Low Energy Probes of Chiral Symmetry

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Strong interactions described by QCD

$$\mathcal{L}_{QCD} = \bar{q}(i\not{D} - m)q - \frac{1}{4}G_{\mu\nu}G^{\mu\nu}$$

However, QCD hard to solve at low energy because

i) "Wrong" degrees of freedom

ii) Nonlinearity

iii) Strong coupling: $\alpha_s \sim 1$

Solution is to utilize chiral symmetry

$$\bar{q}(i \not{D} - m)q = \bar{q}_L i \not{D} q_L + \bar{q}_R i \not{D} q_R - m(\bar{q}_L q_R + \bar{q}_R q_L)$$

Then $SU(3)_L \times SU(3)_R$ (chiral) symmetry in $m = 0$ limit

We discuss three examples which probe chiral symmetry via low energy photon interactions

a) Chiral Anomaly

b) Hadronic Polarizabilities

c) Hadronic Parity Violation

Chiral Anomaly

Weyl: Something is symmetric if you do something to it and it looks the same as before'

Importance in physics is due to Noether's theorem, which says that to each symmetry there is a corresponding conservation law

- Translation symmetry$\leftrightarrow$conservation of momentum
- Time translation symmetry$\leftrightarrow$conservation of energy
- Rotational symmetry$\leftrightarrow$conservation of angular momentum

Usually symmetry is broken—can be in only three ways:

i) Explicit breaking—present in Lagrangian

ii) Spontaneous breaking—Lagrangian is symmetric, but ground state is not

iii Anomalous breaking—classical Lagrangian is symmetric by symmetry is broken by quantization

Consider massless QED

$$\mathcal{L} = \bar{\psi}(i\,\partial\!\!\!/ - e\,A\!\!\!/)\psi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}$$

which is invariant under vector and axial transformations—

$$\psi \to \exp(i\beta)\psi \equiv \psi_V{}' \quad \text{and} \quad \psi \to \exp(i\zeta\gamma_5)\psi \equiv \psi_A{}'$$

which, via Noether's theorem, leads to conservation of both $J_\mu = \bar{\psi}\gamma_\mu\psi$ and $J_{5\mu} = \bar{\psi}\gamma_\mu\gamma_5\psi$—

$$\partial^\mu J_\mu = \partial^\mu J_{5\mu} = 0$$

Then Sutherland, Veltman theorem leads to $\mathrm{Amp}(\pi^0 \to \gamma\gamma) = 0$. Finite quark, pion mass leads to estimate

$$\mathrm{Amp}(m_\pi^2) \sim \frac{e^2}{4\pi F_\pi} \times \frac{m_\pi^2}{\Lambda_\chi^2} \quad \text{and} \quad \tau_{\pi^0} \sim 10^{-13}\,\mathrm{sec}$$

compared to

$$\tau_{\pi^0}^{exp} = (0.84 \pm 0.04) \times 10^{-16}\,\mathrm{sec}$$

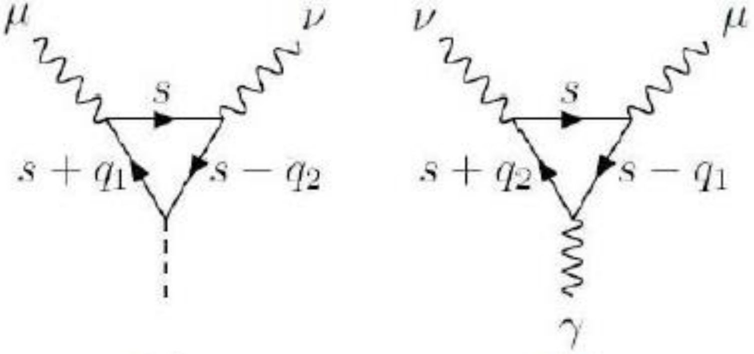
Difference explained by anomaly.

Anomaly is due to modification of short distance properties of QFT. That short distance effects must be handled carefully suggested by commutator

$$\{\psi_a(t,\vec{x}), \psi_b^\dagger(t,\vec{y})\} = \delta^3(\vec{x}-\vec{y})\delta_{ab}$$

Various ways to study short distance behavior. Most intuitive is simple perturbation theory.

Calculate diagrams



(a) (b)

$$T_{\mu\nu\gamma}(q_1, q_2) = U_{\mu\nu\gamma}(q_1, q_2) + U_{\nu\mu\gamma}(q_2, q_1)$$

where

$$U_{\mu\nu\gamma}(q_1, q_2) = -i\frac{e^2 K_F}{2}\int \frac{d^4 s}{(2\pi)^4}\mathrm{Tr}\left(\frac{1}{\not{s}+\not{q}_1}\gamma_\mu \frac{1}{\not{s}}\gamma_\nu \frac{1}{\not{s}-\not{q}_2}\gamma_\gamma \gamma_5\right)$$

Predict

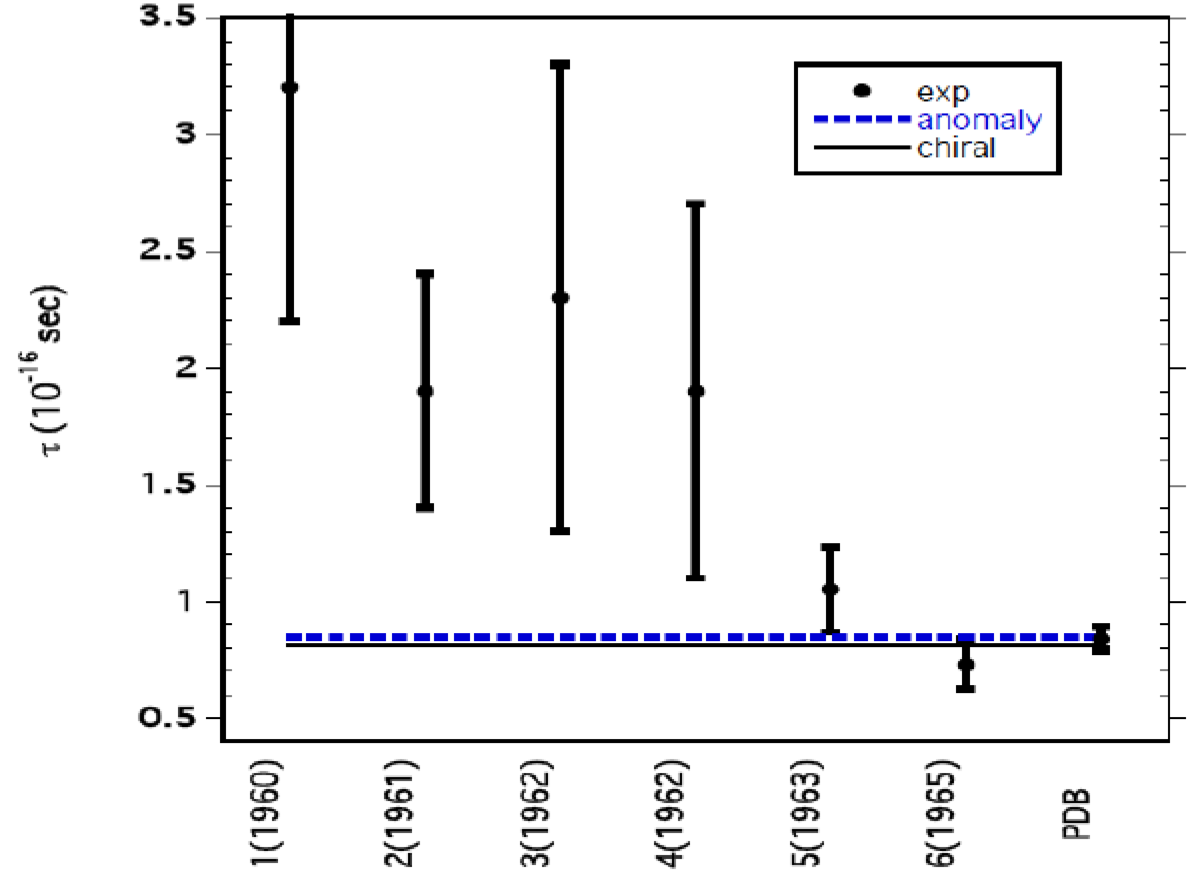
$$< 2\gamma|\partial^\mu J^3_{5\mu}|0 >= F_\pi m_\pi^2 \frac{1}{m_\pi^2} < 2\gamma|\pi^0 >= \frac{e^2}{4\pi^2}\epsilon_{\mu\nu\alpha\beta}q_1^\mu\epsilon_1^\nu q_2^\alpha\epsilon_2^\beta$$

Leads to prediction

$$\Gamma_{\pi\gamma\gamma} = \frac{|A_{\pi\gamma\gamma}|^2 m_\pi^3}{4\pi} = 7.76 \text{ eV}$$

One loop chiral corrections by Goity, BRH, Bernstein, by Moussallam and Anatharayan, by Ioffe, etc. yield

$$\Gamma_{\pi\gamma\gamma} = 8.10\,\text{eV} \pm 1\%$$



3.5
3
2.5
2
1.5
1
0.5
τ (10^-16 sec)
exp
anomaly
chiral
1(1960)
2(1961)
3(1962)
4(1962)
5(1963)
6(1965)
PDB

Low Energy Problem: $\gamma 3\pi$

Chiral symmetry at $\mathcal{O}(p^4)$ makes no-free-parameter predictions which can test QCD—effective Lagrangian is

$$\mathcal{L}_A = -\frac{N_c}{48\pi^2}\epsilon^{\mu\nu\alpha\beta}\left[eA_\mu \mathrm{Tr}(QL_\nu L_\alpha L_\beta - QR_\nu R_\alpha R_\beta) \right.$$

$$\left. + ie^2 F_{\mu\nu}A_\alpha T_\beta\right]$$

with

$$U = \exp(i\vec{\tau}\cdot\vec{\phi}_\pi/F_\pi),$$

$$L_\mu = \partial_\mu U U^\dagger, \qquad R_\mu = \partial_\mu U^\dagger U$$

$$T_\beta = \mathrm{Tr}\left(Q^2 L_\beta - Q^2 R_\beta + \frac{1}{2}QUQU^\dagger L_\beta \right.$$

$$\left. -\frac{1}{2}QU^\dagger QUR_\beta\right)$$

Piece responsible for $\pi^0 \to \gamma\gamma$ is

$$\mathcal{L}_A = \frac{e^2 N_c}{16\pi^2 F_\pi}\mathrm{Tr}(Q^2\tau_3)\epsilon^{\mu\nu\alpha\beta}F_{\mu\nu}A_\alpha\partial_\beta\pi^0$$

$$= \frac{e^2 N_c}{24\pi^2 F_\pi}F_{\mu\nu}\tilde{F}^{\mu\nu}\pi^0$$

Another prediction is for $\gamma 3\pi$ vertex—-

$$\mathrm{Amp}_{\gamma\pi^+\pi^-\pi^0} = -iA_{3\pi}(0)\epsilon^{\alpha\beta\gamma\delta}\epsilon_\alpha p_{+\beta}p_{-\gamma}p_{0\delta}$$

with

$$A_{3\pi}(0) = \frac{eN_C}{12\pi^2 F_\pi^3} = 9.7\,\mathrm{GeV}^{-3}$$

Compare with experiment of Antipov et al. $N\pi \to \pi\pi N$

$$A_{3\pi}^{expt} = (12.9 \pm 0.9 \pm 0.5)\,\mathrm{GeV}^{-3}$$

Apparently 30% too high, but prediction is in chiral limit. Need chiral and radiative corrections to reach real world limit—$s = 4m_\pi^2$—rather than $s = 0$. Theoretical corrections are about 20% in the right direction—

$$A_{\gamma 3\pi}(0) = (10.7 \pm 1.2)\,\mathrm{GeV}^{-3}$$

Another approach is to use γ^* via reaction $e^-\pi^- \to e^-\pi^-\pi^0$ which yields

$$A_{\gamma 3\pi}(0) = (9.9 \pm 1.1)\,\mathrm{GeV}^{-3}$$

Hadronic Polarizabilities

Polarizabilities

What is a polarizability?

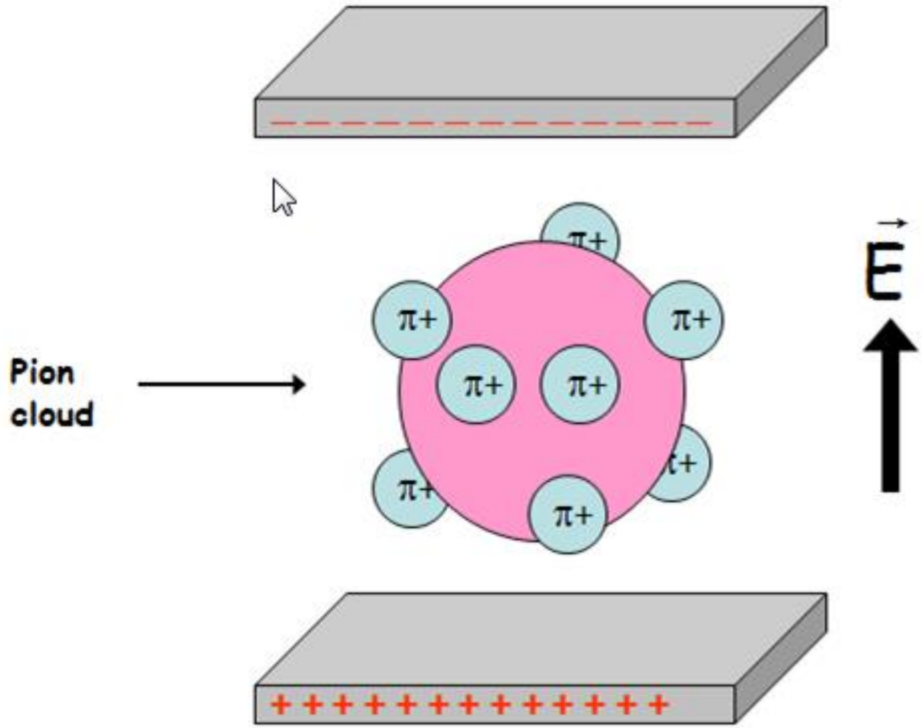
Answer: A measure of response of system to quasi-static electric and/or magnetic field. Simplest example: electric polarizability α_E—applied electric field $\vec{E}$ induces EDM $\vec{p}$

$$\vec{p} = 4\pi\alpha_E\vec{E}$$

Equivalently energy density is

$$u = -\frac{1}{2}4\pi\alpha_E\vec{E}^2$$

Proton electric polarizability



Electric polarizability: proton between charged parallel plates

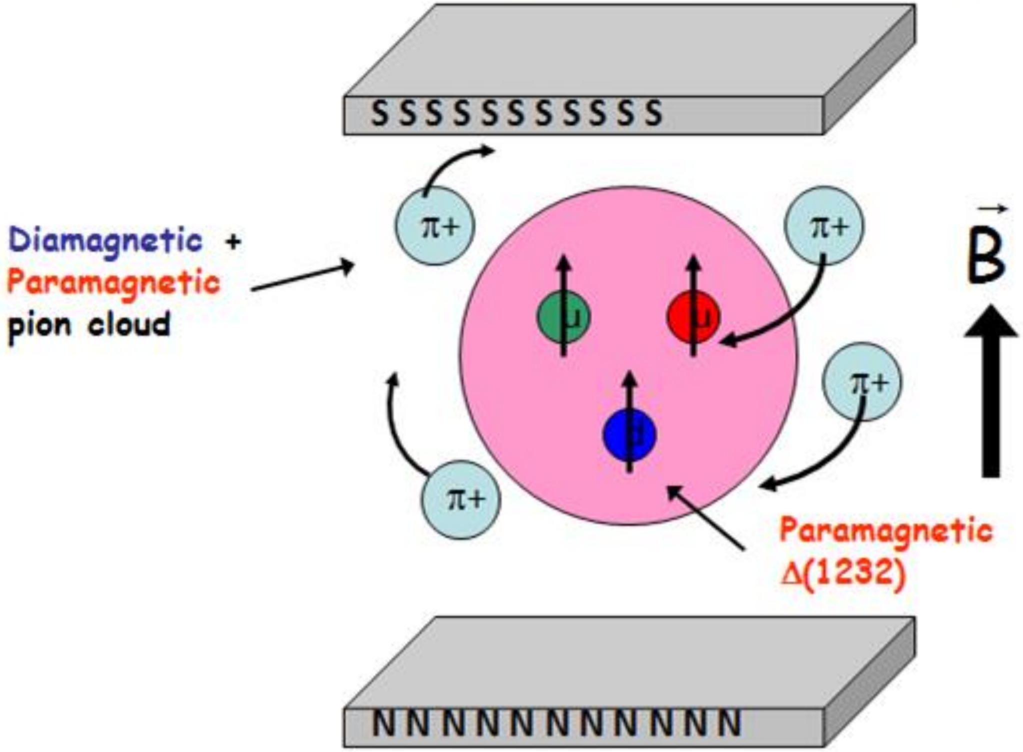
Similarly magnetic polarizability β_M—applied magnetic field $\vec{H}$ induces MDM $\vec{m}$

$$\vec{m} = 4\pi\beta_M\vec{H}$$

with energy density

$$u = -\frac{1}{2}4\pi\beta_M\vec{H}^2$$

Proton magnetic polarizability



Magnetic polarizability: proton between poles of a magnetic

How to measure? Compton scattering—if

$$H = \frac{(\vec{p} - e\vec{A})^2}{2m} - \frac{1}{2}4\pi\alpha_E\vec{E}^2 - \frac{1}{2}4\pi\beta_M\vec{H}^2$$

with

$$\vec{E} = -\frac{\partial}{\partial t}\vec{A}, \qquad \vec{B} = \vec{\nabla}\times\vec{A}$$

then

$$T = \hat{\epsilon}\cdot\hat{\epsilon}'\left(-\frac{Q^2}{m} + \omega\omega' 4\pi\alpha_E\right) + \hat{\epsilon}\times\vec{k}\cdot\hat{\epsilon}'\times\vec{k}' 4\pi\beta_M$$

and

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{m^2}\left(\frac{\omega'}{\omega}\right)^2 \left(\frac{1}{2}(1+\cos^2\theta) - \frac{m\omega\omega'}{\alpha}\right.$$

$$\left.\cdot\left[\frac{\alpha_E+\beta_M}{2}(1+\cos\theta)^2 + \frac{\alpha_E-\beta_M}{2}(1-\cos\theta)^2\right] + \ldots\right)$$

Results from MAMI (TAPS), Illinois, Saskatoon for proton

$$\alpha_E^p = (12.0\pm 0.6)\times 10^{-4}\,\mathrm{fm}^3, \quad \beta_M = (1.9\mp 0.6)\times 10^{-4}\,\mathrm{fm}^3$$

Estimate via chiral perturbation theory (Bernard, Kaiser, Meissner):

$$\alpha_E^p = \frac{\alpha g_A^2}{48\pi^2 F_\pi^2 M_n}\left[\frac{5\pi}{2\mu} + 18\log\mu + \frac{33}{2} + \mathcal{O}(\mu)\right] = 7.4$$

$$\beta_M^p = \frac{\alpha g_A^2}{48\pi^2 F_\pi^2 M_n}\left[\frac{\pi}{4\mu} + 18\log\mu + \frac{63}{2} + \mathcal{O}(\mu)\right] = -2.0$$

with $\mu = m_\pi/m_n$ in units of $10^{-4}\,\mathrm{fm}^3$.

If take only leading term ($\mathcal{O}(q^3)$ HBChipt), then

$$\alpha_E^p = 10\beta_M^p = \frac{5g_A^2}{96\pi F_\pi^2 m_\pi} = 12.2$$

in perfect agreement with experiment. Clearly accidental since $\mathcal{O}(q^4)$ calculation gives

$$\alpha_E^p = 10.5 \pm 2.0 \qquad \text{and} \qquad \beta_M^p = 3.5 \pm 3.6$$

However, see also Pascalutsa and Lensky.

Neutron measurements much more difficult since

i) no neutron target available

ii) no Thomson term to interfere with

Has been done via

n-Pb scattering, d,^{3}He targets, etc.

All agree that $\alpha_E^n \simeq \alpha_E^p$ and $\beta_M^n \simeq \beta_M^p$. Schumacher recommends values

$$\alpha_E^n = 12.5 \pm 1.8 \qquad \text{and} \qquad \beta_M = 2.7 \mp 1.8$$

in good agreement with HBchipt prediction that primarily isoscalar.

Charged Pion Polarizabilities

Another interesting case is charged pion. Here lowest order chiral symmetry $\mathcal{O}(p^4)$ gives

$$\alpha_E^{\pi^+} + \beta_M^{\pi^+} = 0 \quad \text{and} \quad \alpha_E^{\pi^+} - \beta_M^{\pi^+} = 5.4 \times 10^{-4}\,\text{fm}^3$$

where here size of polarizability is given by axial term in radiative pion decay—$\pi^+ \rightarrow e^+\nu_e\gamma$. Two loop also done, generating small corrections

$$\alpha_E^{\pi^+} = 2.9 \times 10^{-4}\,\text{fm}^3 \quad \text{and} \quad \beta_M^{\pi^+} = -2.5 \times 10^{-4}\,\text{fm}^3$$

as expected.

Experimental results are varied. Three methods:

i) Primakoff Effect: $\pi^+ + N \to \pi^+ + N + \gamma$ Antipov et al. (1985) $\alpha_E^{\pi^+} = (6.8 \pm 1.4 \pm 1.2) \times 10^{-4}$ fm^3

ii) Pion Pole: $\gamma + N \to \gamma + N + \pi^+$ Aibergenov et al. (1986) $\alpha_E^{\pi^+} = (20 \pm 12) \times 10^{-4}\,\mathrm{fm}^3$

Ahrens et al. (MAMI-2005): $\alpha_E^{\pi^+} - \beta_M^{\pi^+} = (11.6 \pm 1.5 \pm 3.0 \pm 0.5) \times 10^{-4}\,\mathrm{fm}^3$

iii) Annihilation: $\pi^+\pi^- \to \gamma\gamma$

Babusci et al. (1992) $\alpha_E^{\pi^+} = (2.2 \pm 1.6) \times 10^{-4}\,\mathrm{fm}^3$

Future: Spin Polarizabilities

Basic Idea: Higher order in photon energy expansion:

$$\alpha_E = \alpha_{E1E1} \qquad \beta_M = \beta_{M1M1}$$

If include spin then allow terms

$$\gamma_{E1E1}, \gamma_{M1M1}, \gamma_{E1M2}, \gamma_{M1E2}$$

such that

$$H = -\frac{1}{2}4\pi\left[\gamma_{E1E1}\vec{\sigma}\cdot\vec{E}\times\dot{\vec{E}} + \gamma_{M1M1}\vec{\sigma}\cdot\vec{B}\times\dot{\vec{B}}\right.$$

$$\left. +2\gamma_{E1M2}\sigma_i E_j B_{ij} - 2\gamma_{M1E2}\sigma_i B_j E_{ij}\right]$$

with $E_{ij} = \frac{1}{2}(\nabla_i E_j + \nabla_j E_i)$. Note terms are $\mathcal{O}(\omega^3)$ vs. $\mathcal{O}(\omega^2)$ for α_E, β_M

HBchipt predictions at $\mathcal{O}(q^3)$ are

$$\gamma^p_{E1E1} = -5\gamma^p_{M1M1} = 5\gamma^p_{E1M2} = 5\gamma^p_{M1E2} = -\frac{\alpha^p_E}{10\pi m_\pi}$$

At this order there is a large contribution from pion pole graph

$$\gamma^p_{E1E1} = -\gamma^p_{M1M1} = \gamma^p_{E1M2} = -\gamma^p_{M1E2} = \frac{24}{g_A}\frac{\alpha^p_E}{10\pi m_\pi}$$

dominates structure terms.

At present no direct measurements of spin polarizabilities. However, analysis of backward Compton scattering yields

$$LEGS : \gamma_\pi = (-27.1 \pm 2.2 \pm 2.6) \times 10^{-4}\,\mathrm{fm}^4$$

$$MAMI-TAPS : \gamma_\pi = (-35.9 \pm 2.3) \times 10^{-4}\,\mathrm{fm}^4$$

$$MAMI-LARA : \gamma_\pi = (-40.9 \pm 0.6 \pm 2.2) \times 10^{-4}\,\mathrm{fm}^4$$

Schumacher recommends $\gamma_\pi = (-38.7 \pm 1.8) \times 10^{-4}\,\mathrm{fm}^4$ and dispersion relation (higher order GDH) gives value in forward direction

$$\gamma_0 = (-1.0 \pm 0.8 \pm 0.10) \times 10^{-4}\,\mathrm{fm}^4$$

Note $|\gamma_\pi^p| >> |\gamma_0^p|$. Reason is

$$\gamma_0^p = -\gamma_{E1E1}^p - \gamma_{M1M1}^p - \gamma_{E1M2}^p - \gamma_{M1E2}^p$$

so large pion pole contribution cancels, while

$$\gamma_\pi^p = -\gamma_{E1E1}^p + \gamma_{M1M1}^p - \gamma_{E1M2}^p + \gamma_{M1E2}^p$$

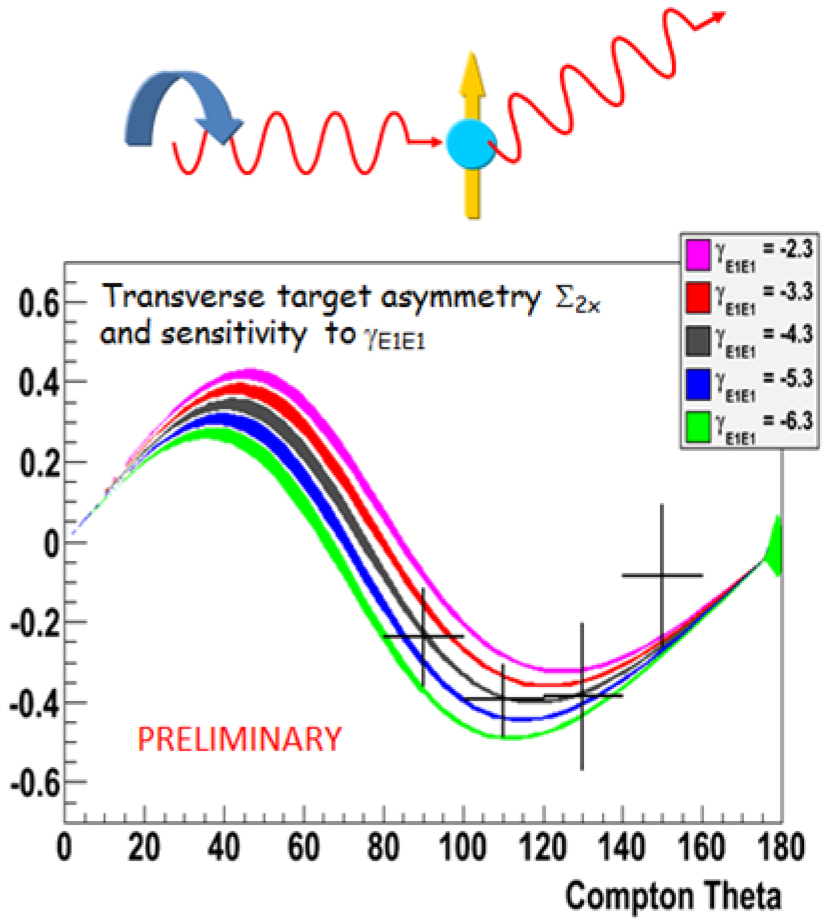
wherein pion pole does NOT cancel—

$$^{pole}\gamma_\pi^p = -\frac{9.6\alpha_E^p}{\pi m_\pi g_A} = -42.7 \times 10^{-4}\,\mathrm{fm}^4$$

Clearly pole dominates γ_π^p. Lowest order chiral prediction for

$$\gamma_0^p = \frac{6}{5\pi m_\pi}\alpha_E^p = +6.2 \times 10^{-4}\,\mathrm{fm}^4$$

Wrong sign, so higher order terms must be important.



Transverse target asymmetry Σ_{2x} and sensitivity to γ_{E1E1}
γ_{E1E1} = -2.3
γ_{E1E1} = -3.3
γ_{E1E1} = -4.3
γ_{E1E1} = -5.3
γ_{E1E1} = -6.3
PRELIMINARY
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
0 20 40 60 80 100 120 140 160 180
Compton Theta

Hadronic Parity Violation

1980: DDH Approach

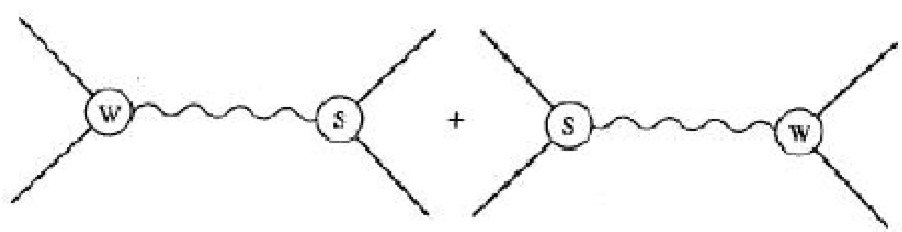
Basic idea:

Meson exchange gives good picture of PC NN interaction, with

$$\mathcal{H}_{\rm st} = ig_{\pi NN}\bar{N}\gamma_5\tau\cdot\pi N + g_\rho \bar{N}\left(\gamma_\mu + i\frac{\mu_V}{2M}\sigma_{\mu\nu}k^\nu\right)\tau\cdot\rho^\mu N$$

$$+g_\omega \bar{N}\left(\gamma_\mu + i\frac{\mu_S}{2M}\sigma_{\mu\nu}k^\nu\right)\omega^\mu N$$

so use for PV NN



Then define general PV weak couplings:

$$\mathcal{H}_{\rm wk} = \frac{f_\pi^1}{\sqrt{2}}\bar{N}(\tau\times\pi)_3 N$$

$$+\bar{N}\left(h_\rho^0\tau\cdot\rho^\mu + h_\rho^1\rho_3^\mu + \frac{h_\rho^2}{2\sqrt{6}}(3\tau_3\rho_3^\mu - \tau\cdot\rho^\mu)\right)\gamma_\mu\gamma_5 N$$

$$+\bar{N}(h_\omega^0\omega^\mu + h_\omega^1\tau_3\omega^\mu)\gamma_\mu\gamma_5 N - h_\rho^{'1}\bar{N}(\tau\times\rho^\mu)_3\frac{\sigma_{\mu\nu}k^\nu}{2M}\gamma_5 N$$

Yields two-body PV NN potential

$$V^{\rm PNC} = i\frac{f_\pi^1 g_{\pi NN}}{\sqrt{2}}\left(\frac{\tau_1 \times \tau_2}{2}\right)_3 (\sigma_1 + \sigma_2)\cdot\left[\frac{\mathbf{p}_1 - \mathbf{p}_2}{2M}, f_\pi(r)\right]$$

$$-g_\rho\left(h_\rho^0 \tau_1\cdot\tau_2 + h_\rho^1\left(\frac{\tau_1 + \tau_2}{2}\right)_3 + h_\rho^2\frac{(3\tau_1^3\tau_2^3 - \tau_1\cdot\tau_2)}{2\sqrt{6}}\right)$$

$$\times((\sigma_1 - \sigma_2)\cdot\left\{\frac{\mathbf{p}_1 - \mathbf{p}_2}{2M}, f_\rho(r)\right\}$$

$$+i(1+\chi_V)\sigma_1 \times \sigma_2\cdot\left[\frac{\mathbf{p}_1 - \mathbf{p}_2}{2M}, f_\rho(r)\right])$$

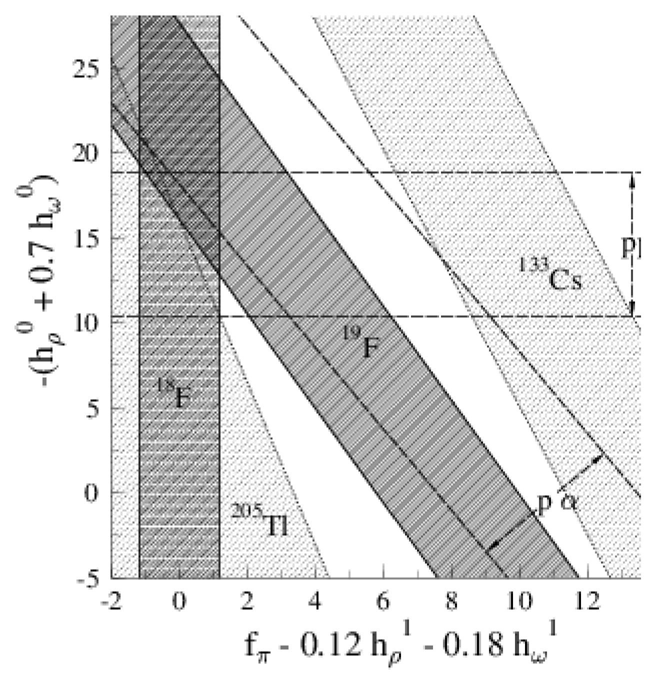
$$-g_\omega\left(h_\omega^0 + h_\omega^1\left(\frac{\tau_1 + \tau_2}{2}\right)_3\right)$$

$$\times((\sigma_1 - \sigma_2)\cdot\left\{\frac{\mathbf{p}_1 - \mathbf{p}_2}{2M}, f_\omega(r)\right\}$$

$$+i(1+\chi_S)\sigma_1 \times \sigma_2\cdot\left[\frac{\mathbf{p}_1 - \mathbf{p}_2}{2M}, f_\omega(r)\right])$$

$$-(g_\omega h_\omega^1 - g_\rho h_\rho^1)\left(\frac{\tau_1 - \tau_2}{2}\right)_3 (\sigma_1 + \sigma_2)\cdot\left\{\frac{\mathbf{p}_1 - \mathbf{p}_2}{2M}, f_\rho(r)\right\}$$

$$-g_\rho h_\rho^{1'} i\left(\frac{\tau_1 \times \tau_2}{2}\right)_3 (\sigma_1 + \sigma_2)\cdot\left[\frac{\mathbf{p}_1 - \mathbf{p}_2}{2M}, f_\rho(r)\right]$$



-(h_ρ^0 + 0.7 h_ω^0)
f_π - 0.12 h_ρ^1 - 0.18 h_ω^1
18F
19F
205Tl
133Cs
p α
pl

Represent in terms of "Reasonable Range" and "Best Value"

Coupling	DDH Reasonable Range	DDH "Best" Value
h_π	$0 \to 30$	12
h_ρ^0	$30 \to -81$	-30
h_ρ^1	$-1 \to 0$	-0.5
h_ρ^2	$-20 \to -29$	-25
h_ω^0	$15 \to -27$	-5
h_ω^1	$-5 \to -2$	-3

all times "sum rule value" 3.8×10^{-8}

Size of leading PV amplitude h_π^1 seems at variance with DDH best estimate $h_\pi^1(DDH) = 4.3 \times 10^{-7}$ vs. ^{18}F value $h_\pi^1 < 1.3 \times 10^{-7}$. How to resolve:

i) A_γ in $\vec{n}p \rightarrow d\gamma$ underway at SNS

ii) B_γ in $\vec{\gamma}p \rightarrow \pi^+ n$

Chen and Ji obtained LO result

$$B_\gamma^{LO} = \frac{\sqrt{2} F_\pi h_\pi^1 (\mu_p - \mu_n)}{g_A M_N}$$

NLO effects by Puglia, BRH, and Ramsey-Musolf

$$B_\gamma^{NLO} = \frac{\sqrt{2} F_\pi h_\pi^1}{g_A M_N}[\mu_p - \mu_n(1 + \frac{m_\pi}{M_N})] - \frac{4\sqrt{2} m_\pi}{g_A \Lambda_\chi}\bar{C}$$

Both A_γ and B_γ sensitive (almost completely) to h_π^1. However, can also measure asymmetry in

$$\vec{\gamma} d \to np$$

which is also of great interest and sensitive to short distance pieces.

$$C_\gamma \simeq= -0.16\lambda_s^0 + 0.13\lambda_s^2 + 0.63\lambda_t$$

where λ_i are so-called Danilov parameters.

Conclusion

Probes of chiral symmetry possible with low energy electron machine. We have examined three such

a) Chiral Anomaly

b) Hadronic Polarizabilities

c) Hadronic Parity Violation

but many others possible.