MASSACHUSETTS INSTITUTE OF TECHNOLOGY

Department of Physics

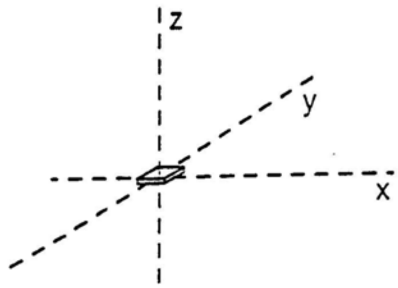
8.03 Vibrations and Waves — Fall Term 2006

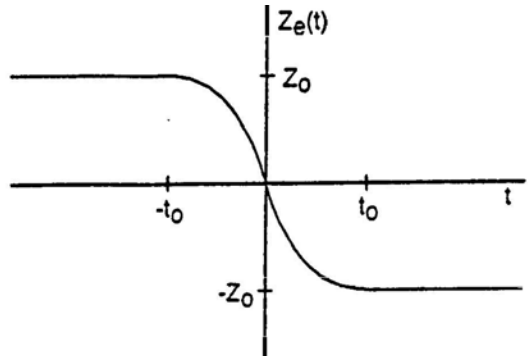
Problem Set #8

Due: 4:30 PM, Thursday, November 9

Homework Bins, 3rd Floor between buildings 8 and 16

Problem 1: Avalanche Breakdown





Avalanche breakdown in a semiconductor can cause a sudden motion of charge across a thin chip of material. In the example illustrated above, a small sample is located at $\vec{r} = 0$. N electrons of charge $-|e|$ flow across a distance of $2z_0$ in a time $2t_0$.

The position of any one of the electrons is given by

$$z_e(t) = z_0 \qquad t < -t_0$$

$$z_e(t) = \frac{1}{6}\beta t^3 - \frac{1}{2}\beta t_0^2 t \qquad t_0 < t < -t_0$$

$$z_e(t) = -z_0 \qquad t > t_0$$

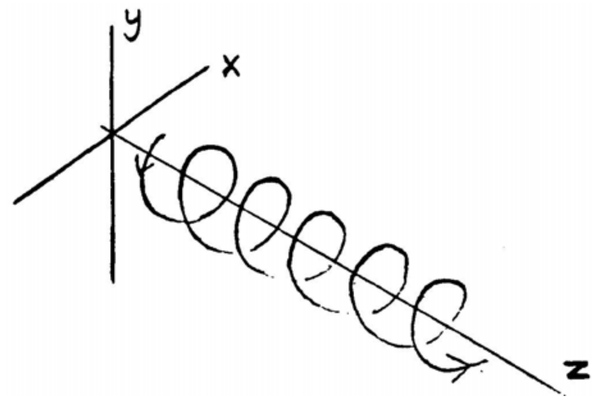
The radiated electric field $\vec{E}_{rad}$ is observed at the position $\vec{r} = R\left(\frac{\sqrt{3}}{2}\hat{x} + \frac{1}{2}\hat{z}\right)$, where $R >> ct_0$. Neglect the non-zero extent of the sample in calculating the radiated fields.

a) When is $\vec{E}_{rad}$ non-zero at this position?

b) Find an expression for $\vec{E}_{rad}$ at this position. Be sure to indicate the direction of the vector as well as its dependence on the given parameters and t. Make a sketch of $\vec{E}_{rad}$ as a function of time.

c) Find the radiated magnetic field $\vec{B}_{rad}$ observed at the same position.

Problem 2: Synchrotron Radiation



In a distant part of our galaxy an electron executes spiral motion in a magnetic field as shown above. Its speed is non-relativistic. The position of the electron is given by

$$\vec{r}(t) = \hat{x}\alpha\cos(\beta t) + \hat{y}\alpha\sin(\beta t) + \hat{z}\gamma t$$

where α, β, and γ are constants.

a) Give the frequency ν (in Hertz), the wavevector $\vec{k}$ (a vector), and the polarization state of the radiation observed by a radio astronomer located far way in the following locations (x, y, z).

1) $(0,0,R)$
2) $(R/\sqrt{2},0,R/\sqrt{2})$
3) $(R,0,0)$

b) Give an expression for the total energy radiated per second by the electron in terms of α, β, γ and any primary physical constants.

c) On a sphere of radius R about the source, what is the ratio between the largest and smallest value of the time averaged Poynting vector?

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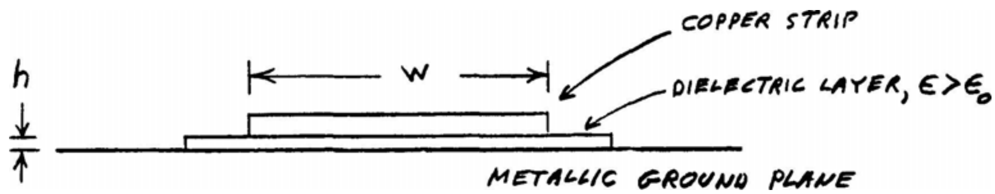
Problem 3: Microstrip Transmission Line



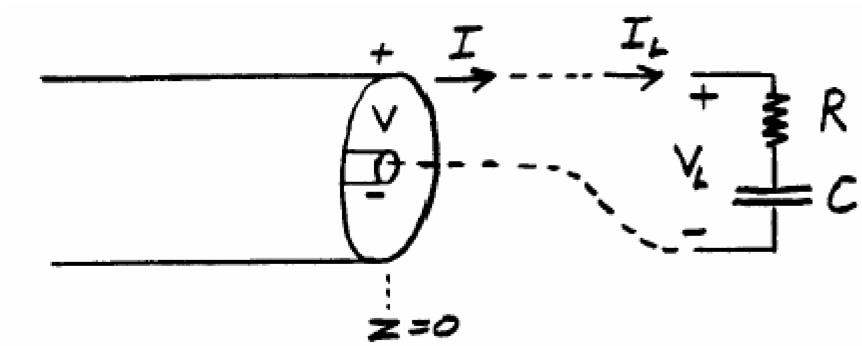
The diagram shows the cross-section of a microstrip transmission line that is used to conduct high frequency signals around circuit boards in many avionics applications.

a) Sketch the electric and magnetic field lines in a plane perpendicular to the direction of propagation, that is, in the plane of the paper for the above diagram. For clarity, you may wish to exaggerate the thickness h relative to the width **w.**

b) Assume that w >>h so that one can safely ignore fringing fields. Calculate the capacitance and the inductance per unit length of the structure.

c) Find expressions for the velocity and the characteristic impedance of the line in terms of h, w, $\varepsilon/\varepsilon_0$, the velocity of light in a vacuum, c, and the impedance of the vacuum, $\sqrt{\mu_0/\varepsilon_0}$ = 377 ohms.

Problem 4: Reflection at a Non-ohmic Load



A coaxial cable with characteristic impedance Z_o is terminated by a series combination of a resistor and a capacitor as shown above. If a harmonic voltage wave is incident from the left, a reflected wave will be set up by the load. The resulting total voltage on the line will have the

$$\tilde{V}(z,t) = \tilde{V}_i e^{j(\omega t - kz)} + \tilde{V}_r e^{j(\omega t + kz)}.$$

a) Write down an expression for the complex current $\tilde{I}(z,t)$ on the line.

b) Find the relation between the complex voltage across the load, $\tilde{V}_L$ and the complex current into it $\tilde{I}_L$.

c) By matching the boundary conditions on voltage and current, find $\tilde{V}_r$ in terms of $\tilde{V}_i$, ω, R, C, and Z_o. Note that the relation will be complex, indicating that the load can change the phase as well as the amplitude of the reflected wave.

d) Check the result of your calculation in c) by using the general expression for the voltage reflection coefficient given in lecture.

e) Sketch the amplitude and the phase of the reflected wave as a function of frequency for the special case $R = Z_o$.

Problem 5: Take Home Experiment, Scattering of Light

Do the take home experiment, Scattering of Light. Write up your observations and comments. Be brief. One page should be enough, perhaps two if you include sketches.

Problem 6: Take Home Experiment, Liquid Prism

The previous experiment will give you some experience using the sealant RTV. This is a good time to build the cell used in the Liquid Prism experiment. Be neat, take your time, and do not be discouraged if it leaks the first time. Just build and test the cell this week. This is the time consuming part. In the next problem set, I will have you do the experiment and write up your observations.