Optimum Design of the Shunt-Series Feedback Pair with a Maximally Flat Magnitude Response*

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Summary—**In this paper, analysis and design of the shunt-series pair feedback amplifier are given. The design achieves optimum gain-bandwidth product with a maximally flat magnitude transmission for a specified gain or desensitivity. Phantom zero is used to avoid degradation of feedback bandwidth and to obtain a filter type transient response. Experimental verification of the theory is given with a medium frequency alloyed-junction transistors and HF mesa transistors. The agreement between the theoretical and experimental work is shown to be good in all cases, both in time and in frequency domains.**

Introduction

EXTENSIVE studies have been made on feedback amplifiers using three transistor stages. To make available more forward gain, three stages are used rather than two; a larger amount of forward gain, in turn, provides a large closed-loop gain and/or amount of feedback. However, in a three-stage feedback amplifier, if proper attention is not given in the design, the amplifier will oscillate. To avoid oscillations, interstage networks are used to shape the loop transmission function properly. Unavoidable reduction of gain results because of the introduction of lossy passive shaping networks. In some cases the gain is reduced so considerably that a feedback pair can become competitive.

Fig. 1 shows a shunt-series feedback pair with resistive feedback,[1,2] the configuration of interest in this paper; a somewhat more complicated feedback circuit will be discussed later. Biasing and coupling circuitry are omitted for simplicity.

The circuit of Fig. 1 will be analyzed by using the simple unilateral equivalent circuit in Fig. 2.[3] In order to obtain a high gain in the forward path, a large value for R_I must be chosen. For R_i and R_I much larger than r_b', one can ignore r_b' without any significant error. The load for the first stage, then, is approximately a parallel RC network. The CE stage with a parallel RC load can be represented by the equivalent circuit of Fig. 3. The analysis of the over-all circuit is then simple, and design information can be obtained easily.

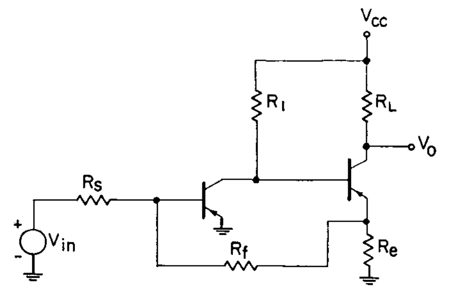

Fig. 1—Shunt-series feedback pair with resistive feedback.

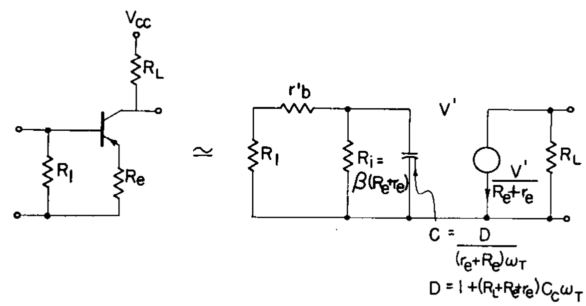

Fig. 2—Equivalent circuit for a common-emitter transistor amplifier with resistive load.

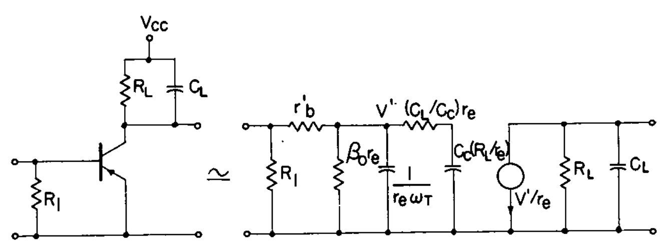

Fig. 3—Equivalent circuit for a common-emitter transistor amplifier with an RC load.

Analysis

A general analysis for feedback amplifiers is presented elsewhere,[4] and it is shown therein how a feedback amplifier circuit can be separated into a forward circuit μ and a feedback circuit λ. The closed loop gain function can be then written as:

$$A = \frac{\mu(p)}{1 - \mu(p)\lambda(p)}. \tag{1}$$

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[1] M. S. Ghausi, "Analysis and Design of the Shunt-Series Feedback Pair," Electronic Research Lab., University of California, Berkeley, Ser. No. 60, Issue No. 307, Rept. No. 74; August 16, 1960.

[2] D. O. Oram, "Simplification of Silicon Diffused-Base-Transistor Equivalent Circuit; Its Use in the Design of Wide-Temperature Range Video Amplifier," presented at 1960 Northeast Electronics Research and Engineering Meeting, Boston, Mass.; November 15–17.

[3] D. O. Pederson, "A Note on Simplified Transistor Equivalent Circuits," Electronics Research Lab., University of California, Berkeley, Series No. 60, Issue No. 229, Rept. No. 81; February 5, 1959.

[4] M. S. Ghausi and D. O. Pederson, "A New Design Approach for Feedback Amplifiers," Electronics Research Lab., University of California, Berkeley, Series No. 60, Issue No. 274, Rept. No. 104, April 12, 1960; presented at Symp. on Active Networks and Feedback System, Polytech. Inst. Brooklyn, Brooklyn, N. Y., April 19–21, 1960.

The complete forward circuit for Fig. 1 is shown in Fig. 4. Since R_I, $R_{i2} \gg r_b'$, r_b' of the second stage is thus ignored.

The forward circuit current gain is:

$$\mu = \frac{I_L}{I_S} = \frac{1}{(R_e + r_{e2})C_{i2}r_{e1}C_{i1}(1+\delta)} \cdot \left[\frac{1}{p^2 + p\left(p_{i1} + p_{i2} + \frac{C_{e1}}{C_{i1}C_{i2}r_{e1}} + \frac{1}{R_pC_{i1}(1+\delta)}\right) + p_{i2}\left(p_{i1} + \frac{1}{R_pC_{i1}(1+\delta)}\right)}\right], \quad (2)$$

where $p = \sigma + j\omega$ and

$$C_{i2} = \frac{D}{(r_{e2} + R_e)\omega_{T2}} = \frac{1 + (R_L + r_{e2} + R_e)C_{e2}\omega_{T2}}{(R_e + r_{e2})\omega_{T2}}$$

$$C_{i1} = \frac{1 + r_{e1}C_{e1}\omega_{T1}}{r_{e1}\omega_{T1}}$$

$$p_{i1} = \frac{1}{R_{i1}C_{i1}}, \qquad R_{i1} = \beta_{01}r_{e1}$$

$$p_{i2} = \frac{R_I + R_{i2}}{R_IR_{i2}C_{i2}} \qquad R_{i2} = \beta_{02}(r_{e2} + R_e)$$

$$R_p = \frac{(R_e + R_f)R_S}{R_S + R_e + R_f}$$

$$\delta = \frac{r_b'}{R_p},$$

or one can rewrite (2) as:

$$\mu = \frac{\mu_0 p_1 p_2}{(p + p_1)(p + p_2)}, \quad (3)$$

where p_1 and p_2 are the poles of the forward circuit and are both real and negative, and

$$\mu_0 = \beta_{01}\left(\frac{R_p}{R_p + r_b' + R_{i1}}\right)\beta_{02}\left(\frac{R_I}{R_I + R_{i2}}\right). \quad (4)$$

The feedback factor is:

$$\lambda = \frac{I_{fb}}{I_L} = -\frac{R_e}{R_e + R_f} = -\lambda_0. \quad (5)$$

The closed-loop current gain is then given by (1):

$$A_I = \frac{\mu}{1 - \mu\lambda} = \frac{\mu_0 p_1 p_2}{p^2 + p(p_1 + p_2) + p_1p_2(1 + \mu_0\lambda_0)}. \quad (6)$$

For

$$\mu\lambda \gg 1, \qquad A_I \simeq -\frac{1}{\lambda} = \frac{R_e + R_f}{R_e}. \quad (7)$$

The root locus for (6) is shown in Fig. 5.

For the closed loop to have a maximally flat magnitude (MFM) transmission function, the closed-loop poles must be on the 45° radial line [point P in Fig. 5(a)]. If a maximally flat delay (MFD) transmission is desired, the closed-loop poles must be on the 30° radial line [point P' in Fig. 5(b)].

For $\psi = 45$ the following relationship must hold between the coefficients in (6):

$$p_1 + p_2 = \sqrt{2p_1p_2(1 + \mu_0\lambda_0)}, \quad (8)$$

and the closed-loop bandwidth is then given by

$$\omega_{3\,\text{db}} = \frac{p_1 + p_2}{\sqrt{2}}. \quad (9)$$

For purely resistive feedback, and large amounts of feedback, the closed-loop poles may have an angle ψ larger than 45°, which would result in frequency peaking, hence a large overshoot in time domain. To obtain an MFM response, one may either narrow band a stage by a capacitor (this would diminish the feedback bandwidth) or one may introduce a capacitance C_f across R_f. (See Fig. 6.) The latter provides a zero in the feedback function without a loss in feedback bandwidth. We shall obviously prefer the latter case.

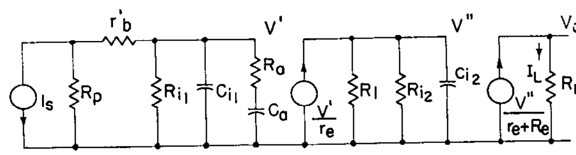
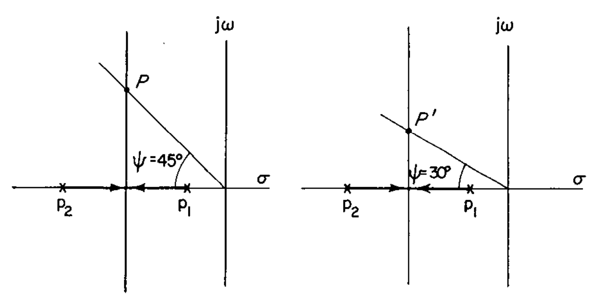


Fig. 4—Forward circuit for the shunt-series feedback pair.



(a) (b)

Fig. 5—Root locus. (a) Closed-loop pole locations for a MFM response. (b) Closed-loop pole locations for a MFD response.

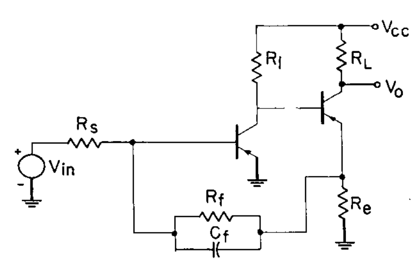


Fig. 6—Shunt-series feedback pair with a phantom zero.

Phantom Zero

A better design would result if a phantom zero (zero in the feedback path) is used by shunting the feedback resistance R_f by a capacitance C_f as shown in Fig. 6. For most practical cases the amplifier gain specified is large, $A_I \geq 10$. For convenience we shall use this restriction in order to simplify the expressions in the analysis and design. Furthermore, for large bandwidths $C_f \ll C_i$; hence we shall ignore C_f in the forward circuit for the same reasons.

The resulting forward circuit is then the same as in Fig. 4 but the feedback function is

$$\lambda = -\frac{\lambda_0}{z}(p + z), \tag{10}$$

where

$$z = \frac{1}{R_f C_f} \quad \text{and} \quad \lambda_0 \simeq \frac{R_e}{R_f}.$$

The loop transmission function then has a real zero and two real poles:

$$T = T'_{(0)} \frac{(p + z)}{(p + p_1)(p + p_2)}. \tag{11}$$

To get a large bandwidth, we select the zero to be larger than p_1 and/or p_2. The root locus is a circle as shown in Fig. 7, whose equation is given by[5]

$$\omega^2 + (\sigma + z)^2 = z^2 - z(p_1 + p_2) + p_1 p_2. \tag{12}$$

The center of the circle is at $(0, -z)$, and its radius is at $\rho = (z^2 - z(p_1 + p_2) + p_1 p_2)^{1/2}$. The closed-loop function for this case is:

$$A_I = \frac{\mu}{1 - \mu\lambda}$$

$$= \frac{\mu_0 p_1 p_2}{p^2 + p\left(p_1 + p_2 + \dfrac{\mu_0 \lambda_0 p_1 p_2}{z}\right) + p_1 p_2 (1 + \mu_0 \lambda_0)}. \tag{13}$$

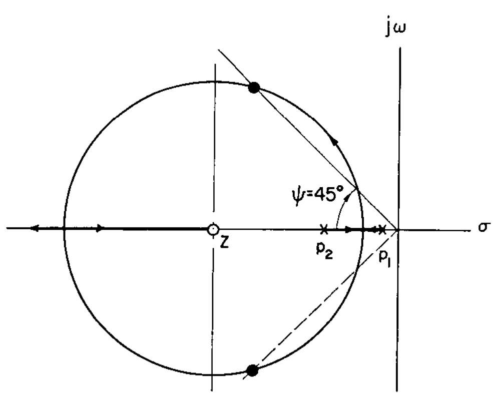


Fig. 7—Root locus for two poles and one zero.

[5] See Appendix I.

Notice that since the zero is in the feedback path, it does not appear as a zero in the closed-loop transfer function; hence, it is called a "phantom zero."

For the response (13) to be MFM, the roots of the closed-loop system must lie on the 45° radials, which implies that

$$p_1 + p_2 + \frac{\mu_0 \lambda_0 p_1 p_2}{z_1} = \sqrt{2 p_1 p_2 (1 + \mu_0 \lambda_0)}. \tag{14}$$

The 3 db bandwidth is then given by

$$\omega_{3\,\text{db}} = \sqrt{p_1 p_2 (1 + \mu_0 \lambda_0)}. \tag{15}$$

In terms of the transistor parameters

$$\omega_{3\,\text{db}} = \sqrt{\frac{\omega_{T1}\omega_{T2}}{A_I\, D_2} \left[\frac{1}{1 + \dfrac{R_{i1} + r_b'}{R_p}}\right] \left[\frac{1}{1 + \dfrac{R_{i2} + r_b'}{R_I}}\right]}. \tag{16}$$

Since the gain-bandwidth product vs bandwidth decreases for resistive broadbanding, one has for

$$R_I \gg R_{i2} + r_b' \quad \text{anb} \quad R_p \gg R_{i1} + r_b',$$

$$(\omega_{3\,\text{db}})_{\max} \approx \sqrt{\frac{\omega_{T1}\omega_{T2}}{A_I\, D_2\left(1 + \dfrac{r_b'}{R_S}\right)}}. \tag{17}$$

The desensitivity or amount of feedback $T(0) \approx \mu(0)/A_I(0)$, and the feedback bandwidth is approximately equal to the dominant pole p_1 of the loop transmission function (11).

It should be remembered that if any overshoot of less than one per cent is desired, the angle ψ should be reduced to 30° for a MFD response [Fig. 5(b)]. Obviously (8) and (9) will no longer hold and the corresponding equations can be easily obtained.

Design

Let us consider first the problem of attaining maximum closed-loop bandwidth for a specified A_I with given transistors. In deciding on the values to be used for the various elements, it is noted from the previous section that in order to have a large gain in the forward circuit, R_I should be as large as possible. Even though a small value of R_I would broad band the second stage, it would also decrease the forward gain μ_0. The gain-bandwidth product vs bandwidth decreases in resistive broad banding, however, and thus the design would be better if R_I were as large as possible. For practical reasons, there is an upper limit to R_I; a reasonable design choice is to select R_I of the same order of magnitude as R_{i2} which itself has a large value due to the external emitter lead resistance R_e. To get a small D factor for the second stage, a low value of R_e is desirable. A specified value of closed-loop gain ($A_I \geq 10$) would also require a low value of R_f. However, R_f appears as an interstage broad-banding resistance in the forward circuit, and by the same line of reasoning as above, a large value of R_f is desirable. There exists then an optimum value of R_f (hence R_e) which will result in a maximum closed-loop bandwidth for a given gain. The expression for optimum value of R_f is[6]

$$(R_f)_{\text{opt.}} = \sqrt{A_I\left(\frac{r_b' R_s}{R_s + r_b'}\right)\left(\frac{1 + (R_L + r_e)C_{c2}\omega_{T2}}{C_{c2}\omega_{T2}}\right)}. \qquad (18)$$

A logical basis for the design of the MFM feedback pair for given A_I and maximum bandwidth and large desensitivity may now be formulated as follows: For the given transistor parameters and specified closed-loop gain[7] ($A_I \geq 10$), determine R_f from (18). From the gain expression

$$A_I = \frac{R_e + R_f}{R_e} \approx \frac{R_f}{R_e},$$

R_e is determined. R_I can be chosen to be of the same order of magnitude as R_{i2}, say, equal to $R_{i2} = \beta_0(R_e + r_{e2})$. From (14) one can determine z and thus C_f. Usually z is of the same order of magnitude as $\omega_{3\text{db}}$. [See (6).] Experimentally it may be expedient to select such a value of z and then adjust C_f for the desired response shape.

Illustrative Examples

Example 1

Consider that an amplifier is to be designed using alloyed-junction transistor with the following parameters:

	Transistor No. 1	Transistor No. 2
f_T	11 Mc	13 Mc
β_0	61	49
C_{cb}	18 pf	18 pf
r_b'	100 ohms	100 omhs

(V_c = 6 v and I_c = 2.5 ma)

A closed-loop gain of 20(26 db) is desired with source and load resistances of $R_S = 10K$, and $R_L = 50$ ohms, respectively. From (18) $R_f \simeq 1K$. For a gain of 20, (7) yields:

$$R_e = 50 \text{ ohms}.$$

Let

$$R_I = 2.5K \approx \beta_0 R_e.$$

From (14), $z = 2\pi(3.2 \times 10^6)$; hence $C_f = 50$ pf.

The calculated and experimental response curves are shown in Fig. 8 (next page). The dotted curves are experimental response for resistive feedback ($C_f = 0$) and with C_f larger than needed for an MFM response. The performance values are listed in Table I.

[6] See Appendix II.

[7] If the desensitivity is specified with the transistors, the design would proceed as follows: Since $\mu(0) \simeq \beta_{01}\beta_{02}/2$ (The factor 2 appears because of $R_I = R_{i2}$, R_s, R_f, $\gg R_{i1}$.), specifications on $T(0)$, *i.e.*, $\mu(0)/A_I(0)$, will then determine A_I and then follow as though gain were specified.

Example 2

Consider that an amplifier is to be designed using HF mesa transistors (*e.g.*, TI 2N1 143) with the following parameters:

	Transistor No. 1	Transistor No. 2
f_T	420 Mc	450 Mc
β_0	57	50
C_{cb}	2 pf	2 pf
r_b'	75 ohms	75 ohms

(V_c = 6 v and I_c = 5 ma)

A closed-loop gain of 20 (26 db) is desired with source and load resistances of $R_S = 1K$ and $R_L = 500$ ohms, respectively. From (17), $R_f = 1K$. For a gain of 20, (7) yields

$$R_e = 50 \text{ ohms}.$$

Let

$$R_I = 2.7K \approx \beta_0(R_e + r_e).$$

From (14), $z = 2\pi(121 \times 10^6)$; hence $C_f = 1.3$ pf.

The calculated and experimental response curves are shown in Fig. 9. The dotted curves are again experimental response for $C_f = 0$ and $C_f = 5$ pf. The performance values are listed in Table II.

The agreements in both cases are good, and further improvement can be made if some of the simplifying assumptions are removed and a more elaborate equivalent circuit is used, especially in the case of mesa transistors. Figs. 10 and 11 show photographs of the amplifier response in frequency and time domains designed on the above basis for a 6-nsec rise time. A Telonic Sweep Generator was used for the frequency domain, and a Tektronix Type N sampling 'scope was used for time domain measurements.

Conclusion

A simple analysis and design procedure is given for the shunt-series feedback pair transistor amplifier. The design achieves optimum gain-bandwidth product for a specified gain or desensitivity. It is shown that a simple analysis and design can predict the pertinent performance of the amplifier with a good degree of accuracy. The feedback pair, due to its large output impedance and small input impedance, can be cascaded with negligible interaction. By stagger tuning two such pairs, we can easily design an amplifier with a 4-pole MFM or MFD response. The feedback pair is most desirable for applications where the source impedance is large and the load impedance is small. A comparison of this structure with those of three-stage amplifiers would show that the feedback pair designed on the optimum basis developed herein is competitive with triples in many cases, especially considering also that only two transistors are used instead of three.

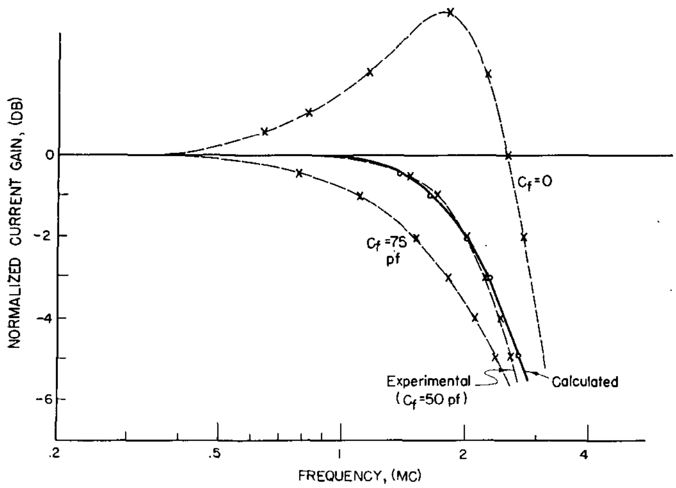


Fig. 8—Frequency response of the amplifier of example 1.

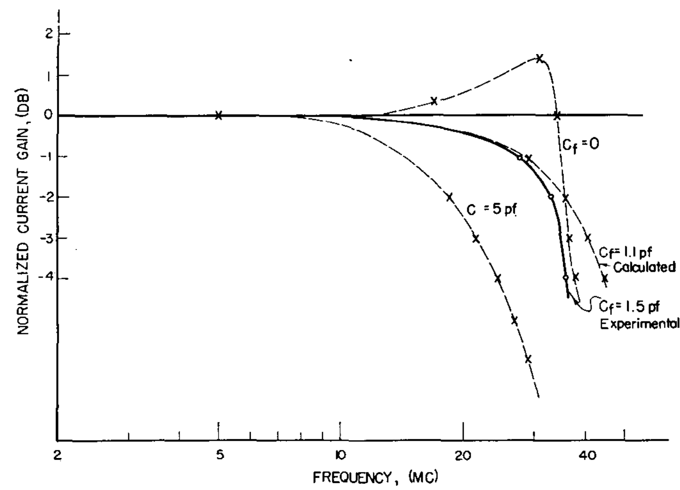


Fig. 9—Frequency response of the amplifier of Example 2.

TABLE I

	Calculated Values	Experimental Values
Closed-loop gain $A_I(0)$	20 (26 db)	20 (26 db)
Closed-loop bandwidth f_{3db}	2.29 Mc	2.25 Mc
Amount of feedback $T(0)$	37.6 (32.7 db)	38 (33 db)
Feedback bandwidth f_f	95 kc	94 kc

TABLE II

	Calculated Values	Experimental Values
Closed-loop gain $A_I(0)$	20 (26 db)	20.5 (26.2 db)
Closed-loop bandwidth f_{3db}	41 Mc	35 Mc
Amount of feedback $T(0)$	39 (33.9 db)	35 (30.9 db)
Feedback bandwidth f_f	1.1 Mc	1.3 Mc

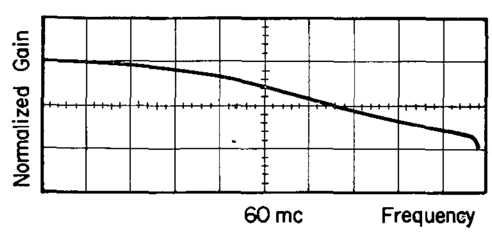


Fig. 10—Frequency response of a designed amplifier observed on 'scope.

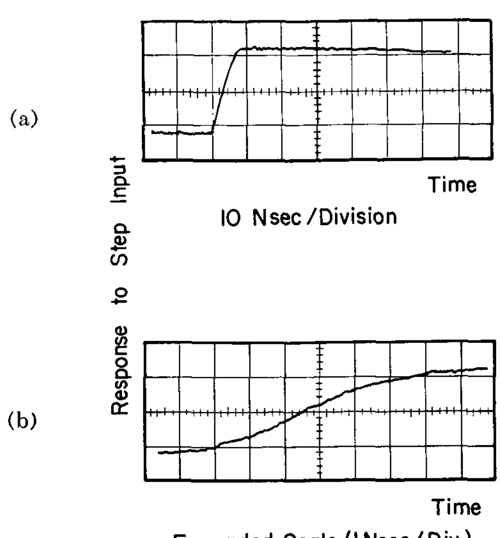


Fig. 11—Step input transient response of the amplifier of Fig. 10.

Appendix I

Consider a loop transmission function with two real poles p_1 and p_2 and a zero z. Let $z \gg p_1, p_2$.

$$T = T_0' \frac{(p + z)}{(p + p_1)(p + p_2)}. \quad (19)$$

To find the root locus, we set

$$\text{Arg}\,(p + z) - [\text{Arg}\,(p + p_1) + \text{Arg}\,(p + p_2)] = \pm 180 \quad (20)$$

$$(\omega_{3db})^2 = \frac{\mu_0}{A_I} p_1 p_2 = K \frac{R_f}{[R_f(R_s + r_b') + r_b' R_s]\left\{\frac{R_I \beta_{02}}{\omega_{T2}}[1 + (R_L + r_e)C_{c2}\omega_{T2}] + \left(\frac{\beta_{02} R_I C_{c2}}{A_I}\right) R_f\right\}}. \quad (25)$$

Substituting $(\sigma + j\omega)$ for p in Eq. (20),

$$\tan^{-1}\frac{\omega}{\sigma + z} - \left[\tan^{-1}\frac{\omega}{\sigma + p_1} + \tan^{-1}\frac{\omega}{\sigma + p_2}\right] = 180°, \quad (21)$$

or

$$\frac{\omega}{\sigma + z} - \frac{\frac{\omega}{\sigma + p_1} + \frac{\omega}{\sigma + p_2}}{1 - \frac{\omega}{\sigma + p_1}\frac{\omega}{\sigma + p_2}} = 0. \quad (22)$$

Simplifying and rearranging (22), we have

$$\omega^2 + (\sigma + z)^2 = z^2 - z(p_1 + p_2) + p_1 p_2. \quad (23)$$

Eq. (23) is recognized to be the equation of a circle with center at $(0, -z)$ and radius $\rho = (z^2 - z(p_1 + p_2) + p_1 p_2)^{1/2}$.

Appendix II

For large amounts of feedback where the condition of (14) is satisfied,[8] the bandwidth is given by (15) (rewritten here for convenience):

$$\omega_{3\,db} = \sqrt{p_1 p_2 (1 + \mu_0 \lambda_0)}, \quad (24)$$

where p_1, p_2, and $\mu_0\lambda_0$ are all defined in the text of this paper. Specification of gain determines R_e in terms of R_f (7). By substituting the expressions of p_1, p_2, and $\mu_0\lambda_0$ in (24), and upon simplification, one obtains: (25)

To maximize $(\omega_{3db})^2$, we set $= d(\omega_{3db})^2/dR_f$ equal to zero, and obtain:

$$(R_f)_{\text{opt}} = \sqrt{\frac{A_I R_S r_b'}{(R_S + r_b')}\left[\frac{1 + (R_L + r_e)C_{c2}\omega_{T2}}{C_{c2}\omega_{T2}}\right]}. \quad (26)$$

For a current source (26) reduces to:

$$(R_f)_{\text{opt}} = \sqrt{\frac{r_b' A_I}{C_{c2}\omega_{T2}}[1 + (R_L + r_e)C_{c2}\omega_{T2}]}. \quad (27)$$

Acknowledgment

The author wishes to acknowledge the helpful discussions with Dr. D. O. Pederson and D. Lynn of the University of California. He also extends his appreciation to Dr. J. H. Mulligan, Jr., for carefully reading the paper.

[8] If the amount of feedback is not large enough such that (14) is satisfied, *i.e.*, if $(p_1 + p_2) > \sqrt{2p_1p_2(1 + \mu_0\lambda_0)}$, $z < 0$. Hence no C_f is needed, and the bandwidth is given by:

$$\omega_{3db} = \sqrt{p_1 p_2 T_0 - \frac{(p_1 + p_2)^2}{2} + \sqrt{2p_1 p_2 T_0[p_1 p_2 T_0 - (p_1 + p_2)^2] + \frac{(p_1 + p_2)^4}{4}}},$$

where $T_0 = \mu_0\lambda_0$.