Low Energy Probes of Chiral Symmetry

Barry R. Holstein
UMass
Strong interactions described by QCD

\[ \mathcal{L}_{QCD} = \bar{q}(i \not{D} - m)q - \frac{1}{4}G_{\mu\nu}G^{\mu\nu} \]

However, QCD hard to solve at low energy because

i) "Wrong" degrees of freedom

ii) Nonlinearity

iii) Strong coupling: \( \alpha_s \sim 1 \)
Solution is to utilize chiral symmetry

$$\bar{q}(i \not{D} - m)q = \bar{q}_L i \not{D} q_L + \bar{q}_R i \not{D} q_R - m(\bar{q}_L q_R + \bar{q}_R q_L)$$

Then $SU(3)_L \times SU(3)_R$ (chiral) symmetry in $m = 0$ limit

We discuss three examples which probe chiral symmetry via low energy photon interactions

a) Chiral Anomaly

b) Hadronic Polarizabilities

c) Hadronic Parity Violation
Chiral Anomaly
Weyl: Something is symmetric if you do something to it and it looks the same as before.

Importance in physics is due to Noether’s theorem, which says that to each symmetry there is a corresponding conservation law:

- Translation symmetry $\leftrightarrow$ conservation of momentum
- Time translation symmetry $\leftrightarrow$ conservation of energy
- Rotational symmetry $\leftrightarrow$ conservation of angular momentum
Usually symmetry is broken—can be in only three ways:

i) Explicit breaking—present in Lagrangian

ii) Spontaneous breaking—Lagrangian is symmetric, but ground state is not

iii) Anomalous breaking—classical Lagrangian is symmetric by symmetry is broken by quantization
Consider massless QED

\[ \mathcal{L} = \bar{\psi} (i \not{\partial} - e \not{A}) \psi - \frac{1}{4} F_{\mu \nu} F^{\mu \nu} \]

which is invariant under vector and axial transformations—

\[ \psi \rightarrow \exp(i \beta) \psi \equiv \psi_V' \quad \text{and} \quad \psi \rightarrow \exp(i \zeta \gamma_5) \psi \equiv \psi_A' \]

which, via Noether's theorem, leads to conservation of both \( J_\mu = \bar{\psi} \gamma_\mu \psi \) and \( J_{5\mu} = \bar{\psi} \gamma_\mu \gamma_5 \psi \)—

\[ \partial^\mu J_\mu = \partial^\mu J_{5\mu} = 0 \]
Then Sutherland, Veltman theorem leads to \( \text{Amp}(\pi^0 \rightarrow \gamma\gamma) = 0 \). Finite quark, pion mass leads to estimate

\[
\text{Amp}(m_{\pi}^2) \sim \frac{e^2}{4\pi F_{\pi}} \times \frac{m_{\pi}^2}{\Lambda_{\chi}^2} \quad \text{and} \quad \tau_{\pi^0} \sim 10^{-13} \text{ sec}
\]

compared to

\[
\tau_{\pi^0}^{exp} = (0.84 \pm 0.04) \times 10^{-16} \text{ sec}
\]

Difference explained by anomaly.
Anomaly is due to modification of short distance properties of QFT. That short distance effects must be handled carefully suggested by commutator

\[ \{ \psi_a(t, \vec{x}), \psi^\dagger_b(t, \vec{y}) \} = \delta^3(\vec{x} - \vec{y})\delta_{ab} \]

Various ways to study short distance behavior. Most intuitive is simple perturbation theory.
**Calculate diagrams**

\[ T_{\mu\nu\gamma}(q_1, q_2) = U_{\mu\nu\gamma}(q_1, q_2) + U_{\nu\mu\gamma}(q_2, q_1) \]

where

\[ U_{\mu\nu\gamma}(q_1, q_2) = -i \frac{e^2 K_F}{2} \int \frac{d^4s}{(2\pi)^4} \text{Tr} \left( \frac{1}{s + q_1} \gamma_\mu \frac{1}{s} \gamma_\nu \frac{1}{s - q_2} \gamma_\gamma \gamma_5 \right) \]
Predict

\[ <2\gamma|\partial^\mu J_{5\mu}^3|0> = F_\pi m_\pi^2 \frac{1}{m_\pi^2} <2\gamma|\pi^0> = \frac{e^2}{4\pi^2} \epsilon_{\mu\nu\alpha\beta} q_1^\mu \epsilon_1^\nu q_2^\alpha \epsilon_2^\beta \]

Leads to prediction

\[ \Gamma_{\pi\gamma\gamma} = \frac{|A_{\pi\gamma\gamma}|^2 m_\pi^3}{4\pi} = 7.76 \text{ eV} \]

One loop chiral corrections by Goity, BRH, Bernstein, by Moussallam and Anatharayan, by Ioffe, etc. yield

\[ \Gamma_{\pi\gamma\gamma} = 8.10 \text{ eV} \pm 1\% \]
Low Energy Problem: $\gamma^3\pi$

Chiral symmetry at $O(p^4)$ makes no-free-parameter predictions which can test QCD—effective Lagrangian is

$$\mathcal{L}_A = - \frac{N_c}{48\pi^2} \epsilon^{\mu\nu\alpha\beta} [e A_\mu \text{Tr}(QL_\nu L_\alpha L_\beta - QR_\nu R_\alpha R_\beta)$$

$$+ ie^2 F_{\mu\nu} A_\alpha T_\beta]$$

with
\[ U = \exp(i \vec{r} \cdot \vec{\phi}_\pi / F_\pi), \]
\[ L_\mu = \partial_\mu U U^\dagger, \quad R_\mu = \partial_\mu U^\dagger U \]
\[ T_\beta = \text{Tr} \left( Q^2 L_\beta - Q^2 R_\beta + \frac{1}{2} Q U Q U^\dagger L_\beta \right. \]
\[ \left. - \frac{1}{2} Q U^\dagger Q U R_\beta \right) \]

Piece responsible for \( \pi^0 \rightarrow \gamma \gamma \) is

\[ \mathcal{L}_A = \frac{e^2 N_c}{16 \pi^2 F_\pi} \text{Tr}(Q^2 \tau_3) \epsilon^{\mu \nu \alpha \beta} F_{\mu \nu} A_\alpha \partial_\beta \pi^0 \]
\[ = \frac{e^2 N_c}{24 \pi^2 F_\pi} F_{\mu \nu} \tilde{F}^{\mu \nu} \pi^0 \]
Another prediction is for $\gamma 3\pi$ vertex—

$$\text{Amp}_{\gamma \pi^+ \pi^- \pi^0} = -i A_{3\pi}(0) \epsilon^\alpha \epsilon^\beta \epsilon^\gamma \epsilon^\delta \epsilon_{\alpha p+} \epsilon_{\beta p-} \epsilon_{\gamma p0} \epsilon_{\delta}$$

with

$$A_{3\pi}(0) = \frac{eN_C}{12\pi^2 F_\pi^3} = 9.7 \text{ GeV}^{-3}$$

Compare with experiment of Antipov et al. $N\pi \rightarrow \pi\pi N$

$$A_{3\pi}^{exp} = (12.9 \pm 0.9 \pm 0.5) \text{ GeV}^{-3}$$
Apparently 30% too high, but prediction is in chiral limit. Need chiral and radiative corrections to reach real world limit—$s = 4m^2_\pi$—rather than $s = 0$. Theoretical corrections are about 20% in the right direction—

$$A_{\gamma_3\pi}(0) = (10.7 \pm 1.2) \text{ GeV}^{-3}$$

Another approach is to use $\gamma^*$ via reaction $e^-\pi^- \rightarrow e^-\pi^-\pi^0$ which yields

$$A_{\gamma_3\pi}(0) = (9.9 \pm 1.1) \text{ GeV}^{-3}$$
Hadronic Polarizabilities
Polarizabilities

What is a polarizability?

Answer: A measure of response of system to quasi-static electric and/or magnetic field. Simplest example: electric polarizability $\alpha_E$—applied electric field $\vec{E}$ induces EDM $\vec{p}$

$$\vec{p} = 4\pi \alpha_E \vec{E}$$

Equivalently energy density is

$$u = -\frac{1}{2} 4\pi \alpha_E \vec{E}^2$$
Proton electric polarizability

Electric polarizability: proton between charged parallel plates
Similarly magnetic polarizability $\beta_M$—applied magnetic field $\vec{H}$ induces MDM $\vec{m}$

$$\vec{m} = 4\pi \beta_M \vec{H}$$

with energy density

$$u = -\frac{1}{2} 4\pi \beta_M \vec{H}^2$$
Proton magnetic polarizability

Magnetic polarizability: proton between poles of a magnetic field
How to measure? Compton scattering—if

\[ H = \frac{(\vec{p} - e\vec{A})^2}{2m} - \frac{1}{2} 4\pi\alpha_E \vec{E}^2 - \frac{1}{2} 4\pi\beta_M \vec{H}^2 \]

with

\[ \vec{E} = -\frac{\partial}{\partial t} \vec{A}, \quad \vec{B} = \vec{\nabla} \times \vec{A} \]

then

\[ T = \hat{\epsilon} \cdot \hat{\epsilon}' \left( -\frac{Q^2}{m} + \omega\omega' 4\pi\alpha_E \right) + \hat{\epsilon} \times \vec{k} \cdot \hat{\epsilon}' \times \vec{k}' 4\pi\beta_M \]

and
\[
\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{m^2} \left( \frac{\omega'}{\omega} \right)^2 \left( \frac{1}{2} (1 + \cos^2 \theta) - \frac{m\omega\omega'}{\alpha} \right) \\
\cdot \left[ \frac{\alpha_E + \beta_M}{2} (1 + \cos \theta)^2 + \frac{\alpha_E - \beta_M}{2} (1 - \cos \theta)^2 \right] + \ldots
\]

Results from MAMI (TAPS), Illinois, Saskatoon for proton

\[\alpha_E^P = (12.0 \pm 0.6) \times 10^{-4} \text{ fm}^3, \quad \beta_M = (1.9 \pm 0.6) \times 10^{-4} \text{ fm}^3\]
Estimate via chiral perturbation theory (Bernard, Kaiser, Meissner):

\[ \alpha_E^p = \frac{\alpha g_A^2}{48\pi^2 F_\pi^2 M_n} \left[ \frac{5\pi}{2\mu} + 18 \log \mu + \frac{33}{2} + \mathcal{O}(\mu) \right] = 7.4 \]

\[ \beta_M^p = \frac{\alpha g_A^2}{48\pi^2 F_\pi^2 M_n} \left[ \frac{\pi}{4\mu} + 18 \log \mu + \frac{63}{2} + \mathcal{O}(\mu) \right] = -2.0 \]

with \( \mu = m_\pi / m_n \) in units of \( 10^{-4} \text{ fm}^3 \).
If take only leading term (\(\mathcal{O}(q^3)\) HBChipt), then

\[
\alpha^p_E = 10 \beta^p_M = \frac{5g_A^2}{96\pi F_\pi^2 m_\pi} = 12.2
\]

in perfect agreement with experiment. Clearly accidental since \(\mathcal{O}(q^4)\) calculation gives

\[
\alpha^p_E = 10.5 \pm 2.0 \quad \text{and} \quad \beta^p_M = 3.5 \pm 3.6
\]

However, see also Pascalutsa and Lensky.
Neutron measurements much more difficult since

i) no neutron target available

ii) no Thomson term to interfere with

Has been done via

n-Pb scattering, d,\(^3\)He targets, etc.
All agree that $\alpha^n_E \sim \alpha^p_E$ and $\beta^n_M \sim \beta^p_M$. Schumacher recommends values

$$\alpha^n_E = 12.5 \pm 1.8 \quad \text{and} \quad \beta^p_M = 2.7 \mp 1.8$$

in good agreement with HBchipt prediction that primarily isoscalar.
Charged Pion Polarizabilities

Another interesting case is charged pion. Here lowest order chiral symmetry $\mathcal{O}(p^4)$ gives

$$\alpha_E^{\pi^+} + \beta_M^{\pi^+} = 0 \quad \text{and} \quad \alpha_E^{\pi^+} - \beta_M^{\pi^+} = 5.4 \times 10^{-4} \text{ fm}^3$$

where here size of polarizability is given by axial term in radiative pion decay—$\pi^+ \rightarrow e^+\nu_e\gamma$. Two loop also done, generating small corrections

$$\alpha_E^{\pi^+} = 2.9 \times 10^{-4} \text{ fm}^3 \quad \text{and} \quad \beta_M^{\pi^+} = -2.5 \times 10^{-4} \text{ fm}^3$$

as expected.

Experimental results are varied. Three methods:
i) Primakoff Effect: \( \pi^+ + N \rightarrow \pi^+ + N + \gamma \) Antipov et al. (1985) \( \alpha_{E}^{\pi^+} = (6.8 \pm 1.4 \pm 1.2) \times 10^{-4} \text{ fm}^3 \)

ii) Pion Pole: \( \gamma + N \rightarrow \gamma + N + \pi^+ \) Aibergenov et al. (1986) \( \alpha_{E}^{\pi^+} = (20 \pm 12) \times 10^{-4} \text{ fm}^3 \)

Ahrens et al. (MAMI-2005): \( \alpha_{E}^{\pi^+} - \beta_{M}^{\pi^+} = (11.6 \pm 1.5 \pm 3.0 \pm 0.5) \times 10^{-4} \text{ fm}^3 \)

iii) Annihilation: \( \pi^+ \pi^- \rightarrow \gamma \gamma \)

Babusci et al. (1992) \( \alpha_{E}^{\pi^+} = (2.2 \pm 1.6) \times 10^{-4} \text{ fm}^3 \)
Future: Spin Polarizabilities

Basic Idea: Higher order in photon energy expansion:

\[ \alpha_E = \alpha_{E1E1} \quad \beta_M = \beta_{M1M1} \]

If include spin then allow terms

\[ \gamma_{E1E1}, \gamma_{M1M1}, \gamma_{E1M2}, \gamma_{M1E2} \]

such that

\[ H = -\frac{1}{2} 4\pi \left[ \gamma_{E1E1} \vec{\sigma} \cdot \vec{E} \times \dot{\vec{E}} + \gamma_{M1M1} \vec{\sigma} \cdot \vec{B} \times \dot{\vec{B}} \right. \]

\[ \left. + 2\gamma_{E1M2} \sigma_i E_j B_{ij} - 2\gamma_{M1E2} \sigma_i B_j E_{ij} \right] \]

with \( E_{ij} = \frac{1}{2}(\nabla_i E_j + \nabla_j E_i) \). Note terms are \( \mathcal{O}(\omega^3) \) vs. \( \mathcal{O}(\omega^2) \) for \( \alpha_E, \beta_M \)
HBchipt predictions at $O(q^3)$ are

$$
\gamma_{E1E1}^p = -5\gamma_{M1M1}^p = 5\gamma_{E1M2}^p = 5\gamma_{M1E2}^p = -\frac{\alpha_E^p}{10\pi m_\pi}
$$

At this order there is a large contribution from pion pole graph

$$
\gamma_{E1E1}^p = -\gamma_{M1M1}^p = \gamma_{E1M2}^p = -\gamma_{M1E2}^p = \frac{24}{g_A} \frac{\alpha_E^p}{10\pi m_\pi}
$$

dominates structure terms.

At present no direct measurements of spin polarizabilities. However, analysis of backward Compton scattering yields

$\text{LEGS} : \gamma_\pi = (-27.1 \pm 2.2 \pm 2.6) \times 10^{-4} \text{ fm}^4$

$\text{MAMI - TAPS} : \gamma_\pi = (-35.9 \pm 2.3) \times 10^{-4} \text{ fm}^4$

$\text{MAMI - LARA} : \gamma_\pi = (-40.9 \pm 0.6 \pm 2.2) \times 10^{-4} \text{ fm}^4$
Schumacher recommends \( \gamma_\pi = (-38.7 \pm 1.8) \times 10^{-4} \text{fm}^4 \) and dispersion relation (higher order GDH) gives value in forward direction

\[
\gamma_0 = (-1.0 \pm 0.8 \pm 0.10) \times 10^{-4} \text{fm}^4
\]

Note \( |\gamma_\pi^p| \gg |\gamma_0^p| \). Reason is

\[
\gamma_0^p = -\gamma_{E1E1}^p - \gamma_{M1M1}^p - \gamma_{E1M2}^p - \gamma_{M1E2}^p
\]

so large pion pole contribution cancels, while

\[
\gamma_\pi^p = -\gamma_{E1E1}^p + \gamma_{M1M1}^p - \gamma_{E1M2}^p + \gamma_{M1E2}^p
\]

wherein pion pole does NOT cancel—

\[
pole \gamma_\pi^p = -\frac{9.6 \alpha_E^p}{\pi m_\pi g_A} = -42.7 \times 10^{-4} \text{fm}^4
\]
Clearly pole dominates $\gamma_\pi^P$. Lowest order chiral prediction for

$$\gamma_0^P = \frac{6}{5\pi m_\pi} \alpha_E^P = +6.2 \times 10^{-4} \text{ fm}^4$$

Wrong sign, so higher order terms must be important.
Transverse target asymmetry $\Sigma_{2x}$ and sensitivity to $\gamma_{E1E1}$

PRELIMINARY

<table>
<thead>
<tr>
<th>$\gamma_{E1E1}$</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\gamma_{E1E1}$</td>
<td>-2.3</td>
</tr>
<tr>
<td>$\gamma_{E1E1}$</td>
<td>-3.3</td>
</tr>
<tr>
<td>$\gamma_{E1E1}$</td>
<td>-4.3</td>
</tr>
<tr>
<td>$\gamma_{E1E1}$</td>
<td>-5.3</td>
</tr>
<tr>
<td>$\gamma_{E1E1}$</td>
<td>-6.3</td>
</tr>
</tbody>
</table>
Hadronic Parity Violation
1980: DDH Approach

Basic idea:

Meson exchange gives good picture of PC NN interaction, with

$$H_{st} = i g_{\pi NN} \bar{N} \gamma_5 \tau \cdot \pi N + g_{\rho} \bar{N} \left( \gamma_\mu + i \frac{\mu_V}{2M} \sigma_{\mu\nu} k^\nu \right) \tau \cdot \rho^\mu N$$

$$+ g_\omega \bar{N} \left( \gamma_\mu + i \frac{\mu_S}{2M} \sigma_{\mu\nu} k^\nu \right) \omega^\mu N$$

so use for PV NN
Then define general PV weak couplings:

\[ \mathcal{H}_{\text{wk}} = \frac{f_\pi^1}{\sqrt{2}} \bar{N}(\tau \times \pi)_3 N \]

\[ + \bar{N} \left( h^0_\rho \tau \cdot \rho^\mu + h^1_\rho \rho^\mu_3 + \frac{h^2_\rho}{2\sqrt{6}} (3\tau_3 \rho^\mu_3 - \tau \cdot \rho^\mu) \right) \gamma_\mu \gamma_5 N \]

\[ + \bar{N} (h^0_\omega \omega^\mu + h^1_\omega \tau_3 \omega^\mu) \gamma_\mu \gamma_5 N - h^1_\rho \bar{N} (\tau \times \rho^\mu)_3 \frac{\sigma_{\mu\nu} k^\nu}{2M} \gamma_5 N \]

Yields two-body PV NN potential
\[ V^{\text{PNC}} = i \frac{f_{\pi} g_{\pi NN}}{\sqrt{2}} \left( \frac{\tau_1 \times \tau_2}{2} \right) \left( \sigma_1 + \sigma_2 \right) \cdot \left\{ \frac{p_1 - p_2}{2M}, f_\pi(r) \right\} \]

\[ -g_\rho \left( h_\rho^0 \tau_1 \cdot \tau_2 + h_\rho^1 \left( \frac{\tau_1 + \tau_2}{2} \right) + \frac{3}{2\sqrt{6}} h_\rho^2 \left( 3\tau_1 \tau_2^3 - \tau_1 \cdot \tau_2 \right) \right) \]

\[ \times \left( (\sigma_1 - \sigma_2) \cdot \left\{ \frac{p_1 - p_2}{2M}, f_\rho(r) \right\} \right) \]

\[ + i (1 + \chi_V) \sigma_1 \times \sigma_2 \cdot \left\{ \frac{p_1 - p_2}{2M}, f_\omega(r) \right\} \]

\[ -g_\omega \left( h_\omega^0 + h_\omega^1 \left( \frac{\tau_1 + \tau_2}{2} \right) \right) \]

\[ \times \left( (\sigma_1 - \sigma_2) \cdot \left\{ \frac{p_1 - p_2}{2M}, f_\omega(r) \right\} \right) \]

\[ + i (1 + \chi_S) \sigma_1 \times \sigma_2 \cdot \left\{ \frac{p_1 - p_2}{2M}, f_\omega(r) \right\} \]

\[ -(g_\omega h_\omega^1 - g_\rho h_\rho^1') \left( \frac{\tau_1 - \tau_2}{2} \right) \left( \sigma_1 + \sigma_2 \right) \cdot \left\{ \frac{p_1 - p_2}{2M}, f_\rho(r) \right\} \]

\[ -g_\rho h_\rho^1' i \left( \frac{\tau_1 \times \tau_2}{2} \right) \left( \sigma_1 + \sigma_2 \right) \cdot \left\{ \frac{p_1 - p_2}{2M}, f_\rho(r) \right\} \]
Represent in terms of "Reasonable Range" and "Best Value"

<table>
<thead>
<tr>
<th>Coupling</th>
<th>DDH Reasonable Range</th>
<th>DDH &quot;Best&quot; Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$h^0_\pi$</td>
<td>0 → 30</td>
<td>12</td>
</tr>
<tr>
<td>$h^0_\rho$</td>
<td>30 → −81</td>
<td>−30</td>
</tr>
<tr>
<td>$h^1_\rho$</td>
<td>−1 → 0</td>
<td>−0.5</td>
</tr>
<tr>
<td>$h^2_\rho$</td>
<td>−20 → −29</td>
<td>−25</td>
</tr>
<tr>
<td>$h^0_\omega$</td>
<td>15 → −27</td>
<td>−5</td>
</tr>
<tr>
<td>$h^1_\omega$</td>
<td>−5 → −2</td>
<td>−3</td>
</tr>
</tbody>
</table>

all times "sum rule value" $3.8 \times 10^{-8}$
Size of leading PV amplitude $h_{\pi}^1$ seems at variance with DDH best estimate $h_{\pi}^1(DDH) = 4.3 \times 10^{-7}$ vs. $^{18}$F value $h_{\pi}^1 < 1.3 \times 10^{-7}$. How to resolve:

i) $A_{\gamma}$ in $\vec{n}p \rightarrow d\gamma$ underway at SNS

ii) $B_{\gamma}$ in $\vec{\gamma}p \rightarrow \pi^+ n$
Chen and Ji obtained LO result

\[ B_{\gamma}^{LO} = \frac{\sqrt{2} F_\pi h_\pi^1 (\mu_p - \mu_n)}{g_A M_N} \]

NLO effects by Puglia, BRH, and Ramsey-Musolf

\[ B_{\gamma}^{NLO} = \frac{\sqrt{2} F_\pi h_\pi^1}{g_A M_N} [\mu_p - \mu_n (1 + \frac{m_\pi}{M_N})] - \frac{4\sqrt{2} m_\pi}{g_A \Lambda_\chi} \bar{C} \]
Both $A_\gamma$ and $B_\gamma$ sensitive (almost completely) to $h_{\pi}^1$. However, can also measure asymmetry in

$$\gamma d \rightarrow np$$

which is also of great interest and sensitive to short distance pieces.

$$C_\gamma \simeq -0.16\lambda_s^0 + 0.13\lambda_s^2 + 0.63\lambda_t$$

where $\lambda_i$ are so-called Danilov parameters.
Conclusion
Probes of chiral symmetry possible with low energy electron machine. We have examined three such

a) Chiral Anomaly

b) Hadronic Polarizabilities

c) Hadronic Parity Violation

but many others possible.