future generations during the search process. The NSGA-II is used as a selection method to keep competitive candidate solutions and transfer those to next generation of the genetic iterations [14]. NS2-IRR GA uses crowding distances to calculate a fitness value of each individual to consider density of solutions and then to remove converging to local optimal solutions. The crowding distance estimating the density of individuals surrounding a particular individual in the phenotype non-dominated Pareto front, is calculated as an average distance of the two individuals on either side of this point along each of the objectives based on following equation

$$I[i]_{\text{distance}} = I[i]_{\text{distance}} + (I[i + 1]_{\text{distance}} - I[i - 1]_{\text{distance}}) \cdot m$$ (14.11)

where $m$ is the number of objectives and $I[i]_{\text{distance}}$ is the $m$th objective function value of the $i$th individual in the set $I$. Thus, current population is sorted according to a crowded comparison operator. Thereafter, only $N$ number (i.e., population size) of individuals will be selected for the next population. The crowded comparison operator is presented as following equation,

$$i \geq n \ j \ \text{if} \ (i_{\text{rank}} < j_{\text{rank}}) \ \text{or} \ ((i_{\text{rank}} = j_{\text{rank}} \ \text{and} \ i_{\text{distance}} > j_{\text{distance}}))$$ (14.12)

where $i_{\text{rank}}$ and $j_{\text{rank}}$ are non-dominated rank of the $i$ and $j$th individuals, respectively, and $i_{\text{distance}}$ and $j_{\text{distance}}$ are local crowding distance of the $i$ and $j$th individuals, respectively. The selection, crossover, and mutation operator are carried out to create a new child population from the current population. The binary tournament selection is carried out to create the child population, with predefined size $N$. These steps are continued until the defined criteria or generation is satisfied.
The hybrid multi-objective NS2-IRR GA is used in this study to detect structural damages for a 3-D structure. The change of the modified MSE between the damage induced structure and the damage simulated structure by GA process is expressed as objective functions of the NS2-IRR GA. The proposed GA minimizes the objective functions expressed as Eq. (14.6). To calculate objective functions from modified MSEs, incomplete mode shapes are used after removing rotational components which are difficult to measure. Therefore, to detect induced damage scenarios, the objective functions as a manner of multi-objective optimization are defined as follows:

\[
\text{min} \ (\text{Objective}_1, \ \text{Objective}_2) \quad (14.13)
\]

where \(\text{Objective}_1\) and \(\text{Objective}_2\) are

\[
\text{Objective}_1 = \sum_{m/s/2} \sum_{el} \left| \Phi_i^T \begin{bmatrix} K_j & \Phi_j^T \end{bmatrix} \Phi_i \right| \quad (14.14)
\]

\[
\text{Objective}_2 = \sum_{m/s/2+1} \sum_{el} \left| \Phi_i^T \begin{bmatrix} K_j & \Phi_j^T \end{bmatrix} \Phi_i \right| \quad (14.15)
\]

where \(m/s\) is the selected number of incomplete mode shapes. The rotational components of the mode shapes are difficult to measure, thus, in this study, the translational components of the mode shapes are only used to calculate change of the modal strain energy.

### 14.3 Application to Modular Steel Structures

The proposed algorithm is evaluated on synthetic data from a FEM model. To ensure that the dynamics represented by this model are realistic, the FEM model is based on a physical lab structure. The properties of this structure are as follows: The detailed material properties are given in Table 14.1. The column section is 0.25 in. (0.00635 m) \(\times\) 2 in. (0.0508 m) with a length of 24 in. (0.6096 m), and the frame section is 0.00635 m \(\times\) 0.0508 m with a length of 0.6096 m in each bay.

As a prototype structure model, a numerical 4-story 2-bay by 1-bay scale-model steel frame building structure with dimensions 0.6096 m \(\times\) 1.2192 m wide and 2.4384 m high is used. The structure model is developed using FEM with 12 \(\times\) 12 stiffness matrix and consistent mass matrix. The first ten natural frequencies for the designed structures are calculated using SAP2000 (2.2022, 4.2186, 5.5575, 6.9149, 12.078, 16.422 and 27.038 Hz) and using MATLAB (2.2086, 4.5295, 5.6076, 7.0849, 12.8728, 18.5872, and 27.8020 Hz). Those separately calculated natural frequencies showed good-agreement with each other.

#### 14.3.1 Multiple Minor Damage Scenario Case 1

In order to compare the performance of the single-objective GA and multi-objective GA approaches, a damage scenario having multiple damages in multiple locations is generated. A minor damage, which causes only small effects of change to the global mode shapes of the structure, is hard to detect using traditional modal property based damage detection methods. Five percent of the stiffness reduction of the structural elements 1 and 10 and structural elements 1, 10, 17, 33, and 52 is generated by reducing Young’s modulus using Eq. (14.8) in the prototype 4-story structure as shown in Fig. 14.2a, b, respectively. The italic and non-italic numbers are the node numbers and element numbers of the structure, respectively. Induced damages in structural elements are expressed as thick dashed red lines. In order to calculate modified MSE, and to eventually calculate predefined objective functions [i.e., Eqs. (14.10, 14.14, and 14.15)], first two mode shapes with

<table>
<thead>
<tr>
<th>Table 14.1 Member material properties</th>
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<tbody>
<tr>
<td>Structural element</td>
</tr>
<tr>
<td>---------------------</td>
</tr>
<tr>
<td>Frame for 2-bay</td>
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<tr>
<td>Column</td>
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For the single-objective GA, IRR GA is chosen because of its effective performance in detection of various damages in 2-D plain steel structures and its superiority compared to other simple GAs [7].

The IRR GA does not give exact locations and extent of the induced damages from several trials by changing GA properties such as population size and tournament selection sizes. Figure 14.3 shows the trials of the single-objective IRR GA and its best fit solutions during generations are plotted. Even though the IRR GA continued to more than 400 generations with a population size of 500, it does not give optimal solutions as shown in Fig. 14.4. In order to obtain an optimal solution from single-objective IRR GA, the best fitness of the objective function, expressed as Eq. (14.10), should be zero value which means that the simulated damages using GA are the same as the induced damages. The final damages detected from the IRR GA using 500 for the population size and 2 for the size of the tournament selection are plotted in Fig. 14.4. Even though the locations of the induced damages are at structural element 1 and 10 with each 5% reductions of the member stiffness, the single-objective GA failed to detect locations and extent of those induced damages with large errors and noises.
However, using a multi-objective NS2-IRR GA approach, all the induced damages are detected with correct locations and extent of the damages as shown in Fig. 14.5. The multi-objective NS2-IRR GA used 500 for population size, which is the same as that of single-objective IRR GA. It used a total of 130 generations to detect all induced structural damages. It should be noted that the trials of the single-objective IRR GA are $500 \times 400 = 200,000$, and the trials of the multi-objective NS2-IRR GA are $500 \times 130 = 65,000$. The total possible number of damage scenarios of the optimization domain is $101^{52}$ because each structural element has 101 cases of the damage scenarios from 0 to 100% damage scenarios with 1% discrete unit, and the 4-story prototype structure has 52 structural elements, expressed as non-italic numbers in Fig. 14.2. Thus, it should be noted that the multi-objective NS2-IRR GA uses a much smaller number of trials and offers exact solutions. Multi-objective NS2-IRR GA tried only $65,000/101^{52}$ cases (i.e., $3.8744e-098$% of the total possible damage scenarios). It is worth mentioning that the suggested multi-objective NS2-IRR GA has significant performance in solving this nonlinear optimization problem. The multi-objective NS2-IRR GA uses a crowding distance schema [i.e., Eq. (14.11)] and crowded operation selection method [i.e., Eq. (14.12)] in the step of assigning fitness values for the next generation of the GA to each individual and selection step to keep well fit individuals (i.e., solutions). These advanced fitness assigning methods and selection schema may prevent individuals of the population in the multi-objective NS2-IRR GA from premature convergence to local optimal solutions by keeping various individuals which have competitive design variables of the optimization [i.e., $\alpha_1 \alpha_1 \ldots \alpha_{el}$ from Eq. (14.8)].

14.3.2 Multiple Minor Damage Scenario Case 2

In order to do extensive investigations of the suggested methodologies of the structural damage detection, multiple minor damage scenario having damages in five different locations is examined as shown in Fig. 14.2b. All damages are expressed as thick dashed red lines in the structure. To investigate the robustness of the suggested damage detection methodology, damage locations are well scattered in the entire structures.

For the 4-story prototype structure, multi-objective NS2-IRR GA is applied with a population size of 1,000, where tournament selection size is 2, crossover rate is 0.9, and mutation rate is 0.01. In order to calculate two objectives of the hybrid multi-objective NS2-IRR GA, the first eight mode shapes are used with only global translational X and Y components of the mode shapes. For the Objective 1, the first, third, fifth and seventh mode shapes are used, and for the Objective 2 the second, fourth, sixth, and eighth mode shapes are used. Figure 14.6 shows near optimized population in the 1,000th generation. The near optimized individuals are well converged to zero values of the both objectives. Then, the $\alpha$ of the individual, which has minimum summation values of the both objectives, is plotted as locations and extent of the detected damages. Even though 1% errors in structural elements 1, 10, and 17 are found and three 1% noises are also found as shown in Fig. 14.7, the overall induced damages are well detected. In order to remove these negligible minor errors and noises, the optimization problem domain is reduced to maximum 10% possible damages based on currently detected damage information. With this reduced optimization domain, the population is fast converged to zero values of the both objectives, and one of the individuals is converged to zero values of the both objectives as shown in Fig. 14.8. The $\alpha$ of the fully converged individual to zero values for the both objectives is plotted in Fig. 14.9. The hybrid multi-objective NS2-IRR GA with same GA properties, which are used in previous full damage domain, detected all the induced multiple minor damages, as shown in Fig. 14.9.
14.4 Conclusion

Modal strain energy as a damage index is widely used to detect structural damages. To localize and quantify damages, single-objective genetic algorithms have been used as a method of inverse problem solving. However, these approaches have provided less satisfying results to detect damages in 3-dimensional structures. In this paper hybrid multi-objective genetic algorithms are proposed as a damage detection method by solving inverse problem to minimize change of the modified modal strain energy in each structural element. In order to investigate the performance of the proposed method, 3-dimensional 2-bay
4-story modular steel structures are designed and numerically modeled using finite element methods. The performance of the proposed approach is compared to those of the traditional approach using single-objective genetic algorithm. The newly proposed approach showed significantly better performance in detecting multiple minor damages, which have little effect to change of the modal properties of the structure. The newly proposed approach used incomplete mode shapes containing only global translational components to remove difficulties of measuring rotational components of the mode shapes. For the future study, physical laboratory experimental model will be used to validate proposed damage detection method using various damage scenarios.

Acknowledgement The authors acknowledge the support provided by Royal Dutch Shell through the MIT Energy Initiative, and thank chief scientists Dr. Dirk Smit, Dr. Sergio Kapusta, project manager Dr. Yile Li, and Shell-MIT liaison Dr. Jonathan Kane for their oversight of this work. Thanks are also due to James Long for his help with finalizing this paper.

References