# Information Inaccuracy in Inventory Systems — Stock Loss and Stockout

Yun Kang* and Stanley B. Gershwin†

August 23, 2004

**Abstract**

Many companies have automated their inventory management processes and rely on an information system in critical decision making. However, if the information is inaccurate, the ability of the system to provide high availability of products at the minimal operating cost can be compromised. In this paper, analytical and simulation modelling demonstrate that even a small rate of stock loss undetected by the information system can lead to inventory inaccuracy that disrupts the replenishment process and creates severe out-of-stocks. In fact, revenue losses due to out-of-stocks can far outweigh the stock losses themselves. This sensitivity of performance to the inventory inaccuracy becomes even higher in systems operating in lean environments.

Motivated by an automatic product identification technology under development at the Auto-ID Center, various methods of compensating for the inventory inaccuracy are presented and evaluated. Comparisons of the methods reveal that the inventory inaccuracy problem can be effectively treated even without automatic product identification technologies in some situations.

## 1 Introduction

FOR MANY COMPANIES that operate inventory-carrying facilities, providing high product availability to customers at minimal operation costs is one of the key factors that determine the success of their businesses. Especially in industries where the competition is fierce and profit margins are thin, companies have automated the inventory management processes to better meet customer demand and reduce operational costs. For example, many retailers use an automatic replenishment system which tracks the number of products in the store and place an order to the supplier in a timely fashion with minimal human intervention.

*Massachusetts Institute of Technology, Department of Mechanical Engineering, Auto-ID Center, ykang@mit.edu.

†Massachusetts Institute of Technology, Department of Mechanical Engineering, Laboratory for Manufacturing and Productivity, gershwin@mit.edu.

By doing so, the companies depend on the accuracy of the computerized information system for critical decision making. Information regarding what products are where and in what quantity must be provided accurately to effectively coordinate the movement of the goods. However, if the information provided by the computer system is incorrect, the ability to provide the product to the consumers at the minimal operation cost is compromised. For example, if the computer's record of stock quantity in the facility does not agree with the actual physical stock, orders may not be placed to the supplier in time, or the facility could be carrying unnecessary inventory.

This research investigates the problems related to the information inaccuracy in inventory systems — what the inaccuracy is, what the causes are, and what impact it has on the performance of the inventory system. In addition to quantifying the costs of inaccuracy, this research also addresses various ways the inaccuracy can be mitigated to improve the system performance.

## 1.1 Inventory Inaccuracy

The issues discussed here became apparent due to the work of the Auto-ID Center. The Auto-ID Center, founded in 1999 at the Massachusetts Institute of Technology, is sponsored by over 100 global companies, many of whom are leaders in their industries. Its aim is to create an automatic product identification system that can potentially replace bar-code technology. A radio frequency identification (RFID) tag, which is a microchip with an antenna, would be placed on physical objects in trade — a soda bottle, a pair of jeans, a car engine, etc. By placing the RFID readers that sense the presence of tagged objects throughout key locations in the supply chain, the objects can be tracked from the point of manufacture to and beyond the point of consumption. The Auto-ID Center is engaged in designing and deploying a global infrastructure that will make it possible for computers to provide accurate, real-time identification and location of objects.

In the midst of working with a number of select sponsors to understand the potential applications of the Auto-ID Center technology, we learned something that is contrary to a popular belief. That is, retailers are not very good at knowing how many products they have in the stores.

Consider a global retailer who will be referred to as Company A for confidentiality. Each store carries thousands of product lines (also known as SKUs — stock keeping units), and as a common practice for any inventory-carrying facility, it conducts a physical count of all the items at least once a year for financial reporting purposes. After the manual inventory verification, the stores are able to compare the stock quantity in the inventory record (which is stored in the computer information system) and the actual stock quantity. For each store, the percentage of SKUs whose inventory record matches the actual stock perfectly is calculated. Define this as the *perfect inventory accuracy* of a store. Figure 1A summarizes the perfect inventory accuracy for a large subset of Company A's stores.

According to the histogram, the best performing store is the one in which only 70%-75% of its inventory records match the actual inventory. In one store, two thirds of its inventory records are inaccurate. On average, the inventory accuracy of Company A stores is only 51%. In other words, only about a half the SKUs have perfectly accurate inventory records.

Another measure of the inventory accuracy can be obtained by relaxing the requirement and allowing the inventory record of a SKU be considered accurate if it agrees with the actual stock

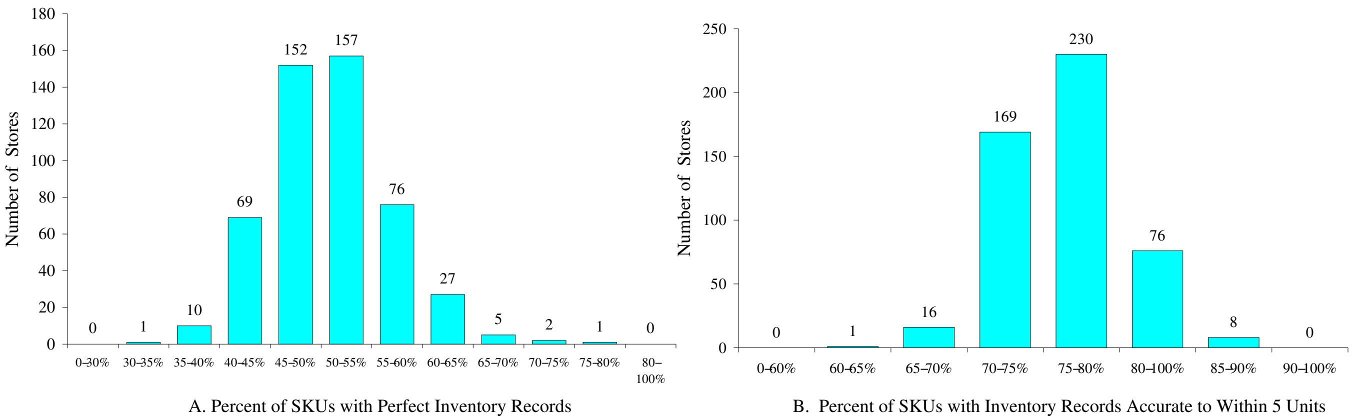


Figure 1: Inventory accuracy in Company A stores

within ±5 items. A histogram for this definition is shown in Figure 1B. Under this definition, the average accuracy of Company A stores rises to 76%. What this means is that on average, the inventory record for one out of four SKUs in the store deviates from the actual stock by six or more items.

The impact of inaccurate inventory records on the performance of retailers like Company A can be severe because the stores rely on the inventory record to make important operations decisions. Since Company A stores carry thousands of SKUs, tracking the inventory record of every SKU manually is very time-consuming. Instead, the stores use an automatic replenishment system in which the inventory record of each SKU is monitored and the computer system determines the order quantity based on the inventory record readings. If there is an error in the inventory record, items may not be ordered in a timely fashion, resulting in out-of-stocks or excess inventory.

Raman et al. reports similar findings from a study done with a leading retailer. Out of close to 370,000 SKUs investigated, more than 65% of the inventory records did not match the physical inventory at the store-SKU level. Moreover, 20% of the inventory records differed from the physical stock by six or more items. The retailer in the report also used information technology extensively to automate the replenishment processes (Raman, DeHoratius, and Ton 2001).

## 1.2 Causes of Inventory Inaccuracy

These findings indicate that perfect inventory records are difficult to maintain. In the midst of the many activities taking place in the stores, the inventory record is very likely to be incorrect. The causes of discrepancies in the records are many, and some of the commonly observed ones are discussed here: stock loss, transaction error, inaccessible inventory, and incorrect product identification.

Stock loss, also known as *shrinkage* in industry, includes all forms of loss of the products available for sale. One common example is theft, which can be committed by both shoppers (external theft) and employees (internal theft). It also includes collusion between customers and staff and the unauthorized consumption (such as eating) of the stock by both shoppers and employees. In addition, the vendors can also steal merchandise while in the store performing replenishment duties for their merchandise. Stock loss can also occur when products are rendered unavailable for sale by becoming out of date, damaged, or spoiled.

Stock loss can be categorized into known and unknown stock loss. The former refers to all losses that are identified by the store personnel and reflected in the computer inventory record (such as out-of-date products that are taken off the shelf and written off the books). The latter refers to the rest of the losses not detected and thus not updated into the record. Undetected theft, for example, would fall under this category. It is the unknown stock loss that creates inventory record inaccuracy.

Transaction error occurs typically at the inbound and outbound sides of the facility. At the inbound side, shipments that arrive from the suppliers have to be registered into the store information system. There may be discrepancy between the shipment record and the actual shipment, and if it goes unnoticed by the receiving clerk, the inventory record will not reflect the actual stock accurately. On the outbound side, the checkout registers are not exempt from contributing to the inventory record errors. Typically, the cashiers are rewarded based on the speed of checkouts, and when a shopper brings similar products with identical price, they may choose to scan only one of the products and process them as identical SKUs. The result is that the inventory record of the scanned product decreases more than it should, while that of other products is left unchanged.

Inaccessible inventory refers to products that are somewhere in the facility but are not available because they cannot be found. This can happen when a consumer takes a product from the shelf and places it at another location. It can also happen in the back room or any other storage area in the store. The inaccessible inventory will eventually be found and made ready for sale. However, a long time may pass until this happens, and until then, the inaccessible products are no different from being nonexistent as far as revenue is concerned.

Incorrect product identification can occur in several different ways. Wrong labels can be placed on the products by both the suppliers and the stores. When the bar-codes on these labels are scanned during receiving or checkout, the inventory record for wrong items will change. Incorrect identification can also happen during manual inventory counts.

What makes inventory inaccuracy seem like an insurmountable problem is the sheer volume of the products handled in the stores. Typical retail stores, being at the far end of the supply chain, are the merge points of thousands of products that come in all different categories, shapes, and sizes, and tens of thousands of items may come in and go out of the store in a single day. For this reason, keeping track of the location of every item and making sure the inventory record agrees with the actual stock quantity is a daunting task.

## 1.3 The Stock Loss Problem

Determining which causes contribute to inventory record error and in what proportion is no less difficult than maintaining the accuracy of the inventory record itself. While the stores admit the gravity of inventory inaccuracy problems and consider it to be one of the major obstacles to the successful execution of their operations, they often do not know when and where it occurs and in what magnitude. However, of all the inventory error causes discussed, industry findings suggest that the unknown stock loss can be a dominant factor for many SKUs.

What makes the unknown stock loss differ from the other causes discussed here is the direction of the inventory record error. Since the loss of the physical items are not reported in the record, the inventory record overestimates the stock. On the other hand, the other causes — transaction errors, inaccessible inventory, and incorrect product identification — can make the error either positive or negative for a given SKU. While it would be almost impossible to break down the inventory error into individual causes, the results of manual inventory counts can reveal some truth about the extent to which unknown stock loss contributes to the inventory inaccuracy. If the inventory record overestimates the actual stock persistently, it is likely that unknown stock loss is the dominant cause of the inaccuracy.

Consider again Company A whose stores carry brands from Company B, who is a global consumer goods manufacturer. To understand the extent of the inventory inaccuracy problem, the two companies decided to pick the topmost selling product from Company B and monitor how the inventory record and the actual inventory change over the period of eight weeks. Dozens of Company A's stores were selected in several regions of North America, and field observers visited the stores once a week and manually counted the stock quantity of the product. At the outset of this testing, the inventory record was set to exactly match the actual inventory. At the end of the testing, however, the actual inventory was less than the inventory record, and the total adjustment was 5% of sales quantity on average over the stores tested. In a thin margin retail industry, this figure is a substantial loss in the bottom line profit.

Company C is a leading supermarket chain who also uses automatic replenishment system for its stores, and in a recent year reported combined known and unknown stock loss of 1.14% of sales in monetary value. Among the product categories that have the highest rates of stock loss were batteries and razor blades, whose stock loss equaled 8% and 5% of sales, respectively. Both of these are products characterized by high value and small size, and thus it was believed that theft accounted for most of the losses.

There are also few industry reports that shed light on the magnitude of the stock loss at the macroscopic level. An extensive study on the magnitude of stock loss was conducted by ECR Europe. Based on a sampling of 200 companies with dominant share of the consumer goods industry in Europe, the study reports that stock loss amounts to 1.75% of sales annually for the retailers. This figure translates to 13.4 billion euros annually. Of this, 59% (or, 1% of total sales) was unknown to the retailers — meaning that the stores did not know where or how the products were lost (ECREurope 2001).

Every year, the University of Florida publishes a similar industry-wide empirical research on retail inventory shrinkage in the US (Hollinger 2003). In the most recent report, 118 retailers from 22 different retail markets reported an average stock loss equaling 1.7% of total annual sales, a figure very close to the result from the ECR Europe. It further reports that the retailers perceive theft by the shoppers, employees, and vendors account for 80% of the total stock loss.

Since the stock loss figures are typically obtained by comparing the manual count of all inventories and the store inventory records, these findings suggest that overall in the retail industry, the inventory record error tends to have nonzero mean. The magnitude of this error, however, can vary significantly from one product to another, and the stores are able to estimate this figure for all of its SKUs at the end of yearly audit.

For these reasons, we focus on stock loss as a primary cause of inventory record error throughout the paper.

## 1.4 Literature Review

The literature in the field of inventory management is vast. Here we summarize the published documents most closely related to this work.

Iglehart et al. (1972) considers a reorder-point stocking policy subjected to random demand and inventory record error. Assuming the stocking policy is designed to protect only against variations in the demand and lead time, the optimal combination of additional safety stock and frequency of cycle count is obtained. This optimal combination minimizes the sum of the inventory holding cost and the counting cost, subject to the service level meeting a desired target.

Morey (1985) also investigates reorder-point based policies and develops a closed-form expression that relates the service level and three factors that affect it: frequency of cycle count, safety stock level, and the magnitude of inventory record error. This formulation is intended to serve as a very conservative, 'back-of-the-envelope' calculation tool for inventory managers to estimate the service level improvement due to combination of one or more of the three options.

Morey (1986) calculates the minimum required frequency between audits that maintains an inventory record accuracy (not the service level) in pre-specified limits. The optimal audit frequency is determined for two types of audits: perfect audits which eliminate all discrepancies between the book and actual inventory, and imperfect audits which leave errors in records.

A number of works have appeared that address the effective timing of cycle counts in multiple SKU environments. Cantwell (1985), Edelman (1984), and Reddock (1984) discuss the ABC analysis which assigns differing tolerances in inventory accuracy depending on the proportion of the total sales of the products. The size of tolerance would be directly related to the frequency of cycle counts. Neely (1987) proposes a few more methods of determining when to count, including increasing the cycle count frequency for high-activity SKUs.

In environments where there are many SKUs and the cost of manually counting the entire inventory becomes prohibitive, the inventory managers have the option of choosing and counting only a portion of the SKUs. Various sampling techniques exist to perform this task, and are explored in Buck and Sadowski (1983), Dalenius and Hodges (1959), Cochran (1977), Arens and Loebecke (1981), and Martin and Goodrich (1987).

Bernard (1985) and Graff (1987) discuss managerial steps that can be taken to make the cycle counts more effective and to improve the inventory record accuracy in multi-item production environment. Graff (1987) also emphasizes that cycle count merely provides a measurement of the inventory, and it alone is inadequate to control or improve the accuracy.

Various definitions and measures of inventory accuracy are presented in Ernst et al. (1984), Buker (1984), Chopra (1986) and Young (1986). Ernst et al. (1984) also proposes using a control chart to monitor the changes in the inventory accuracy. It serves as a tool for the inventory manager to identify when to look for non-random variability in the inventory accuracy. Hart (1998) provides a case study of a company that used a control chart.

A few works address inventory inaccuracy in MRP (Manufacturing Resource Planning) systems. French (1980) identifies numerous sources of work-in-process inventory inaccuracy. Krajewski (1987) uses a large-scale simulation to assess which factors in a MRP-based production environment (inventory inaccuracy being one of them) have the biggest impact on performance. Brown (2001) also uses simulation to investigate the impact of not only the frequency of error, but the magnitude of error and the location in the bill of material structure where the error takes place.

In surveying the literature, we have found that almost all of the research that address inventory policies assume that perfect knowledge of the inventory is available. There is a scarcity of works that address the causes and consequences of inventory error.

### 1.5 Outline

In this paper, the research work in inventory inaccuracy is largely divided into two parts. The first part (Section 2) investigates what happens when the inventory record error created by unknown stock loss is left uncorrected and how much the system performance is degraded as a result. Specifically, inventory systems operating under the (Q,R) policy is studied. The second part (Section 3) primarily addresses the question of what can be done to deal with the inventory record error and thereby improve the performance of the system. Various compensation methods are discussed and modeled, including the Auto-ID Center technology which has motivated this research.

## 2 Inventory Inaccuracy in the (Q,R) Policy

### 2.1 Mechanisms of the (Q,R) Policy

The (Q,R) policy is a commonly-used inventory policy in which the inventory is monitored continuously[1] and an order of fixed quantity $Q$ is placed to the supplier if the sum of inventory on-hand (quantity in the facility available for sale) and on-order (quantity ordered but not yet received) is less than or equal to the reorder point $R$. The time between placing an order and its arrival is called *lead time*.

The reorder point is set so that when an order is placed, enough inventory exists in the facility to meet the demand until the order arrives. Thus the reorder point has a critical bearing on the performance of this policy. If it is set too low, inventory will be depleted frequently and out-of-stocks will occur. If it is set too high, then the facility will be carrying unnecessary inventory.

The reorder point is often explained as consisting of two parts: the expected value of total demand during lead time and safety stock:

$$R = (\text{expected demand during lead time}) + (\text{safety stock}). \tag{1}$$

If the demand is known and constant, then setting the reorder point equal to the total expected demand during lead time would ensure that all demand would be satisfied. However, if there is

[1] In practice, monitoring is often done daily.

randomness in the system — such as in the demand or supplier lead time — then the reorder point will have to be higher to cover the uncertainties. This extra inventory is safety stock.

The (Q,R) policy is effective in its timing of orders and thus provides high availability of the stock at the minimal inventory level, provided that the on-hand inventory information used during the review is accurate. In reality, however, the exact value of on-hand inventory is often unknown, and many stores estimate the on-hand inventory based primarily on two measurements that they have access to: the incoming shipments and outgoing sales. The data for the former is obtained either through order transaction records or shipment verifications, and the latter through a technology commonly used that keeps track of bar-code-scanned sales at the checkout registers (called POS — *Point of Sales*). By updating the computerized inventory record whenever these two events are observed[2], the stores are able to automate the inventory review and order placement processes with minimal human intervention.

According to this method, the inventory record at the beginning of period $k+1$, denoted $\tilde{x}_{k+1}$, is determined from the inventory record at the beginning of the previous period, $\tilde{x}_k$, the quantity received in the previous period, $h_k$, and the quantity sold in the previous period, $a_k$, through the relationship

$$\tilde{x}_{k+1} = \tilde{x}_k + h_k - a_k. \tag{2}$$

In reality, the inventory record suffers from accuracy problems even if the incoming shipment and sales are known exactly. As discussed earlier, the unknown stock loss is an example of the causes of the error.

Throughout the research, we make a fundamental assumption that differs from those of traditional inventory models: stores do not know the exact value of on-hand inventory at the time of ordering. Therefore, our models distinguish between the inventory record and the actual inventory, and recognize the discrepancy between these two caused by stock loss.

## 2.2 Stochastic Simulation Model

To see how the stock loss, by creating a discrepancy between the actual inventory and the inventory record, can affect the performance of the (Q,R) policy, consider a single-item inventory model with the following assumptions:

- Demand for purchase during each period $k$, $w_k$, is assumed to be independent and distributed according to a truncated normal distribution with mean $\mu_w$ and standard deviation $\sigma_w$. (That is, we used a normal distribution with these parameters and discarded negative demands.)
- Demand for stock loss in period $k$, $v_k$, is also independent and identically distributed, and is generated from a Poisson distribution with mean $\lambda$.
- Lead time is known and fixed at $L$.
- Demand occurring at zero actual on-hand inventory is lost (no backlog).

A Poisson distribution is chosen for stock loss to prevent assigning negative values when the mean of the distribution is small.

The sequence of events in each period is assumed to be as follows:

[2] The industry terminology for this inventory record is *perpetual inventory*.

1. The inventory record is reviewed and an order is placed to the supplier.
2. The incoming order is received.
3. Sales and stock loss take place.

Denote by $x_k$ the actual inventory at the beginning of period $k$. According to this sequence, there is $x_k + h_k$ available for meeting the demand for purchase and stock loss in period $k$. When the sum of demand for purchase and stock loss exceeds the available inventory, the available inventory is divided proportionately to meet the the two demands. The sales in period $k$ is then

$$a_k = \begin{cases} w_k & \text{if } w_k + v_k \leq x_k + h_k, \\ (x_k + h_k)\frac{w_k}{w_k+v_k} & \text{otherwise.} \end{cases} \tag{3}$$

Since sales can only take on integer values, the quantity in the second line is rounded to the nearest integer. The changes in actual inventory and the inventory record are then

$$\tilde{x}_{k+1} = \tilde{x}_k + h_k - a_k \tag{4}$$

$$x_{k+1} = x_k + h_k - a_k - \min(v_k, x_k + h_k - a_k). \tag{5}$$

The *min* term represents the *actual* stock loss in period $k$: it is the smaller of the stock loss demand $v_k$ (sufficient on-hand inventory) and the difference between the available inventory $x_k + h_k$ and sales $a_k$ (insufficient on-hand). Since stock loss is not seen by the inventory record, it is not included in Equation (4).

Figure 2 shows the evolution of the actual inventory and the inventory record in a sample simulation run. The average demand $\mu_w$ is 10 and the standard deviation $\sigma_w$ is 2. The average daily stock loss $\lambda$ is 0.2, which is 2% of the average demand. Lead time $L$ is 3 periods and the fixed order quantity $Q$ is 50. The operation end time $t_f$ is chosen to be 365 periods since the standard procedure in the industry is to conduct a physical count of the stock at least once a year and reconcile the inventory record. The initial inventory is $R + Q - \mu_w L$ (chosen to be consistent with the deterministic model to follow this section). It was found through many simulation repetitions that in the absence of stock loss, a reorder point $R$ of 41 produced stockout — defined as the total lost sales as a percentage of total demand over the operating period — of 0.5%. We assume that this is the desired target stockout and use the corresponding reorder point $R = 41$ when simulating cases in which the stock loss occurs.

Initially, the inventory record and actual inventory are equal. However, the inventory record is not aware of the stock loss and starts to diverge from the actual inventory. As the gap between the two curves widens, the actual inventory curve hits zero more frequently, creating lost sales. Initially, there would be partial out-of-stocks (i.e., a portion of the daily demand during a period would be lost). However, as the inventory error grows further, out-of-stock worsens and the periods in which the entire demand is lost start to appear. This is seen by the flat portions of the actual inventory curve lying on the $x$-axis. On average, the duration of this complete out-of-stock gets longer with time.

We also observe a continual rise in the inventory record cycles. Each time stock loss occurs, the gap between the two curves grows. Since the actual inventory cannot be zero and the replenishment

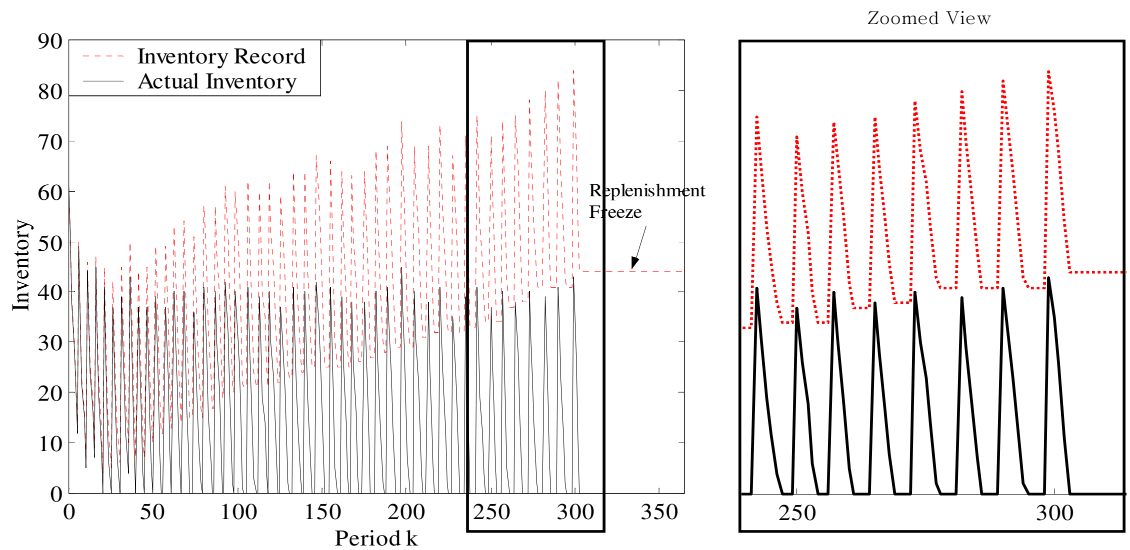


Figure 2: A sample simulation run showing inventory evolution of (Q,R) system when subjected to stock loss

quantity is fixed at $Q$, the accumulation of error makes the inventory record cycles rise over time. In fact, when the inventory record cycles continue to rise even further, the system eventually reaches a point where the inventory record stays above the reorder point and no order is placed. Such 'freezing' of replenishment is undesirable since it leads to extremely high lost sales.

Figure 3 shows how stockout is impacted by stock loss as the average stock loss demand $\lambda$ is varied from 0% to 7% of average demand. Each data point is the average of 500 independent simulation runs.

When there is no stock loss ($\lambda = 0$), the inventory system achieves the target stockout of 0.5%. As the stock loss increases, the error in the inventory record grows and cumulative lost sales rises. Shortly after the stock loss of 1%, the inventory system experiences freezing of replenishment and the stockout curve becomes steeper. What is of interest is how fast the stockout rises with the stock loss in the system. Even when the stock loss is as small as 1% of average demand (i.e., 1 item disappears for every 100 items in demand by shoppers on average), the error accumulating in the inventory record is large enough to disrupt the replenishment process and make 17% of the total demand from the shoppers lost due to out-of-stocks. When the average stock loss is at 2.4%, more than half of the demand ends up as lost sales. It does not require a high level of stock loss to degrade the in-stock performance of the system. Therefore, when nothing is done to correct the inventory error, stockout is highly sensitive to the inventory inaccuracy created by stock loss.

The results also convey a compelling managerial insight. Items lost to shoplifters, for example, are direct loss to the retailer, but the chain reaction created by shoplifting — error in the inventory record, untimely replenishment, and out-of-stocks — creates lost sales substantially greater than the items stolen. Results show that the lost sales quantity can be ten to twenty times higher. Even

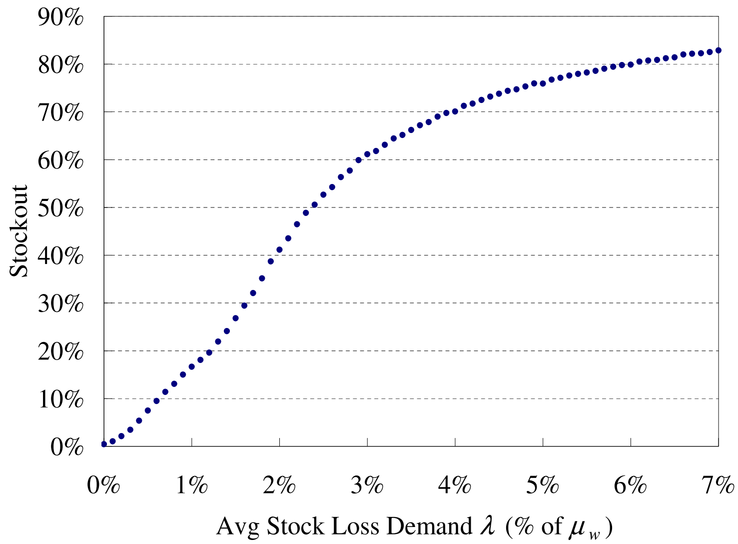


Figure 3: Simulation data points for stockout vs. average stock loss demand

if the comparison is made in bottom line monetary values[3], the unrealized revenue due to lost sales is substantial in highly competitive retail environments. This means that to effectively control the stock loss problem, management needs to pay close attention to maintaining inventory accuracy.

## 2.3 Deterministic Model

Whereas in the previous section the demand for purchase and stock loss were assumed to be stochastic and discrete, in this section they are treated as constant and continuous. With this simplification, we look for a closed-form solution for the system performance given the parameters of the (Q,R) policy. Moreover, by developing a model with deterministic demand and stock loss, the role that randomness plays in the inventory inaccuracy problem can also be examined.

Assume demand for purchase and stock loss occur at a rate of $w$ and $v$ units per time, respectively. The lead time $L$ is again fixed and known, and the assumption regarding excess demand (lost sales and no backlog) remains unchanged from the previous model. Also, the ordering decision is made in accordance with the (Q,R) policy. Figure 4 shows how the inventory record and actual inventory evolve over time.

The deterministic model exhibits all the essential features seen in the stochastic simulation model — the growing gap between the recorded and actual inventories, the continual rise of the inventory record cycles, and the eventual freezing of replenishment as the inventory record stays above the reorder point.

For convenience, we break the inventory evolution into two time intervals. Let Region A consist

[3]Since stock loss is lost property and the lost sales is lost revenue, comparison of the impact on profit would require the profit margin of the product. The margin, however, varies widely based on the pricing strategies of the retailers, and could range from a small percentage to multiple times the product cost.

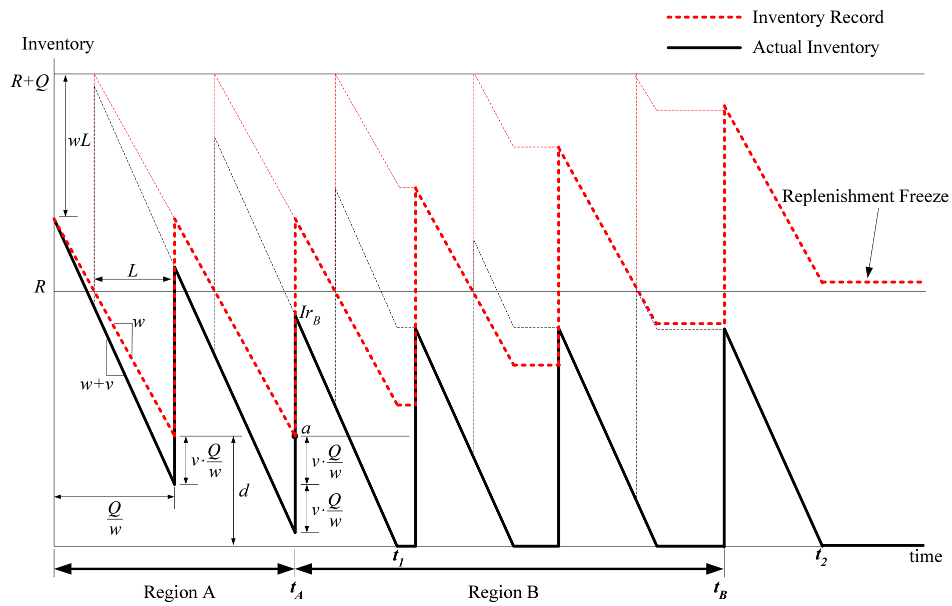


Figure 4: (Q,R) Policy subjected to stock loss under deterministic demand

of the group of cycles in which no out-of-stock occurs. The time $t_A$ marks the end of this region. Region B consists of the group of complete cycles that fall between $t_A$ and the time of last order arrival prior to system freezing. Let $t_B$ denote the end of this region.

We can compute various performance measures of the system — the time of first out-of-stock $t_1$, the time of replenishment freeze $t_2$, and the stockout $S_{out}$. The exact calculations for these quantities can be obtained. However, by making the plausible assumptions that the stock loss rate $v$ is small and there are many replenishment cycles before the end of operation $t_f$, we arrive at expressions that are much simpler and yet able to approximate the exact calculation very closely. Appendix 5 describes in detail how the expressions for both exact and approximate values are found. Approximations for $t_1$ and $t_2$ are

$$t_1 \approx \frac{R - wL}{v} + \frac{Q}{w+v} \tag{6}$$

$$t_2 \approx t_1 + \frac{L(w+v)}{v} + \frac{L}{2}\left(\frac{wL(W+v)}{vQ} - 1\right) + \frac{Q}{w+v}. \tag{7}$$

The stockout $S_{out}$ is determined by adding all the horizontal, flat portions of actual inventory curve in Figure 4 and dividing it by the operating time $t_f$. It is approximated by

$$S_{out} \approx \begin{cases} 0 & \text{if } t_f < t_1, \\ \frac{1}{t_f}\left[\frac{m(m+1)}{2}\frac{vQ}{w(w+v)}\right] & \text{if } t_1 \le t_f < t_2, \\ \frac{1}{t_f}\left[\frac{L}{2}\left(\frac{wL(w+v)}{vQ} + 1\right) + t_f - t_2\right] & \text{if } t_f \ge t_2, \end{cases} \tag{8}$$

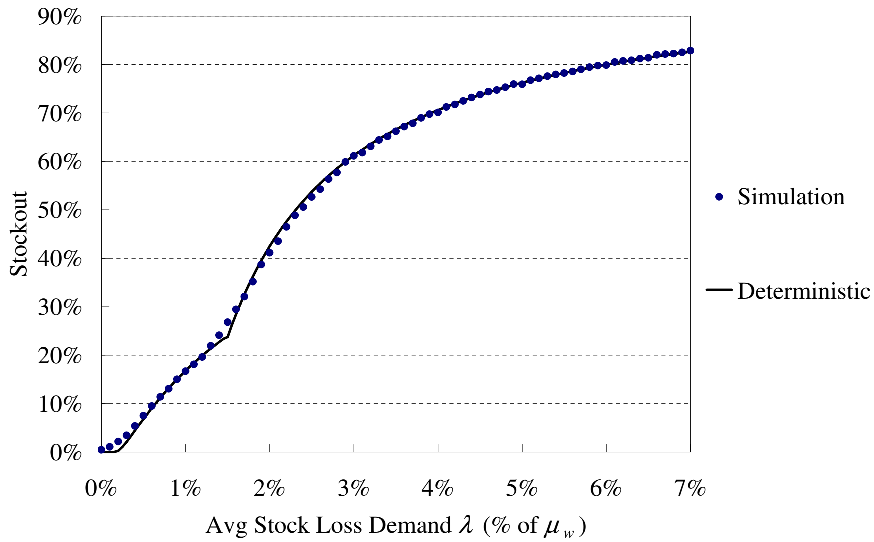


Figure 5: Comparison of stockout calculations from deterministic and simulation model

where $m$ appearing in the second expression is the number of complete cycles between $t_1$ and $t_f$ if $t_f$ is located in Region B, approximated by

$$m \approx \frac{2w+v}{2v} + \sqrt{\left(\frac{2w+v}{2v}\right)^2 + 2\frac{w(w+v)}{vQ}(t_f - t_1)}. \tag{9}$$

In Figure 5 is shown the approximation for stockout calculated in the deterministic model (solid line), along with the simulation points (from Figure 3). The close agreement of the deterministic model calculation with the simulation confirms the finding that when the inventory error is left untreated, system performance is highly sensitive to the inventory inaccuracy created by stock loss. Furthermore, the randomness in the model behavior is not what causes the inventory inaccuracy problem.

## 2.4 Sensitivity Analysis

In this section, we investigate in what circumstances the stock loss impacts the system performance the most. Using the simulation model, we conduct a parametric analysis by observing how stockout is affected when the lead time $L$ and order quantity $Q$ are varied.

Figure 6 illustrates the effect of varying lead time on the stockout when the system is subjected to average stock loss demand at 1% of the average purchase demand. The parameters are set to be consistent with what is presented in Section 2.2: $\mu_w = 10, \sigma_w = 2, Q = 50$, and $t_f = 365$. Note that along with the lead time, the reorder point $R$ is also set to provide the same target stockout of 0.5% *in the absence of stock loss*. Thus, we assume that the inventory manager, either unaware of the stock loss or ignoring it, simply sets the reorder point based on the purchase demand characteristics and lead time. We have already mentioned that $R = 41$ provided the target stockout when the lead

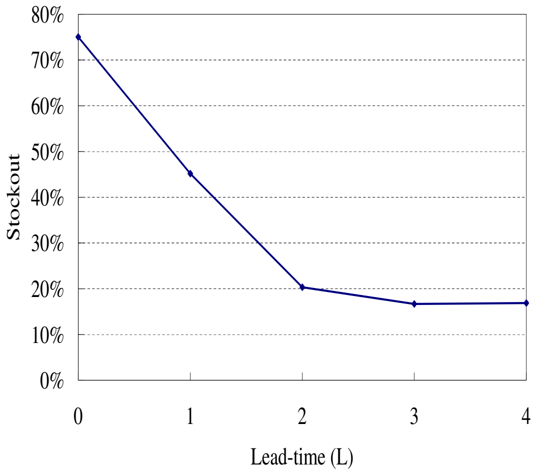


Figure 6: Stockout vs. $L$

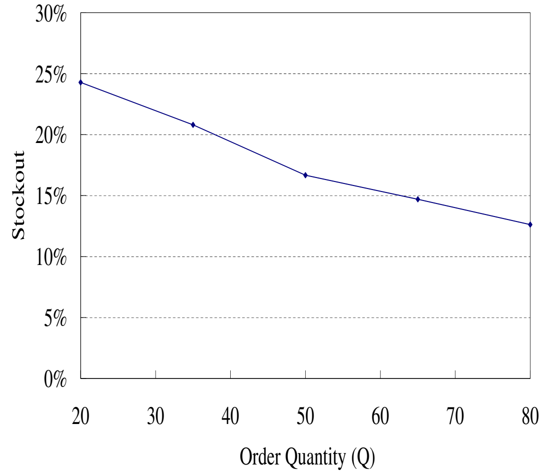


Figure 7: Stockout vs. $Q$

time $L$ is 3. For smaller lead times, the variability of the lead time demand is also smaller, and thus the safety stock component of the reorder point can be reduced and still achieve the same target stockout — again, provided there is no stock loss. Similarly, for longer lead times, the reorder point will have to increase. However, once we allow uncompensated stock loss, the reduced safety stock associated with shorter lead times leads to much worse performance.

We have seen that the performance of a system with $L = 3$ is highly sensitive to unaccounted stock loss. At shorter lead times, this sensitivity becomes greater. In a system where ordered products are delivered instantly ($L = 0$), it only takes an average stock loss demand of 1% to render three quarters of the total purchase demand unfulfilled. The reason why such an extreme out-of-stock condition is created is because with zero lead time, Region B in Figure 4 does not exist. Instead, in the first cycle after Region A, the system directly enters the replenishment freeze zone. The time of replenishment freeze is on average 95 when $L = 0$, 225 when $L = 1$, and 349 when $L = 2$.

Figure 7 is the result of same simulation runs, this time holding $L = 3$ and varying the order quantity $Q$ from 20 to 80. Having a large order quantity reduces stockout since the actual inventory is higher on average.

These observations demonstrate the severe consequences of inventory inaccuracy on lean systems characterized by short lead times and frequent ordering of small quantities. At shorter lead times, the desired product availability can be achieved with smaller safety stock if there is no stock loss (thus allowing $R$ to be reduced). However, small safety stock provides little protection against unexpected disturbances in the system. Inventory inaccuracy, which is considered an uncertainty in the system, is likely to wreak far greater havoc on lean systems, and thus maintaining accurate inventory record is critical to reap the benefits lean systems have to offer.

# 3 Compensation Methods for Inventory Inaccuracy

In the previous section, one underlying assumption used in the models was that nothing is done to correct the inventory record error. The management may not be aware of the stock loss, or may simply choose to ignore it in designing the inventory policy. In this section, we examine various techniques inventory managers can use to compensate for the inventory record error. The methods of controlling the error are many, but we describe some of the representative ones here and assess the improvements each method can make in bringing the in-stock performance to the desired level.

Consider the simulation exercise used in Section 2.2 to assess the impact of unknown stock loss on the performance of the (Q,R) policy if no corrective actions are taken. We use this model as a basis for testing how well each error-adjustment method performs in compensating for the inventory error. By using the same set of assumptions, we can examine how much improvement is made from the no-correction case by each compensation technique.

## 3.1 Compensation Methods

### 3.1.1 Safety Stock

Safety stock is often used as a protection against uncertainties in variables in inventory operations, such as the demand and supplier lead time. It can be extended to serve as a buffer against uncertainty in the inventory record.

In the (Q,R) policy, the level of safety stock is determined by setting the reorder point $R$ (Section 2.1). Since the reorder point consists of the expected demand during lead time and safety stock, to provide a buffer against inventory error would require increasing the reorder point to a level higher than that needed to cover the variability in purchase demand. In the numerical example shown in Section 2.2, the reorder point of 41 achieved 0.5% stockout when there is no stock loss occurring in the system. Since the expected purchase demand during lead time is 30 ($\mu_w L = 10 \cdot 3 = 30$), a safety stock of $41 - 30 = 11$ units was required to provide this target stockout. To cover the additional uncertainty in the inventory record, a higher safety stock would be required. Thus, to see the benefit of carrying higher safety stock, we simulate the (Q,R) policy with $R$ higher than 41.

### 3.1.2 Manual Inventory Verification

One of the most commonly used techniques for mitigating the inventory error is manually counting the items in the facility and correcting the inventory record. The inventory managers can choose to verify the inventory for a part of the entire SKU more frequently than the required yearly audit. This frequency may depend on various elements, such as the availability of the labor and product characteristics, including the profit margin, sales velocity, and whether the products are highly prone to stock loss and other causes of inventory error.

We assume manual verification is done at predetermined, regular intervals, such as every month or every six months. In the simulation, the inventory record is set to equal to the actual on-hand at the end of the period when verification is done. It is assumed that manual count is done with perfect accuracy.

#### 3.1.3 Manual Reset of the Inventory Record

If a direct measurement of the on-hand inventory is not available, inventory managers can gather and monitor the available data and search for any patterns that may be indicative of the presence of serious inventory error. In the (Q,R) policy, for example, we saw that if the inventory error grows enough, it will eventually reach a point where the inventory record stays above the reorder point and no replenishment is made. In that situation, the daily POS (Point-of-Sales) reading will simply be zero every day. Knowing that this is an unlikely event under normal operations, the inventory manager can choose to manually reset the inventory record to zero, thereby allowing the automated replenishment system to start placing orders again.

To simulate this compensation method, we set the inventory record to zero at the end of each period whenever zero sales is observed. Since the probability of zero demand is very small ( $7.4 \cdot 10^{-7}$) in the truncated normal distribution for purchase demand with $\mu_w = 10$ and $\sigma = 2$, zero sales would be a strong indication of the existence of an out-of-stock condition.

#### 3.1.4 Constant Decrement of the Inventory Record

If the inventory manager is aware of the presence of stock loss and also knows its stochastic behavior, another way to compensate for the error is to decrement the inventory record by the average stock loss demand each period. Since the actual value of the stock loss at each period is unknown, simply decrementing the record will still not eliminate the error in the inventory record. However, over time, this corrective action can be expected to perform better than leaving the inventory record unadjusted.

In the simulation, an additional step at the end of each period is added to decrease the inventory record by the estimated daily stock loss demand $\lambda$. The actual inventory and record now change according to

$$\widetilde{x}_{k+1} = \widetilde{x}_k + h_k - a_k - \lambda \tag{10}$$

$$x_{k+1} = x_k + h_k - a_k - \min(v_k, x_k + h_k - a_k). \tag{11}$$

#### 3.1.5 Auto-ID

The technology under development at the Auto-ID Center differs fundamentally from the current inventory systems in that it provides a direct measurement of the stock quantity using RFID readers and tags. To preserve generality, we refer as 'Auto-ID' all means of automatically obtaining the direct measurement of the stock quantity without having to count the items manually. Here we assume the Auto-ID provides a perfectly accurate measurement of the actual inventory and examine how it improves the inventory system performance. An analysis that accounts for Auto-ID with imperfect measurements can be found in Kang (2003).

Auto-ID is simulated by setting the inventory record to equal to the actual inventory at the end of each period. Thus, the ordering decisions are made with the perfect knowledge of the on-hand quantity. The equations describing the inventory estimate and the actual inventory are

$$\widetilde{x}_{k+1} = x_{k+1} \tag{12}$$
$$x_{k+1} = x_k + h_k - a_k - \min(v_k, x_k + h_k - a_k). \tag{13}$$

It should be noted that new compensation methods can be created by combining two or more of the techniques described above. For instance, manual verification of the inventory can be conducted along with carrying a higher safety stock.

## 3.2 Results and Discussion

### 3.2.1 Performance comparison of the compensations methods

Figure 8 shows how the inventory system performance changes when the compensation methods are implemented. The same parameters from the numerical example in Section 2.2 are used: $\mu_w = 10, \sigma_w = 2, L = 3$, and $Q = 50$. The average stock loss $\lambda$ is held constant at 0.1, which is 1% of average demand for purchase. The reorder point $R$ is varied in steps of 2 around the base value of 41 (which produces the target stockout of 0.5% in the absence of stock loss) for each compensation. The rationale behind varying the reorder point is that in the (Q,R) policy, $R$ is the decision parameter, and it is the responsibility of the inventory manager to select the $R$ that produces the most desirable performance for each compensation technique.

The figure plots stockout against average inventory for each compensation method. Since the desired goal is to obtain a low stockout at minimal inventory, the stockout-inventory pair is chosen as the performance measure. The vertical distance between the curves is the difference in average inventory required to attain a particular stockout. Therefore, for a given stockout, the compensation technique with the lowest inventory would be the best-performing one. Notice that by increasing the reorder point higher than the base value 41, we are also testing how each compensation technique performs in conjunction with carrying higher safety stock.

The 'No Compensate' curve represents the case in which nothing is done to correct the inventory error caused by stock loss other than varying $R$ (Section 3.1.1). This curve thus serves as a basis from which improvements made by each compensation method can be observed. The rightmost data point in this curve corresponds to the lowest reorder point, and thus has the highest stockout. As $R$ increases, stockout improves at the expense of inventory. The convex shape of the curve is observed for all compensation methods.

In the manual inventory verification method (Section 3.1.2 — represented by 'Verify Twice'), counting is assumed to be conducted twice a year. The result shows that even the infrequent inventory record reconciliation of every six months improves the performance dramatically.

The 'Reset Record' curve is the result of resetting the inventory record to zero when sales are zero (Section 3.1.3). Notice that the vertical distance from the 'No Compensate' curve is large for low reorder points but is almost zero for high reorder points. This is because at low reorder points, inventory is small on average and zero sales occur frequently. Thus, the POS provides useful

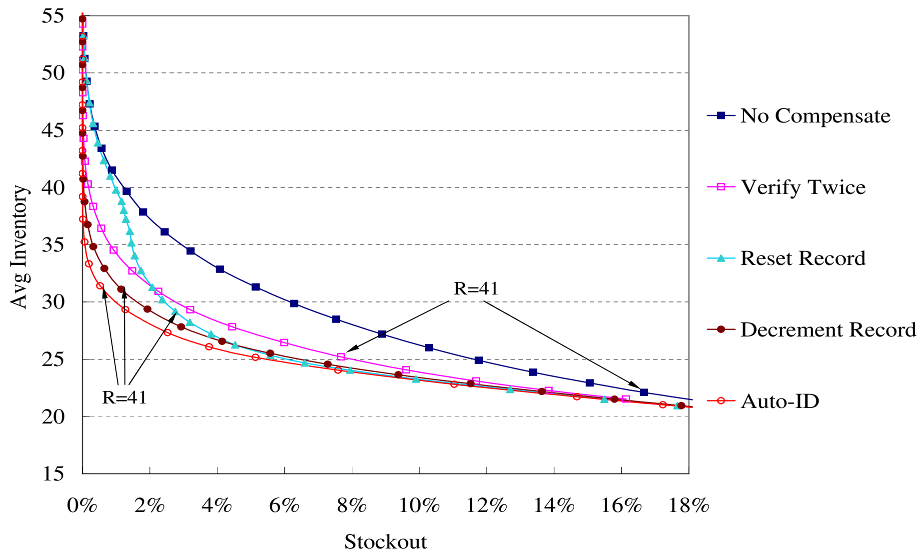


Figure 8: Stockout and average inventory for various compensation methods

information needed to correct the inventory record error. At high reorder points, zero sales are infrequent, and the system behaves close to the 'No Compensate' case.

The strategy of decrementing the inventory record daily by the average stock loss (Section 3.1.4 — shown by the 'Decrement Record' curve) performs remarkably well in improving the stockout-inventory compromise from the no compensation case. Simply reducing the inventory record value by a constant amount each period still leaves errors in the record, but over time the record is able to track the actual inventory much more closely and keep the out-of-stocks low.

As expected, having a perfectly accurate knowledge of the on-hand inventory (Section 3.1.5 — the 'Auto-ID' curve) achieves the best stockout-inventory compromise: Auto-ID is able to attain the lowest inventory for any given stockout. The benefit of having the accurate knowledge of on-hand inventory becomes greater as the desired target stockout becomes smaller.

The effect of carrying higher safety stock can be observed from the 'No Compensate' curve. Since in the absence of stock loss the minimum reorder point required to achieve 0.5% stockout is 41, any reorder point higher than this can be considered safety stock for protecting the system from inventory record error. When the stock loss demand is 1% of average purchase demand, the reorder point must be increased to at least 73 to maintain the stockout at 0.5%. This means the safety stock will have to increase by more than three days' worth of average purchase demand. Starting the inventory operation with higher reorder point allows more time for the actual inventory to stay above zero. However, as the gap between the actual inventory and inventory record grows and out-of-stocks begin to occur, this compensation method takes no further action to correct the error. In fact, when the stock loss is higher at 3%, the reorder point must be much higher at 145 (including a safety stock of more than eleven days' worth of average demand). This indicates that at high stock losses, the inventory required to maintain the low target stockout becomes prohibitive. Therefore, merely stocking up the facility with extra inventory to provide a buffer against uncertainty in inventory accuracy is not an effective way to treat the problem.

### 3.2.2 Limitations of Each Method

The results reveal that if the stochastic behavior of stock loss is known, a significant improvement in performance can be achieved by compensating for the inventory record error. We also have seen that in some instances, such as decrementing the inventory record by average stock loss, a dramatic improvement can be made even without Auto-ID. However, the stockout-inventory performance is not the only measure that has to be taken into account in selecting the appropriate compensation method.

Higher safety stock, as we have seen, keeps the lost sales to the minimum only for very small stock losses, and does so at the price of carrying high inventory. For inventory inaccuracy caused by nonzero-mean error such as the stock loss considered here, this is not a desirable solution.

Manual verification of the inventory record has a number of disadvantages as well. It is costly to implement, especially in low-margin, high-competition environments where the availability of workforce is limited. In addition, manually verifying the entire facility requires shut down of the operation, which leads to loss of revenue. Targeting only a portion of the entire SKUs and cycle counting them is an alternative, but often items cannot be found in the designated locations when they are misplaced by shoppers or employees. In mass merchandise retailing environments where there are hundreds of thousands of individual items at any time, finding the items of interest during the cycle count alone becomes a challenging task. If the possibilities of mis-labeling and mis-counts are also considered, there is no guarantee that the manual counts will accurately reflect the true on-hand inventory.

The method of resetting the inventory record to zero bears the danger of false positives. This is true especially for low demand products, for which zero sales does not necessarily mean zero inventory. Incorrectly setting the inventory record to zero results in over-stocking the inventory. In our example, the use of this compensation makes sense since the probability of zero purchase demand is extremely small. For products with much lower demand, the inventory record should be reset only if a number of consecutive zero sales days are observed. However, determining the number of such days to wait until reset requires a sophisticated analysis.

Decrementing the inventory record, while simple in concept and effective in keeping the stockouts low in our model, presents a few disadvantages as well. First of all, implementing this method can be expected to face cultural barriers in the organizations. The perpetual inventory has always been discrete, nonnegative integers. Under this method, however, the computer record could be negative and non-integer depending on how it is implemented.

More important is the sensitivity of the system performance to the stock loss demand estimate used in decrementing the inventory record. In Section 2, it was pointed out that even a small level of stock loss can create high stockouts. This is tantamount to saying that if the estimated stock loss demand is slightly lower than the actual stock loss demand, the stockout will also be high. To study this sensitivity, another set of simulations were run. Whereas in the previous simulation the average stock loss demand was assumed to be known exactly, now we assume the inventory manager makes an incorrect estimate of the true stock loss demand. Figure 9 shows how the stockout and average

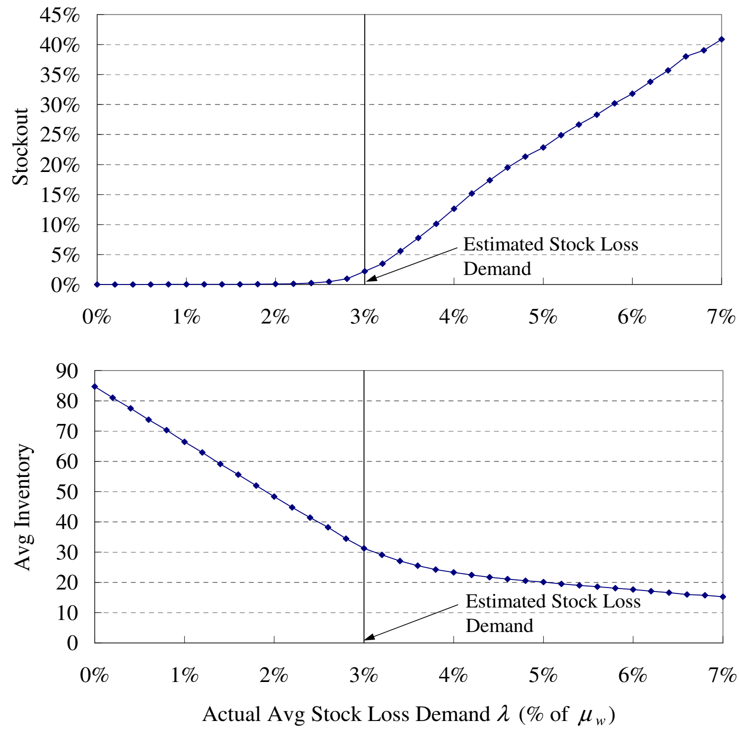


Figure 9: Stockout and average inventory when the estimated stock loss is incorrect in using the inventory record decrement strategy

inventory change when the average stock loss demand is estimated to be 3% of average demand but the true average stock loss demand varies from 0 to 7%.

If the estimated average stock loss demand of 3% is equal to the actual stock loss demand occurring in the facility, then this compensation method performs well in adjusting the inventory error and achieves a relatively low stockout rate of 2.2%. However, as the amount by which the estimated value underestimates the actual stock loss demand grows, stockout rises rapidly. In other words, to the right of 3% stock loss, stockout performance exhibits sensitivity similar to what is observed in situations where the inventory error is left uncorrected. There is a difference in how the stockout rises, however. In the case where no compensation is applied, stockout rises even more sharply when the stock loss is high enough to create replenishment freeze. Here, we do not observe such change in the stockout curve slope: the stockout rises more or less at a constant rate with increasing actual stock loss demand. This is because even when the system freezing takes place, decrementing the inventory record daily will eventually bring the inventory record below the reorder point, thus setting the replenishment back into action. Therefore, another benefit of the decrement strategy is that it prevents the replenishment freeze from taking place, and thus eliminates the extreme out-of-stocks.

The performance of the system suffers if the estimated stock loss demand is higher than the actual stock loss demand as well. In this case, stockout drops to zero, but the average inventory in the system rises rapidly as actual stock loss decreases. When the estimate is off by 2%, the average inventory is more than twice as high. Therefore, the ability of the inventory record decrement strategy to effectively compensate for the inventory error depends critically on the accuracy of the stock loss estimate. Even a small deviation from the actual stock loss demand will result in either high stockout or high unnecessary inventory in the facility.

Auto-ID requires high up-front investment in RFID readers and tags, in addition to the costs involved in design and execution of real-time inventory tracking algorithm and software. Moreover, being an emerging technology still under development, there is no guarantee that Auto-ID will work perfectly and provide an exact account of the actual stock quantity in the store.

# 4 Conclusions

This research was motivated by the potential ability of the Auto-ID Center's product identification technology to address one of the greatest obstacles in successful inventory management — inventory inaccuracy. The simulation and analytical work described here reveal that when no corrective action is taken, even a small rate of stock loss can disrupt the replenishment process and create severe out-of-stocks. In fact, the lost sales due to stock loss can be substantially higher than the stock loss itself. Furthermore, the harmful effect of stock loss is greater in lean environments characterized by short lead times and small order quantities.

Upon investigating various ways to deal with the inventory error, it was found that even without the sophisticated identification technology like Auto-ID, the inventory inaccuracy problem can be effectively controlled if the stochastic behavior of the stock loss is known. However, each compensation method has limitations.

# Acknowledgments

This research was partly supported by the MIT Auto-ID Center and partly by the Singapore-MIT Alliance.

# Appendix

# 5 Calculations for Deterministic Model — (Q,R) Policy

Calculating performance measures of the inventory system of Section 2.3 requires computing the ending times $t_A$ and $t_B$ of Regions A and B of Figure 4.

## 5.1 Exact Calculations

### 5.1.1 $t_1$

The analysis of Region A and the calculation of times $t_A$ and $t_1$ require a focus on the actual inventory, since they are defined by the actual inventory reaching zero.

$t_A$ can be determined by finding the number of cycles in Region A, denoted $n_A$, and the length of each cycle. In the first cycle, the initial actual inventory and the initial inventory record are both $R + Q - wL$. When the inventory record reaches the reorder point $R$, it has decreased by $(R + Q - wL) - R = Q - wL$. The inventory record further decreases by $wL$ until just before the first order arrives. The inventory record is then $R - wL$ so the total decrease in the first cycle is $(R + Q - wL) - (R - wL) = Q$. When the order of amount $Q$ arrives, the inventory record jumps back up to $R + Q - wL$. This cycle in the inventory record repeats as long as the real inventory is above zero. The length of a cycle is the time required for the demand (at rate $w$) to consume the amount $Q$, or $Q/w$.

When the actual inventory reaches zero, sales are interrupted and a new kind of behavior begins. The actual inventory, since it decreases at the rate $v$ faster than the inventory record, drops by an additional amount, $v \cdot (\text{length of a cycle}) = v(Q/w)$, in the first cycle. That is, the actual inventory decreases by $Q(1+v/w)$ in the first cycle. The net change, after the order arrives, is then a decrease of $v(Q/w)$.

As long as the demand and stock loss rate are constant, the actual inventory decreases by this amount during each cycle. However, when the actual inventory reaches zero, the slope of the two inventory curves change — in fact, they go to zero. The number of cycles until this happens, i.e., the number of cycles in Region A, is the largest integer number of times the quantity $v(Q/w)$ can fit into the inventory record at the end of the first cycle. This value is $d$ in Figure 4, and $d = R - wL$. $n_A$ is then

$$n_A = I\left(\frac{R - wL}{v\frac{Q}{w}}\right) \tag{14}$$

where $I(x)$ is the largest integer less than $x$. Then $t_A$ is $n_A$ times the length of a cycle, or

$$t_A = n_A Q/w = I\left(\frac{R - wL}{v\frac{Q}{w}}\right)\frac{Q}{w}. \tag{15}$$

The value of the actual inventory at time $t_A$, denoted $I_{r_B}$, is the value of the actual inventory at time 0 minus the stock loss since time 0. The stock loss since $t = 0$ is $vt_A$, so

$$I_{r_B} = R + Q - wL - vt_A = R + Q - wL - vI\left(\frac{R - wL}{v\frac{Q}{w}}\right)\frac{Q}{w}. \tag{16}$$

The remaining time after $t_A$ until the actual inventory reaches zero is $I_{r_B}$ divided by $v + w$, the rate that actual inventory actually decreases. The time of the first out-of-stock, $t_1$, is $t_A$ plus the time after $t_A$ that the actual inventory reaches zero, or

$$\begin{aligned} t_1 =& t_A + \frac{I_{r_B}}{v+w} \\ =& I\left(\frac{R-wL}{v\frac{Q}{w}}\right)\frac{Q}{w} + \frac{I_{r_B}}{v+w}. \end{aligned} \tag{17}$$

#### 5.1.2 $t_2$

The analysis of Region B and the calculation of times $t_B$ and $t_2$ require a shift of focus to the inventory record, since they are defined by the inventory record exceeding $R$.

In this section, we determine $n_B$, the number of cycles in Region B, and the length of each cycle. To calculate $n_B$, we make use of the fact that in Region B, the ending value of an inventory record cycle is higher than that of the previous cycle.

**First cycle in Region B** The inventory record is $R + Q - wL$ at time $t_A$, and it drops at rate $w$ as long as the actual inventory remains strictly positive. As we have shown, the actual inventory remains positive for a time period of length $I_{r_B}/(v + w)$, so the inventory record drops by $wI_{r_B}/(v + w)$ between $t_A$ and $t_1$. Note that this decrease is *less* than the order amount $Q$ since the time between $t_A$ and $t_1$ is less than a full Region A cycle.

Since the actual inventory is zero immediately after $t_1$, sales are zero and the inventory record remains constant until the order arrives. The inventory record, between $t_1$ and when the order arrives, stays at $R + Q - wL - wI_{r_B}/(v + w)$. Since the value of the inventory record just before the order arrives in Region A is $R - wL$, the increase in the inventory record just before the order arrives at the end of the first cycle in Region B is $(R + Q - wL - wI_{r_B}/(v + w)) - (R - wL) = Q - wI_{r_B}/(v + w)$. When the order arrives, both the inventory record and the actual inventory jump by $Q$.

**Later cycles** In later cycles, the actual inventory always starts at exactly $Q$. The time required for the actual inventory to reach zero is $Q/(v + w)$, and it remains at zero until the next order arrives. The inventory record decreases by $wQ/(v + w)$ while the actual inventory is positive, and then, after a period of no sales, it jumps by $Q$. Therefore, the net change in the inventory record during a Region B cycle other than the first is $Q - wQ/(v + w) = vQ/(v + w)$.

To summarize, the value of the inventory record at $t_1$, the end of the first cycle of Region B, is

$$R + Q - wL - \frac{wI_{r_B}}{v+w} \tag{18}$$

and the inventory record at the end of the $i^{\text{th}}$ cycle is

$$R + Q - wL - \frac{wI_{r_B}}{v+w} + (i-1)\left(\frac{vQ}{v+w}\right). \tag{19}$$

**Number of cycles** Let $n_B$ be the number of Region B cycles until the inventory record is greater than or equal to $R$ at the lowest point of a cycle. For there to be at least one cycle, we must have

$$R + Q - wL - \frac{wI_{r_B}}{v+w} \geq R$$

or

$$Q - \frac{wI_{r_B}}{v+w} \geq wL.$$

Then $n_B$ is given by

$$n_B = \begin{cases} 0 & \text{if } Q - \dfrac{wI_{r_B}}{v+w} < wL \\ 1 + \min n & \text{otherwise} \end{cases}$$

where $n$ is an integer such that

$$R - wL + \left(Q - \frac{wI_{r_B}}{v+w}\right) + n\left(Q - \frac{wQ}{v+w}\right) \geq R$$

or

$$n \geq \frac{wL - \left(Q - \frac{wI_{r_B}}{v+w}\right)}{\left(\dfrac{vQ}{v+w}\right)}.$$

Therefore, $n_B$ is given by

$$n_B = \begin{cases} 0 & \text{if } Q - \dfrac{wI_{r_B}}{v+w} < wL \\ 1 + I\left(\dfrac{wL - \left(Q - \dfrac{wI_{r_B}}{v+w}\right)}{\left(\dfrac{vQ}{v+w}\right)}\right) & \text{otherwise.} \end{cases}$$

**Length of Region B** Unlike the Region A cycles, the duration of the Region B cycles are not all the same. To determine the total length of Region B, we need to compute the length of its cycles. The length of each cycle is the time it takes for the inventory record to reach the reorder point from the start of the cycle plus the lead time $L$.

That is, if the inventory record at the start of a cycle is $x$ (which includes the order of size $Q$ that just arrived), the time until the inventory record reaches $R$ is $(x - R)/w$ and the length of the cycle is

$$(x - R)/w + L. \tag{20}$$

At the start of the first cycle, the inventory level is the same as in every cycle in Region A: $R + Q - wL$. Therefore, the length of the first cycle is $Q/w$.

To determine the length of the second cycle, we recall from (18) that the inventory record at time $t_1$ is $R+Q-wL-wI_{r_B}/(v+w)$. Therefore, the inventory record, at the start of the second cycle, just after the order arrives, is $R+Q-wL-wI_{r_B}/(v+w)+Q$. The length of the second cycle is then $(Q-wL-wI_{r_B}/(v+w)+Q)/w+L$ or $Q/w+Q/w-I_{r_B}/(v+w)$.

More generally, (19) implies that the inventory record at the start of the $i^{\text{th}}$ cycle, for $i \geq 2$, is

$$R+2Q-wL-\frac{wI_{r_B}}{v+w}+(i-2)\left(\frac{vQ}{v+w}\right)$$

so the length of the $i^{\text{th}}$ cycle is, according to (20),

$$\frac{1}{w}\left(2Q-\frac{wI_{r_B}}{v+w}+(i-2)\left(\frac{vQ}{v+w}\right)\right). \tag{21}$$

The total length of Region B, for $n_B \geq 2$, is therefore

$$\frac{Q}{w}+\sum_{i=2}^{n_B}\frac{1}{w}\left(2Q-\frac{wI_{r_B}}{v+w}+(i-2)\left(\frac{vQ}{v+w}\right)\right).$$

Carrying out the summation above and simplifying, the total length of Region B can be summarized as

$$\begin{cases} 0 & \text{if } n_B = 0, \\ \frac{Q}{w} & \text{if } n_B = 1, \\ (n_B-1)\left(\frac{2Q}{w}-\frac{I_{r_B}}{v+w}\right)+(n_B-1)(n_B-2)\frac{vQ}{2w(v+w)} & \text{if } n_B \geq 2. \end{cases} \tag{22}$$

$t_2$ is now the sum of $t_A$, the total length of Region B, and the last in-stock duration that exists immediately after Region B, which is $\frac{I_{r_B}}{v+w}$ if $n_B = 0$ and $\frac{Q}{v+w}$ otherwise. This is

$$t_2 = \begin{cases} t_A + \frac{I_{r_B}}{v+w} & \text{if } n_B = 0, \\ t_A + \frac{Q}{w} + \frac{Q}{v+w} & \text{if } n_B = 1, \\ t_A + \frac{Q}{w} + (n_B-1)\left(\frac{2Q}{w}-\frac{I_{r_B}}{v+w}\right)+(n_B-1)(n_B-2)\frac{vQ}{2w(v+w)}+\frac{Q}{v+w} & \text{if } n_B \geq 2. \end{cases} \tag{23}$$

### 5.1.3 $S_{out}$

We compute the stockout $S_{out}$ to be the fraction of the entire operation time, $t_f$, occupied by the flat portions of the actual inventory curve in Region B. The length of the flat line in each Region B cycle is found by subtracting from the length of each cycle (which has already been determined in the previous section) the in-stock duration of each cycle, which is $\frac{I_{r_B}}{v+w}$ for the first cycle and $\frac{Q}{v+w}$ thereafter. Using (21), the length of the flat line in the $i^{\text{th}}$ cycle is

$$\frac{1}{w}\left(2Q-\frac{wI_{r_B}}{v+w}+(i-2)\left(\frac{vQ}{v+w}\right)\right)-\frac{Q}{v+w}=\left(\frac{Q}{w}-\frac{I_{r_B}}{v+w}\right)+(i-1)\frac{Qv}{w(v+w)}.$$

If $t_f \geq t_2$, then the stockout is the sum of the lengths of the flat lines in the above expression with $i = n_B$ and the amount by which $t_f$ exceeds $t_2$, which is

$$\begin{aligned} S_{out} =& \frac{1}{t_f}\left[\sum_{i=1}^{n_B}\Big(\frac{Q}{w} - \frac{I_{r_B}}{v+w}\Big) + (i+1)\frac{Qv}{w(v+w)} + t_f - t_2\right] \\ =& \frac{1}{t_f}\left[n_B\Big(\frac{Q}{w} - \frac{I_{r_B}}{v+w}\Big) + (n_B)(n_B-1)\frac{Qv}{2w(v+w)} + t_f - t_2\right]. \end{aligned} \tag{24}$$

If $t_1 \leq t_f < t_2$, then stockout takes the form similar to the previous expression, except $n_B$ is replaced by the number of complete Region B cycles that exist prior to the finishing time $t_f$, denoted by $m$. Also, the last two terms $t_f - t_2$ are replaced by length of the remaining flat line that may exist between the completion of $m$ cycles and $t_f$. If $m$ is zero — meaning $t_f$ is located between $t_A$ and the end of the first cycle in Region B — then this remaining flat line is the greater of zero or the quantity that remains when $t_A$ and the in-stock duration $\frac{I_{r_B}}{v+w}$ are taken away from $t_f$. Otherwise, it is the greater of zero or the quantity that remains when the total length of $m$ cycles and the in-stock duration $\frac{Q}{v+w}$ is taken away from $t_f$. Using (22) and (24), we write this expression as

$$S_{out} = \begin{cases} \frac{1}{t_f}\max\Big[0, t_f - t_A - \frac{I_{r_B}}{v+w}\Big] & \text{if } m = 0, \\ \frac{1}{t_f}\Big[\Big(\frac{Q}{w} - \frac{I_{r_B}}{v+w}\Big) + \max\Big[0, t_f - t_A - \frac{Q}{w} - \frac{Q}{v+w}\Big] & \text{if } m = 1, \\ \frac{1}{t_f}\Big[m\Big(\frac{Q}{w} - \frac{I_{r_B}}{v+w}\Big) + (m)(m-1)\frac{Qv}{2w(v+w)} & \\ + \max\Big[0, t_f - t_A - (m-1)\left(\frac{2Q}{w} - \frac{I_{r_B}}{v+w}\right) - (m-1)(m-2)\frac{vQ}{2w(v+w)} - \frac{Q}{v+w}\Big]\Big] & \text{if } m \geq 2 \end{cases} \tag{25}$$

What remains is the expression for $m$. First, we look for the number of intervals (a real number), denoted by $m'$, that lie between $t_A$ and $t_f$. Using (22), we can write the quadratic equation

$$(m'-1)\left(\frac{2Q}{w} - \frac{I_{r_B}}{v+w}\right) + (m'-1)(m'-2)\frac{vQ}{2w(v+w)} = t_f - t_A. \tag{26}$$

Solving this quadratic equation for $m'$, and taking its integer portion (since we are looking for integer number of complete cycles), we obtain for $m$

$$m = I\Big(m'\Big) = I\left(\frac{-b + \sqrt{b^2 - 4ac}}{2a}\right) \tag{27}$$

where

$$\begin{aligned} a &= \frac{Qv}{w(v+w)} \\ b &= 4\frac{Q}{w} - 2\frac{I_{r_B}}{v+w} - 3\frac{Qv}{w(v+w)} \\ c &= 2\Big(\frac{Qv}{w(v+w)} - \frac{Q}{w} - \frac{I_{r_B}}{v+w} - (t_f - t_A)\Big). \end{aligned}$$

We complete the calculation of stockout by noting that when $t_f < t_A$, $S_{out} = 0$.

## 5.2 Approximate Calculations

The exact calculations shown in the previous section can be simplified significantly by making a set of appropriate approximations. The first approximation begins with the number of cycles in Region A, $n_A$. Whereas in the exact calculation $n_A$ had to be an integer, we now relax this constraint and use the approximation

$$I\left(\frac{R - wL}{v\frac{Q}{w}}\right) \approx \frac{R - wL}{v\frac{Q}{w}} \tag{28}$$

in Equation (14). This approximation works well when the stock loss rate $v$ is small and the cycle length $\frac{Q}{w}$ is much smaller than $t_f$. If this is true, the argument in $I(\cdot)$ in the left side of equation will be large and taking only the integer portion of the argument will be close to the argument itself. $n_A$ and $t_A$ then becomes

$$n_A \approx \frac{R-wL}{v\frac{Q}{w}} \tag{29}$$

$$t_A \approx \frac{R-wL}{v}. \tag{30}$$

In determining $t_1$, we make an additional assumption that the beginning actual inventory in the first cycle of Region B, $I_{r_B}$, is equal to $Q$. Again, this works well for small $v$ because the amount by which the actual inventory decreases more than the inventory record in each cycle will be small. With this approximation, the expression for $t_1$ becomes

$$t_1 \approx \frac{R - wL}{v} + \frac{Q}{v + w}. \tag{31}$$

Applying this approximation also to $n_B$, and once again relaxing the constraint that $n_B$ has to be an integer, we obtain

$$n_B \approx \frac{wL(v + w)}{vQ}. \tag{32}$$

$t_2$ now simplifies to

$$\begin{aligned} t_2 \approx & t_A + \frac{Q}{w} + (n_B - 1)\left(\frac{2Q}{w} - \frac{I_{r_B}}{v + w}\right) + (n_B - 1)(n_B - 2)\frac{vQ}{2w(v + w)} + \frac{Q}{v + w} \\ = & t_A + L\left(\frac{1}{2} + \frac{w}{v}\right) + \frac{wL^2(v + w)}{2vQ} + \frac{Q}{v + w}. \end{aligned} \tag{33}$$

We proceed further by carrying out these approximations to calculation of stockout and arrive at

$$S_{out} \approx \begin{cases} 0 & \text{if } t_f < t_1, \\ \frac{1}{t_f}\left[m(m+1)\frac{vQ}{2w(v+w)}\right] & \text{if } t_1 \le t_f < t_2, \\ \frac{1}{t_f}\left[\frac{L}{2}\left(\frac{wL(v+w)}{vQ} + 1\right) + t_f - t_2\right] & \text{if } t_f \ge t_2. \end{cases} \tag{34}$$

where $m$ also changes by the approximations to

$$m \approx \frac{2w+v}{2v} + \sqrt{\Big(\frac{2w+v}{2v}\Big)^2 + 2\frac{w(v+w)}{Qv}(t_f - t_1)}. \tag{35}$$

Note that in approximating $S_{out}$ for the case when $t_1 \leq t_f < t_2$, we also assumed any flat line that may exist beyond $m$ cycles is negligible.

# References

Arens, A. A. and J. K. Loebbecke (1981). *Applications of Statistical Sampling to Auditing*. Englewood Cliffs, NJ: Prentice-Hall.

Bernard, P. A. (1985). Cycle counting: The missing link. *Production and Inventory Management 25*(4), 27–40.

Brown, K. L., R. A. Inman, and J. A. Calloway (2001). Measuring t ande effects of inventory inaccuracy in mrp inventory and delivery performance. *Production Planning and Control 12*(1), 46–57.

Buck, J. R. and R. P. Sadowski (1983). Optimum stratified sampling. *IIE Transactions 15*, 119–126.

Buker, D. W. (1984). Inventory accuracy. *Inventory Management Reprints APICS*, 221–223.

Cantwell, J. (1985). The how and why of cycle counting; the abc method. *Production and Inventory Management Journal Second Quarter*.

Chopra, V. (1986). How to set up an effective inventory accuracy program. *29th Annual APICS International Conference Proceedings*, 238.

Cochran, W. (1977). *Sampling Techniques* (3 ed.). New York: John Wiley and Sons.

Dalenius, T. and J. Hodges (1959). Minimum variance stratification. *American Statistical Journal*, 198–203.

ECREurope (2001). *Shrinkage: Introducing a Collaborative Approach to Reducing Stock Loss in the Supply Chain*. Brussels, Belgium: ECR Europe.

Edelman, M. P. (1984). Cycle counting for distribution inventories. *Readings in Production and Inventory Control and Planning APICS*, 29.

Ernst, R., J.-L. Guerrero, and A. Roshwalb (1993). A quality control approach for monitoring inventory stock levels. *J. Opl Res. Soc. 44*(11), 1115–1127.

French, R. L. (1980). Accurate wip inventory — a critical mrp system requirement. *Production and Inventory Management 21*, 17–22.

Graff, R. T. (1987). The limitations of cycle counting. *Production and Inventory Management Journal Fourth Quarter*, 39–43.

Hart, M. K. (1998). Improving inventory accuracy using control charts. *Production and Inventory Management Third Quarter*(11), 44–48.

Hollinger, R. C. (2003). *2002 National Retail Security Survey Final Report.* Gainesville, Florida: University of Florida.

Iglehart, D. L. and R. C. Morey (1972). Inventory systems with imperfect asset information. *Management Science 18*(8), 388–394.

Kang, Y. (2003). Information inaccuracy in information systems. *Ph.D. Dissertation. Massachusetts Institute of Technology*.

Krajewski, L. J., B. E. King, L. P. Ritzman, and D. S. Wong (1987). Kanban, mrp, and shaping the manufacturing environment. *Management Science 33*(1), 39–57.

Martin, J. W. and P. S. Goodrich (1987). Minimizing sample size for given accuracy in cycle counting. *Production and Inventory Management Journal Fourth Quarter*, 24–27.

Morey, R. C. (1985). Estimating service level impacts from changes in cycle count, buffer stock, or corrective action. *Journal of Operations Management 5*(4), 411–418.

Morey, R. C. and D. A. Dittman (1986). Optimal timing of account audits in internal control. *Management Science 32*(3), 272–282.

Neely, P. (1987). Simple mathematics applied to inventory accuracy. *Production and Inventory Management 28*(3), 64–68.

Raman, A., N. DeHoratius, and Z. Ton (2001). Execution: The missing link in retail operations. *California Management Review 43*(3), 136–152.

Reddock, J. G. (1984). How to count a cycle. *Readings in Production and Inventory Control and Planning APICS*, 28.

Young, J. B. (1986). The limits of cycle counting. *29th Annual APICS International Conference Proceedings APICS*, 241.