On the Effect of Processing Energy on Multiple Access Channels*

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Abstract

Energy efficiency is an important issue in mobile wireless networks since the battery life of mobile terminals is limited. We are considering, besides the energy consumed for transmission, the energy consumed by being in the ‘on’ state, which we term ‘processing energy’. This energy constitutes a considerable fraction of the total energy available to battery-limited wireless devices, hence it is capital to take it into account in the capacity calculations. In [1], we have discussed the impact of processing energy on the capacity of a single user Additive White Gaussian Noise (AWGN) channel and proved that burstiness of signaling can be capacity achieving. We first show that in low SNR regime, there is an equivalence between the effect of channel variability and that of processing energy on capacity. Then, we extend the analysis of [1] to the AWGN multiple access channel with \( M \) senders and a single receiver. We, then, show that Time Division Multiple Access outperforms other multiple access techniques for the purpose of maximizing the sum rate. We prove that, for the same purpose, burstiness is capacity achieving in the low SNR regime when the sum of the ratios of total energy to processing energy is less than unity; in this case, the 2-user capacity region is rectangular. Finally, we present numerical results that show the shape of the general 2-user achievable capacity region under a processing energy constraint and we compare it to the TDMA curve and the Cover-Wyner region obtained when no burstiness is allowed.

1 Introduction

1.1 Processing Energy in the Single User Case

We have studied in [1] the capacity of a single user AWGN channel in the presence of a processing cost, which we have denoted by \( \epsilon \). We let \( \mathcal{E} \) be the total energy available to the transmitter. The question we ask is the following: Given a certain total amount of energy \( \mathcal{E} \), how would an increase of the processing energy, \( \epsilon \), affect the optimal transmission mode? Clearly, since the optimal strategy should avoid paying too much overhead owing to the processing energy cost, sending more bursty signals would result in higher rates.

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On the other hand, transmission should not be too bursty, since putting all the energy in one slot may result in a loss of rate. Therefore, there is a certain tradeoff between sending bursty signals and adopting a continuous transmission strategy. This tradeoff is controlled by the values of $\mathcal{E}$ vs. $\epsilon$.

For low SNR, we proved in [1] that

$$\mathcal{E} \approx \Theta_{opt} \sqrt{2\epsilon}. \quad (1)$$

where $\Theta_{opt}$ is the optimal burstiness of signaling, i.e. that would maximize the transmission rate. When $\Theta_{opt} = 1$, a constant transmission strategy is being used; when $\Theta_{opt} < 1$, bursty signals are being transmitted. The smaller the $\Theta_{opt}$, the burstier the transmission.

Then, we proved that, in low SNR regime, burstiness is capacity achieving when the total energy available to the sender is less than the square root of double the processing energy, i.e. when

$$\mathcal{E} \leq \sqrt{2\epsilon}. \quad (2)$$

Since we have normalized the noise variance, $\mathcal{E}$ will be interpreted as the Signal to Noise Ratio (SNR).

### 1.2 Background on Non-Coherent Channels

This section refers to some of the results in [6], [7] and [8]. We consider wideband wireless communication over a channel where the transmitter has no channel state information (CSI). The coherent model assumes perfect channel knowledge; and the non-coherent i.i.d. fading model assumes not only no CSI at the receiver, but also that the channel changes so rapidly that there is no hope at all to obtain even partial channel knowledge based on the previously received signal.

From a signaling point-of-view, at low SNR, the estimation of the fading coefficients becomes difficult. It is therefore desirable to use peaky signaling, that is, to use only a small fraction of the available degrees of freedom, and avoid estimating the other fading coefficients. In contrast, if the fading coefficients are known, it is desirable to spread the transmitted energy over all the available time-frequency slots, in order to reduce the SNR per degree of freedom and obtain the optimal energy efficiency. Thus, the choice of the optimal peakiness of the input signals, which we denote by $\delta$, is a key difference between the coherent and the non-coherent cases. Then, as in [6],

$$C(SNR) = SNR - \Delta(SNR) \quad (3)$$

Intuitively, in the case that partial channel information is available, the channel estimation becomes easier; when the channel changes over time slowly, a channel estimate can be used for a longer time. The effect of channel variation on the relation between capacity and SNR has been shown in [6] to be captured by a coefficient, $\beta$, which is termed coherence level. The coherent channel with perfect CSI at the receiver corresponds to the case that $\beta = 1$, and the i.i.d. non-coherent channel corresponds to the case that $\beta = 0$. We consider block fading channels with an intermediate coherence level $\beta \in [0, 1]$.

The rate of channel time variation is characterized by the coherence time, denoted by $T$. In a block fading model, the channel fading coefficients are assumed to remain constant within a block of $T$ symbol periods. The parameter $T$ is therefore used to indicate how slow the channel changes over time.
It has been shown in [6], that for the special case of $\beta = 1$, $T = \text{SNR}^{-2}$. It has also been shown that for any $\beta \in [0, 1]$,

$$T = \text{SNR}^{-2\beta} \quad \text{and} \quad \delta = \text{SNR}^{1-\beta}. \quad (4)$$

As $\beta$ decreases, transmission must become increasingly peaky to be capacity achieving.

2 Correspondence Between Processing Energy and Channel Variability

In 1.1 and in 1.2, we have shown that processing energy and channel variability both require peaky distribution for achieving capacity. In this section, we show that, in the low SNR regime, there is a simple correspondence between processing energy and channel variability. When $T$ is large, the lack of knowledge is small and the cost we pay for not knowing the channel is small. Inversely, when $T$ is small, the channel changes more often and the cost to pay increases. Therefore, the changing rate of the channel can be quantified by $1/T$. We can picture the lack of knowledge of the channel as an extra cost to pay and, most importantly, represent this cost as an energy penalty.

Let us consider side-by-side two channels, channel 1 and channel 2. The first is characterized by a processing energy $\epsilon$. The second is characterized by a level of coherence $\beta$ and a coherence time $T$. Both channels have the same SNR. Let us assume that

$$\frac{1}{\epsilon} = T. \quad (5)$$

For channel 1, when $\Theta_{opt} = 1$, we have from (1) $\text{SNR} = \mathcal{E} = \sqrt{2\epsilon}$. If we neglect the $\sqrt{2}$ factor, we obtain $\text{SNR} \approx \sqrt{\epsilon}$. For channel 2, when $\beta = 1$, (i.e. $\delta = 1$), from (4), $T = \text{SNR}^{-2}$, or $\text{SNR} = \frac{1}{\sqrt{T}}$. Consequently,

$$\text{SNR} \approx \sqrt{\epsilon} \quad \text{and} \quad \text{SNR} = \frac{1}{\sqrt{T}}. \quad (6)$$

which, by our assumption of similar SNR in both channels, only holds when (5) holds. On the other hand, from (1), again neglecting the $\sqrt{2}$ factor, we have for channel 1

$$\Theta_{opt} = \frac{\text{SNR}}{\sqrt{2\epsilon}} \approx \frac{\text{SNR}}{\sqrt{\epsilon}}. \quad (7)$$

But, for channel 2, using (5) and (4) when $\beta \in [0, 1],

$$\sqrt{\epsilon} = \sqrt{\frac{1}{T}} = \text{SNR}^\beta \quad (8)$$

Therefore, substituting (8) in (7), we obtain the burstiness of signaling of channel 1 as a function of the level of coherence for channel 2: $\Theta_{opt} = \text{SNR}^{1-\beta}$. Finally, for channel 2, recall that from (4) the optimal peakiness $\delta = \text{SNR}^{1-\beta}$. Hence,

$$\Theta_{opt} = \text{SNR}^{1-\beta} = \delta = \text{SNR}^{1-\beta} \quad (9)$$

So the burstiness of signaling as defined for channels with additional processing energy is the same as the peakiness of signaling as defined for channels with a certain variability or a coherence level. On the other hand, the lack of knowledge of the channel can be interpreted as an energy penalty.
3 Multiple Access Channels Under Processing Cost

3.1 Background on Multiple Access Channels

The capacity region of the time sampled AWGN channel above with energy constraint $E_i$ for user $i$ under no processing energy is the subset of $\mathbb{R}^M$ containing the rate-tuples $(R_1, \ldots, R_M)$ with nonnegative components satisfying

$$\sum_{i \in S} R_i \leq \frac{1}{2} \log(1 + \sum_{i \in S} E_i), \forall S \subseteq \{1, \ldots, M\}.$$ 

Some points in this capacity region are known to be achievable by successive cancellation. These are the points which satisfy

$$R_i \leq \frac{1}{2} \log(1 + E_i \sigma^2).$$

The energy constraint for user $i$ is given by

$$\sum_{k=1}^n E_{i,k} \leq E_i$$

where $E_{i,k} = E[X_i^2]$ and $X_i^k$ is the input to the channel from sender $i$ at time $k$, and $n$ is the largest time index. For the 2-user case, we show the capacity region in figure 1. More background on multiple access channels and intuition about the Cover-Wyner region can be found in [4], [5] and [2].

![Figure 1: The Cover-Wyner Region](image)

3.2 Inclusion of the Processing Energy in the Energy Constraint

Let us first consider the single user case. Since, in [1], we have shown that burstiness can be capacity achieving, we model the channel as $Y = aX + N$, where $a$ is a binary random variable taking the value 1 or 0 according to whether the sender is ‘on’ or ‘off’, all other variables being Gaussian random variables.[3] In order to find the capacity of the channel, we aim to maximize

$$I(a, X; Y) = I(a; Y) + I(X; Y | a).$$ (10)

Let $\Theta = P(a = 1)$. Then,

$$I(X; Y | a) = \Theta I(X; Y | a = 1) + (1 - \Theta)I(X; Y | a = 0) = \Theta I(X; Y | a = 1)$$ (11)

$$I(a, X; Y) = I(a; Y) + \Theta I(X; Y | a = 1)$$ (12)

$$= I(a; Y) + \Theta \log(1 + \frac{E_i}{\Theta} - \epsilon)$$ (13)

$$\leq H(a) + \Theta \log(1 + \frac{E_i}{\Theta} - \epsilon) = -\Theta \log \Theta + \Theta \log(1 + \frac{E_i}{\Theta} - \epsilon).$$ (14)

In the sequel, we neglect the first term, i.e. the incremental information conveyed by the mere fact of transmitting. For high SNR, clearly the second term dominates. For low and regular SNR, a more difficult proof is needed. Therefore, the solution we obtain in
this paper for the capacity region in the multiple access case gives at least the achievable rate region.

Let $\mathcal{A}$ be the set of active users. For the case of two users, $\mathcal{A} = \{00, 01, 10, 11\}$. For instance, '01' means that user 1 is OFF while user 2 is ON.

Let $Q_k^A$ be the indicator function taking the value 1 if at time $k$, the given combination of active users out of the set $\mathcal{A}$ is true. $E^i$ is the total energy available for user $i$ and $P_k^A(i)$ is the actual transmitted energy of this user at time $k$, given the set of active users.

Now, if the processing energy is taken into account and is given by $\epsilon^i$ for user $i$, the system that we would like to solve becomes

$$\max \quad \frac{1}{n} \sum_{k=1}^{n} \sum_{A} Q_k^A \log(1 + \frac{\sum_{i=1}^{M} Q_k^A P_k^A(i)}{\sigma^2})$$

$$\text{s.t.} \quad \frac{1}{n} \sum_{k=1}^{n} \sum_{A} Q_k^A (P_k^A(i) + \epsilon^i) \leq E^i.$$  

Note that in writing (16), we assumed that the transmitter and the receiver have agreed on the time at which transmission should occur.

When we transmit, i.e. when $Q_k^A = 1$, the concavity of the function $\log(1 + x)$ implies that $P_k^A(i)$ should, at any time $k$ for which $Q_k^A = 1$, be equal to a constant, say $\nu^A(i)$.

We denote the burstiness of signaling by

$$\Theta^A = \frac{1}{n} \sum_{k=1}^{n} Q_k^A.$$  

The constraint in (16), relaxed to equality, becomes

$$\sum_{A} \Theta^A (\nu^A(i) + \epsilon^i) = E^i.$$  

4 Superiority of TDMA for Maximizing the Sum Rate

There are several multiple access techniques for accessing a shared AWGN channel. In this section, we will briefly review TDMA, FDMA and CDMA.

In Time Division Multiple Access (TDMA), different users are assigned to different time slots. Let $\alpha$ be the fraction of time that user 1 transmits. User 1 will then transmit with an energy $\frac{E_1^1}{\alpha}$. Therefore, $R_1 = \frac{1}{2} \alpha \log(1 + \frac{E_1^1}{\alpha \sigma^2})$. Similarly, for user 2, $R_2 = \frac{1}{2} (1-\alpha) \log(1 + \frac{E_2^1}{(1-\alpha) \sigma^2})$. The TDMA rates are suboptimal except at one point where the TDMA curve touches the dominant face.

In what follows, we will prove that under processing energy, Time Division Multiple Access (TDMA) achieves a better sum rate than the 45 degree line of the Cover-Wyner region that we obtain by merely removing the processing energy from the total available energy. For simplification, we will consider the two-user case: User 1 will have a rate that he achieves by himself ($R_{alone}$) and a rate achieved when both users are transmitting ($R_{shared}$). These two rates can be expressed as follows:

$$R_{alone} = \theta^{10} \log(1 + \frac{\nu^{10}(1)}{\sigma^2}) \quad \text{and} \quad R_{shared} = \theta^{11} \log(1 + \frac{\nu^{11}(1) + \nu^{11}(2)}{\sigma^2})$$  

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A similar reasoning can be applied to user 2. Therefore the sum rate would be given by:

\[ R_1 + R_2 = \theta_{10} \log(1 + \frac{\nu_{10}(1)}{\sigma^2}) + \theta_{01} \log(1 + \frac{\nu_{01}(1)}{\sigma^2}) + \theta_{11} \log(1 + \frac{\nu_{11}(1) + \nu_{11}(2)}{\sigma^2}) \]  \hspace{1cm} (20)

In order to find the maximum sum rate, we shall solve the following system:

\[
\begin{align*}
\max \theta_{10} \log(1 + \frac{\nu_{10}(1)}{\sigma^2}) & + \theta_{01} \log(1 + \frac{\nu_{01}(1)}{\sigma^2}) \\
& + \theta_{11} \log(1 + \frac{\nu_{11}(1) + \nu_{11}(2)}{\sigma^2}) \\
\text{s.t.} \quad \theta_{10} (\nu_{10}(1) + \epsilon^1) & + \theta_{11} (\nu_{11}(1) + \epsilon^1) \leq \epsilon^1 \\
\theta_{01} (\nu_{01}(2) + \epsilon^2) & + \theta_{11} (\nu_{11}(2) + \epsilon^2) \leq \epsilon^2 \\
\theta_{01} + \theta_{10} + \theta_{11} & \leq 1.
\end{align*}
\]  \hspace{1cm} (21)

Consider the case where both senders are ‘ON’, i.e. when \( \theta_{11} > 0 \). The rate achieved is \( \log(1 + \frac{\nu_{11}(1) + \nu_{11}(2)}{\sigma^2}) \). In the Cover-Wyner region, this rate is the one obtained from the 45 degree line with user \( i \) having an energy of \( \nu_{11}(i) \). At the same time, this rate can be obtained by using TDMA with each sender transmitting a fraction \( \alpha \) of the time, where \( \alpha = \frac{\nu_{01}(1)}{\nu_{10}(2) + \nu_{01}(1)} \). This proof is given in [9]. This argument proves that obtaining this rate does not really necessitate that both users be ‘ON’ since we can alternate their transmissions with the convenient shares of energies and achieve the same total rate. Therefore, we can increase \( \theta_{10} \) and \( \theta_{01} \) and set \( \theta_{11} \) to 0.

Now, we will investigate the effect of processing energy on the maximum sum rate. In this case, TDMA gives an advantage over the non-processing scenario since TDMA will save us part of the processing energy. For simplification, assume unit noise variance and symmetric users having the same \((\epsilon, \epsilon)\). We know that \( R_1 + R_2 = \log(1 + 2\epsilon - 2\epsilon) \). Consider that, instead of allowing the channels to send together, TDMA is used. Then, the rates are as follows

\[ R_1 = \alpha \log(1 + \frac{\epsilon}{\alpha} - \epsilon), \quad R_2 = (1 - \alpha) \log(1 + \frac{\epsilon}{1 - \alpha} - \epsilon). \]  \hspace{1cm} (25)

Since we are in the symmetric case, TDMA achieves the maximum sum rate when \( \alpha = 0.5 \). Therefore, the total sum rate is \( \log(1 + 2\epsilon - \epsilon) \). Comparing the sum rate to the sum of the rates in (25), we see that TDMA achieves a sum rate larger than the one achieved by allowing \( \theta_{11} \) to be non-zero. Therefore, we can always achieve a better sum rate by using TDMA in the presence of processing energy.

In the \( M \)-user case, the more we allow users to send together, i.e. \( \theta_{11} \rightarrow 1 > 0 \), the more we expend unnecessary processing energy. Using TDMA will allow only one user to send at a time and only one processing energy to be expended. Therefore, the benefit of TDMA will increase as more and more users share the medium as it is saving \((M - 1)\) times the processing energy.

At the same time, the effect of processing energy will decrease since whatever the number of users are transmitting, only one processing energy is expended. Recognizing that for the purpose of maximizing the sum rate, \( \theta_{11} \) should be set to 0, we can go back and solve the original system in (21), which is repeated below.

\[
\begin{align*}
\max \theta_{10} \log(1 + \frac{\nu_{10}(1)}{\sigma^2}) & + \theta_{01} \log(1 + \frac{\nu_{01}(1)}{\sigma^2}) \\
\text{s.t.} \quad \theta_{10} (\nu_{10}(1) + \epsilon^1) & \leq \epsilon^1
\end{align*}
\]  \hspace{1cm} (26)

\[ \theta_{01} (\nu_{01}(2) + \epsilon^2) + \theta_{11} (\nu_{11}(2) + \epsilon^2) \leq \epsilon^2 \]  \hspace{1cm} (23)

\[ \theta_{01} + \theta_{10} + \theta_{11} \leq 1. \]  \hspace{1cm} (24)
\[ \theta^{01}(\nu^{01}(2) + \epsilon^2) \leq \mathcal{E}^2 \]  
\[ \theta^{01} + \theta^{10} \leq 1. \]  
(28)  

Using Lagrange multipliers, the first constraints will be biting. Therefore, \( \nu^{10} = \frac{\epsilon_1}{\sigma^1} - \epsilon_1 \) and \( \nu^{01} = \frac{\epsilon_2}{\sigma^2} - \epsilon_2 \). Taking the derivative with respect to \( \theta^{10} \) and \( \theta^{01} \) and setting them to 0, we obtain the optimal burstiness of signaling for each of the users:

\[
\Theta^{10} = \frac{\mathcal{E}^1 W(e^{\frac{\epsilon^1 - \sigma^2}{\sigma^1 e}})}{(\epsilon^1 - \sigma^2)(W(e^{\frac{\epsilon^1 - \sigma^2}{\sigma^1 e}}) + 1)} \quad \text{and} \quad \Theta^{01} = \frac{\mathcal{E}^2 W(e^{\frac{\epsilon^2 - \sigma^2}{\sigma^2 e}})}{(\epsilon^2 - \sigma^2)(W(e^{\frac{\epsilon^2 - \sigma^2}{\sigma^2 e}}) + 1)}.
\]  
(30)

We denoted by \( W \) the Lambert \( W \) function which is defined as:

\[ \text{Lambert} W(x) \times e^{\text{Lambert} W(x)} = x. \]  
(31)

But, from the last constraint, we need to satisfy \( \theta^{01} + \theta^{10} \leq 1 \). Recognizing that,

\[
\lim_{x \to e^{-1}} W(x) = -1, \quad \lim_{x \to 0} W(x - e^{-1}) = -1 \quad \text{and} \quad \lim_{x \to 0} \frac{W(x - e^{-1}) + 1}{\sqrt{x}} = \sqrt{2e},
\]  
(32)

we obtain the condition for burstiness in the low SNR regime, i.e. as \( \epsilon^i \to 0 \), or \( \Theta^i \to 0 \)

\[
\frac{\mathcal{E}^1}{\sqrt{2\epsilon^1}} + \frac{\mathcal{E}^2}{\sqrt{2\epsilon^2}} < 1.
\]  
(33)

This result can be easily generalized to the \( M \)-user case. In the low SNR regime, burstiness becomes capacity achieving when

\[
\sum_{i=1}^{M} \frac{\mathcal{E}^i}{\sqrt{2\epsilon^i}} < 1.
\]  
(34)

Remarks:

- If (34) is verified, then the capacity region is rectangular. Consider a time slot from 0 to 1. Then, if user 1 only needs 25% of the time slots and user 2 needs 45%, and as long as the two percentages sum to less than 1, the two users do not interfere together, as each one of them will occupy different time slots.
- For high SNR, the capacity region without processing is almost triangular, and TDMA asymptotically follows the 45 degree line, covering almost the whole capacity region. So, even with no processing energy, TDMA can be used with almost no loss in capacity. In the presence of processing energy, TDMA will be favored at the maximum sum rate, i.e. for the whole 45 degree line which constitutes most of the capacity region. So taking TDMA to be the capacity region under processing energy is reasonable.

### 4.1 Graphical Illustration

We illustrate the effect of processing energy on the system where all users are only allowed to transmit at all times, under the constraint of additional processing energy, i.e. the Cover-Wyner region obtained by simply subtracting the processing energy from the total energy available. This is the region obtained with maximum rates \( \log(1 + \mathcal{E}^1 - \epsilon^1) \),
Figure 2: TDMA and the Cover-Wyner region with no burstiness allowed under processing energy

\[
\log(1 + E^2 - \epsilon^2) \quad \text{and sum rate} \quad \log(1 + E^1 + E^2 - \epsilon^1 - \epsilon^2).
\]

The TDMA curve is obtained by using \((R_1, R_2) = (\alpha \log(1 + \frac{E^1}{\alpha} - \epsilon^1), (1 - \alpha) \log(1 + \frac{E^2}{1-\alpha} - \epsilon^2))\) and varying \(\alpha\) between 0 and 1. As argued before, the maximum sum rate is achieved by TDMA in the presence of processing energy. This is illustrated in figure (2). In the high-SNR case, TDMA is very close to the Cover-Wyner region and the effect of processing energy is clearly going to make TDMA achieve a higher sum rate.

5 Capacity Region

In this section, we aim to find the shape of the whole capacity region in the presence of processing energy. For this purpose, we fix \(R_2\) and aim to maximize \(R_1\). We know the maximum value that \(R_2\) can take (say \(R_{2\text{max}}\)), which is achieved when user 2 transmits by himself. Let the fixed value of \(R_2\) be \(x\), where \(x < R_{2\text{max}}\). We approximate the rates \(R_1\) and \(R_2\) by

\[
R_1 = \Theta^{10} \log(1 + \nu^{10}(1)) + \beta \Theta^{11} \log(1 + \nu^{11}(1) + \nu^{11}(2))
\]

(35)

\[
R_2 = \Theta^{01} \log(1 + \nu^{01}(2)) + (1 - \beta) \Theta^{11} \log(1 + \nu^{11}(1) + \nu^{11}(2)) = x.
\]

(36)

Now, using (36), we can express \(R_1\) in terms of \(x\) as follows

\[
R_1 = \Theta^{10} \log(1 + \nu^{10}(1)) + \frac{\beta}{1 - \beta} (x - \Theta^{01} \log(1 + \nu^{01}(2))).
\]

(37)

\(R_1\) should be maximized subject to \(\Theta^{01} + \Theta^{10} + \Theta^{11} < 1\) and \(\Theta^{10}(\nu^{10}(1) + \epsilon_1) + \Theta^{11}(\nu^{11}(1) + \epsilon_1) \leq E_1\). Using Lagrange multipliers, this maximization problem can be solved and a closed form solution could be obtained for the maximum \(R_1\) in terms of \(R_2\). The equations , while compact, do not present closed-form evaluations, so we have resorted to numerical methods. We take \(E^1 = 0.8, E^2 = 0.7\) and \(\epsilon^1 = 0.3\) and \(\epsilon^2 = 0.2\). We compare the region obtained with the Cover Wyner region obtained with no burstiness allowed with the new transmission scheme described in this paper and the TDMA as described in 4.1.
Figure 3 shows that the achievable rate region, shown in blue, gives great improvements over the Cover-Wyner region with no burstiness allowed (shown in black). On the other hand, the red curve shows that TDMA allows to achieve the maximum sum rate.

Figure 3: Capacity region with no burstiness allowed, TDMA and the achievable rate region under processing energy constraint

6 Conclusion

We prove that there is a direct correspondence between the processing cost and the variability of the channels, that allows to interpret the lack of perfect coherence as an energy penalty. Moreover, the peakiness of signaling in channels with an intermediate coherence level is the same as the burstiness of signaling in AWGN channels with a cost. The other part of the paper extends the results of the single user AWGN channel with a cost to the Multiple Access Channels. We only consider the mutual information conditioned on the indicator of when we actually transmit although it is in fact possible to use the indicator itself to convey information; this can be done by having a random place
to turn on and off and using the detection of this to carry information. The results we obtain give at least an achievable capacity region which readily gives great improvements over the Cover-Wyner region with no burstiness allowed. We show that TDMA is optimal in achieving the maximum sum rate. We then find the condition for which burstiness is optimal for $M$ users sharing the channel. When this condition is verified, we know the capacity region will be rectangular. Finally, we present numerical results under general conditions and illustrate the shape of the achievable capacity region under processing energy.

References


