

Balanced Model Reduction of Flat-Plate Solar Collector using Descriptor State-Space Formulation

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Abstract—Model Reduction of the flat-plate solar energy collector dynamics is considered, taking into account the continuous variation of the received radiant flux. Models of the fluid and absorber temperature distributions are traditionally determined via a pair of partial differential equations (PDEs). To get a tractable finite-dimensional model instead of the infinite-dimensional PDE model we apply the finite difference method to the PDEs to get a high-order descriptor linear time-invariant state space model. Then, we compute a reduced-order model via the method of balanced truncation for descriptor systems. A substantial order reduction is possible and the obtained reduced-order model is tractable for the purposes of simulation, control and estimation.

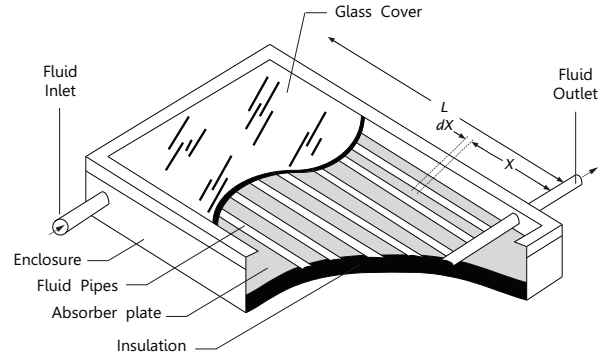


Fig. 1. Schematic of a flat plate solar collector

I. INTRODUCTION

Solar energy technologies account for most of the available renewable energies on earth. The thermal energy coming from the solar radiation can be utilized in wide range of applications such as space and water heating by using solar collectors. Flat plate solar collectors are the most simple ones yet efficient specially for the aforementioned applications. They basically consist of a dark absorber plate and a heat transport fluid that flows through pipes to remove the heat from the absorber. Flat-plate solar collectors have a dynamic behavior in response to the continuous daily variations in the solar radiation received from sunset to sunrise. However, most of the analytical studies that predict the performance or examine the effect of various parameters on the collector behavior were based on convenient steady-state models to avoid complexities inherent in the actual dynamic problem. Some transient and steady-state models were discussed in [1–5]. In this paper, a closed form mathematical expression for the fluid temperature presented in [5] is used. It is a result of solving a pair of PDEs that represents the dynamic collector behavior.

The numerical solution of PDEs requires intensive computations. It is desired to develop a reduced order model that will reduce the computational time and still preserve the accuracy. Furthermore, complexity reduction is also highly desirable for control system design purposes. A control scheme based on the original distributed system is not generally tractable for computer implementation or realization. Furthermore, a controller derived from a high-order model will be of high order and may involve a large amount of computational time for its implementation, thus making it difficult for real time control. A reduced order model of the process is therefore necessary for the development of lower-order controllers.

Model reduction is also a desirable step for simple simulation and easy interpretation of the results.

Many model reduction schemes of state-space systems were introduced in the literature. One of the most famous and most simple schemes is *balanced model reduction*, which has proven to be very efficient in approximating high-order systems by low-order models for a variety of applications [6, 7]. In [8], the authors developed a balanced reduced order model of low order that fits well with the transient behavior of infinite dimensional rod heating system.

However, the application of the finite difference method to the PDEs will not yield a standard state-space system. Instead, we get a *descriptor* or *singular state-space system* [9]. Balanced model reduction has been generalized for descriptor state space systems as in [10, 11]. It has also been applied previously to PDEs in the context of fluid dynamics [12]. In this paper, we apply the balanced truncation method to the descriptor system arising from the semi-discretization of the PDEs. Earlier work on model reduction of this system used Padé approximation techniques [13]. Balanced model reduction has been applied to other infinite dimensional systems [14, 15].

The rest of the paper is organized as follows: in Section II, the analytical solution for the partial differential equations representing the fluid temperature as a function of both the time and the position along the collector is reviewed. The finite difference method is applied to these PDEs in Section III to

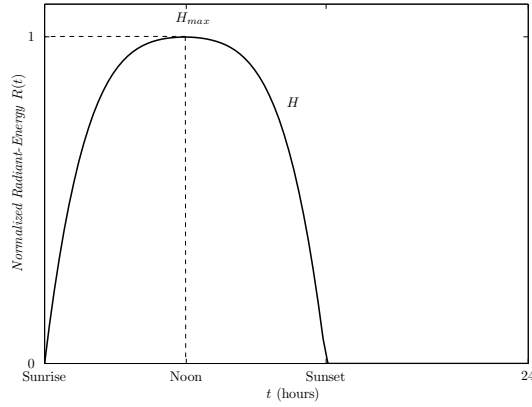


Fig. 2. The daily variation of the absorbed radiant energy.

get a high-order descriptor linear time-invariant state space model. In Section IV, we compute a reduced-order model via the method of balanced truncation for descriptor systems. Finally, both the exact and the reduced models of different orders are compared using MATLAB simulations in Section V. Concluding remarks and future work will be discussed in Section VI.

II. SOLUTION TO THE HEAT TRANSFER PROBLEM

The flat-plate solar collector, shown in Fig.1, can be modeled by the following two first order linear partial differential equations representing the instantaneous thermal balance of a differential element of length along the absorber-plate together with the absorber-to-fluid heat convection process [5]:

$$\frac{\partial T_p}{\partial t}(t, x) = -\frac{\gamma}{N_L} \frac{\partial T_f}{\partial x}(t, x) - \gamma T_p(t, x) + \gamma u(t) \quad (1)$$

$$0 = -\frac{\partial T_f}{\partial x}(t, x) + N_c T_p(t, x) - N_c T_f(t, x) \quad (2)$$

These equations are the compact form of the original PDEs derived in [5] after grouping of numerous variables to yield a small number of dimensionless groups. T is the normalized temperature defined by:

$$T = (\theta - \theta_a) / (\theta_e - \theta_a)$$

θ_e is the equilibrium or stagnation temperature attained under no-flow conditions and continuous exposure to the peak radiant input H_{max} and θ_a is the ambient-air temperature. T_f and T_p are the fluid and plate temperatures respectively.

The input to the collector system $u(t)$ is the radiant energy absorbed by the plate H that follows a pattern similar to that shown in Fig. 2. This represents the chronological variation of the radiant energy flux during the sunlight hours and it is normalized to give the intensity ratio $R(t)$ that may be expressed by an n th order polynomial as follows:

$$R(t) = \begin{cases} \sum_{j=0}^n a_j t^j & 0 \leq t \leq t_{ss}, \\ 0 & t > t_{ss}. \end{cases} \quad (3)$$

According to (3), it is possible to reflect the diurnal heating and the nocturnal cooling periods on the collector behavior.

x is the dimensionless distance along the collector given by:

$$x = X/L, \quad 0 \leq X \leq L.$$

The two dimensionless numbers N_L and N_c are the heat loss number and the convection number respectively and both are defined by:

$$N_L = UWL/\dot{m}c_f$$

and

$$N_c = hWL/\dot{m}c_f$$

γ is defined by:

$$\gamma = UW/m'_p c_p \quad h^{-1}$$

where \dot{m} is the fluid mass flow rate (kg/s), W is the width of the collector, L is length of the collector, c_f is the specific heat of fluid (J/kg °C), c_p is the specific heat of plate (J/kg °C), h is the plate to fluid heat convection coefficient (W/m² °C), U is the overall collector heat-loss coefficient (W/m² °C).

Assuming that the plate temperature was initially equal to the ambient air temperature [5] and by taking the Laplace transform of equations (1) and (2) with respect to t we obtain:

$$\frac{\partial \Upsilon_f}{\partial x} = N_L \left(U(s) - \Upsilon_p - \frac{s}{\gamma} \Upsilon_p \right) \quad (4)$$

$$\frac{\partial \Upsilon_f}{\partial x} = N_c \Upsilon_p - N_c \Upsilon_f \quad (5)$$

By eliminating Υ_p from both (4) and (5) then integrating with respect to x we get:

$$\begin{aligned} \Upsilon_f = \Upsilon_f|_{x=0} e^{-N_c(1+s/\gamma)x/(N+1+s/\gamma)} \\ + \left[1 - e^{-N_c(1+s/\gamma)x/(N+1+s/\gamma)} \right] U(s)/(1+s/\gamma) \end{aligned} \quad (6)$$

where $N = N_c/N_L$. Since both θ_i and θ_a are considered to be constant, then the Laplace transform $\Upsilon_f|_{x=0}$ will be equal to T_i/s [5]. Accordingly:

$$\begin{aligned} \Upsilon_f = T_i/s e^{-N_c(1+s/\gamma)x/(N+1+s/\gamma)} \\ + \left[1 - e^{-N_c(1+s/\gamma)x/(N+1+s/\gamma)} \right] U(s)/(1+s/\gamma) \end{aligned} \quad (7)$$

Taking the Laplace inverse of (7) the exact solution has the form:

$$T_f(t, x) = \mathcal{L}^{-1} \Upsilon_f(s, x)$$

$$T_f(t, x) = T_i e^{-N_c X/(1+N)} \left[1 - e^{-\alpha t} \int_0^\beta e^{-y} I_0(2\sqrt{\alpha y t}) dy \right] + \int_{\bar{t}}^t \left[\gamma e^{-\alpha z} \int_0^{N_c x} e^{-y} I_0(2\sqrt{\gamma N z y}) dy \right] \times \left[\sum_{j=0}^n A_j z^j \right] dz \quad (8)$$

where:

$$\bar{t} = \begin{cases} 0 & 0 \leq t \leq t_{ss}, \\ t - t_{ss} & t > t_{ss}. \end{cases}$$

$$\beta = N_c N x / (1 + N)$$

$$\alpha = \gamma(1 + N)$$

$$A_j = (-1)^j \sum_{r=j}^n \frac{r!}{(r-j)!j!} a_r t^{r-j}$$

Eq. (8) represents the analytical solution of the fluid temperature as a function of the position x and the time t . Different computational methods can be applied to determine the fluid temperature at different positions along the collector and different times during the day. One method is to use MATLAB capabilities to solve numerical integrations and zero order modified Bessel functions of first kind appearing in (8). However, this method have large computational efforts and time which motivated us to come up with a reduced order model that accurately represents the system as summarized in the following sections. The reduced order model can then be used in controller design.

III. FINITE DIFFERENCE APPROXIMATION AND STATE-SPACE FORMULATION

Considering the pair of PDEs (1),(2), we discretize it in space by a set of M grid points $\{x_1, \dots, x_M\}$. We assume uniform grid spacing $\Delta x_k = \frac{1}{M}$.

We can write the following two-point backward difference approximation of the partial differentiation operator:

$$\frac{\partial T_f}{\partial x}(t, x_k) \approx M(T_f(t, x_k) - T_f(t, x_{k-1})), k = 1, \dots, M \quad (9)$$

where $T_f(t, x_{-1}) = 0$. Higher-order backward differences can be constructed similarly [16], but no significant differences were observed in the quality of approximation.

By substituting (9) in (2) we get a system of $2M$ ordinary differential equations:

$$\dot{T}_p(t, x_k) = \frac{\gamma M}{N_L} T_f(t, x_{k-1}) - \frac{\gamma M}{N_L} T_f(t, x_k) - \gamma T_p(t, x_k) + \gamma u(t) \quad (10)$$

$$0 = M T_f(t, x_{k-1}) - (M + N_c) T_f(t, x_k) + N_c T_p(t, x_k) \quad (11)$$

Denote:

$$z_p(t) \triangleq \begin{bmatrix} T_p(t, x_1) \\ \vdots \\ T_p(t, x_M) \end{bmatrix}, z_f(t) \triangleq \begin{bmatrix} T_f(t, x_1) \\ \vdots \\ T_f(t, x_M) \end{bmatrix}$$

Thus we have the following linear time-invariant descriptor system:

$$\begin{bmatrix} I_M & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \dot{z}_p \\ \dot{z}_f \end{bmatrix} = \begin{bmatrix} \gamma I_M & A_{12} \\ N_c I_M & A_{22} \end{bmatrix} \begin{bmatrix} z_p \\ z_f \end{bmatrix} + \begin{bmatrix} B_1 \\ 0 \end{bmatrix} u \quad (12)$$

where:

$$B_1 = \begin{bmatrix} \gamma \\ \vdots \\ \gamma \end{bmatrix}, A_{12} = \frac{M\gamma}{N_L} \begin{bmatrix} -1 & 1 & -1 & & \\ & \ddots & \ddots & \ddots & \\ & & 1 & -1 & \end{bmatrix},$$

$$A_{22} = \begin{bmatrix} -(M + N_c) & & & & \\ M & -(M + N_c) & & & \\ & \ddots & \ddots & \ddots & \\ & & \ddots & \ddots & M & -(M + N_c) \end{bmatrix}$$

If we define our output y to be the fluid outlet temperature $T_f(t, x_M)$, we can write (12) in the following standard form of descriptor systems:

$$E \dot{z} = A z + B u \quad (13)$$

$$y = C z \quad (14)$$

where $C = [0 \dots 0 1]$ and $z = [z_p^T \ z_f^T]^T$.

IV. BALANCED MODEL REDUCTION FOR DESCRIPTOR SYSTEMS

We summarize the balanced model reduction algorithm of descriptor systems presented in [11].

Assume an n^{th} descriptor system is given in the form (13) where the matrix pencil (A, E) is regular and c -stable¹. Let n_f denote the number of finite eigenvalues of the pencil (A, E) , and n_∞ denote the number of infinite eigenvalues. The model reduction of (13) requires finding an approximation of order $\ell \ll n$ in the form:

$$\bar{E} \dot{\bar{z}} = \bar{A} \bar{z} + \bar{B} u \quad (15)$$

$$y = \bar{C} \bar{z} \quad (16)$$

The numbers of finite and infinite eigenvalues of (\bar{A}, \bar{E}) are ℓ_f, ℓ_∞ , respectively.

The model reduction algorithm can be described as follows:

- 1) Compute the generalized real Schur form (or QZ decomposition):

$$E = V \begin{bmatrix} E_f & E_u \\ 0 & E_\infty \end{bmatrix} U^T \text{ and } A = V \begin{bmatrix} A_f & A_u \\ 0 & A_\infty \end{bmatrix} U^T \quad (17)$$

where U, V are orthogonal, E_f is upper triangular non-singular, E_u is upper triangular nilpotent, A_f is upper quasi-triangular and A_u is upper triangular nonsingular.

- 2) Solve the generalized Sylvester equation [17]:

$$E_f Y - Z E_\infty = -E_u \quad (18)$$

$$A_f Y - Z A_\infty = -A_u \quad (19)$$

¹A pencil is said to be regular if $\det(\lambda E - A) \neq 0$ for some $\lambda \in \mathbb{C}$, and it's said to be c -stable if all the finite eigenvalues of the pencil lie in the open left-half plane.

3) Solve the continuous generalized Lyapunov Equations:

$$E_f^T X_{po} A_f + A_f^T X_{po} E_f = -C_f^T C_f \quad (20)$$

$$E_f X_{pc} A_f^T + A_f X_{pc} E_f^T = -(B_u - Z B_\infty)(B_u - Z B_\infty)^T \quad (21)$$

where $[B_u^T B_\infty^T]^T = V^T B$ and $[C_f^T C_u] = C U$.

4) Solve the discrete Lyapunov equations:

$$A_\infty X_{ic} A_\infty^T - E_\infty X_{ic} E_\infty^T = B_\infty B_\infty^T \quad (22)$$

$$A_\infty^T X_{ic} A_\infty - E_\infty^T X_{ic} E_\infty = (C_f Y + C_u)^T (C_f Y + C_u) \quad (23)$$

5) Compute the Cholesky factors $R_f, L_f, R_\infty, L_\infty$ of $X_{pc} = R_f R_f^T$, $X_{po} = L_f^T L_f$, $X_{ic} = R_\infty R_\infty^T$, $X_{io} = L_\infty^T L_\infty$.

6) Compute the thin singular value decomposition:

$$L_f E_f R_f = [U_1 \ U_2] \begin{bmatrix} \Sigma_1 & \\ & \Sigma_2 \end{bmatrix} [V_1 \ V_2]^T$$

where $\Sigma_1 = \text{diag}(\sigma_1, \dots, \sigma_{\ell_f})$, $\Sigma_2 = \text{diag}(\sigma_{\ell_f+1}, \dots, \sigma_r)$ with $r = \text{rank}(L_f E_f R_f)$. The diagonal elements $\{\sigma_1, \dots, \sigma_r\}$ are called the *proper Hankel singular values* and are ordered in a descending order.

7) Compute the thin singular value decomposition: $L_\infty A_\infty R_\infty = U_3 \Sigma_3 V_3^T$, where $\Sigma_3 = \text{diag}(\zeta_1, \dots, \zeta_{\ell_f})$ with $\ell_\infty = \text{rank}(L_\infty A_\infty R_\infty)$. Similarly, $\{\zeta_1, \dots, \zeta_{\ell_f}\}$ are called the *improper singular values*.

8) The reduced order systems matrices are:

$$\bar{E} = \begin{bmatrix} I_{\ell_f} & \\ & W_\infty^T E_\infty T_\infty \end{bmatrix}, \bar{A} = \begin{bmatrix} W_f^T A_f T_f & \\ & I_{\ell_\infty} \end{bmatrix}$$

$$\bar{B} = \begin{bmatrix} W_f^T (B_u - Z B_\infty) \\ W_\infty^T \end{bmatrix}, \bar{C} = [C_f T_f \ (C_f Y + C_u) T_\infty]$$

where $W_f = L_f^T U_1 \Sigma_1^{-1/2}$, $W_\infty = L_\infty^T U_3 \Sigma_3^{-1/2}$, $T_f = R_f V_1 \Sigma_1^{-1/2}$ and $T_\infty = R_\infty V_3 \Sigma_3^{-1/2}$.

V. SIMULATION RESULTS

In this section we show some simulation results demonstrating the efficiency of the proposed method. We use the following radiant-energy input used in [5] throughout the simulation:

$$u(t) = R(t) = 0.5091t - 0.10019t^2 + 0.009118t^3 - 0.000326t^4 \quad (24)$$

We choose our grid size in the finite difference method to be $M = 200$ and we set $T_i = 0$. Consider the set of parameters used in [5]: $N_c = 16.6$; $\gamma = 0.25$; $N_L = 0.5$.

Fig. 3 shows that the four-outputs trajectory corresponding to the descriptor state space (13) is well approximating the analytical solution (8).

We examine the performance of the model reduction scheme with single-output system where the output is outlet temperature ($x = 1$). We have $n_f = 200$ and $n_\infty = 200$. It is observed that the improper Hankel singular values have no effect on the dynamics and hence we have $\ell = \ell_f$.

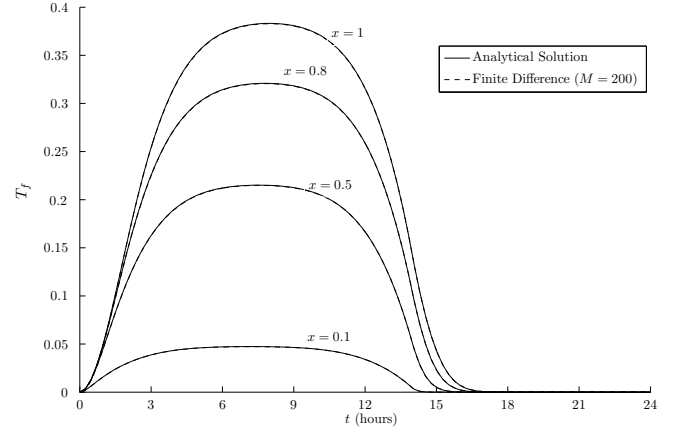


Fig. 3. The resulting finite difference trajectory with four outputs compared with the analytical solution based on (8).

The values of first nine proper Hankel singular values are as follows:

$$\sigma_1 = 2.183 \times 10^{-1}, \sigma_2 = 3.147 \times 10^{-2}, \sigma_3 = 6.076 \times 10^{-3}, \sigma_4 = 9.761 \times 10^{-4}, \sigma_5 = 1.247 \times 10^{-4}, \sigma_6 = 1.257 \times 10^{-5}, \sigma_7 = 1.006 \times 10^{-6}, \sigma_8 = 6.606 \times 10^{-8}, \sigma_9 = 5.304 \times 10^{-9}.$$

The values indicate that a third or fourth order approximation is enough. This was affirmed through simulation in Fig. 4 where the performance of the model reduction schemes with $\ell = 1, 2, 3, 4$ is shown. Note that the second order approximation has little deviations, while the third order approximation represents the trajectory faithfully. The fourth order approximation approximate the system almost perfectly.

The frequency domain performance of the approximation can be checked also. Using (7) with $T_i = 0$ we have the frequency response formula of the original system:

$$\Upsilon_f(j\omega, 1) = \frac{1 - e^{-N_c(1+j\omega/\gamma)/(N+1+j\omega/\gamma)}}{1 + j\omega/\gamma}$$

Fig. 5 shows the comparison result, where it reaffirms that the fourth order is a suitable approximation of the infinite dimensional system.

We assume that we are interested in the temperature of the fluid at different positions. Fig. 6 shows the comparison with four outputs, where it's seen also that $\ell = 3$ or 4 is enough to approximate the system.

In Fig. 7, we consider another set of parameters with three outputs, and it indicates again that the third or fourth order systems are good approximations.

VI. CONCLUSION

In this paper we studied balanced model reduction of the solar collector dynamic. Using a semi-discretization procedure, we obtained a high-order descriptor state-space formulation. We utilized a balanced model reduction scheme to approximate the high-order system.

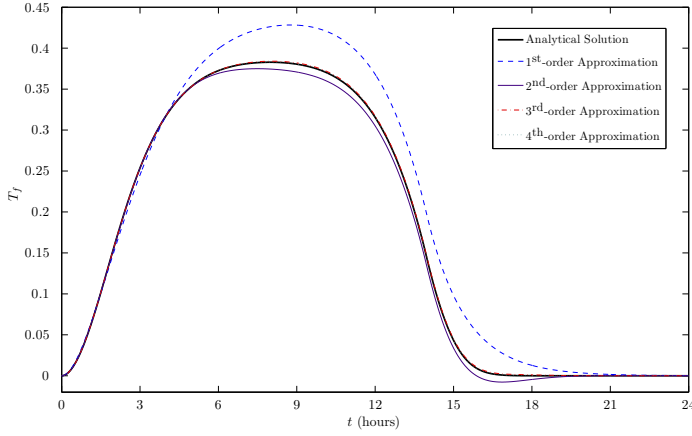


Fig. 4. Comparison of the response of the original system versus the approximations with $N_c = 16.6$; $\gamma = 0.25$; $N_L = 0.5$ and single-output ($x = 1$).

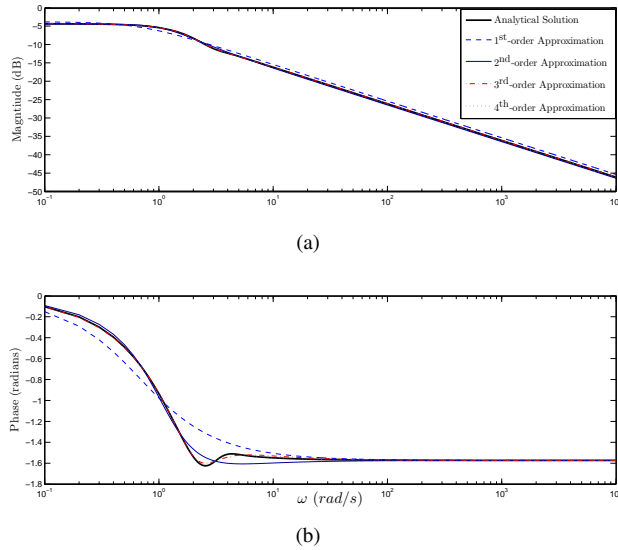


Fig. 5. Comparison of the bode plot of the original system versus the approximations with $N_c = 16.6$; $\gamma = 0.25$; $N_L = 0.5$ and single-output ($x = 1$).

It was observed that a substantial order reduction is possible. Particularly, the infinite-order dynamics are well-approximated by a third or fourth order system only. Such low-order makes the simulation and control of solar collectors more feasible.

In terms of future directions, we can apply model reduction to more complex PDE models. Also, more sophisticated model-reduction techniques can be used such as Hankel-norm [18, 19] and \mathcal{H}_∞ -norm model reductions [20].

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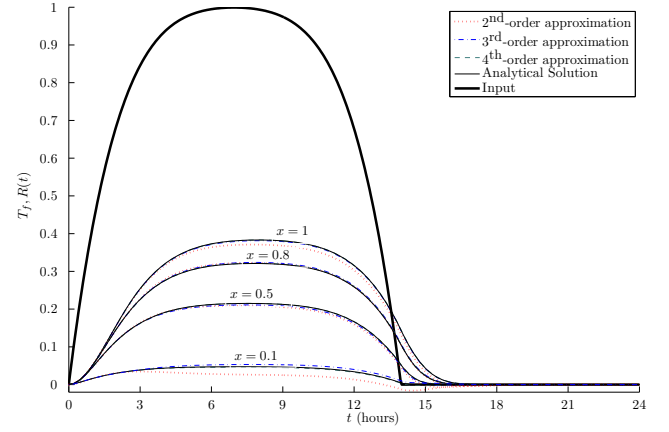


Fig. 6. Comparison of the response of the original system versus the approximations with $N_c = 16.6$; $\gamma = 0.25$; $N_L = 0.5$ and four outputs.

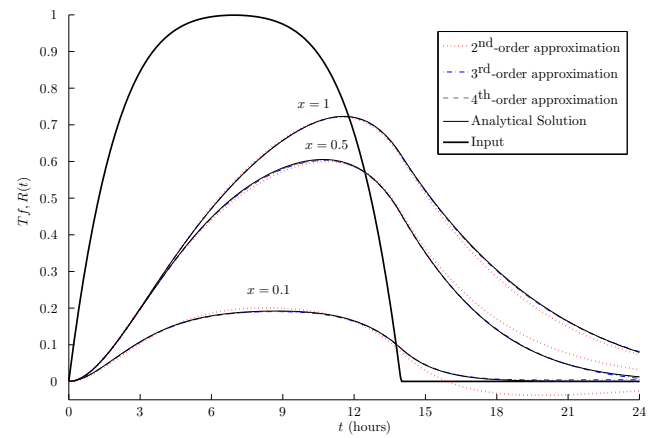


Fig. 7. Comparison of the response of the original system versus the approximations with $N_c = 16.6$; $\gamma = 0.15$; $N_L = 2.5$ and three outputs.

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