Manufacturing SYSTEMS ENGINEERING

by Stanley B. Gershwin

Errata and Other Comments

August 27, 1997

Some of the more important errors of *Manufacturing Systems Engineering* are corrected in the following. Other comments also appear. Many of the errata and comments were contributed by readers, for which I am grateful.

Any other important errors or comments that are reported (and that I am convinced of) will be added to this list. Readers are requested to send them to the e-mail address below.

Stan Gershwin

Chapter 1

Chapter 2

- page 21, 6th line from the bottom: *average* or *expected value* — There should be an "or" between *average* and *expected value*

- page 22, line 9 from the bottom: *vice versa* is ambiguous. The sentence should be

  If C is true, then D must be false, and if D is true, then C must be false.

- page 27: The graph in Figure 2.2 is missing a self-loop at state 0. Its probability is 1.

- page 33, lines 9-10: The sentence should be "If there is only one set C and no set T, the chain is irreducible." Irreducibility is an important attribute because it means that there is a unique steady-state probability distribution. This distribution can be obtained by solving a set of n linear equations in n unknowns, where n is the number of states. Perhaps another term is needed since this is also true (a unique steady-state probability distribution which can be obtained by solving n linear equations in n unknowns) for a Markov chain with a single C and a non-empty T set.

- page 33, 5th line from the bottom: "all $s < \tau$" should be " all $s \leq \tau$"

- page 36: Lines 8 and 9 from the bottom should be "Let $\mathbf{p}(\alpha, t)$ be the probability of finding the system in state α at time t, $\alpha = 0$ or 1."

- page 37, Figure 2.6: In a discrete-state, continuous-time Markov process, *every* node has a self-loop. The labels of edges in the graphs for these Markov processes are rates, not probabilities; and self-loops in these graphs are not drawn.
- page 39, line 5: "... for all $T > 0$."
- page 41: It is useful to point out that the differential equations form a special case of Equation (2.24) on page 35, in which $\lambda_{10} = r$, $\lambda_{00} = -r$, $\lambda_{01} = p$, $\lambda_{11} = -p$.
- page 44, middle of the page: "$s < \tau$" should be "$s \leq \tau$"
- page 44, 8th line from the bottom: "$s < \tau$" should be "$s \leq \tau$"
- page 46, second bullet: $+G_1$ should be -G_1, $+G_2$ should be -G_2, and $+G_n$ should be -G_n.
- page 48: We should explicitly define

$$f(Z, \alpha) = \lim_{x \to Z} f(x, \alpha)$$

- page 49, line 7: After the sentence "Boundary conditions...", insert

  We must have $P(Z, 0) = 0$ because whenever $\alpha = 0$, x is decreasing.
- page 55: Problem 18 may be worded ambiguously. Before answering the first question, determine A. The relationship between C and Z, which is the top equation of page 52, depends on the relationship between A and C.

Chapter 3

- page 64: Footnote 4 should be "Reprinted from Gershwin (1993) with permission from Elsevier Science Publishers."
- page 67, end of Section 3.3.1: Other possibilities, requiring other formulas, can also be imagined. One is a line with mixed ODF and TDF failures. Also, TDF failures are likely to be correlated, which equation (3.3) does not take into account. For example, failures due to power outages are not operation-dependent, but they affect the whole factory at once.
- page 70, line 1: delete "be".
- page 72, after line 7 from the bottom:

  It is convenient to define $n_0(t) > 0$ and $n_k(t) < N_k = \infty$. This is a symbolic way of saying that the first machine is never starved and the last is never blocked.

- page 73, at the end of Item 12: The meaning of equation (3.9) (or equation (3.8) for internal states) should be clarified. It is that in equation (3.11), $T(s, s') = 0$ for every s and s' that do not satisfy (3.9). (Unfortunately, equations (3.10) and (3.11) appear later than they should.) In fact, there should be one more assumption of the following form:

$$\text{prob}(s(t+1) = (n_1, n_2, ..., n_{k-1}, \alpha_1, \alpha_2, ...\alpha_k)|s(t) = (n'_1, n'_2, ..., n'_{k-1}, \alpha'_1, \alpha'_2, ...\alpha'_k))$$

$$= \begin{cases} 0 \text{ if (3.9) is violated} \\ \text{prob}(\alpha_1|n'_0, \alpha'_1, n'_1)\text{prob}(\alpha_2|n'_1, \alpha'_2, n'_2)...\text{prob}(\alpha_k|n'_{k-1}, \alpha'_k, n'_k) \text{ otherwise} \end{cases}$$

  in which these probabilities are given by Assumption 9, and where (for the purpose of these assumptions) n'_0 and n'_k, are defined above.

- page 74: Equations (3.10) and (3.11) should appear earlier. Also, it should be pointed out that (3.11) is nothing other than equation (2.2) in a new guise.

- page 84: The last full paragraph should be

  Now we notice that wherever the first argument of ϕ is 0, its coefficient has r_1 or $1 - r_1$ as a factor, and that wherever the first argument of ϕ is 1, its coefficient has p_1 or $1 - p_1$ as a factor. Similarly, wherever the second argument of ϕ is 0, its coefficient has r_2 or $1 - r_2$ as a factor, and that wherever the second argument of ϕ is 1, its coefficient has p_2 or $1 - p_2$ as a factor. This suggests ...

- page 90, line 5: (3.76) should be (3.80).

- page 90, line 6: $\frac{r_2}{p_2}$ should be $\frac{p_2}{r_2}$.

- page 100: A parenthesis is missing in Equation (3.108). It should be

$$\mathbf{p}(N, 1, 0)r_2 = \mathbf{p}(N-1, 1, 0)\mu_1 + \mathbf{p}(N, 0, 0)r_1 + \mathbf{p}(N, 1, 1)p_2 \tag{3.108}$$

- page 102. (3.119) can be written $r_1 D_1 = p_1 E_1$. It is also true that $r_2 D_2 = p_2 E_2$. Note that D_i is the probability that Machine i is down.

- page 103, lines 9 and 10 from the bottom: "... for $n = N - 1$, add Equations ..."

- page 117: Equations (3.190), (3.195), and (3.196) are not correct. They do not properly account for flow during periods of partial starvation or blockage. As described in more detail in succeeding sections, a buffer may be empty for reasons other than an upstream failure. In particular, if an upstream machine has a smaller μ than a downstream machine, and all machines remain operational for long enough, the buffer just upstream of the faster machine eventually becomes empty. Material still flows through the faster machine when this buffer is empty, but at the rate of the slower machine. This is a partial starvation; a partial blockage is similar. Equations (3.190), (3.195), and (3.196) do not account for the flow during the partial starvation and blockage periods. To do so, additional terms would have to be added to represent the reduced flow during these periods. This is done for two-machine lines in equations (3.238), (3.239), (3.247), and (3.248).

However, this would be difficult for longer lines because the speed of the reduced flow depends on the cause of the partial starvation or blockage. That is, if $\mu_1 = 1.1$, $\mu_2 = 1.2$, $\mu_3 = 1.3$, Machine 3 could operate at any of these speeds: 1.3 when Buffer 2 is not empty and Machine 3 is operational; 1.2 when Buffer 2 is empty, Buffer 1 is not empty, and Machines 2 and 3 are operational; and 1.1 when both buffers are empty and all machines are operational. Modifications of Equations (3.190), (3.195), and (3.196) would have to keep track of the fraction of time that each of these conditions occurred (to say nothing of all other relevant events involving partial blocking).

- page 118, middle of the page: $x - \mu_2\delta t + \mu_2\delta t + \delta x$ should be $x - \mu_1\delta t + \mu_2\delta t + \delta x$
- page 120, Equations (3.204) and (3.206): $o(\delta t)$ should be added to both equations.
- page 121, Equations (3.209) and (3.213): $o(\delta t)$ should be added to both equations.
- page 122, Equation (3.217): The last term should be $+r_2\mathbf{p}(N,1,0)$.
- page 124: Equation (3.234) should be $\mu_1 f(0,1,0) = 0$.

Chapter 4

- page 149, line 2: “is” should be “are”.
- page 150: In the paragraph starting in line 7, “Buffer $i-1$” should be “Buffer i’ in two places. The paragraph should then be:

  To analyze (4.14), recall that $\alpha_i(t) = 0$ implies that $n_i(t-1) < N_i$ since if Buffer i were full, Machine i could not fail, and if Machine i fails, Buffer i could not become full. Also, $\alpha_i(t) = 0$ implies that $n_{i-1}(t-1) > 0$ for similar reasons. Equation (3.8) then implies that $n_{i-1}(t) > n_{i-1}(t-1)$, so $n_{i-1}(t) > 0$. Consequently, (4.14) is equivalent to ...

- page 152, line 4 from the bottom: “onlyq” should be “only”

- page 162, equation (4.29): Sheng Yang of Tsingua University has the following comment.

  On page 162, you say the first term of (4.29):

$$\text{prob}\left[M_i \text{ down at } t+\delta t \middle| M_u(i) \text{ up and } n_i < N_i \text{ at } t\right] = p_i \delta t$$

  but I think this may be wrong. The reason is as follows:

$$\text{prob}\left[M_i \text{ down at } t+\delta t \middle| M_u(i) \text{ up and } n_i < N_i \text{ at } t\right]$$

$$= \frac{\text{prob}\left[M_i \text{ down at } t+\delta t; M_u(i) \text{ up and } n_i < N_i \text{ at } t\right]}{\text{prob}\left[M_u(i) \text{ up and } n_i < N_i \text{ at } t\right]}$$

$$= \frac{\left(\begin{array}{c} \text{prob}\left[M_i \text{ down at } t+\delta t; n_{i-1}=0, M_u(i) \text{ up and } n_i < N_i \text{ at } t\right] \\ +\text{prob}\left[M_i \text{ down at } t+\delta t; n_{i-1}>0, M_u(i) \text{ up and } n_i < N_i \text{ at } t\right] \end{array}\right)}{\text{prob}\left[M_u(i) \text{ up and } n_i < N_i \text{ at } t\right]}$$

$$= \frac{\text{prob}\left[M_i \text{ down at } t+\delta t; n_{i-1}>0, M_u(i) \text{ up and } n_i < N_i \text{ at } t\right]}{\text{prob}\left[M_u(i) \text{ up and } n_i < N_i \text{ at } t\right]}$$

$$= \text{prob}\left[M_i \text{ down at } t+\delta t \middle| n_{i-1}>0, M_u(i) \text{ up and } n_i < N_i \text{ at } t\right] \times \frac{\text{prob}\left[n_{i-1}>0, M_u(i) \text{ up and } n_i < N_i \text{ at } t\right]}{\text{prob}\left[M_u(i) \text{ up and } n_i < N_i \text{ at } t\right]}$$

  Because

$$\text{prob}\left[M_i \text{ down at } t+\delta t \middle| n_{i-1}>0, M_u(i) \text{ up and } n_i < N_i \text{ at } t\right] = p_i \delta t,$$

$$\begin{aligned} &\text{prob}\left[n_{i-1}>0, M_u(i) \text{ up and } n_i < N_i \text{ at } t\right] \\ &= \text{prob}\left[M_u(i) \text{ up and } n_i < N_i \text{ at } t\right] - \text{prob}\left[n_{i-1}=0, M_u(i) \text{ up and } n_i < N_i \text{ at } t\right] \\ &= E_u(i) - p(i-1;0,1,1), \end{aligned}$$

$$\text{prob}\left[M_u(i) \text{ up and } n_i < N_i \text{ at } t\right] = E_u(i),$$

  So

$$\text{prob}\left[M_i \text{ down at } t+\delta t \middle| M_u(i) \text{ up and } n_i < N_i \text{ at } t\right] = p_i \delta t \frac{E_u(i) - p(i-1;0,1,1)}{E_u(i)}.$$

So your approximation may bias the estimate of $p_u(i)$ in a positive direction and result in an lower estimated production rate, especially when the isolated production rate $\rho_i \gg \rho_{i-1}$.

I believe that this comment is correct. Dr. Yang has oberved that I forgot that $M_u(i)$ can be up even when Buffer $i-1$ is empty. The fraction that multiplies $p_i \delta t$ accounts for that. Can this observation lead to more accurate results, or more reliable algorithms?

Mitchell Burman develops a continuous material long line decomposition in his 1995 MIT Ph.D. thesis. He has found a similar adjustment of the corresponding term.

- page 163, Sheng Yang comments:

  At the bottom of page 163, you say

  $$\text{prob}[n_{i-1} = 1 \text{ and } M_u(i-1) \text{ down and } M_d(i-1) \text{ up and } M_u(i) \text{ up and } n_i < N_i] =$$

  $$\text{prob}[n_{i-1} = 1 \text{ and } M_u(i-1) \text{ down and } M_d(i-1) \text{ up}]$$

  I think this is not accurate unless you assume that

  prob$[n_{i-1} = 1$ and $M_u(i-1)$ down and $M_d(i-1)$ up and $M_u(i)$ up and $n_i = N_i] = 0$.

  I believe that it is OK. $M_d(i-1)$ up implies that M_i is up. Therefore, since $\{n_{i-1} = 1$ and $M_d(i-1)$ up$\}$ implies that $M_u(i)$ is up (according to the definition of $M_u(i)$ up on page 160). This means that $\{M_u(i)$ up$\}$ is redundant.

- page 166, Sheng Yang comments:

  On the equation (4.37) of page 166, you say $B(i)$ is approximately equal to $r_i \delta t$. In fact, $B(i) = r_i \delta t$ is accurate without approximation because

  $$\begin{aligned} & B(i) \\ & = \text{prob}\Big[M_i \text{ up and } \{n_{i-1} > 0 \text{ or } (n_{i-1} = 0 \text{ and } M_u(i-1) \text{ up})\} \text{ at } t + dt \Big| \\ & \qquad M_i \text{ down and } n_i < N_i \text{ at } t] \\ & = \text{prob}\Big[M_i \text{ up and } n_{i-1} > 0 \text{ at } t + dt \Big| M_i \text{ down and } n_i < N_i \text{ at } t\Big] \\ & = r_i \delta t. \end{aligned}$$

- page 167, line 6: Replace "Section 3.8.1" with "The first equation in the limit proof on page 104".

- page 168, equation (4.47): The denominator should be $P(i-1)$.

- page 168, Comment by Bob Lipset:

  I am confused by the wording of 2.(a) [Step 1]. You say to evaluate two-machine line $L(i)$, using the most recent values of the parameters for the $(i-1)$st line to produce solution values for $P(i-1)$ and $\mathbf{p}(i-1, 001)$. Is this really an evaluation of $L(i)$, or is that meant to read $L(i-1)$?

  Use the most recent values of all the quantities on the right side of (4.47)-(4.51) to evaluate the left sides of those equations. In particular, this means that, to evaluate $P(i-1)$ and $\mathbf{p}(i-1, 001)$, use the values of $r_u(i-1)$, $p_u(i-1)$, and $\mu_u(i-1)$ that were found recently, and use the values of $r_d(i-1)$, $p_d(i-1)$, $\mu_d(i-1)$ that were found the last time Step 2 was executed. Evaluate (4.47) (after correcting it as indicated above) using the value of $\mu_u(i)$ that was determined the last time that Step 1 was executed. Then evaluate (4.48) using the value of $p_u(i)$ that was calculated in equation (4.47), and using the values of $r_u(i)$ and $\mu_u(i)$ that were found the last time that Step 1 was executed. Finally, evaluate (4.50) and (4.51) using the values of $p_u(i)$ and $r_u(i)$ that were just found in equations (4.47) and (4.49).

  Burman dealt with similar issues in his continuous long line decomposition by treating equations like (4.47)-(4.51) as a set of simultaneous equations in $r_u(i)$, $p_u(i)$, and $\mu_u(i)$. His algorithm behaved better than one in which he followed the procedure of the last paragraph.

  Similar comments apply to (4.52)-(4.56).

- page 170, Figure 4.15: The horizontal axis should be ρ_2, not p_2.

- Sheng Yang comments:

  It is also strange that in Fig. 4.15 on page 170, the decomposition curve is above the simulation curve when p_2 is large. Because according to your approximation:

  $$P_i = e_i u_i (1 - \mathsf{prob}(n_{i-1} = 0) - \mathsf{prob}(n_i = N_i))$$

  So you omit a small positive item prob$(n_{i-1} = 0$ and $n_i = N_i)$, which should result in an lower estimated production rate than the real one (your approximation in example 1 has the same effect). But this is contradictory with Fig. 4.15.

  The observation is certainly correct, but I don't know if the conclusion is. The algorithm is nonlinear, so the approximate FR/IT equation could somehow lead to too-small p_u and p_d, and too-large r_u, r_d, μ_u, and μ_d. Perhaps an expert in algorithms like this might have a comment?

Chapter 5

- page 182, middle of page, end of paragraph 1: $i = d(u)$ should be $i = d(b)$

Chapter 6

- page 219, last line: "... and there are a ..."
- page 221 In Reference [3], "Akyldiz" should be "Akyildiz"

Chapter 7

- page 229, middle of the page: Change "There may be no optimal solutions, or there may be one, or there may be many." to "There may be no optimal solutions, or there may be one, there may be an infinite number of solutions. The solution(s) may be finite or unbounded."

Chapter 8

- page 253, line 3 from the bottom: $\delta x(t_1)$ is not defined. It is

$$\delta x(t_1) = x(t_1 + \delta t) - x(t_1)$$

- page 257, Equation (8.11): $o(\delta t)$ should be added to the equation.

Chapter 9

- page 286, equation (9.20): Mike Veatch has raised questions about whether W is C1 (having a continuous derivative) at the hedging point, and if the W calculated in this section is correct.
- page 268, line 12 and 13 from bottom: The sentence should be

  For each t and $X(t)$, $\{t_1(X(t), t),\ t_2(X(t), t),\ \ldots\ t_n(X(t), t)\}$ is the *tentative schedule.*
- page 290, line 3 from the bottom should be

  if $g_+ - Kb(g_+ + g_-) < 0$
- page 291, line 1 should be

  if $g_+ - Kb(g_+ + g_-) \geq 0$

- page 312: George Liberopoulos comments:

  I have some objections to section 9.4.3. In your description of the "calculation of the projected trajectory" (p. 310) I think that you have the 2-dimensional setting in mind. In your conjecture (p. 312) you say that "Constraints, when added to (9.49), are not deleted." This is NOT true.

  Consider a trajectory in a 3-dimensional setting. First you start with x being in a 3-D region. The optimal production rate is at a corner point of the Ω set. Soon, $x(t)$ hits a 2-D boundary, i.e. a surface where 2 3-D regions meet, or alternatively, where 2 corner points of the Ω set are both optimal. If the boundary is attractive, you add the equation of the boundary to the LP and continue on the boundary. Effectively, the new production rate is on the EDGE of the Ω set connecting the two optimal corners. If the boundary is deflective, you do not add a constraint. Instead, you move on into the next region with production rate $\hat{u}$, as you correctly say on p. 311, where $\hat{u}$ is the production rate across the 2-D boundary, really the new corner point. So far, so good.

  Now, assume the boundary is attractive. Then you move on it (remember that you have now added a constraint) until you hit a 1-D boundary, i.e. a curve where 3 or more 2-D boundaries meet. If the boundary is attractive—and it is not clear what you should do to check attractiveness in this case; checking the velocity across the boundary may not work—you add the equation of the boundary, really the equation of one of the new 2-D surfaces that define it, and continue on the 1-D boundary until you reach the hedging point (heaven), if the demand is feasible, or fall behind to inferno while remaining on the 1-D boundary. There is no purgatory here. Remember that when you reach the 1-D boundary, effectively a 3rd corner solution of the Ω set (two are already optimal) wants to enter the basis. So if the boundary ends up being attractive, all 3 corner solutions are optimal and everything is fine.

  If the boundary is deflective, however, what happens is that $x(t)$ EXITS the 2-D boundary it was galloping on, and instead ENTERS one of the other 2-D surfaces that define the 1-D boundary. Essentially, you DROP a constraint and ADD another constraint. Alternatively, on the Ω set, you jump from an edge to another edge. For instance, if, initally, corners 1 and 2 defined u, now corners 2 and 3 define u.

  Things get complicated because you may have 4 corner solutions that are optimal in a 3-D setting. I'm always thinking about the Ω set as the convex hull of simple machine-specific capacity sets. All this is described in Rishel's paper where he describes what a "regular synthesis" is.

  My 3-D visualization skills are not what they once were, but I think he is right. In addition, I will never misspell his name again. (See below.)

- page 329: The last line should be

$$x(t_1) = \begin{bmatrix} .87 \\ .60 \end{bmatrix}$$

That is, the first component of $x_1(t_1)$ should be +.87.

- page 351, just after equation (9.77): "in which λ^0 and λ^1 are constants."
- page 351: Equation (9.78) should be

$$\min_{u \in \Omega(\alpha(t))} \left(\lambda^1 W + \frac{\partial W}{\partial x}(x, \alpha) \right) u.$$

- "Liberopolous" should be "Liberopoulos"

Chapter 10

Chapter 11

Chapter 12

Chapter 13

To err is human ..., This page will be updated if and when more errors are found. If you find any, please communicate them to `gershwin@mit.edu`. Thank you.

Acknowledgments Errors have been found, or comments suggested, by students who took 2.852 at MIT in 1994 and 1995; Asbjoern Bonvik, Malte Buerger, Harong Li, George Liberopoulos, Bob Lipset, Semyon Meerkov, Jimmy Neudorff, Tullio Tolio, Mike Veatch, Sheng Yang, and lots of other people. Thank you.