Theoretical Analysis of Opposing Air Bearing Concept
This concept utilizes air bearings to constrain five degrees of freedom of the optic as shown in the figure below. Three pairs of inherently compensated opposing air bearings are used to constrain x-translation and y- and z-rotation. Two vacuum preloaded air bearings are used to constrain z-translation and x-rotation.

One of every pair of the inherently compensated opposing air bearings acts as a reference point, three points forming a plane. The goal is to make this plane as close to the vertical as possible to decrease deformation associated with gravity sag. These reference bearings are shown behind the optic in the figure above.

At first, these bearings are in vacuum mode to facilitate the insertion of the optic into the device. At this point, the opposing bearings are approached to contact the optic and the supply pressure is turned on. This provides the opposing bearings with thrust capabilities pushing the bearing away from the stationary optic. This thrust is balanced by an actuating pressure (not shown). The vacuum is removed and the lower vacuum preloaded bearings are turned on, after which the reference, stationary bearings are pressurized to center the optic between the opposing bearings. Meanwhile, the lower bearings allow for the slight motion of the optic to the right.
Figure 2 Steps to insert optic into device for metrology

Top Bearing Sizing

The performance of an air bearing highly depends on the air film thickness. These air bearings are to be placed against the optics measured, which in turn have a warp associated with their surfaces. The bearing outer diameter is
calculated such that the air film thickness variation due to the surface non-flatness of the optics is limited to 3 \( \mu \text{m} \).

The optic is modeled as a curved beam with a warp \( \delta \). For small angles \( \theta \), the

\[
\begin{align*}
\delta &= R(1 - \cos \theta) \\
\sin \theta &= L/2R \\
\text{for small angles:} \\
\delta &= L^2/8R
\end{align*}
\]

<table>
<thead>
<tr>
<th></th>
<th>Borosilicate</th>
<th>Silicon</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \delta )</td>
<td>0.005</td>
<td>0.025</td>
</tr>
<tr>
<td>( L )</td>
<td>100</td>
<td>140</td>
</tr>
<tr>
<td>( R )</td>
<td>4083</td>
<td>6250</td>
</tr>
<tr>
<td>( d )</td>
<td>16.67</td>
<td>6.53</td>
</tr>
</tbody>
</table>

where
L: characteristic length \\
\( \delta \): warp \\
R: radius of curvature \\
\( \theta \): half-angle \\
d: bearing diameter \\
y: vertical distance to bearing, 45 mm \\
space tol: 3 \( \mu \text{m} \) (for constant stiffness)

An outer diameter of 7 mm is chosen for the top bearings. Since this diameter is rather small compared to typical values, several factors must be taken into consideration when optimizing the performance of these bearings. The flow in the air film must be fully developed viscous at pressures higher than ambient for the bearing to have a load carrying capacity. The fully developed boundary layers and the very small viscosity of air result in zero static and negligible dynamic lateral friction associated with air bearings.

It has been shown [3] that increasing the ratio of the inner against the outer diameters of the air bearings decreases entrance effects such as separation and frictional losses due to changes in flow direction and thus enhancing boundary layer development; however, this leads to a decrease in flow resistance, which in turn leads to a drop in stiffness. The inner diameter of these bearings is chosen to be 0.4 mm (a practical value to achieve in the machine shop) to give an inner
to outer diameter ratio of 0.057, which is close to the commonly used ratio of 0.05. It should be verified that the flow out of the restrictor is not choked.

It is desired to work with gaps no less than 9 µm for practicality. The stiffness of these bearings depends on the supply pressure, bearing geometry, and air gap. The highest stiffness for a given bearing is obtained when the feeding parameter defined by the ratio of the resistances of the air gap against the restrictor is between 0.85 and 1.15. Keeping this constraint in mind, the required supply pressure is calculated.

The theoretical load capacity is given by

$$
\zeta = \frac{W}{p_{atm}\pi r_2^2} = \frac{p_1}{p_{atm}} \frac{r_1^2}{r_2^2} e^{\frac{2}{3}} \sqrt{\frac{\pi a}{8}} \left[ \text{erf} \left( \frac{2}{\sqrt{a}} \right) - \text{erf} \left( \frac{p_{atm}}{p_2} \frac{2}{\sqrt{a}} \right) \right]
$$

$$
a = \frac{1 - \frac{p_{atm}^2}{p_1^2}}{\ln \frac{r_2}{r_1}}
$$

where $p_1$: Inlet pressure

$p_{atm}$: Atmospheric pressure

$r_1$: Bearing inner radius

$r_2$: Bearing outer radius

$h$: Air gap size

The supply pressure is chosen to be at 40 psig giving a load capacity of 3.1 N and a stiffness of 0.2 N/µm at an air gap of 10 µm. The corresponding feeding parameter is around 1.5. It should be noted that for the given geometric constraint, further decreasing the feeding parameter for an optimum stiffness requires a substantial increase in the supply pressure, reducing the overall efficiency of the small diameter bearings.

For these bearing parameters the theoretical pressure distribution for a viscous flow in the air gap and the mass flow rate at 10 µm air gap are respectively

$$
p = \left[ 1 - \left( 1 - \frac{p_1^2}{p_{atm}^2} \right) \frac{r_1^2}{r_2^2} \ln \frac{r_2}{r_1} \right]^{1/2}
$$

$$
m = \frac{\pi h^3}{12 \mu RT} \left( \frac{p_1^2}{p_{atm}^2} \frac{r_1^2}{r_2^2} \ln \frac{r_2}{r_1} \right) = 7.93 \times 10^{-6} \text{ Kg / s}
$$

The following two figures show the pressure distribution along the bearing radius and the mass flow rate at different air gaps respectively.
Lower Air Bearings
The two air bearings constraining the z-translation and x-rotation are circular porous pads. The bearings are in an inverted position, in other words, the bearings are stationary and a puck is allowed to float on their surface. This is to avoid forces from tubes in randomly moving the bearings. The main reason air bearings are used in this configuration is to accommodate frictional forces introduced during placing the optic into the device and also, to accommodate for thermal expansion along the x and y directions. Vacuum preloading is required to increase the stiffness of these bearings in the z direction because the load carried by these bearings is on the order of $0.5 \rightarrow 1$ N.

Air Cooling
It is important to avoid the presence of local refrigeration effects on the optic due to the cooling of the emerging air. The two major reasons why air temperature drops are high velocity and change in volume. As the velocity of a compressible fluid increases, its enthalpy drops which is associated with a drop in temperature. Since the flow rate in air bearings is extremely small, air can be modeled as an incompressible medium, where a change in velocity is independent of change in enthalpy, and thus a drop in temperature. Also, as a compressed gas at high pressures is allowed to go through a throttle to expand at lower pressures, its temperature drops. This is known as the Joule-Thompson effect for real gases, where the Joule-Thompson coefficient is defined as

$$\left(\frac{\partial T}{\partial P}\right)_b = \frac{1}{c_p} \left[T\left(\frac{\partial v}{\partial T}\right)_p - v\right] = \mu_{JT}$$

where

- $\mu_{JT}$: Joule-Thompson Coefficient
- $a$: van der Waal cst = 0.1361 J.m$^3$/mole$^2$ for N2
- $b$: van der Waal cst = 3.85x10$^{-5}$ m$^3$/mole for N2
- $R$: Universal gas constant = 8.314 J/mole.K
- $c_p$: 29.1 J/mole.K

Using van der Waal equation of state for $v$

$$\mu_{JT} = \frac{1}{c_p} \left[\frac{2a}{RT} - b\right] = 1.89 \times 10^{-6} K/Pa$$

Therefore, for a pressure drop of 40 psig = 275,862 Pa, the associated temperature drop is anticipated to be 0.52 K. However, this model does not take the viscous heating effects into consideration. In reality, experiments [1] have shown that the air film is isothermal where cooling effects are compensated for by viscous heating effects. In addition, using a metallic surface for the air bearings assures that the heat change is absorbed by the metal rather than the optic.
References: