We propose a theoretically motivated procedure for measuring heterogeneity in firms’ growth opportunities and document its empirical properties. The term “growth opportunities” refers to the component of a firm’s market value that cannot be attributed to its assets in place. This decomposition of firm value underpins many of the theoretical models describing cross-sectional differences in firms’ investment and stock return behavior. However, successful applications of such models depend on the quality of empirical measures of growth opportunities. Our procedure identifies economically significant differences in firms’ growth opportunities which are not captured by the commonly used empirical measures.

We base our approach on a theoretical model incorporating investment specific productivity shocks and heterogenous firms. Productivity shocks in the capital goods sector account for a significant fraction of observed growth variability, according to the literature on the real determinants of economic growth (Jeremy Greenwood, Zvi Hercowitz and Per Krusell 1997, Jonas D. M. Fisher 2006). Greenwood, Hercowitz and Krusell (1997) show that, empirically, investment specific shocks are negatively correlated with aggregate investment, both at business cycle and lower frequencies. Our model predicts that the sensitivity of firm stock returns to investment specific productivity shocks (z-shocks) is greater for firms that derive a relatively large fraction of their market value from growth opportunities (high growth firms).

Our empirical procedure relies on the above theoretical insight. We measure the unobservable asset composition (growth opportunities relative to assets in place) through observed differences in stock price sensitivity to z-shocks. Since the asset composition of a firm can change at medium run frequencies, our procedure allows this sensitivity to vary over time. We do so in a nonparametric fashion using high frequency data. As a result, we cannot use the existing measures of z-shocks, such as the price of new equipment, since these are available only at low frequencies. Instead, following our theoretical model, we construct a portfolio of stocks mimicking the z-shock, specifically a zero investment portfolio long the stocks of investment good producers and short the stocks of consumption good producers (IMC). We then classify firms as those with high or low growth opportunities using their stock return beta with respect to IMC returns.

Since growth opportunities are not observable, we assess the success of our procedure indirectly. In particular, our key metric is the response of firms’ investment to the z-shock. Intuitively, firms with more growth opportunities should invest relatively more in response to a favorable z-shock, since they have more potential projects to invest in. We find that the IMC betas ($\beta_{IMC}$) are able to identify heterogeneity in firms’ investment responses to the z-shocks. High $\beta_{IMC}$ firms not only invest more on average, but their investment increases more in response to a positive z-shock. Economically, these effects are significant. The difference in z-shock sensitivity between the high beta and the low beta firms is larger than sensitivity of an average firm.

The $\beta_{IMC}$ sort creates dispersion in several firm characteristics traditionally associated with differences in growth opportunities. In particular, in most standard models, firms with more growth opportunities tend to have higher Tobin’s $Q$ and higher average investment rates.
Indeed, the average investment rate of low beta firms is 80 percent of that of the high beta firms. High $\beta^{IMC}$ firms also tend to have higher Tobin’s $Q$; however, the investment shock betas contain important incremental information about the firms’ future investment. Moreover, consistent with standard economic intuition, high $\beta^{IMC}$ firms hold more cash, pay less in dividends, and invest more in research and development (R&D).

I. The Model

There are two sectors in our model, the consumption good sector, and the investment good sector. Investment specific shocks affect the relative price of the investment good. We focus on heterogeneity in growth opportunities among consumption good producers.

A. Consumption Good Sector

There is a continuum of measure one of infinitely lived firms producing a homogeneous consumption good. Firms behave competitively, and there is no explicit entry or exit in this sector. We assume that all firms are financed by equity.

Each firm starts at time 0 with a single project, producing a flow of future output equal to the aggregate productivity process $x_t$. We assume that $x_t$ follows a Geometric Brownian motion process under the risk neutral probability measure:

$$\frac{dx_t}{x_t} = \mu_x dt + \sigma_x dB_{xt},$$

where $B_{xt}$ is a Brownian motion.

Each firm has an opportunity to invest in a single additional project. This investment opportunity arrives exogenously, at a random time $\tau$, which is distributed exponentially with the firm specific arrival rate $\lambda_f$. When the firm invests in the new project, it must choose the associated amount of physical capital $K$ and pay the investment cost $z_{\tau} x_{\tau} K$. The cost of capital relative to its productivity is the investment specific productivity process $z_{\tau}$, which we assume to follow a Geometric Brownian motion:

$$\frac{dz_{\tau}}{z_{\tau}} = \mu_z dt + \sigma_z dB_{z\tau},$$

where $B_{z\tau}$ is a Brownian motion independent of $B_{xt}$. The new project produces a perpetual stream of cash flows equal to $x_{\tau} K^\alpha$.

All firms in our model are initially endowed with the same project and hence have the same value of assets in place. However, because the arrival rate of new projects ($\lambda_f$) is firm specific, growth opportunities exhibit cross-sectional heterogeneity.

The instantaneous risk free rate ($r$) in our economy is constant. All cash flows are priced by their expected discounted value under the risk neutral probability measure. Then, the time 0 value of the firms’ assets in place is given by

$$E_0 \left[ \int_0^\infty e^{-r\tau} x_{\tau} d\tau \right] = (r - \mu_x)^{-1} x_0,$$

where $E[\cdot]$ denotes the risk neutral expectation.

Firms’ investment decisions are based on a tradeoff between the market value of the new project and the cost of physical capital at the time of the project’s arrival. Assuming that $r > \mu_x$, the time $\tau$ market value of cash flows from the new project is $(r - \mu_x)^{-1} x_{\tau} K^\alpha$. Thus, firms choose the amount of capital $K$ to invest in the new project to maximize

$$K^* (z_{\tau}) = \left[ (r - \mu_x) \alpha^{-1} z_{\tau} \right]^{1/(\alpha-1)}.$$

At time $t = 0$, the value of each firm can be computed as a sum of market values of its existing project and its growth opportunities. The former equals the present value of cash flows generated by the existing project. The latter equals the discounted expected value generated by future investment. Following the standard convention, we call the first component of firm value the value of assets in place, $V_{f0}^{AP}$, and the second component the present value of growth opportunities, $V_{f0}^{GO}$. Thus, under suitable restrictions on the model parameters, the value of the firm equals

$$V_{f0} = V_{f0}^{AP} + V_{f0}^{GO},$$
where

\[ V_{f0}^{AP} = (r - \mu_x)^{-1}x_0, \]

and

\[ V_{f0}^{GO} = E_0 \left[ \int_0^\infty \lambda_f e^{-\lambda_f t} \right. \]
\[ \times \left\{ \left[ K^*(z_t) \right] \frac{\alpha x_t}{(r - \mu_x)} - z_t x_t K^*(z_t) \right\} dt \]
\[ = \lambda_f a \frac{\alpha}{\alpha - 1} x_0, \quad a > 0. \]

The constant \( a \) depends on the model parameters.

### B. Investment Good Sector

There is a representative firm producing new capital goods at the current unit price \( z_r \). We assume that profits of the investment good firm are a fraction \( \phi \) of total sales of new capital goods. Consequently, profits accrue to the investment firm at the rate of \( \Pi_i = \phi z_r x_i \bar{\lambda} K^*(z_i) \), where \( \bar{\lambda} \) is the average arrival rate of new projects among consumption good producers. Then, under suitable restrictions on the model parameters, the time 0 market value of the investment sector firm is given by

\[ V_0^I = E_0 \left[ \int_0^\infty e^{-\alpha \Pi_i} dt \right] = b x_0 \frac{\alpha}{\alpha - 1}, \quad b > 0. \]

### C. Growth Opportunities and Stock Returns

We define the IMC portfolio as the zero-net-investment portfolio long the stocks of the investment sector firms and short the stocks of the consumption sector firms. It is easy to see from equations (3), (8), and (9) that IMC returns are perfectly positively correlated with the investment specific productivity shock \( z \). Thus, IMC portfolio mimics investment specific productivity shocks.

According to equations (3) and (8), the beta of stock returns of each firm in the consumption good sector with respect to the investment specific shock is proportional to the weight of growth opportunities in the firm value. We can thus relate the firm’s beta with respect to the IMC portfolio to the firm’s asset composition. A straightforward calculation shows that the time 0 beta of firm \( f \)'s returns with respect to the IMC portfolio is given by

\[ \beta_{f0}^{IMC} = (r - \mu_x) \frac{V_0 V_{f0}^{GO}}{V_{f0}}, \]

where \( V_0 \) denotes the average firm value in the consumption sector. Equation (10) is the basis of our empirical approach to measuring growth opportunities.

Since growth opportunities are not observable directly, we base our empirical tests on observable differences between firms with high and low growth opportunities. In particular, equations (6), (7), (8), and (10) imply that \( \beta_{f0}^{IMC} \) is increasing in \( \lambda_f \). In addition, the expected investment rate of a firm at time \( t = 0 \) is \( \lambda_f K^*(z_0) \). Thus, investment of firms with high growth opportunities (high \( \lambda_f \)) is more sensitive to the \( z \)-shock. These two statements together imply that the investment rate of high \( \beta_{f0}^{IMC} \) firms is more sensitive to \( z \)-shocks than that of low \( \beta_{f0}^{IMC} \) firms. This prediction is likely to hold in much greater generality than suggested by our stylized model.

### II. Empirical Evidence

#### A. Estimation of \( \beta^{IMC} \)

We construct the IMC portfolio empirically by classifying industries as producing either investment or consumption goods according to the National Income and Product Accounts Input-Output Tables. We then match firms to industries according to their North American Industry Classification System codes. Joao F. Gomes, Leonid Kogan, and Motohiro Yogo (2009) and Dimitris Papanikolaou (2008) describe the details of this classification.

For every firm in Compustat with sufficient stock return data, we estimate a time series of \( \left( \beta_{f0}^{IMC} \right) \) by regressing weekly firm stock returns on IMC portfolio returns over a one-year window.

We restrict our analysis to consumption producing firms and apply standard filters to remove outliers and firms with insufficient data. Variable definitions and additional details are reported in Kogan and Papanikolaou (2009). Our final sample contains 6,831 firms and 62,495 firm-year observations and covers the 1965–2007 period.
B. Empirical Findings

Table 1 reports time series averages of the median characteristics of firms in different $\beta^{\text{IMC}}$ deciles. High $\beta^{\text{IMC}}$ firms tend to have higher investment rates, higher Tobin’s $Q$, higher R&D expenditures as a fraction of sales, and lower total payout to shareholders (dividends plus stock repurchases) as a fraction of cash flows (operating income plus depreciation) than low $\beta^{\text{IMC}}$ firms. The extreme $\beta^{\text{IMC}}$ deciles tend to be populated by smaller firms, with firm size measured by the book value of firm’s capital or its market capitalization, each normalized by the corresponding cross-sectional average. Finally, there is no systematic relationship between leverage and $\beta^{\text{IMC}}$, suggesting that our procedure leads to a similar ordering of asset betas, rather than just equity betas. Overall, these patterns are indicative of an upward sloping profile of growth opportunities across the $\beta^{\text{IMC}}$ deciles.

We estimate the sensitivity of firms’ investment to $z$-shocks using the following econometric specification:

\begin{equation}
    i_{ft} = a_t + \sum_{d=2}^{5} a_d D_d + cX_{ft-1} + \gamma_f
    + b_1 R_{t}^{\text{IMC}} + \sum_{d=2}^{5} b_d D_d \times R_{t-1}^{\text{IMC}} + u_t
\end{equation}

where $i_{ft}$ is the firm’s annual investment rate, defined as capital expenditures over property plant and equipment, $R_{t}^{\text{IMC}} = R_{t-1}^{\text{IMC}} + R_{t}^{\text{IMC}}$ denotes the cumulative log returns on the IMC portfolio and $D_d$ is the $\beta^{\text{IMC}}$ quintile dummy variable ($D_n = 1$ if the firm’s $\beta^{\text{IMC}}$ belongs to the quintile $n$ in year $t - 1$). $X_{ft-1}$ is the vector of controls, which includes the firm’s Tobin’s $Q$, its lagged investment rate, financial leverage, ratio of cash flows to capital, and log of its capital stock relative to the aggregate capital stock. Depending on the specification, we include firm or industry level fixed effects ($\gamma_f$). We standardize all variables to zero mean and unit standard deviation. The sample covers the 1962–2007 period. We cluster standard errors by both firm and time.

As we discussed at the end of Section IC, we focus on the interaction between $\beta^{\text{IMC}}$ and $z$-shocks. We report the results in Table 2.

The investment rate of firms with high $\beta^{\text{IMC}}$ responds more to an investment specific shock. A single–standard deviation IMC return shock changes firm level investment by 0.096 standard deviations on average. This number varies between 0.053 and 0.176 for the low $\beta^{\text{IMC}}$ and high $\beta^{\text{IMC}}$ firms respectively. The spread between quintiles is economically significant. Following a single–standard deviation IMC return shock, the investment rate of low $\beta^{\text{IMC}}$ firms changes by 0.9 percent, compared to 3.1 percent for the high $\beta^{\text{IMC}}$ firms. Fluctuations of this magnitude are substantial compared to the unconditional volatility of the aggregate investment rate changes in our sample, which is 2.5 percent.
Figure 1 illustrates the magnitude of the empirical patterns in investment rates by contrasting the sensitivity of the aggregate investment rate to IMC returns with the analogous plot for the difference in investment rates between the extreme $\beta_{IMC}$ quintile portfolios. We compute aggregate and portfolio level investment rates by dividing the total investment of the corresponding set of firms by their total capital stock.

**REFERENCES**


**Notes:** Top panel plots the aggregate investment rate versus the lagged IMC returns. Bottom panel shows the difference in investment rates between the high and low $\beta_{IMC}$ portfolios.