

# Asset Prices and Real Investment

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Final Draft: April, 2004

## Abstract

Firm investment activity and firm characteristics, particularly the market-to-book ratio or  $q$ , are functions of the state of the economy and therefore contain information about the dynamic behavior of stock returns. This paper develops a model of a production economy in which real investment is irreversible and subject to convex adjustment costs. During low- $q$  (high- $q$ ) periods when the irreversibility constraint (constraint on the rate of investment) is binding, conditional volatility and expected returns on one hand, and market-to-book ratios on the other, should be negatively (positively) related. Empirical tests based on industry portfolios support these predictions for conditional volatility but not for expected returns.

JEL classification: G1, D5, E2, C0

Keywords: Investment; Irreversibility; General equilibrium; Leverage effect; Book-to-market

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## 1. Introduction

Basic features of the real investment process can determine the dynamic properties of stock returns. One of the best known characteristics of real investment is that it is often irreversible. New capital is industry-specific and little value can be recovered after the original investment has been made. Investment also does not occur instantaneously and costlessly; various frictions and physical limitations restrict the investment rate. I study how these constraints on the investment process determine firms' investment decisions, their output dynamics, and ultimately the behavior of their stock prices. Intuitively, the properties of stock returns depend on the state of the real side of the economy. Measures of firms' real activity, such as their investment rate, and firm characteristics, such as Tobin's  $q$  or the market-to-book ratio, are informative about the state of the economy and emerge as natural predictors of the dynamic behavior of stock returns. My analysis suggests that investment frictions can naturally generate a highly nonlinear pattern of stock return behavior. The sign of the relation between the conditional moments of returns and the market-to-book ratio changes from negative to positive depending on whether the latter is relatively low or high. The same is true for the relation between returns and investment. This suggests that one may need to move beyond the commonly used linear empirical specifications to capture some important conditional effects in returns. My empirical tests support this conclusion.

To capture the effect of investment frictions on asset prices, I model capital accumulation within an equilibrium model in which real investment is irreversible and the investment rate is bounded. Specifically, I adopt the general equilibrium model of investment developed in Kogan (2001). It is a two-sector model, in which one of the sectors is subject to constraints on investment. The behavior of this constrained sector provides insight into the impact of investment frictions on stock prices. The model incorporates two types of investment constraints in a tractable manner: investment is restricted to be irreversible and the rate of investment is bounded from above. The former constraint is standard in the investment literature (e.g., Dixit and Pindyck, 1994). The latter is a special case of the standard convex adjustment cost specification and ensures that capital adjustments cannot occur instantaneously.

In contrast to the traditional exchange-economy setting, the supply of capital in my model is endogenously determined and its elasticity depends on economic conditions. This, in turn, determines the time-varying properties of stock returns. To ensure market clearing in equilibrium, either supply or prices must adjust in response to a demand shock. If supply is inelastic, e.g., due to investment frictions, then prices must change. Thus, during periods when the capital stock of the second sector significantly exceeds the optimal level and the irreversibility constraint is particularly severe, the supply of new capital is relatively inelastic and prices of financial assets must adjust to absorb the shocks. Therefore, asset prices are relatively volatile and sensitive to market-wide uncertainty, i.e., returns have higher systematic risk. On the other hand, when investment is about to take place, the supply of capital is relatively elastic and is capable of partially absorbing the shocks, reducing the volatility and systematic risk of stock returns. The second constraint on the investment rate has the opposite effect, following similar logic. When the optimal level of the capital stock exceeds the current level, capital accumulation is hindered by the investment constraint and therefore is relatively inelastic, increasing the variability of stock returns.

While one might not directly observe the difference between the actual and optimal levels of the capital stock or which of the investment constraints is binding, one can infer this to some extent from firm's real economic activity, e.g., from the rate of real investment. Firm characteristics also provide information about the state of the economy. Specifically, within the model, Tobin's  $q$ , or the market-to-book ratio, is a sufficient statistic for the state of the economy and the investment policy. The irreversibility constraint is binding when  $q$  is less than one and the instantaneous investment rate is equal to zero. In this low- $q$  regime, the model implies a negative relation between the conditional moments of returns on one hand and the investment rate and the market-to-book ratio on the other hand. This relation changes sign in the high- $q$  regime, when investment takes place but the investment rate is constrained.

The empirical properties of stock returns and the informative role of firm characteristics, such as the market-to-book ratio, have been extensively analyzed in the literature. However, our understanding of the observed empirical relations has been relatively limited. In one of the few attempts to formally explain the predictive ability of firm characteristics, Berk

(1995) argues that one can observe a mechanical relation between the market-to-book ratio and expected returns due to the effect of expected returns on the stock price (or, put simply, because both variables contain the stock price in their definitions). However, his model is static in nature and does not point to a specific economic mechanism behind time-variation in expected returns. Moreover, it imposes assumptions on the cross-sectional correlation between cash flows and expected returns, which need not hold when both variables are determined endogenously in a fully specified equilibrium setting, as indicated by the conditional nature of my results. In this paper, I suggest a simple economic mechanism generating the informativeness of the market-to-book ratio. My model also predicts the relations between the market-to-book ratio and conditional volatility and between conditional moments of stock returns and investment, which cannot be obtained within the simple setting of Berk (1995).

This paper is also closely related to the work by Berk, Green, and Naik (1999), who investigate the role of firm size and market-to-book ratios in explaining the cross-sectional patterns in stock returns by explicitly modelling firms' real activity. Theirs is a partial equilibrium model, in which the composition of a firm's assets affects the risk of its stock returns and consequently links expected stock returns to firm characteristics. In their model, firms with riskier projects have higher expected stock returns and lower market-to-book ratios. As the authors readily admit, their model ignores the equilibrium feedback of firms' activities on market prices. Such an equilibrium effect is precisely the driving force behind my results. Gomes, Kogan, and Zhang (2001) analyze the cross-sectional properties of stock returns in a general equilibrium setting and trace the link between market-to-book ratios and expected returns to firms' idiosyncratic profitability characteristics. This paper identifies a different reason for the informativeness of the book-to-market ratio by endogenizing the time-varying supply of capital at the industry level. In a recent paper, Zhang (2003) extends the model with adjustment costs by incorporating idiosyncratic productivity shocks at the firm level, overcoming the aggregation challenge using modern numerical techniques. He argues that the spread between conditional betas of value and growth stocks behaves countercyclically and therefore is positively correlated with the market risk premium, which is also countercyclical. This helps in explaining the value premium in stock returns quantitatively. Two

recent papers by Cooper (2003) and Carlson, Fisher, and Giammarino (2003) also attempt to explain the value premium in stock returns using an investment-based model. Both papers link  $q$  to expected returns through operating leverage. This mechanism is quite distinct from the logic of this paper. My model does not rely on operating leverage, since there are no production costs, and instead investigates the effect of investment frictions on equilibrium asset prices.

There is a small finance literature examining the effects of convex adjustment costs on the behavior of prices of financial assets. This includes Basu and Chib (1985), Huffman (1985), Basu (1987), Balvers, Cosimano, and McDonald (1990), Dow and Olson (1992), Basu and Vinod (1994), Naik (1994), Rouwenhorst (1995), Benavie, Grinols, and Turnovsky (1996), and others. These are single-sector general-equilibrium models aiming to address the behavior of the aggregate stock market. Out of these, Naik (1994) is the model best suited to incorporate irreversibility of investment. His focus, however, is on the effects of exogenous changes in output uncertainty on the price of aggregate capital and the aggregate risk premium. Coleman (1997) works out a discrete-time, general-equilibrium model with two sectors and irreversible investment. He concentrates on the dynamic behavior of the short-term interest rate and its relation to sectoral shocks.

The link between real investment and stock prices is also implicit in many macroeconomic models, but is rarely explored. In a recent paper, Hall (1999) stresses the importance of modeling endogenous capital accumulation and uses data on stock prices to extract information about the stock of capital in the economy. Cochrane (1991, 1996) considers a partial-equilibrium model of production and performs an empirical analysis based on producers' first-order conditions. In his model, investment is reversible subject to quadratic adjustment costs. In the first paper, he uses arbitrage arguments to impose restrictions on investment returns and tests the model empirically. His second paper provides some supporting empirical evidence on the extent to which investment returns can explain the variation in expected returns on financial assets in a conditional dynamic asset-pricing model.

The relations between stock returns and firm properties have been relatively overlooked in the literature on real investment, which is concerned with the determinants of real investment. Many of the existing models designed to capture the impact of uncertainty on

investment are of partial equilibrium nature. Such models assume constant volatility of output prices or demand shocks and perform comparative statics experiments in which volatility changes across the economies.<sup>1</sup> To test such comparative statics results empirically, one must implicitly assume that a similar time-series relation between investment and uncertainty holds. Motivated by these theoretical results, Leahy and Whited (1996) establish empirical links between investment, market-to-book ratios, and the volatility of stock returns. Because changes in uncertainty about a firm's future economic environment are not directly observable, they use the conditional volatility of stock returns as a convenient empirical proxy for the uncertainty of the firm's environment. In contrast, I analyze a dynamic equilibrium model in which both the conditional volatility of stock returns and investment are determined endogenously. The conditional volatility of stock returns is naturally time-varying as a function of the current state of the economy. My model has a single state variable which is not directly observable, but which can be replaced by empirically observable proxies, such as the market-to-book ratio or the investment rate. Therefore, my model gives rise to explicit and theoretically consistent dynamic relations between these real variables and the conditional volatility and systematic risk of stock returns.

The paper is organized as follows. In Section 2, I describe the main features of the equilibrium in an economy with irreversible investment and develop asset-pricing implications of irreversibility. In Section 3, I extend the basic setup to incorporate additional constraints on investment. In Section 4, I evaluate the properties of the model using numerical simulation and conduct empirical tests. Section 5 is the conclusion.

## 2. The basic model

In this section I describe the main features of the model. I start with the basic version of the model, incorporating only investment irreversibility and ignoring other adjustment frictions, solved in Kogan (2001). This setup allows for a simple explicit characterization of equilibrium price dynamics. I then introduce convex adjustment costs in the form of an upper bound on the investment rate.

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<sup>1</sup>See, for example, Abel and Eberly (1994), Caballero (1991), Dixit (1991), and Pindyck (1988). Dixit and Pindyck (1994) provide a comprehensive review of the literature.

The focus of my analysis is on an industry that is subject to constraints on investment. To this end, I consider a two-sector production economy. Each of the sectors has its own capital good and produces a sector-specific perishable consumption good. The technology of the first sector exhibits constant returns to scale: a unit of capital good at time  $t$  is transformed into  $1 + \alpha dt + \sigma dW_t$  units at time  $t + dt$ . There is no distinction between the capital good and the consumption good within the first sector. The first consumption good can be used for production within the first sector, for consumption, and for investment into the second sector. I assume that investment into the second sector is irreversible: once a unit of capital has been transferred from the first to the second sector, it can only be used for production within the second sector. Thus, the capital stock of the first sector changes according to

$$dK_{1,t} = (\alpha K_{1,t} - c_{1,t})dt + \sigma K_{1,t}dW_t - dI_t, \quad (1)$$

where  $c_{1,t}$  is the consumption rate of the first good and  $dI_t$  denotes the investment process. By assumption of irreversibility,  $dI_t \geq 0$ .

The capital stock of the second sector changes only due to investment and depreciation, i.e.,

$$dK_{2,t} = -\delta K_{2,t} dt + dI_t, \quad (2)$$

where  $\delta$  is the rate of depreciation. Thus, by assumption, one unit of the first good can be used to create one unit of the second capital good. The latter, on the other hand, can only be used to produce the second consumption good. In the benchmark model, the investment rate  $dI_t/K_{2,t}$  is unconstrained. In the full version of the model below, I impose an additional constraint on the rate of investment,  $dI_t/dt \in [0, i_{\max} K_{2,t}]$ , which is a special case of a standard convex adjustment cost specification. I assume a constant-returns-to-scale technology, so that the total output of the second sector is given by  $c_{2,t} = XK_{2,t}$ , where  $X$  is a productivity parameter. I set  $X$  to one without loss of generality. The first sector of the model can be interpreted as the bulk of the economy, which in the aggregate behaves as if investment were perfectly reversible. The second sector in the model captures investment constraints at the level of a single industry. To simplify the analysis, I assume that the technology of the second sector is riskless, which by construction implies that the

two sectors in the economy exhibit different levels of risk. This is not problematic since the focus of this analysis is on the implications of the model for time-variation in returns of the second sector. The cross-sector comparison of the dynamic properties of returns is still meaningful, while the cross-sector differences in the level of volatility and expected returns reflect the fact that the two sectors operate different technologies.

The second sector consists of a large number of competitive firms. These firms own their capital, and they all use identical production technology and make investment decisions to maximize their market value. I assume that all firms are financed entirely by equity and the total number of shares outstanding in the second sector is normalized to one.  $P_t$  denotes the corresponding share price at time  $t$ , measured in terms of units of the first consumption good. It is not necessary to model firms within the first sector explicitly. Instead, I assume that households are allowed to invest directly into the production technology of the first sector. Thus, even though most of the capital in the economy might belong to the first sector, when discussing stock prices I will focus on the firms in the second sector, which are facing investment constraints.

Households can invest in stocks of firms in the second sector, as well as into the production technology of the first sector and a single locally risk-free asset. They choose their consumption-portfolio plans to maximize the expected lifetime utility of consumption

$$E_0 \left[ \int_0^{\infty} e^{-\rho t} \left( \frac{1}{1-\gamma} c_{1,t}^{1-\gamma} + \frac{b}{1-\gamma} c_{2,t}^{1-\gamma} \right) dt \right].$$

### 2.1. The competitive equilibrium

The equilibrium in this economy is well defined as long as the model parameters satisfy the following technical conditions (see Kogan, 2001, Proposition 1):

$$\alpha(\gamma - 1) + \rho > 0, \quad \gamma < 1, \quad (3)$$

$$\min \left( \alpha(\gamma - 1) - \frac{\sigma^2}{2} \gamma(\gamma - 1) + \rho, \rho - \delta(\gamma - 1) \right) > 0, \quad \gamma \geq 1. \quad (4)$$



An additional restriction

$$\alpha + \delta - \left( \frac{\gamma - 1}{\gamma} \alpha - \frac{\sigma^2}{2} (\gamma - 1) + \frac{\rho}{\gamma} \right) - \frac{\sigma^2}{2} > 0 \quad (5)$$

ensures stationarity of the key economic variables in the model (see Kogan, 2001, Proposition 2).

A closed-form characterization of the equilibrium is not available. However, assuming that the preference parameter  $b$  is sufficiently small, one can derive explicit asymptotic approximations to equilibrium prices and policies. Under this assumption, the second sector is relatively small compared to the rest of the economy most of the time. To interpret the results of the model under this simplifying assumption, one should think of the second sector as a particular industry, which is relatively small compared to the rest of the economy. This is not the only industry for which investment constraints could be relevant, i.e., irreversibility does not have to be limited to a small subset of firms in the economy. However, to keep the model tractable, I only model one such industry explicitly, and capture the bulk of the economy as the first sector with very simple production and investment technologies; see Kogan (2001) for further discussion. Note also that while the explicit expressions for equilibrium prices are valid asymptotically, when the second sector is relatively small, the qualitative properties of stock returns are based on the equilibrium conditions for optimality of firm investment and market clearing, and are therefore more general.

Finally, note that when the second sector is relatively small, it is natural to measure asset prices in terms of units of the first consumption good, as assumed above.

I now summarize the main properties of the equilibrium; the details of the asymptotic solution method and the properties of the equilibrium investment policy can be found in Kogan (2001). The state of the economy can be characterized in terms of a single state variable  $\Xi_t \equiv b^{-1/\gamma} K_{2,t}/K_{1,t}$ , or  $\xi_t \equiv \ln(\Xi_t)$ . Investment takes place when the capital stock of the second sector falls sufficiently low relative to the capital stock of the first sector. Equivalently, investment becomes optimal when  $\Xi_t$  reaches the critical value  $\Xi^*$  and investment prevents the state variable  $\Xi_t$  from falling below the threshold  $\Xi^*$ . Asymptotically, the equilibrium

investment threshold is given by

$$\begin{aligned}
\Xi^* &= \Xi_{(0)} + O(b^{1/\gamma}), \\
\Xi_{(0)} &= \left( \frac{\lambda_1}{\lambda_2} \frac{\kappa - 1}{\kappa - (1 - \gamma)} \right)^{-1/\gamma}, \\
\kappa &= \frac{-p_1 - \sqrt{p_1^2 - 4p_2p_0}}{2p_2}, \\
\lambda_1 &= \left( \alpha \frac{\gamma - 1}{\gamma} - \frac{\sigma^2}{2}(\gamma - 1) + \frac{\rho}{\gamma} \right)^{-\gamma}, \\
\lambda_2 &= (-\delta(\gamma - 1) + \rho)^{-1}, \\
p_0 &= -\lambda_1^{-1/\gamma}, \quad p_1 = -\alpha - \delta + \frac{2\gamma - 1}{2}\sigma^2 + \lambda_1^{-1/\gamma}, \quad p_2 = \frac{\sigma^2}{2}
\end{aligned}$$

and the dynamics of the state variable is given by

$$d\xi_t = \left( -\frac{\alpha}{\gamma} - \delta + \frac{\rho}{\gamma} + \sigma^2 \left( 1 - \frac{\gamma}{2} \right) \right) dt - \sigma dW_t + O(b^{1/\gamma}).$$

## 2.2. Stock returns, $q$ , and investment

One can characterize the equilibrium investment process in terms of marginal  $q$ , defined as the ratio of the market value of the marginal unit of capital to its replacement cost. In equilibrium, investment takes place when  $q_t$  reaches one, otherwise investment is not optimal. Since all firms are competitive and invest whenever  $q_t \geq 1$ ,  $q$  never exceeds one in equilibrium, otherwise cumulative aggregate investment would become infinite and markets wouldn't clear. Thus,  $q_t \leq 1$  and investment takes place only when  $q_t = 1$ . In my model, marginal  $q$  coincides with Tobin's  $q$ , or average  $q$ , which is equal to the ratio of the market value of a firm to the replacement cost of its capital (empirically approximated by the market-to-book ratio). Therefore, the value of any firm in the second sector is given by the product of  $q$  and its capital stock.

To understand how irreversibility affects the prices of financial assets, consider a single firm in the second sector. By definition, the market value of the firm can be computed as a product of its average  $q$  and the replacement cost of its capital:  $P_t = q_t K_{2,t}$ . Over an infinitesimal time interval  $dt$ , the cumulative return to the firm's owners can be represented

as

$$\frac{\pi_t dt + q_t dK_{2,t} + K_{2,t} dq_t - dI_t}{P_t} = \frac{\pi_t dt}{P_t} + \frac{dK_{2,t}}{K_{2,t}} + \frac{dq_t}{q_t} - \frac{dI_t}{q_t K_{2,t}}, \quad (6)$$

where  $\pi_t$  denotes the rate of cash flows from the sales of the firm's output,  $dK_{2,t}$  and  $dq_{2,t}$  denote changes in the firm's capital stock and  $q$ , respectively, and  $dI_t$  is the cumulative investment over the time interval  $dt$ . Eq. (6) uses the fact that changes in the firm's capital stock are instantaneously deterministic. In general, one should use Itô's formula to derive the analogous equation. Using (2), one can reduce (6) to

$$\frac{\pi_t dt}{P_t} - \delta dt + \frac{dq_t}{q_t} + \frac{dI_t}{K_{2,t}} - \frac{dI_t}{q_t K_{2,t}}.$$

Since investment takes place only when  $q_t = 1$ , this in turn equals

$$\left( \frac{\pi_t}{P_t} - \delta \right) dt + \frac{dq_t}{q_t}. \quad (7)$$

Because all firms in the second sector behave competitively, if investment were perfectly reversible, the market value of capital would be identical to its replacement cost and the last term in eq. (7) would be absent, which is the case for the first sector in the economy. Because investment in the second sector is irreversible,  $q_t$  can deviate from one, directly affecting stock returns.

One effect of irreversibility can be seen in the time-variation of the first term in (7). A firm's profits depend on business conditions in general and on the market price of its output in particular. Given the downward-sloping demand curve, the market price of a unit of output is a decreasing function of the total output of the industry, which in turn depends on the amount of capital in the second sector. Since the process of capital accumulation is constrained by firms' inability to disinvest, so is total industry output and, ultimately, firms' profits.

The volatility and risk of stock returns are driven by the second term in (7). Because of irreversibility, the volatility of the market value of capital changes over time. This is the main implication of the model. Intuitively, when the demand for new capital rises, i.e., the industry experiences a positive demand shock, either the price of capital has to increase to

offset an increase in demand, or the supply of new capital must increase, i.e., firms must invest. When the current market value of capital is much less than its replacement cost, i.e.,  $q \ll 1$ , it is not optimal for firms to invest and therefore demand shocks must be absorbed by prices, i.e., stock prices must be volatile. Thus, qualitatively, the state space can be partitioned into two regions:

**“Non-binding” irreversibility constraint.**  $q \simeq 1$ . An increase in  $q$  would prompt firms to invest, therefore the supply of risky assets (capital) is relatively elastic. As a result,  $q$  is not very sensitive to shocks and the magnitude of the last term in (7) can be expected to be relatively low. This implies that the conditional volatility of stock returns is relatively low. Moreover, due to low sensitivity of  $q$  to systematic market-wide shocks, systematic risk and expected stock returns are relatively low.

**“Binding” irreversibility constraint.**  $q \ll 1$ . Firms are unlikely to invest in the short run and irreversibility prevents them from disinvesting. Thus, the supply of capital is relatively inelastic, leading the price of installed capital to absorb the shocks and making it more variable.  $q$  is sensitive to market-wide shocks, implying higher systematic risk and expected returns.

Thus, a relation emerges between  $q$ , real investment, and returns. In times when  $q$  is low, the irreversibility constraint is particularly severe and investment is unlikely to take place. Such periods are associated with relatively low elasticity of supply of new capital and hence with relatively high volatility and systematic risk of returns. Furthermore, since expected stock returns in my model are directly linked to their systematic risk, this gives rise to a negative time-series relation between  $q$  and expected returns. The informativeness of  $q$  (or the market-to-book ratio) stems from the fact that it is a sufficient statistic for the state of the economy in my model.

The above argument ties the properties of stock returns to the behavior of  $q$ . Formally, optimality conditions for the firms’ problem imply that  $q(\xi^*) = 1$ , i.e., firms invest whenever the market value of capital exceeds its replacement cost. Optimality of firm behavior together with the assumption of instantaneous upward adjustment of the capital stock imply that  $q'(\xi^*) = 0$ . To see this, start with the fact that  $q \leq 1$  in equilibrium, because all firms find it optimal to invest at an unbounded rate whenever  $q \geq 1$ . Now, imagine that  $q'(\xi^*)$  is not

equal to zero. Since  $q$  is equal to one at  $\xi^*$  and cannot exceed one for  $\xi \geq \xi^*$ , it must be that  $q'(\xi^*) \leq 0$ . If a firm invests whenever  $\xi = \xi^*$ , it cannot expect its  $q$  to increase in the near future, since its  $q$  is already equal to one. But were the state variable  $\xi$  to decline in the next instant, the firm's market value would fall, since  $q'(\xi^*)$  is negative. Such an expected decline in the market value of the firm would dominate any benefit from profits collected over a short period of time (formally, this is because over a short time period  $\Delta t$ , cumulative profits are of order  $\Delta t$ , while the capital gains/losses are of order  $\sqrt{\Delta t}$ ). Under such conditions a firm would not find it optimal to invest at  $\xi^*$ , which confirms that  $q'(\xi^*)$  must be equal to zero.

The above argument also suggests that even if it were not possible to invest at an infinite rate, the downward pressure on  $q$  would still imply that the upside due to capital gains is limited, hence the downside and the slope of  $q(\xi)$  at  $\xi^*$  would have to be limited accordingly. Thus, in a full version of the model incorporating constraints on the investment rate, the variability of  $q$  and returns would still be negatively related to  $q$  for values of  $q$  less than one. Nor does the above argument rely on the one-factor structure of the model either. Even if the state of the economy were driven by several state variables,  $q$  would affect stock returns in a similar fashion: the variability of  $q$ , and hence the conditional volatility and systematic risk of stock returns, would be higher for relatively low values of  $q$ .

### 2.3. *Conditional volatility and $q$*

One can now characterize the properties of the conditional volatility and other moments of returns. Note that in my model there is no distinction between systematic and idiosyncratic volatility, and model implications apply to both types of risk. For instance, in the model, the conditional market beta of returns is asymptotically proportional to the conditional volatility. The conditional mean of stock returns is in turn asymptotically proportional to the conditional market beta. Thus, formally, all of the results for the conditional volatility of returns can be directly applied to the behavior of the conditional expected returns. Note, however, that while the properties of the conditional volatility of returns are driven primarily by the structure of investment frictions, the implications of the model for the expected returns are sensitive to several simplifying assumptions, e.g., that the conditional moments of

aggregate stock market returns are asymptotically constant (and equal to the corresponding parameters of the production process of the first sector) and that aggregate market returns are asymptotically perfectly correlated with aggregate consumption growth. In summary, the strongest and most direct implications of the model are for the properties of the conditional volatility of returns. Similar implications for expected returns are secondary, more fragile, and harder to test, due to their dependence on the stylized nature of the model and the well-known empirical challenges in estimating expected returns. Therefore, below I focus mostly on the properties of the conditional volatility of returns.

The stock price of firms in the second sector is equal to  $P = q K_2$ , where

$$q = \frac{\lambda_2}{\lambda_1} e^{-\gamma \xi} + \frac{\kappa}{\lambda_1} \frac{A_{(0)}}{1 - \gamma} e^{(\kappa-1)\xi - \kappa \xi^*} + O(b^{1/\gamma}), \quad \xi^* = \ln(\Xi^*), \quad (8)$$

$$A_{(0)} = \frac{\lambda_2 \gamma (1 - \gamma)}{\kappa(\kappa - 1)} \left( \frac{\lambda_1}{\lambda_2} \frac{\kappa - 1}{\kappa - (1 - \gamma)} \right)^{1-1/\gamma}.$$

The conditional volatility of stock returns  $\sigma_R$  is given by  $\sigma |q'(\xi)/q(\xi)|$ . According to (8),

$$\sigma_R = \gamma \sigma \frac{e^{(1-\gamma)(\xi-\xi^*)} - e^{\kappa(\xi-\xi^*)}}{e^{(1-\gamma)(\xi-\xi^*)} + \frac{\gamma}{\kappa-1} e^{\kappa(\xi-\xi^*)}} + O(b^{1/\gamma}). \quad (9)$$

The relation between the conditional volatility of returns and  $q$ , given by (8), is negative. One can interpret (9) as a familiar “leverage effect” (e.g., Black, 1976): a decline in stock prices leads to increased volatility of returns. Of course, since firms in the model are financed entirely by equity, this theoretical result is not driven by financial leverage. Instead, the model suggests that the negative relation between lagged returns and volatility can arise as a result of the negative contemporaneous relation between the conditional volatility and  $q$ , as discussed above. To illustrate the relation in (9) graphically, I plot the conditional volatility against  $q$  in Fig. 1.

I use the same set of parameters as in Kogan (2001). The technological parameters of the first sector are set to  $\alpha = 0.07$  and  $\sigma = 0.17$ . When the first sector represents the bulk of the economy, these parameter values imply that the first two moments of aggregate stock market returns match their historical values (see Campbell, Lo, and MacKinlay, 1997, Table 8.1). Following Cooley and Prescott (1995), I set the depreciation rate of capital  $\delta$  to 5%

and the subjective time-preference parameter  $\rho$  to 5%. For the preference parameter  $\gamma$ , I consider a range of values:  $1/2$ ,  $1$ , and  $3/2$ . These values satisfy the technical conditions (3–5), under which the equilibrium in the economy is well defined and stationary. I assume that the preference parameter  $b$  is sufficiently small, so that the leading terms in asymptotic expressions provide accurate approximations to the true values; see Kogan (2001, Section 2.3.3) for a numerical assessment of the accuracy of the asymptotic expansions). Therefore, I do not specify the exact value for  $b$ .

Fig. 1 shows a monotonic negative relation between conditional volatility and  $q$ , while Fig. 2 shows a monotonic positive relation between the investment rate and  $q$ . The model suggests several testable implications for the behavior of the conditional volatility of returns and investment:

- (i) firms'  $q$  should be negatively related to the instantaneous conditional volatility of returns and should serve as a predictor of return volatility in the near future;
- (ii) a similar relation should hold between  $q$  and conditional expected returns;
- (iii) a relatively low investment rate should be contemporaneously associated with a relatively high volatility of realized returns and should predict high volatility of returns.

The third prediction of the model can be related to the empirical findings of Leahy and Whited (1996), who investigate the dependence of the investment rate on the uncertainty faced by the firm. In particular, they regress the rate of investment and the market-to-book ratio on the predictable component of return variance, which in turn is defined as a one-year-ahead forecast of variance. Leahy and Whited report that a 10% increase in the predicted return variance leads to a 1.7% fall in the rate of investment and a 5% decline in  $q$ .<sup>2</sup> They then conclude that firms respond to an increase in uncertainty about the future economic environment by reducing investment. Note, however, that using the volatility of stock prices as a proxy for underlying uncertainty can be problematic. My analysis provides an alternative interpretation of their findings. In my equilibrium model, the investment rate declines not because the uncertainty faced by the firm increases (the volatility of output prices is approximately constant for small values of  $b$ ), but rather in response to a fall in  $q$ .

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<sup>2</sup>Leahy and Whited (1996) adjust their measures of volatility for financial leverage. This is consistent with the definitions in the model, where firms are financed entirely by equity.

At the same time, a decline in  $q$  leads to an increased volatility of stock returns, generating a negative predictive relation between uncertainty and investment.

Below, I concentrate on the first two implications, that when the market-to-book ratio is relatively low, it should be negatively related to conditional volatility and expected returns. Before testing these implications empirically, I present a full version of the model, on which an empirical formulation is then based.

### 3. The full model

In this section I modify the basic model by relaxing the assumption of an instantaneous rate of adjustment of the capital stock. This extension of the basic model is considered in Kogan (1999, Section 3) and represents a particularly tractable case of convex adjustment costs. Specifically, assume that the investment rate is bounded from above. Specifically, capital stocks follow

$$dK_{1,t} = (\alpha K_{1,t} - c_{1,t})dt + \sigma K_{1,t}dW_t - i_t dt, \quad (10)$$

$$dK_{2,t} = -\delta K_{2,t}dt + i_t dt, \quad (11)$$

where  $i_t \in [0, i_{\max} K_{2,t}]$ . Under the optimal policy, investment takes place at the highest possible rate when the capital stock in the industry falls below the critical threshold, i.e.,  $i_t = i_{\max}$  when  $\xi_t \leq \xi^*$ , otherwise investment rate is equal to zero,  $i_t = 0$ . This policy is qualitatively similar to the investment policy of the original model, as shown in Fig. 3.

Another deviation from the basic case is that  $q$  can now exceed one. This is due to the fact that when  $q$  exceeds one, firms can only invest at rate  $i_{\max}$ , therefore  $q$  does not revert to one instantaneously. At the same time, when  $q$  is below one, its dynamic properties are driven by the same economic mechanism as in the basic model. To illustrate the resulting behavior of stock returns, I plot the conditional volatility of returns as a function of  $q$  in Fig. 4. Note that the behavior of conditional volatility for  $q \leq 1$  closely resembles that in the basic case, Fig. 1. The main difference is that now  $q$  can exceed one and volatility is no longer a monotonic function of  $q$ . Qualitatively, the state space can be partitioned into the following three regions.



**First region.** Low values of  $q$ :  $q \ll 1$ . Firms do not invest and irreversibility prevents them from disinvesting. Thus, the elasticity of supply is relatively low and stock returns are relatively volatile.

**Second region.** Intermediate values of  $q$ :  $q \simeq 1$ . Firms are either about to invest, following an increase in  $q$ , or are already investing at the maximum possible rate and are about to stop, following a decline in  $q$ . The elasticity of supply is relatively high and, as a result,  $q$  is not sensitive to shocks and does not contribute much to stock returns.

**Third region.** High values of  $q$ :  $q \gg 1$ . The industry is expanding. Firms are investing at the maximum possible rate and are likely to continue investing during an extended period of time. Demand shocks do not immediately change the rate of entry into the industry, the elasticity of supply is low, and demand shocks are offset mostly by changes in the output price. Thus,  $q$  is relatively volatile and so are stock returns.

When the rate of investment is allowed to be very high,  $q$  rarely exceeds one. Thus, the third regime can be observed only infrequently, during periods of active growth in the industry. In the extreme case of instantaneous adjustment, as in the basic model, this regime is completely absent. The first two regimes, however, can still be identified.

In summary, the model with adjustment costs in addition to irreversibility predicts that there must be a nonmonotonic relation between  $q$  and conditional moments of returns. When  $q$  is relatively low, the relation should be negative, but it should change to positive when  $q$  is relatively high.

## 4. Empirical analysis

### 4.1. Data

I perform my analysis at the level of industry portfolios.<sup>3</sup> The portfolios are formed monthly from May 1963 through December 2001, for a time series of 464 observations. The industry and size portfolios consist of all NYSE, Amex, and Nasdaq stocks on the Center for Research in Security Prices (CRSP) tapes. Stocks are sorted into 13 industry portfolios based on two-digit Standard Industrial Classification (SIC) codes as reported by CRSP. As

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<sup>3</sup>I thank Jonathan Lewellen for sharing this data. See Lewellen (1999) for a detailed account of data construction.

an empirical proxy for the industry  $q$ , I use the market-to-book (M/B) ratio. Specifically, for each industry portfolio, value-weighted returns are calculated using all stocks with CRSP data, and M/B ratios are calculated as an inverse of the value-weighted B/M ratios, which in turn are calculated from the subset of stocks with Compustat data. In my regressions, I use monthly portfolio returns and the natural logarithm of the M/B ratio.

#### 4.2. Conditional volatility

In light of the discussion in the previous section, the econometric procedure used for estimation must be sufficiently flexible to allow the sign of the relation between  $q$  and conditional moments of returns to change with  $q$ . Therefore, to estimate the relation between conditional volatility and market-to-book, I specify the following time-series model:

$$|R_{i,t} - \bar{R}_i| = a_{i,0} + a_{i,1}(M/B)_{i,t-1} + a_{i,2}(M/B)_{i,t-1}^2 + \epsilon_{i,t}. \quad (12)$$

Here  $R_{i,t}$  denotes the excess monthly return on the industry portfolio  $i$ , obtained by subtracting the one-month T-bill rate from the portfolio return,  $\bar{R}_i$  denotes the sample mean, and the market-to-book ratio is measured as a deviation from its mean. At a high frequency of observation, the conditional expectation of the absolute value of return should provide a reasonable measure of the conditional volatility of returns. By allowing for a second-order term in the dependence on M/B, one can test the implication of the model for the conditional relation between return volatility and  $q$ . The model predicts that  $a_{i,2} > 0$ .

For a slightly different perspective, I also consider a piece-wise linear specification of conditional volatility:

$$|R_{i,t} - \bar{R}_i| = a_{i,0} + a_{i,1}(M/B)_{i,t-1}^- + a_{i,2}(M/B)_{i,t-1}^+ + \epsilon_{i,t}, \quad (13)$$

where  $(M/B)^-$  and  $(M/B)^+$  denote the negative and positive parts of M/B respectively. This specification captures the conditional relation between return volatility and  $q$  by allowing the slope on M/B to depend on whether the latter is above or below its mean. Note that I have assumed that the slope of the relation between volatility and M/B changes at the mean

value of  $M/B$ . As long as this is a satisfactory assumption, the model predicts that  $a_{i,1} < 0$  and  $a_{i,2} > 0$ .

Table 1 reports the estimates of the coefficients of the time-series model (12). All point estimates of the coefficients  $a_{i,2}$  are positive. A joint test rejects the null hypothesis that all the coefficients  $a_{i,2}$  are equal to zero and the average value of the coefficients across the 13 industries is positive and significant. Thus, the relation between the conditional volatility and the  $M/B$  ratio indeed appears to be nonlinear in the way suggested by the theoretical model.

I also estimate an alternative specification, given by (13). The advantage of this specification over (12) is that it can be used to test explicitly for whether the relation between conditional volatility and  $q$  changes sign, not simply whether it is convex. The drawback is that the specification (13) assumes that the sign change must occur close to the mean value of  $M/B$ . If this assumption is violated, the model might be misspecified. The results reported in Table 2 are again supportive of the theoretical predictions. All the point estimates of the coefficient  $a_{i,1}$  are negative. The average value of the estimates is significant and the null that all the coefficients are equal to zero is rejected at conventional levels. This implies that when  $M/B$  is relatively low, the relation between conditional volatility and  $M/B$  is negative. The latter changes sign for relatively large values of  $M/B$ . The point estimates of the coefficients  $a_{i,2}$  are mostly positive, their average value is positive and significant, and the null that all the values are equal to zero is rejected. The magnitudes of the effects are also economically significant. The standard deviation of the long-run distribution of the  $M/B$  ratios is estimated to be between 0.3 and 0.5 for the 13 industries. Thus, as the  $M/B$  ratio deviates from its mean by one standard deviation, the conditional expectation of the absolute value of returns is expected to change by 0.3–1% for an average portfolio. The unconditional expectation of the absolute value of returns is close to 4% for most portfolios. Thus, the estimates in Table 2 indicate that the conditional volatility of returns varies substantially in response to changes in the  $M/B$  ratio.<sup>4</sup>

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<sup>4</sup>As a robustness check, I control for the GARCH(1,1) estimates of return volatility when estimating relations (12) or (13). The qualitative results remain unchanged and magnitudes of the effects are still statistically and economically significant. This suggests that the  $M/B$  ratio contains nontrivial information about return volatility, above and beyond what can be inferred from the return series itself using a basic

### 4.3. Conditional expected returns

To test the predictions of the theoretical model for conditional expected returns, I estimate the following time-series model:

$$R_{i,t} = a_{i,0} + a_{i,1}(M/B)_{i,t-1} + a_{i,2}(M/B)_{i,t-1}^2 + \epsilon_{i,t}, \quad (14)$$

where  $R_{f,t}$  is the one-month T-bill rate. As in (12), I allow the dependence on M/B to be nonmonotonic. According to the model,  $a_{i,2} > 0$ . It is well known that predictive regressions of this type may be biased in small samples, as long as the regressor is highly correlated with the independent variable. Lewellen (1999) reports the results for a linear regression of excess stock returns on the M/B ratio, which is equivalent to restricting the coefficient  $a_{i,2}$  to zero in (14). He adjusts for the finite-sample bias both analytically and using Monte Carlo simulation and finds at best weak evidence for a negative relation between expected returns and the M/B ratio. Since changes in  $(M/B)^2$  are practically uncorrelated with portfolio returns, I report unadjusted point estimates in Table 3. I find no pattern in the sign of the coefficients  $a_{i,2}$  across the portfolios, the average value is statistically indistinguishable from zero and one cannot reject the null hypothesis that all coefficients are simultaneously equal to zero. Thus, the data does not seem to support the model's prediction of a non-monotone relation between expected returns and the M/B ratio.

## 5. Conclusion

In this paper I analyze the effects of irreversibility of real investment on the behavior of financial asset prices within a general equilibrium model. The interaction between the demand side and the supply side of the economy leads to a structural relation between real economic variables and the properties of stock returns. As a result, the conditional volatility of stock returns is stochastic and is a function of the state of the economy. Because of investment frictions, the market-to-book ratio (Tobin's  $q$ ) plays an important role within the model. It serves as a proxy for the state of the economy and is informative about the

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GARCH model. Details are available upon request.

conditional moments of stock returns. In particular, irreversibility and adjustment costs lead to a nonmonotonic relation between conditional volatility and the market-to-book ratio. This relation is negative when the market-to-book ratio is relatively low, and is positive when the latter is relatively high. I test the predictions of the model empirically and find that its implications for the conditional volatility of returns are supported by the data, while there is no evidence for the predicted patterns in expected returns.

Table 1

Conditional volatility, quadratic specification. Estimates of the coefficients of the model  $|R_{i,t} - \bar{R}_i| = a_{i,0} + a_{i,1}(M/B)_{i,t-1} + a_{i,2}(M/B)_{i,t-1}^2 + \epsilon_{i,t}$  based on the May 1963 – December 2001 sample period and White's standard errors.  $R_{i,t}$  denotes the excess monthly return on the industry portfolio  $i$  (in percent) and  $(M/B)_{i,t-1}$  is the natural log of the market-to-book ratio of the portfolio at the end of the previous month, measured as a deviation from its mean. Estimates and test statistics significant at 5% level are marked by a star.  $\chi^2 = \hat{a}_2^\top \Sigma^{-1} \hat{a}_2$ , where  $\Sigma$  is the White's estimate of the covariance matrix of  $\hat{a}_2$ . Under the null hypothesis that  $a_2 = 0$ , this statistic is asymptotically distributed as  $\chi^2$  with 13 degrees of freedom.

Portfolio	$a_{i,1}$	(Std. err.)	$a_{i,2}$	(Std. err.)
Nat. resources	-0.27	(0.73)	3.17	(2.22)
Construction	-1.14*	(0.48)	2.91*	(1.34)
Food, tobacco	-0.00	(0.39)	1.63*	(0.82)
Consumer products	-0.78	(0.57)	2.26	(1.56)
Logging, paper	-0.36	(0.31)	0.85	(0.49)
Chemicals	-1.29*	(0.45)	2.02*	(0.94)
Petroleum	0.36	(0.46)	2.77*	(0.82)
Mach., equipment	-0.40	(0.47)	4.41*	(0.96)
Transportation	-1.20*	(0.46)	0.34	(1.28)
Utilities, telecom.	0.02	(0.33)	1.91*	(0.74)
Trade	-0.67	(0.46)	1.32	(1.14)
Financial	-1.29*	(0.54)	4.46*	(1.55)
Services, other	0.69	(0.45)	2.42*	(0.95)
Average (Std.Err.)			2.34* (0.65)	
$\chi^2$ ( $p$ -value)			37.98* (0.0003)	

Table 2

Conditional volatility, piece-wise linear specification. Estimates of the coefficients of the model  $|R_{i,t} - \bar{R}_i| = a_{i,0} + a_{i,1}(\text{M/B})_{i,t-1}^- + a_{i,2}(\text{M/B})_{i,t-1}^+ + \epsilon_{i,t}$  based on the May 1963 – December 2001 sample period and White's standard errors.  $R_{i,t}$  denotes the excess monthly return on the industry portfolio  $i$  (in percent) and  $(\text{M/B})_{i,t-1}$  is the natural log of the market-to-book ratio of the portfolio at the end of the previous month, measured as a deviation from its mean. Estimates and test statistics significant at 5% level are marked by a star.  $\chi^2 = \hat{c}^\top \Sigma^{-1} \hat{c}$ , where  $c$  is the estimate of a coefficient and  $\Sigma$  is the White's estimate of the covariance matrix of  $c$ . Under the null hypothesis that  $c = 0$ , this statistic is asymptotically distributed as  $\chi^2$  with 13 degrees of freedom.

Portfolio	$a_{i,1}$	(Std. err.)	$a_{i,2}$	(Std. err.)
Nat. resources	-2.10	(1.41)	1.37	(1.46)
Construction	-2.67*	(1.05)	0.33	(0.95)
Food, tobacco	-1.31	(0.94)	1.46*	(0.60)
Consumer products	-2.21*	(0.98)	-0.05	(1.29)
Logging, paper	-1.07	(0.60)	0.54	(0.46)
Chemicals	-2.35*	(0.99)	0.19	(0.66)
Petroleum	-2.39*	(1.11)	3.67*	(0.95)
Mach., equipment	-3.85*	(1.14)	3.92*	(1.01)
Transportation	-1.48	(1.05)	-0.94	(0.86)
Utilities, telecom.	-0.79	(0.72)	1.07	(0.68)
Trade	-1.42	(0.95)	-0.08	(0.93)
Financial	-3.49*	(1.23)	1.33	(0.96)
Services, other	-0.91	(0.94)	2.60*	(1.00)
Average	-2.00*		1.19*	
(Std.Err.)	(0.68)		(0.44)	
$\chi^2$	29.33*		31.80*	
( $p$ -value)	(0.0059)		(0.0026)	
Average ( $a_{i,2} - a_{i,1}$ )	3.19*			
(Std.Err.)	(0.98)			

Table 3

Conditional expected return, quadratic specification. Estimates of the coefficients of the model  $R_{i,t} = a_{i,0} + a_{i,1}(M/B)_{i,t-1} + a_{i,2}(M/B)_{i,t-1}^2 + \epsilon_{i,t}$  based on the May 1963 – December 2001 sample period and White's standard errors.  $R_{i,t}$  denotes the excess monthly return on the industry portfolio  $i$  (in percent) and  $(M/B)_{i,t-1}$  is the natural log of the market-to-book ratio of the portfolio at the end of the previous month, measured as a deviation from its mean. Estimates and test statistics significant at 5% level are marked by a star.  $\chi^2 = \hat{a}_2^\top \Sigma^{-1} \hat{a}_2$ , where  $\Sigma$  is an estimate of the covariance matrix of  $\hat{a}_2$ , robust with respect to conditional heteroskedasticity. Under the null hypothesis that  $a_2 = 0$ , this statistic is asymptotically distributed as  $\chi^2$  with 13 degrees of freedom.

Portfolio	$a_{i,2}$	(Std. err.)
Nat. resources	-1.43	(3.48)
Construction	0.61	(2.19)
Food, tobacco	-0.96	(1.19)
Consumer products	-0.05	(2.39)
Logging, paper	-0.14	(0.79)
Chemicals	1.35	(1.47)
Petroleum	0.36	(1.45)
Mach., equipment	-0.35	(1.79)
Transportation	2.90	(2.06)
Utilities, telecom.	0.89	(1.25)
Trade	-0.24	(1.73)
Financial	1.66	(2.34)
Services, other	-0.07	(1.60)
Average (Std.Err.)	0.35 (1.15)	
$\chi^2$ ( $p$ -value)	13.11 (0.4393)	



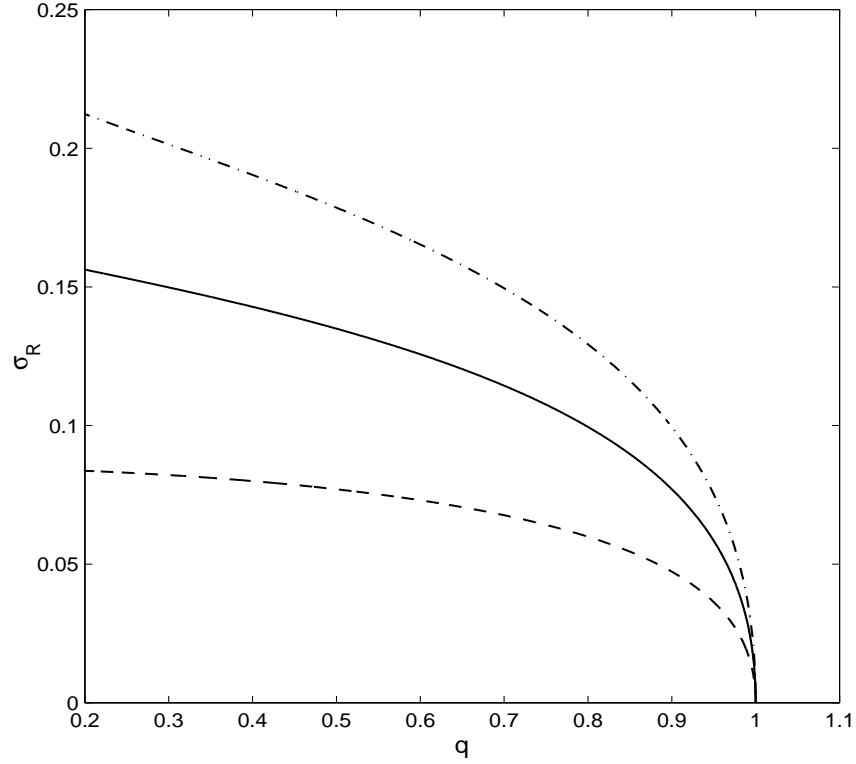


Fig. 1. Conditional volatility of stock returns.  $\sigma_R$  denotes the conditional volatility of stock returns of firms in the second sector. The argument is the average  $q$  of firms in the second sector, defined as the ratio of their market value to the replacement cost of their capital. The subjective time preference rate is  $\rho = 0.05$ , the expected return and the volatility of the production technology of the first sector are given by  $\alpha = 0.07$  and  $\sigma = 0.17$  respectively, and the depreciation rate of capital in the second sector is  $\delta = 0.05$ . The preference parameter  $b$  is assumed to be small. The three lines correspond to the preference parameter  $\gamma$  of  $1/2$  (dash),  $1$  (solid), and  $3/2$  (dash-dot).

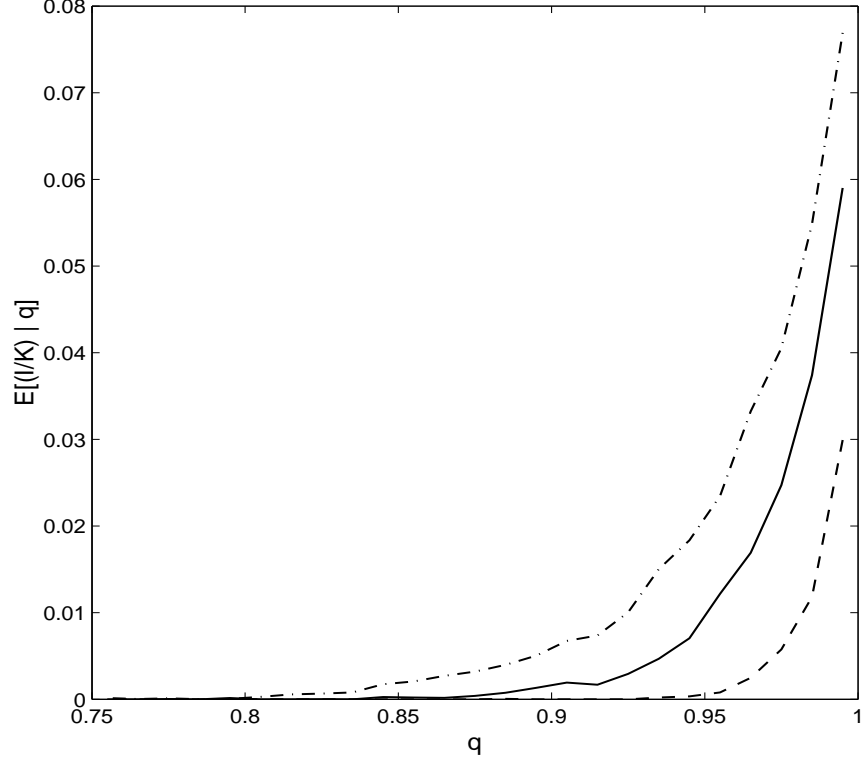


Fig. 2. Conditional investment-to-capital ratio.  $I/K$  denotes the ratio of cumulative investment over a one-year period to the beginning-of-the-year capital stock. The argument is the average  $q$  of firms in the second sector, defined as the ratio of their market value to the replacement cost of their capital. Conditional investment-to-capital ratio  $E[(I/K)|q]$  is estimated using kernel regression based on 100,000 years of simulated data. The subjective time preference rate is  $\rho = 0.05$ , the expected return and the volatility of the production technology of the first sector are given by  $\alpha = 0.07$  and  $\sigma = 0.17$  respectively, and the depreciation rate of capital in the second sector is  $\delta = 0.05$ . The preference parameter  $b$  is assumed to be small. The three lines correspond to three values of the risk-aversion parameter  $\gamma$ :  $1/2$  (dash),  $1$  (solid), and  $3/2$  (dash-dot).

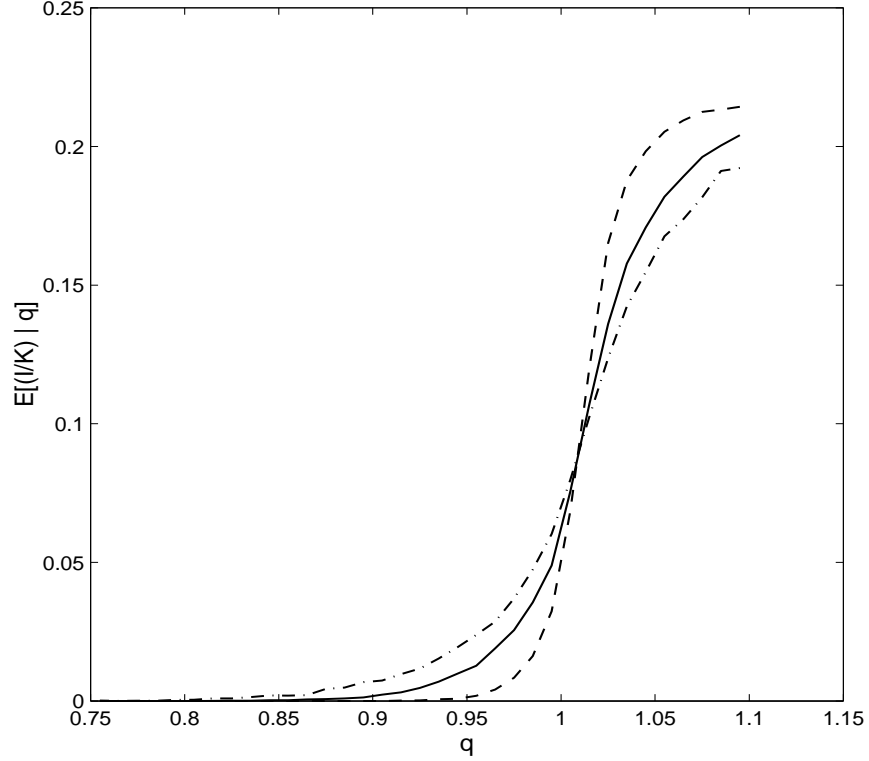


Fig. 3. Conditional investment-to-capital ratio, bounded investment rate.  $I/K$  denotes the ratio of cumulative investment over a one-year period to the beginning-of-the-year capital stock. The argument is the average  $q$  of firms in the second sector, defined as the ratio of their market value to the replacement cost of their capital. Conditional investment-to-capital ratio  $E[(I/K)|q]$  is estimated using kernel regression based on 100,000 years of simulated data. The subjective time preference rate is  $\rho = 0.05$ , the expected return and the volatility of the production technology of the first sector are given by  $\alpha = 0.07$  and  $\sigma = 0.17$  respectively, and the depreciation rate of capital in the second sector is  $\delta = 0.05$ . The maximum investment rate is  $i_{\max} = 0.2$ . The preference parameter  $b$  is assumed to be small. The three lines correspond to three values of the risk-aversion parameter  $\gamma$ :  $1/2$  (dash),  $1$  (solid), and  $3/2$  (dash-dot).

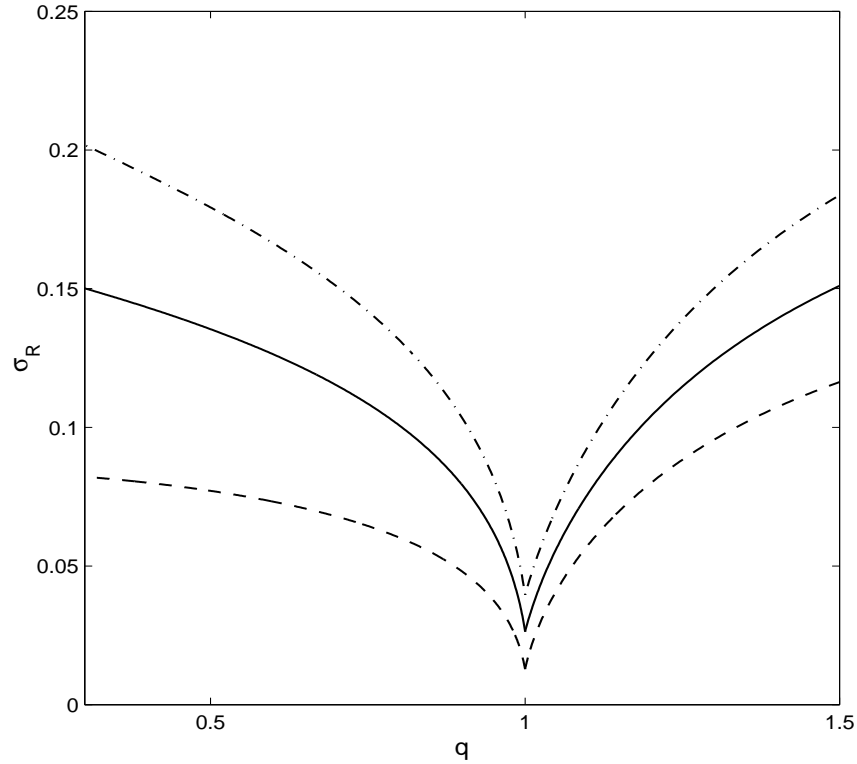


Fig. 4. Conditional volatility of stock returns, bounded investment rate.  $\sigma_R$  denotes the conditional volatility of stock returns of firms in the second sector. The argument is the average  $q$  of firms in the second sector, defined as the ratio of their market value to the replacement cost of their capital. The subjective time preference rate is  $\rho = 0.05$ , the expected return and the volatility of the production technology of the first sector are given by  $\alpha = 0.07$  and  $\sigma = 0.17$  respectively, and the depreciation rate of capital in the second sector is  $\delta = 0.05$ . The maximum investment rate is  $i_{\max} = 0.2$ . The preference parameter  $b$  is assumed to be small. The three lines correspond to three values of the risk-aversion parameter  $\gamma$ :  $1/2$  (dash),  $1$  (solid), and  $3/2$  (dash-dot).

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