Qualitative Observations on Production Line Behavior —

Intuitive and Counter-Intuitive Explanations

Stanley B. Gershwin

Department of Mechanical Engineering
Massachusetts Institute of Technology

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Outline

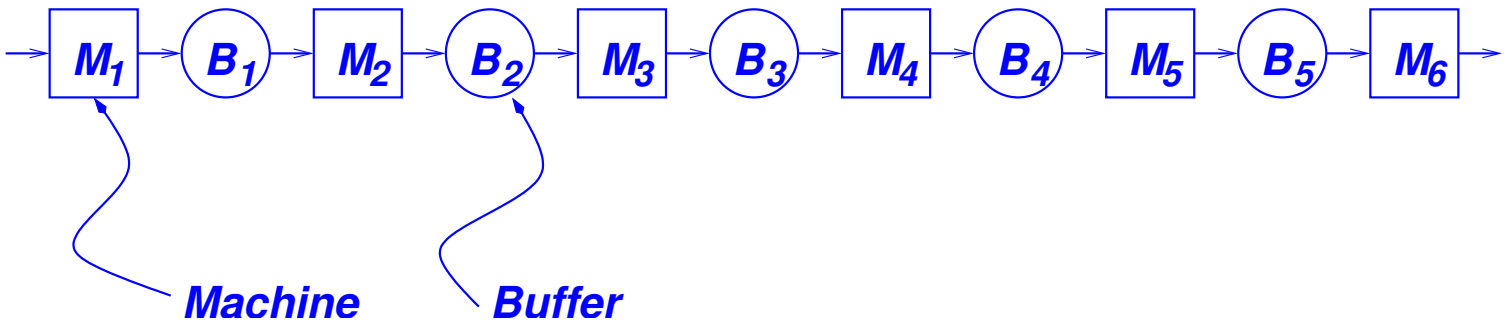
- Introduction
- General Observations
- Infinite Buffers
- Two-Machine Lines
- Long Lines — Behavior of $P(\mathbf{N})$ and $\bar{n}_i(\mathbf{N})$
- Long Lines — Optimization
- Conclusions

Introduction

Objectives:

- To describe interesting behaviors, properties, and phenomena of flow line models.
- To propose explanations.
- To provide intuition.

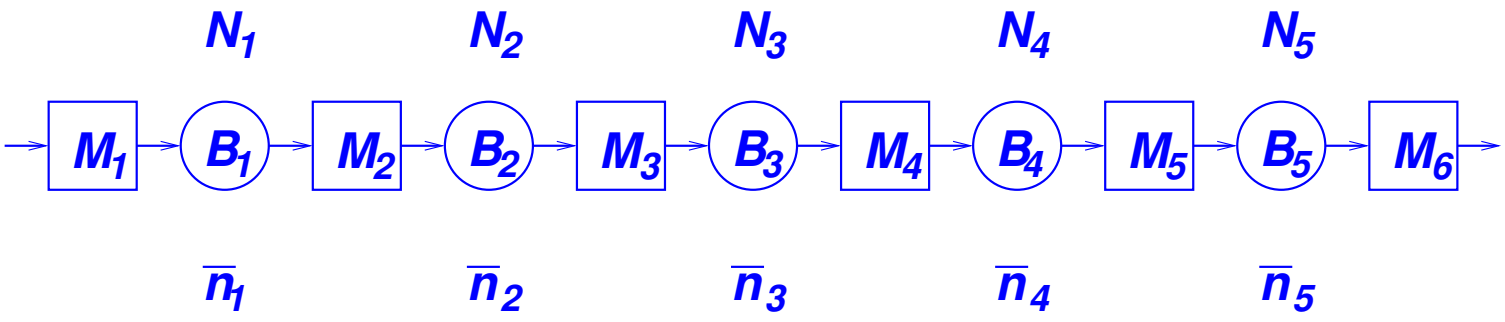
Kinds of Systems



Focus:

- Flow lines
- Randomness due to unreliable machines
- Finite buffers
- Propagation of failures through starvation and blockage

Kinds of Systems



Performance measures:

- Production rate P, a benefit
  - *Production rate increases with buffer sizes.*
- Average in-process inventory $\bar{n}_i$, a cost
  - *In-process inventory varies with buffer sizes.*
- Other costs: machines and buffer space

Methodology

Models:

- Lines are modeled as Markov processes.

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Models:

- Lines are modeled as Markov processes.
- Material and time can be discrete or continuous.
- Machines fail and are repaired at random times.
- Models used here have geometric or exponential distributions of up- and down-time.
- Notation:
  - p=1/MTTF is failure probability or failure rate.
  - r=1/MTTR is repair probability or repair rate.

Methodology

Performance evaluation:

- Two-machine lines are evaluated analytically, exactly. We obtain the steady-state probability distribution and use it to determine the average production rate and buffer level.

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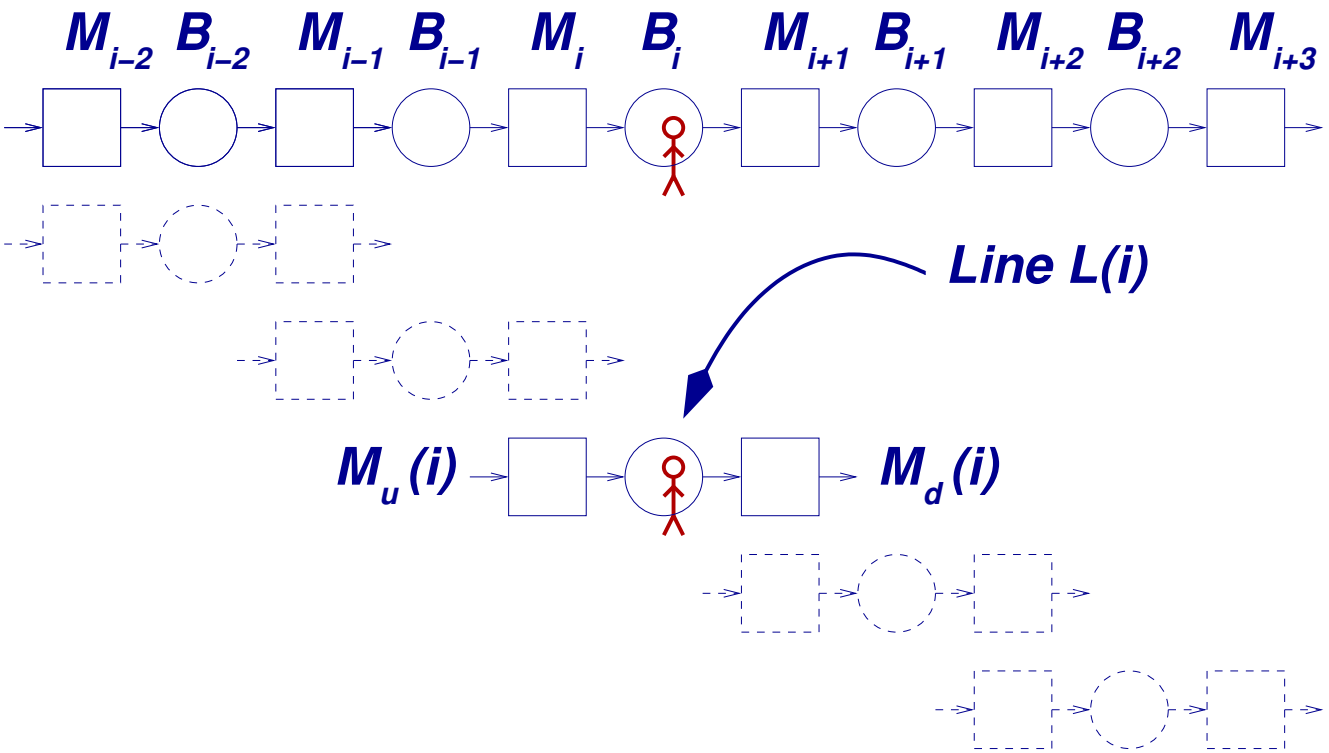
Performance evaluation:

- Two-machine lines are evaluated analytically, exactly. We obtain the steady-state probability distribution and use it to determine the average production rate and buffer level.
- *Short lines can be evaluated numerically, exactly.*
- Long lines are evaluated analytically, approximately.

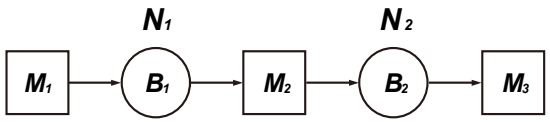
Methodology

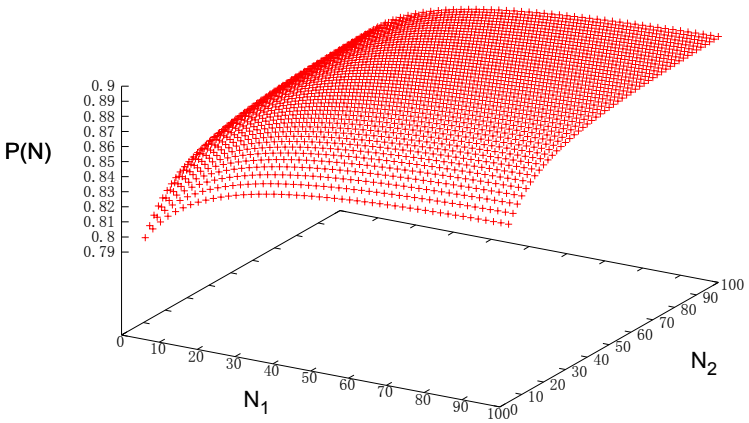


- Long lines are approximately evaluated by *decomposition* .

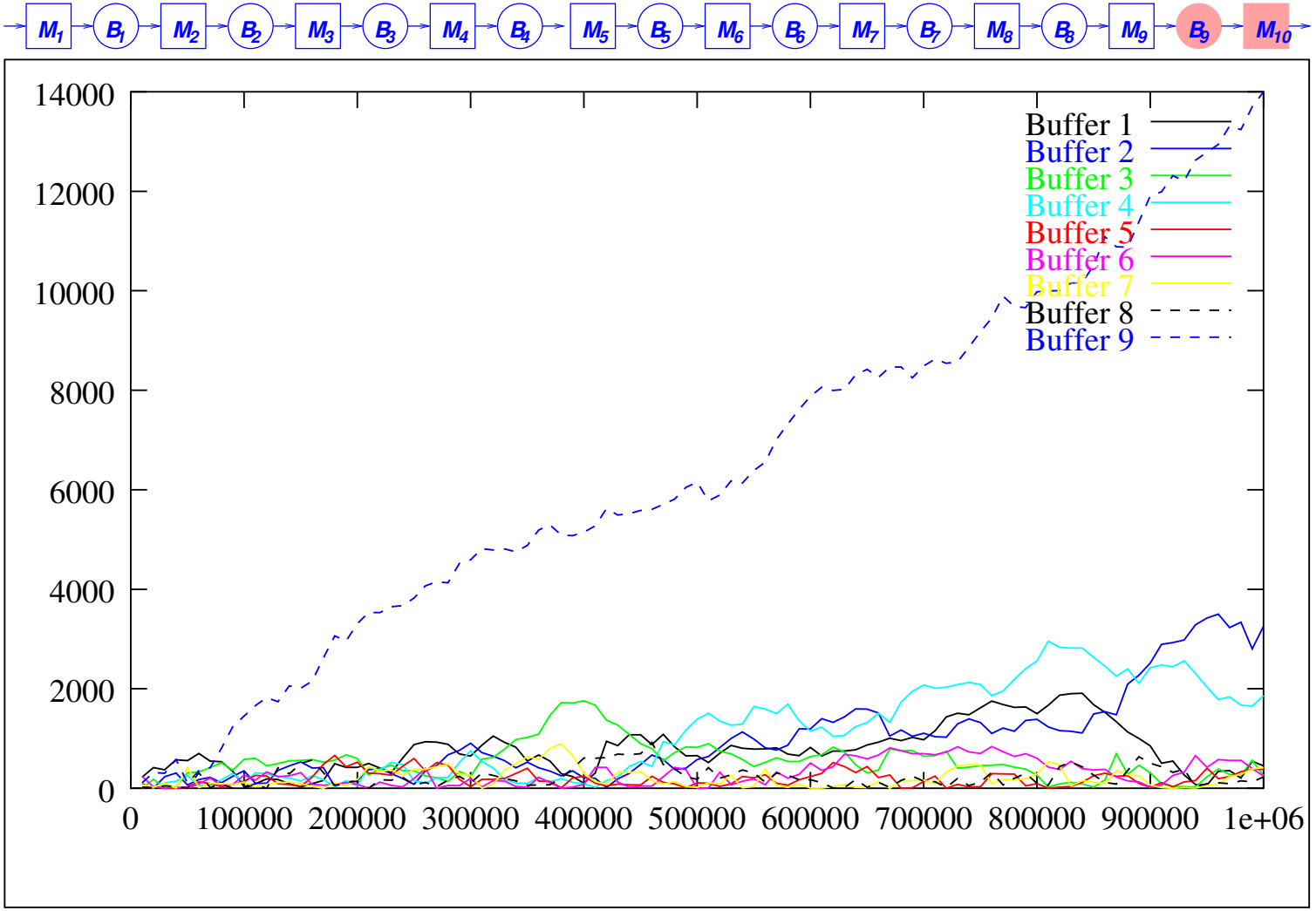
General Observations

- In discrete models, N_i can be treated as continuous because $P(\mathbf{N})$ and $\bar{\mathbf{n}}(\mathbf{N})$ change only a little when N_i changes by 1.
- Monotonicity and concavity have been proven for some 2-machine line models.
- $P(\mathbf{N})$ appears to be monotonic and concave for longer lines.





Infinite Buffers

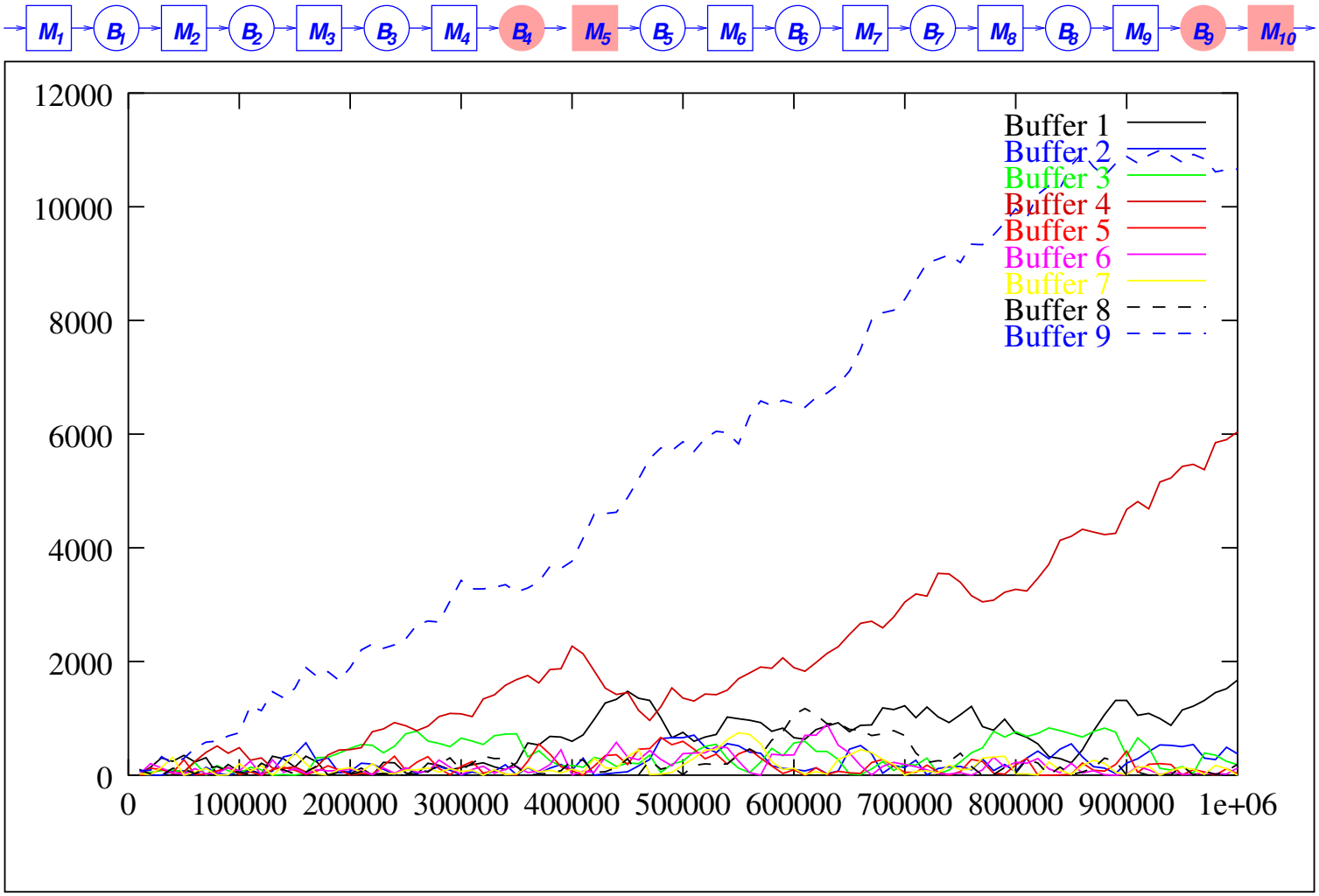


Single bottleneck

Infinite Buffers

- The system is not in steady state.
- An infinite amount of inventory accumulates in the buffer upstream of the bottleneck.
- A finite amount of inventory appears downstream of the bottleneck.

Infinite Buffers

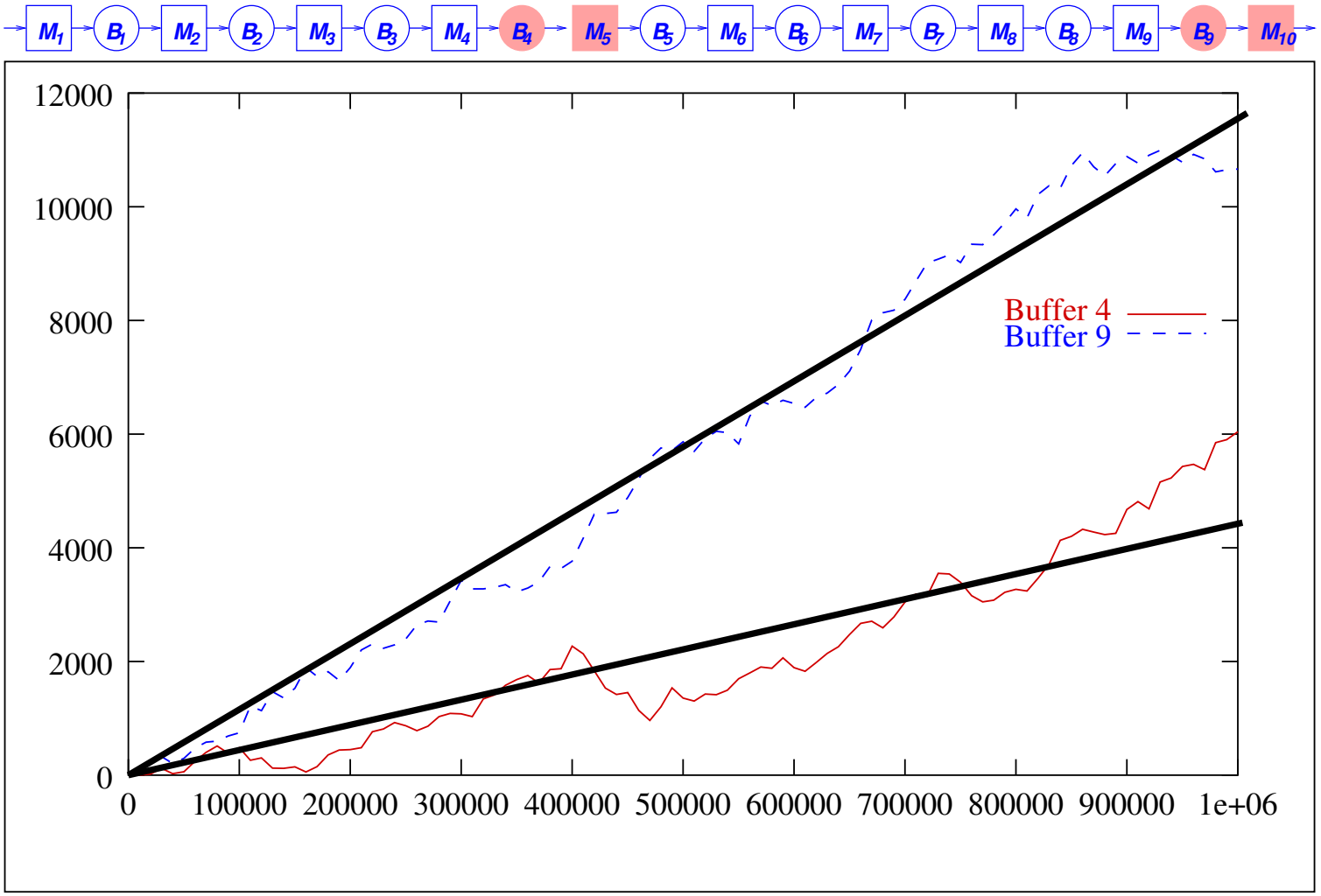


Two bottlenecks

Infinite Buffers

- The *second bottleneck* is the slowest machine upstream of the bottleneck. An infinite amount of inventory accumulates just upstream of it.
- A finite amount of inventory appears between the second bottleneck and the machine upstream of the first bottleneck.
- Et cetera.

Infinite Buffers



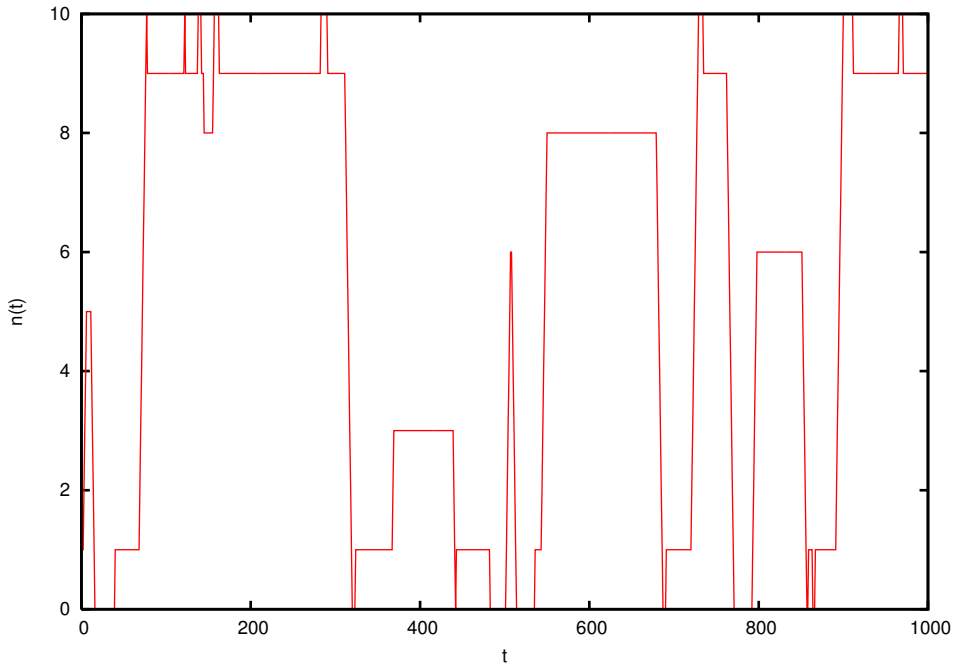
Two bottlenecks

Infinite Buffers

- Each slope is the *difference* between the production rate of the immediate downstream bottleneck and the next upstream bottleneck.
- The largest accumulation of inventory is located at the largest difference, not (necessarily) the first bottleneck.

Two-Machine Lines

Simulations

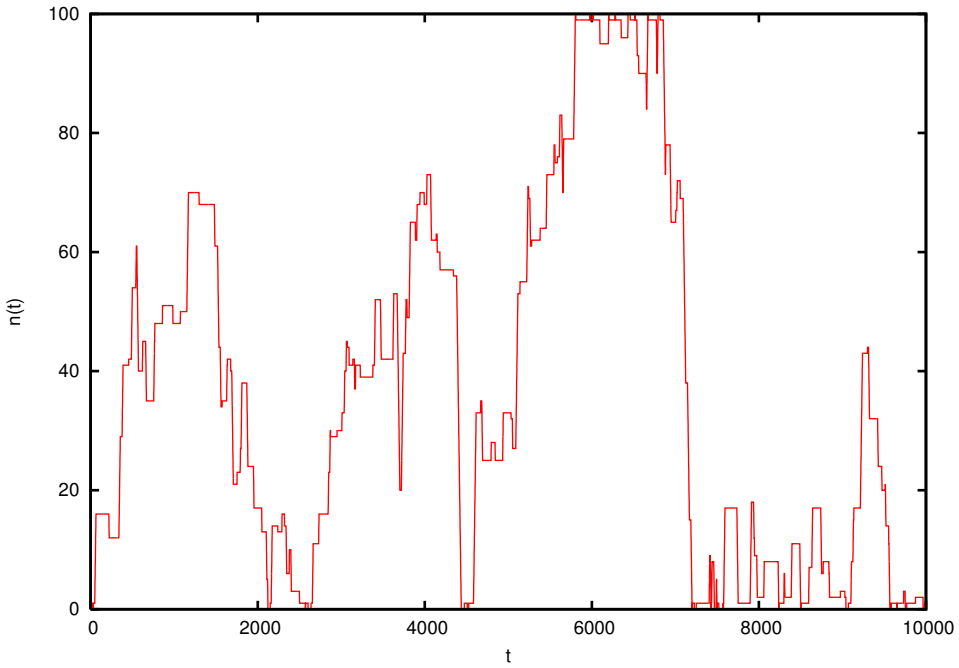


$$r_1 = .1, p_1 = .01, r_2 = .1, p_2 = .01, N = 10$$

Buffer size is the same as MTTR. Blockage and starvation are frequent.

Two-Machine Lines

Simulations

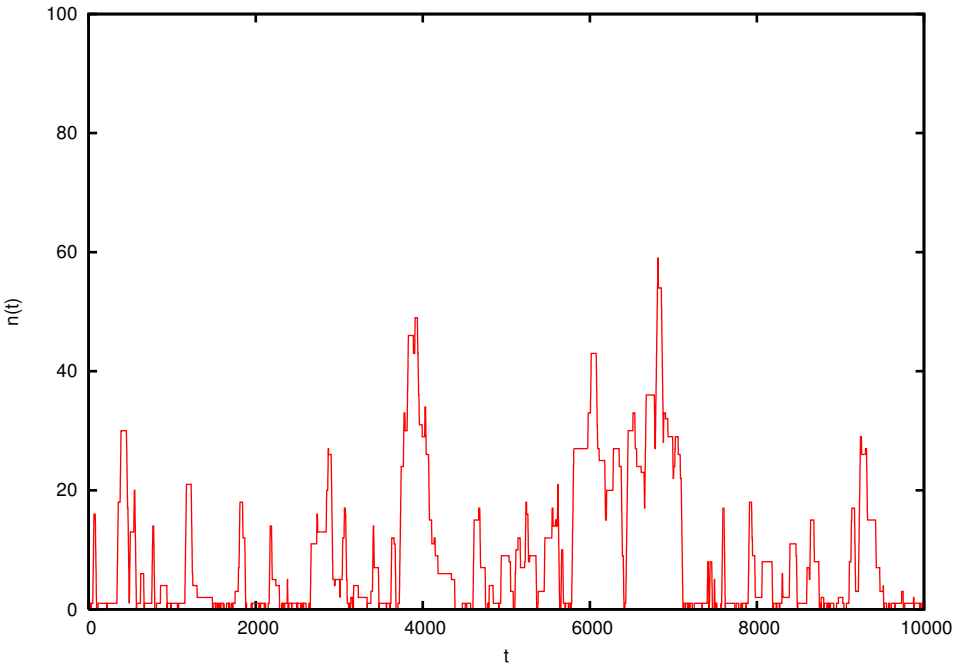


$$r_1 = .1, p_1 = .01, r_2 = .1, p_2 = .01, N = 100$$

Buffer size is 10 $\times$ MTTR. Blockage and starvation are infrequent.

Two-Machine Lines

Simulations

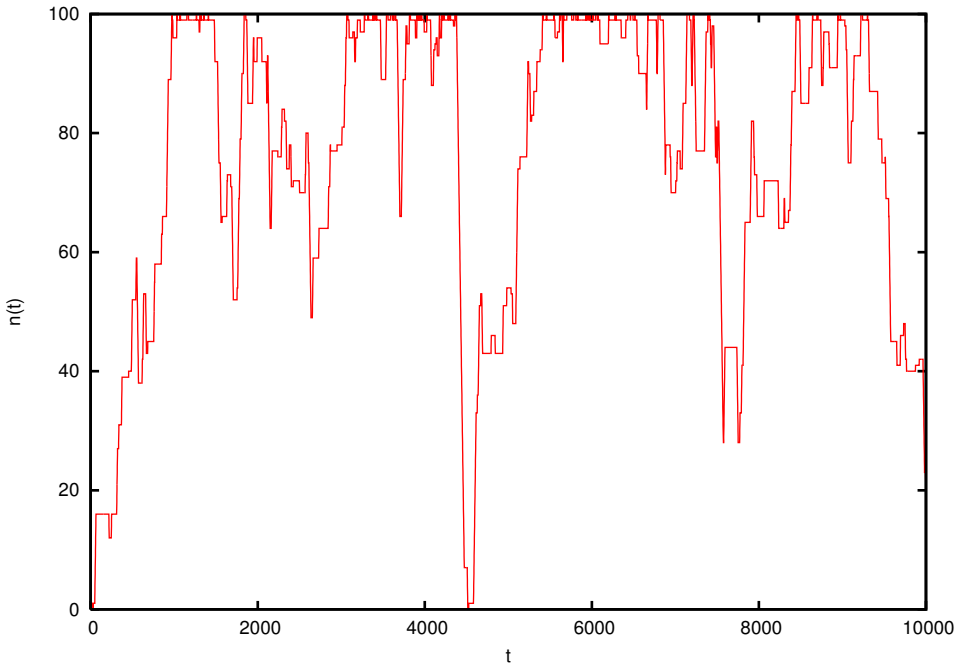


$$r_i = .1, i = 1, 2, p_1 = .02, p_2 = .01, N = 100$$

First machine is a bottleneck. Blockage is infrequent. Starvation is frequent.

Two-Machine Lines

Simulations



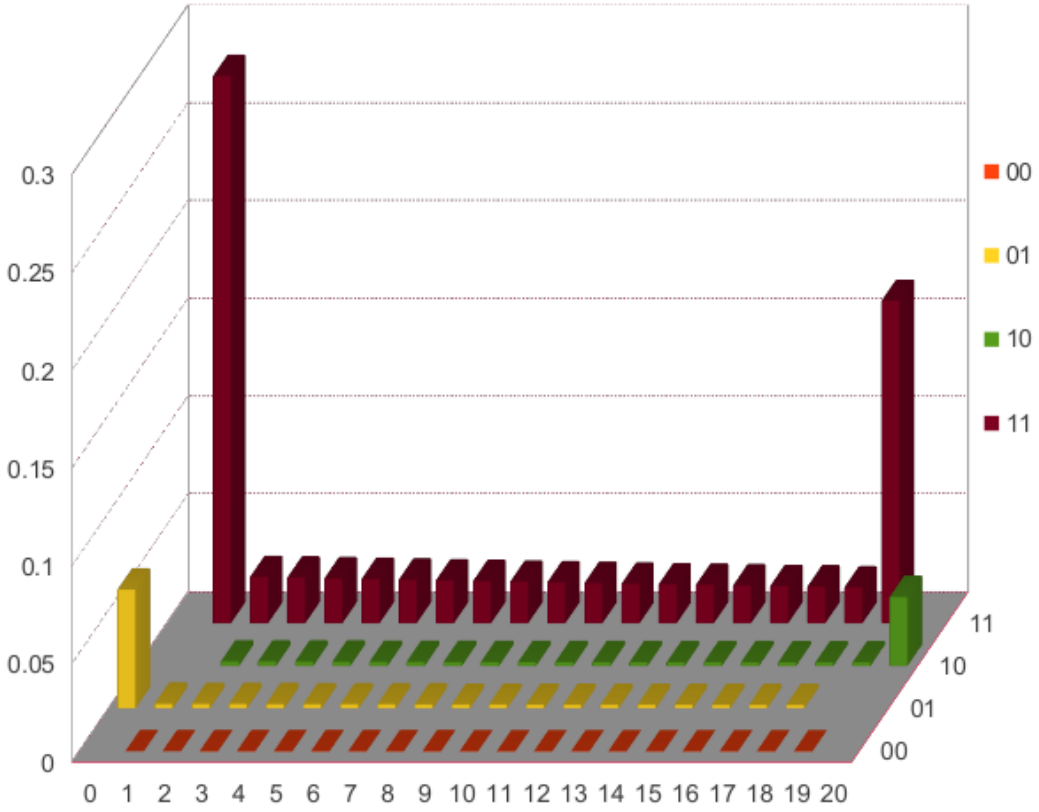
$$r_i = .1, i = 1, 2, p_1 = .01, p_2 = .02, N = 100$$

Second machine is a bottleneck. Starvation is infrequent. Blockage is frequent.

Two-Machine Lines

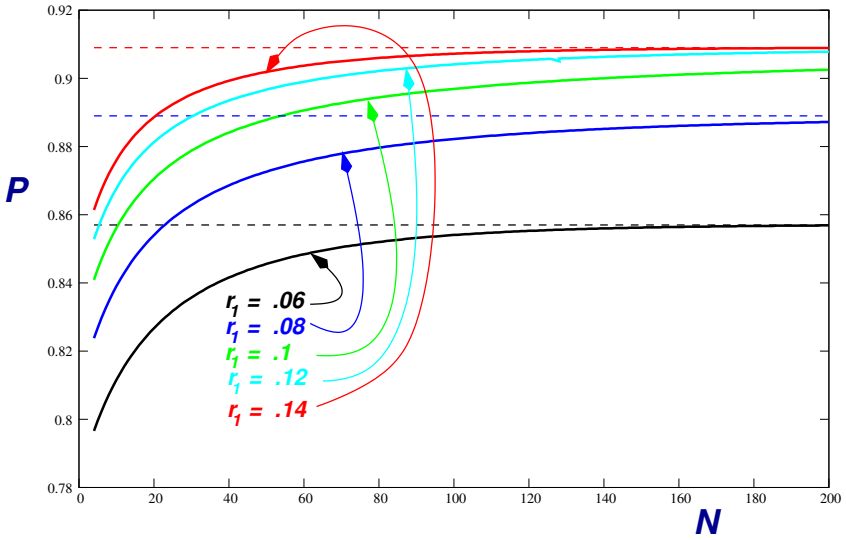
Analytic Numerical Results

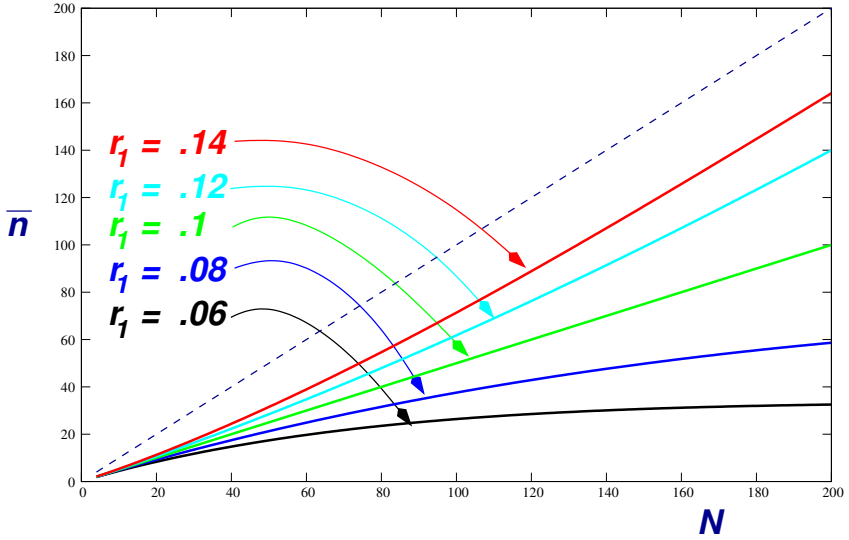
Probabilities of the buffer being empty or full are much larger than probabilities of intermediate buffer levels.



Two-Machine Lines

Analytic Numerical Results





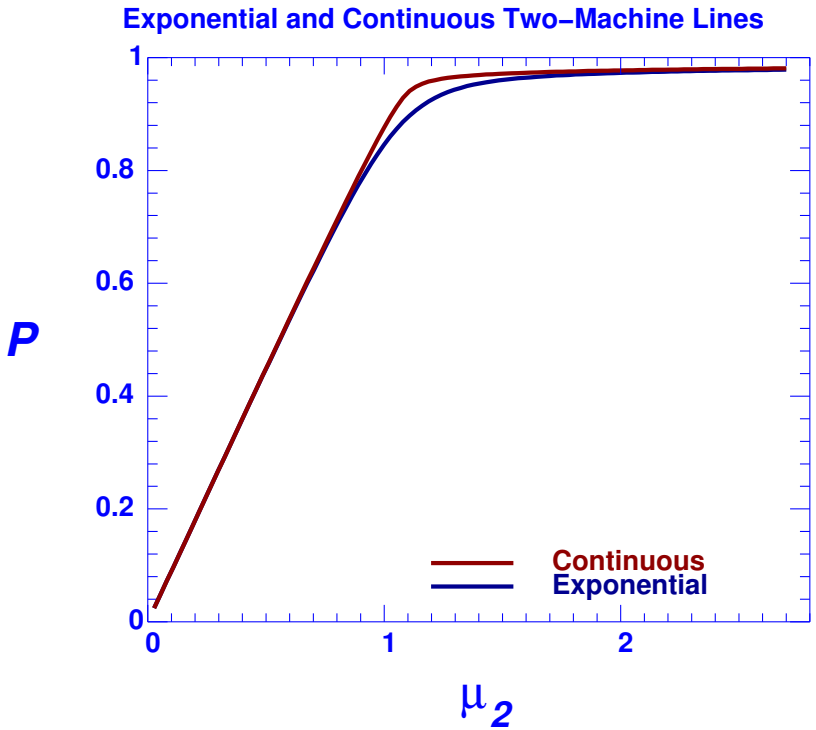
As $N \to \infty$,

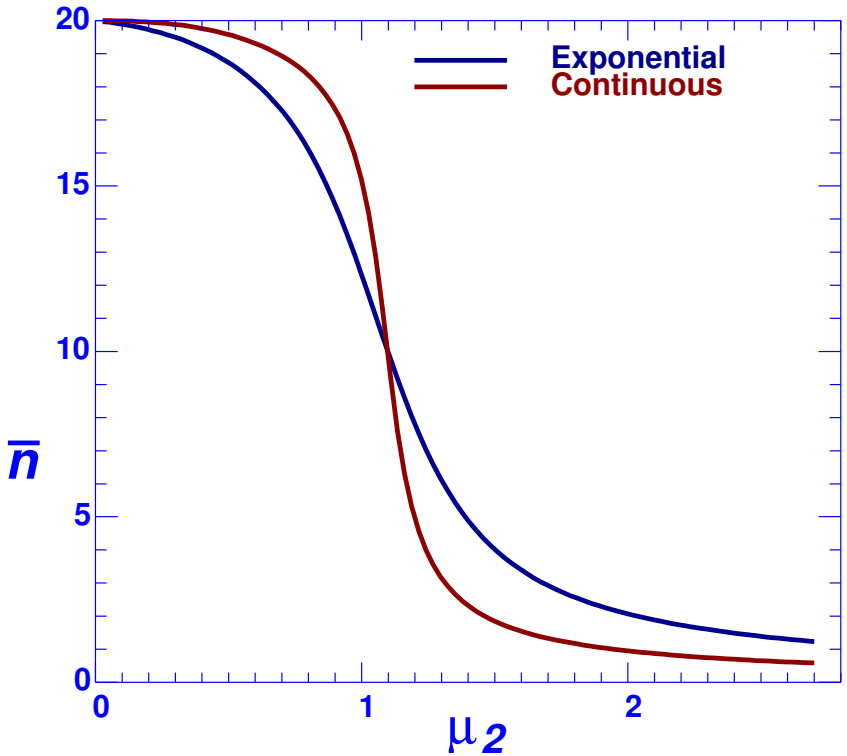
- Production rate approaches an upper limit.
- If the first machine is a bottleneck, average inventory $\bar{n}$ approaches an upper limit.
- If the second machine is a bottleneck, $N - \bar{n}$ approaches an upper limit.
- If the machines are identical, $\bar{n} = N/2$.

$\bar{n}$ increases as the first machine becomes faster (i.e., more productive).

Two-Machine Lines

Analytic Numerical Results

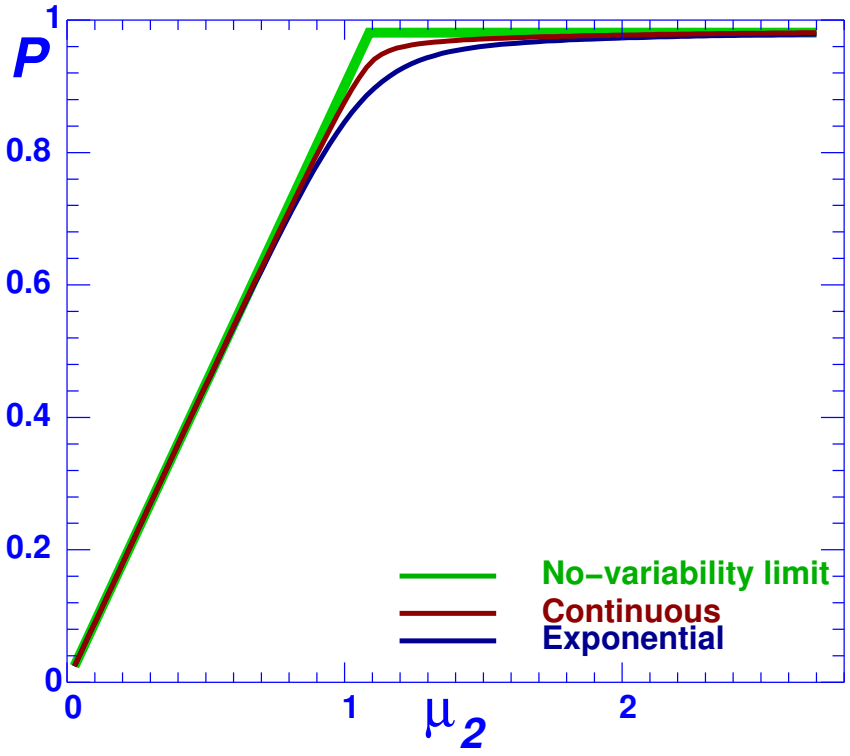


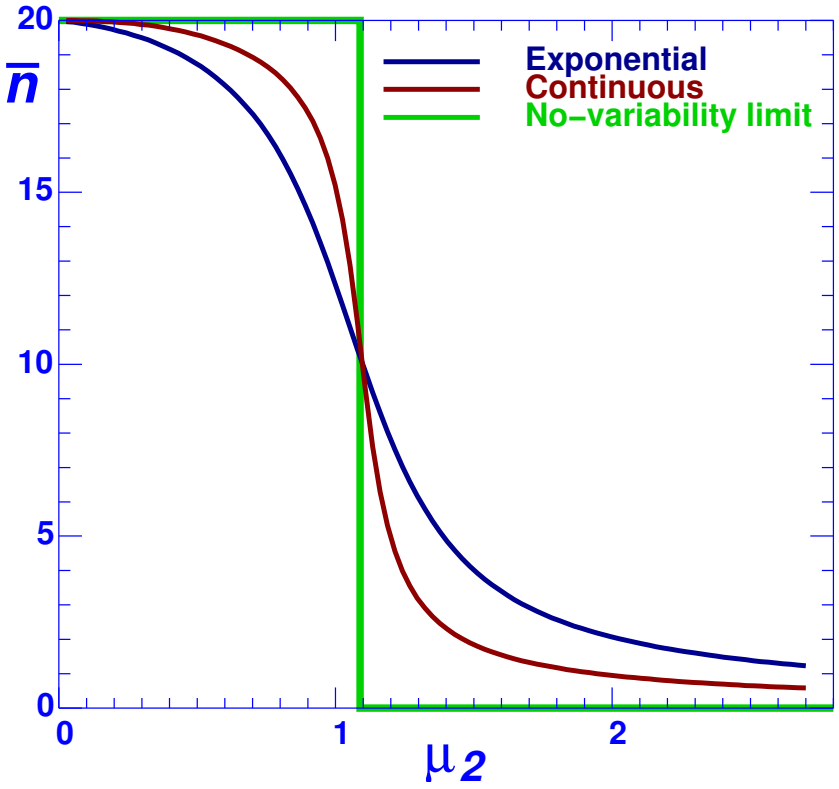


- Two models. Both have exponentially distributed repair and failure times.
- *Exponential:* discrete material, exponentially distributed operating times (mean operation time$=\mu$).
- *Continuous:* continuous material, constant flow through machines when operational (flow rate$=\mu$).
- Parameters of the models are the same.

Two-Machine Lines

Analytic Numerical Results





- Third model, *No-variability limit* .
- Same model as continuous but perfectly reliable and μ adjusted so that isolated production rates of corresponding machines are the same.

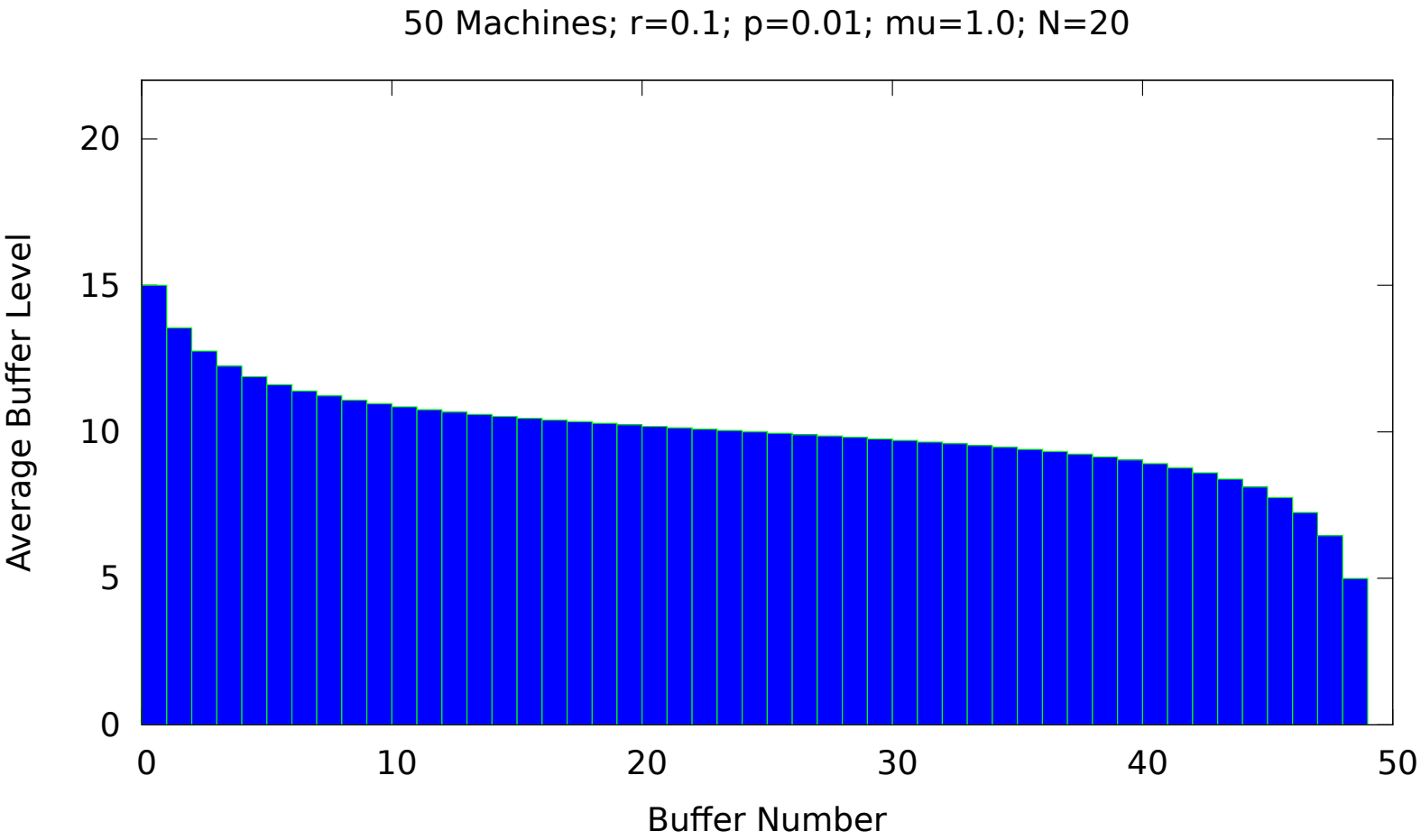
Two-Machine Lines

Analytic Numerical Results

Observations:

- Variability of exponential model $>$ variability of continuous model $>$ variability of no-variability limit.
- Production rate is greatest when variability is least.
- Average inventory is closer to 0 or N when variability is least.

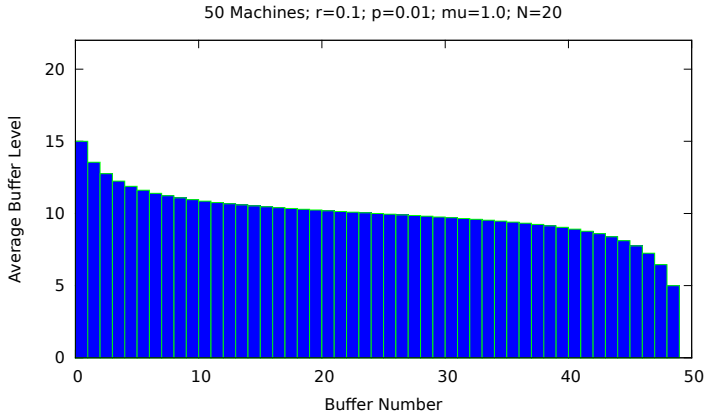
Long Lines — Behavior of $P(\mathbf{N})$ and $\bar{n}_i(\mathbf{N})$



Distribution of average inventory in a line with identical machines and buffers.

P= .79; Total average inventory= 490

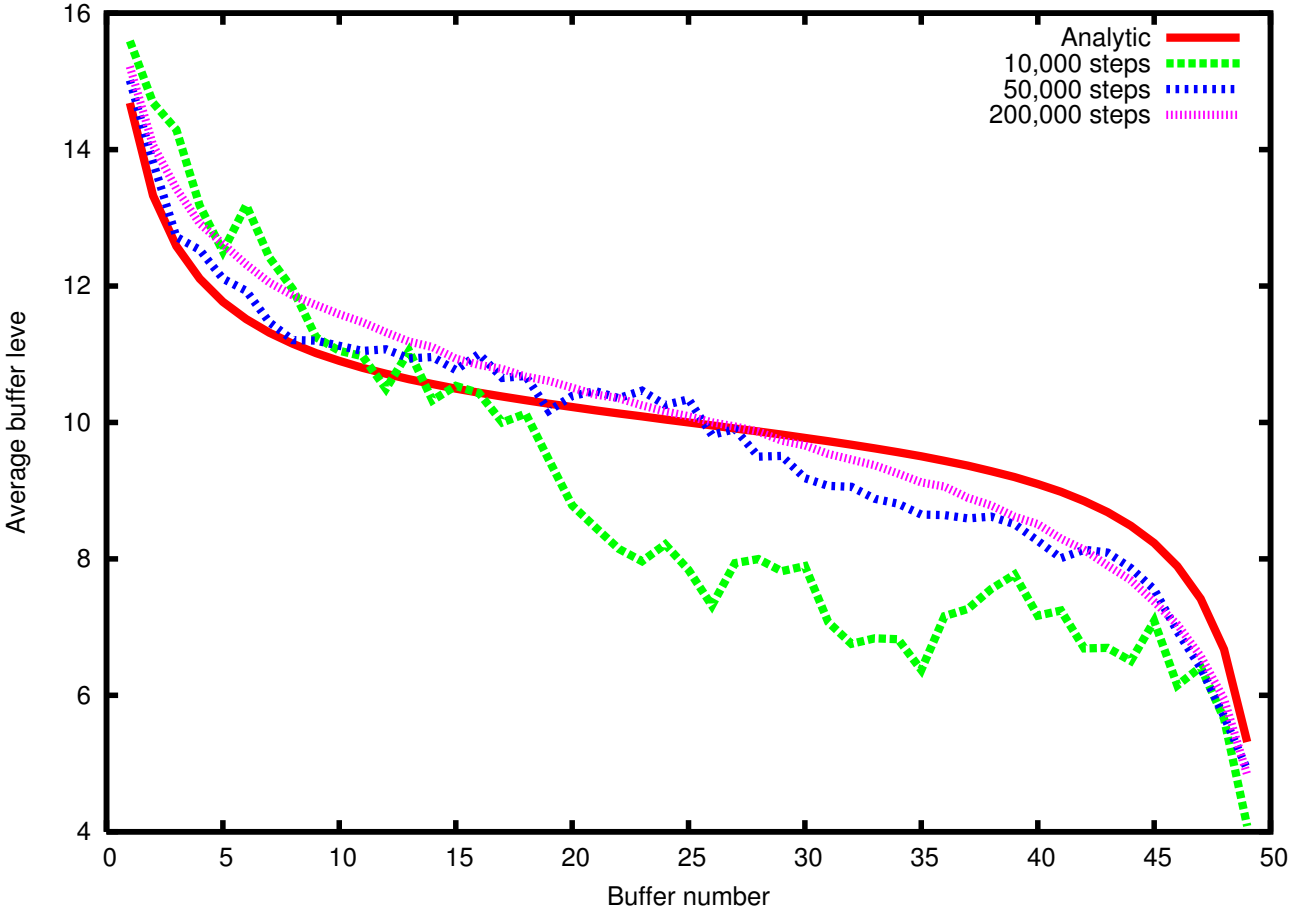
Long Lines — Behavior of $P(\mathbf{N})$ and $\bar{n}_i(\mathbf{N})$



- "Lazy integral"
- Inventory distribution is anti-symmetric: $\bar{n}_i = 20 - \bar{n}_{50-i}$.
- More inventory at the upstream end because
  - each buffer near the beginning of the line has few machines upstream and many downstream.
  - This is like a 2-machine line with a faster upstream machine and a slower downstream machine.
- Less inventory at the downstream end for a similar reason.
- The buffers in the middle are close to half full because they have close to the same number of machines upstream and downstream.

Long Lines — Behavior of $P(\mathbf{N})$ and $\bar{n}_i(\mathbf{N})$

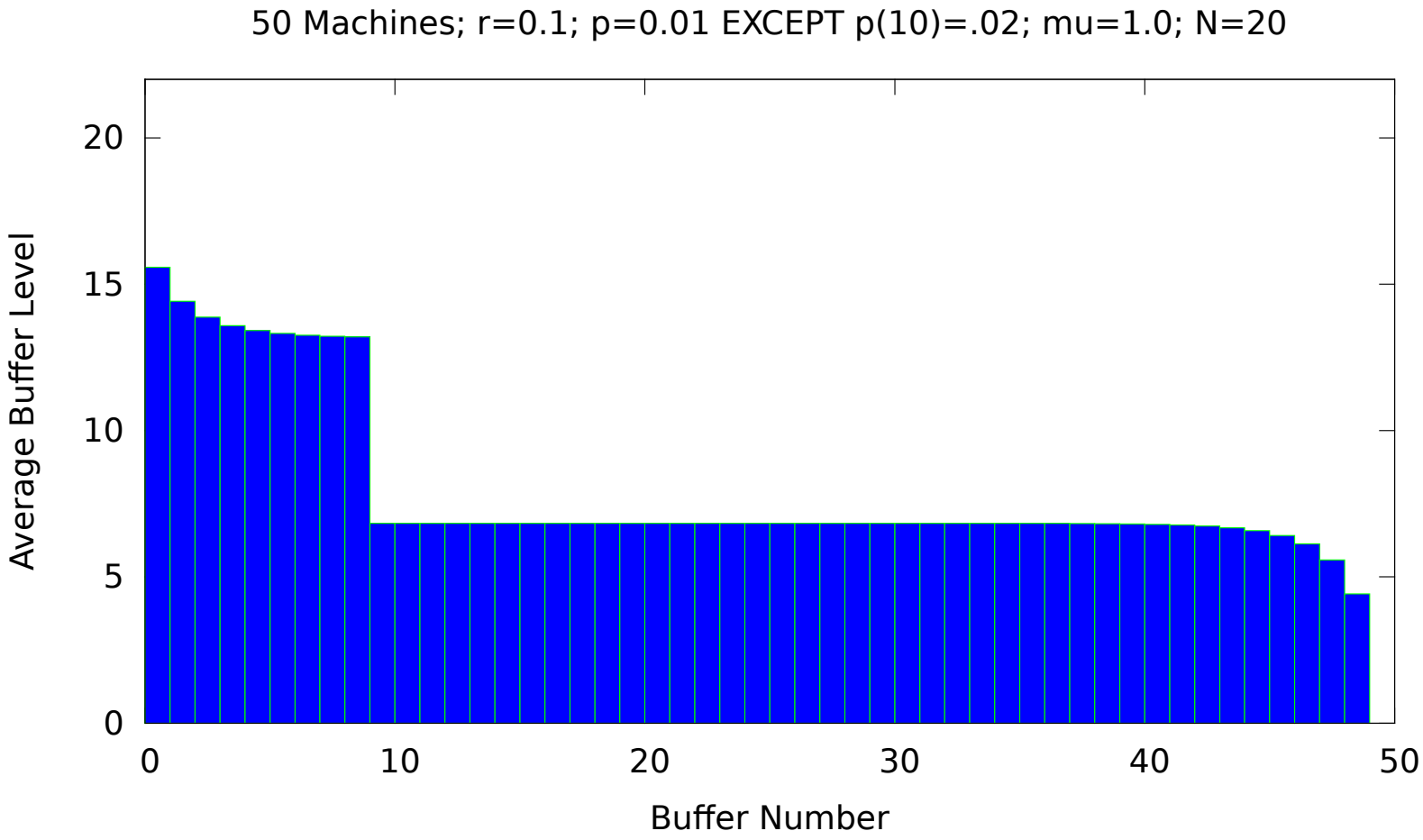
Analytical vs simulation



Time steps	*Decomp*	10,000	50,000	200,000
Production rate	0.786	0.740	0.751	0.750

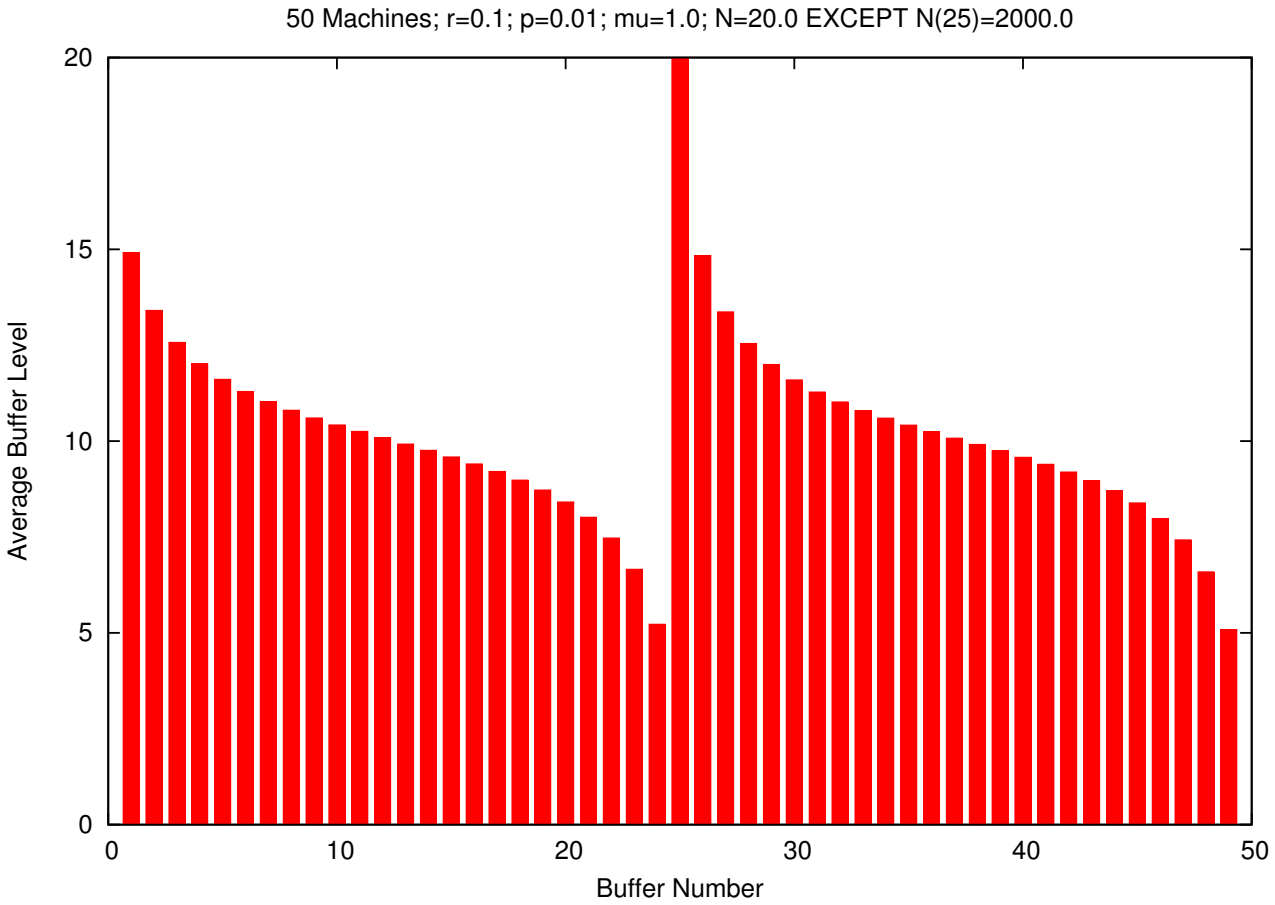
Long Lines — Behavior of $P(\mathbf{N})$ and $\bar{n}_i(\mathbf{N})$

Effects of a Bottleneck



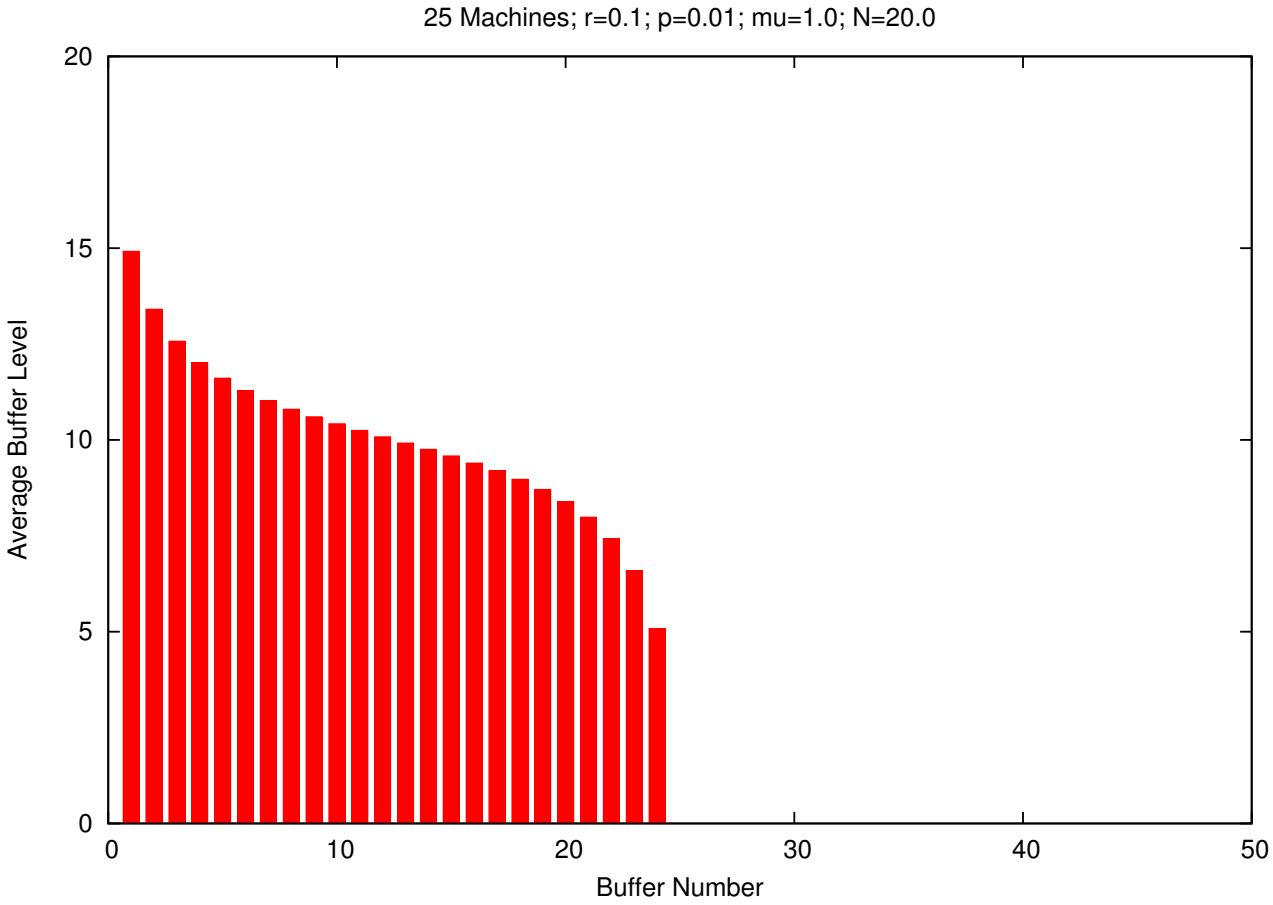
Long Lines — Behavior of $P(\mathbf{N})$ and $\bar{n}_i(\mathbf{N})$

Effects of Very Large Buffers



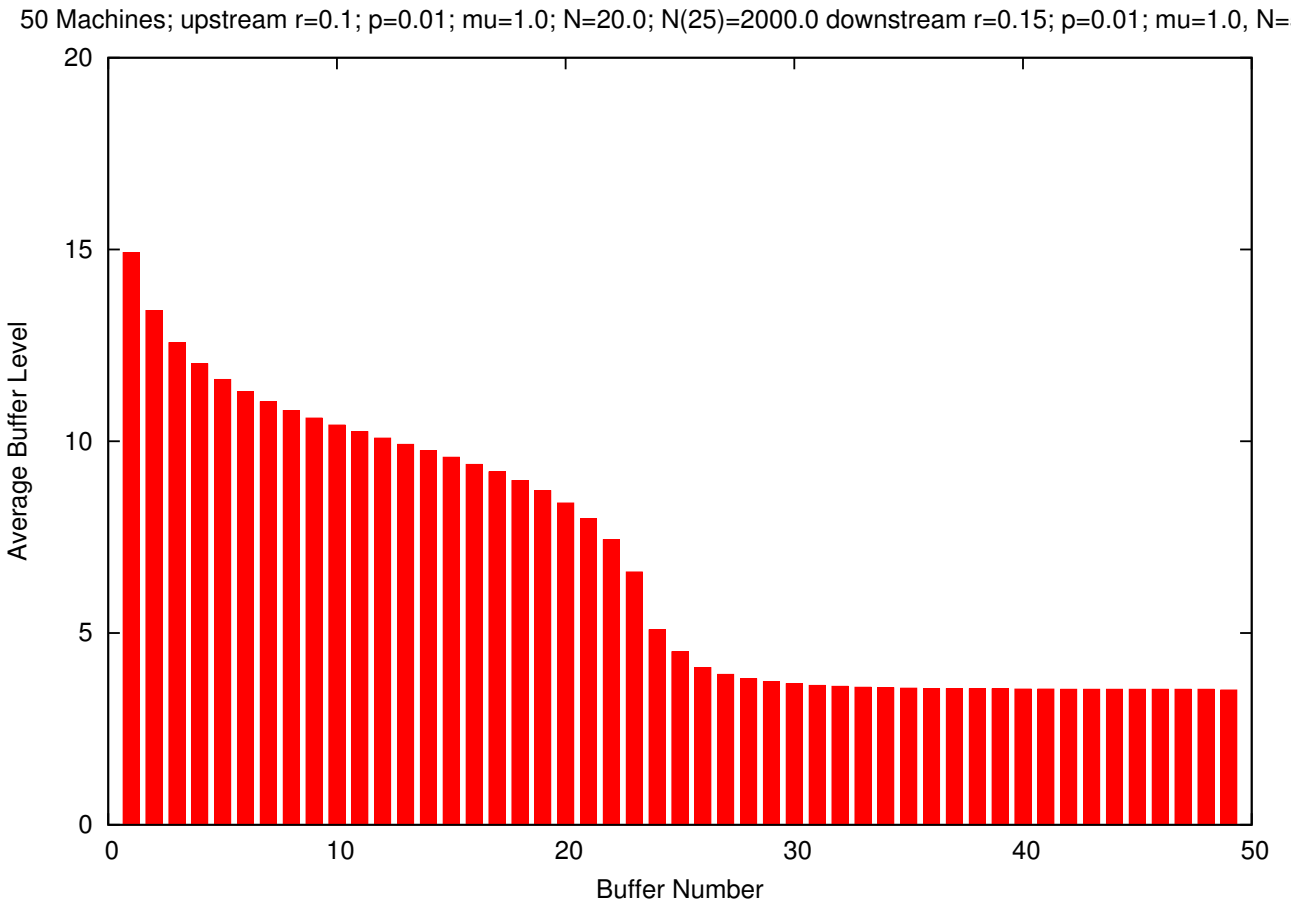
** A very large buffer breaks a line into two smaller lines.

Long Lines — Behavior of $P(\mathbf{N})$ and $\bar{n}_i(\mathbf{N})$



This line is the same as the first half of **.

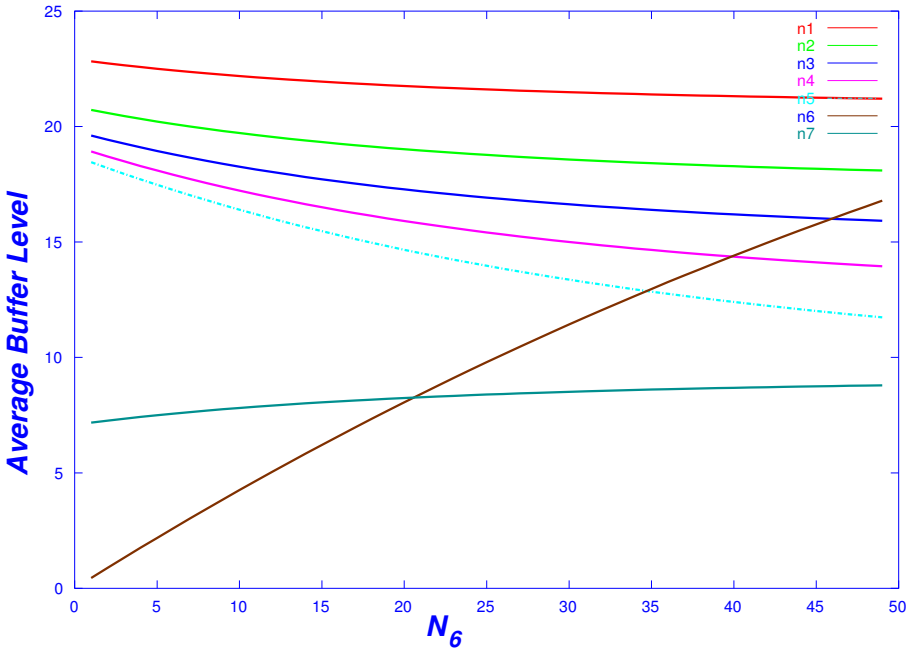
Long Lines — Behavior of $P(\mathbf{N})$ and $\bar{n}_i(\mathbf{N})$



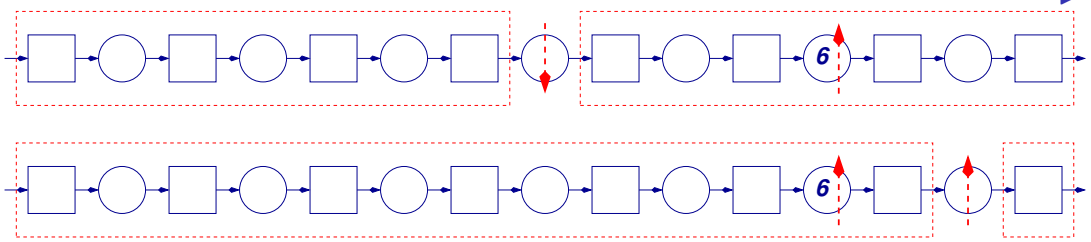
Upstream same as **; downstream faster.

Long Lines — Behavior of $P(\mathbf{N})$ and $\bar{n}_i(\mathbf{N})$

Effect of one buffer's size on the average inventory in others



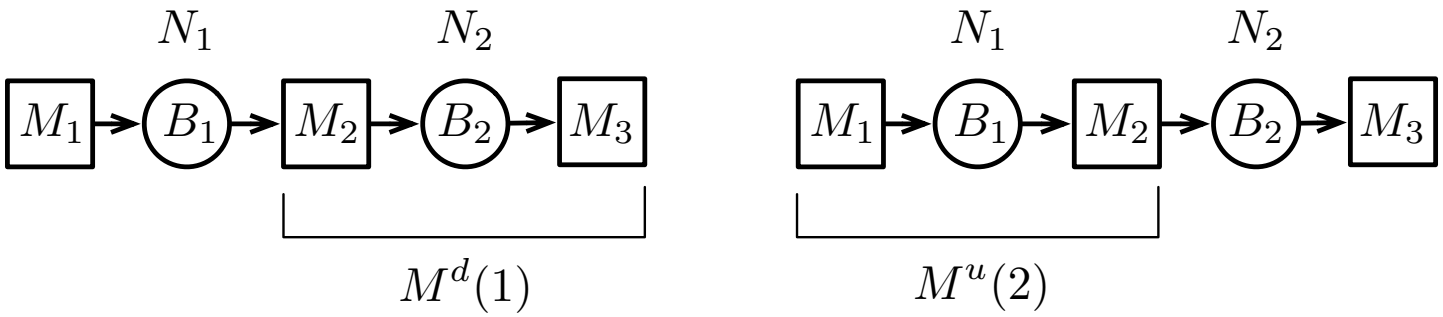
- 8-machine line; we vary the size of Buffer 6, N_6
- *Observation:* as N_6 increases, upstream inventories decrease, downstream inventory increase.
- *Explanation:* Any Buffer i divides the line in two parts.
  - If Buffer i is upstream of Buffer 6, the increase of N_6 makes the *downstream* part of the line faster.
  - If Buffer i is downstream of Buffer 6, the increase of N_6 makes the *upstream* part of the line faster.



Long Lines — Behavior of $P(\mathbf{N})$ and $\bar{n}_i(\mathbf{N})$

Effect of one buffer's size on the average inventory in others

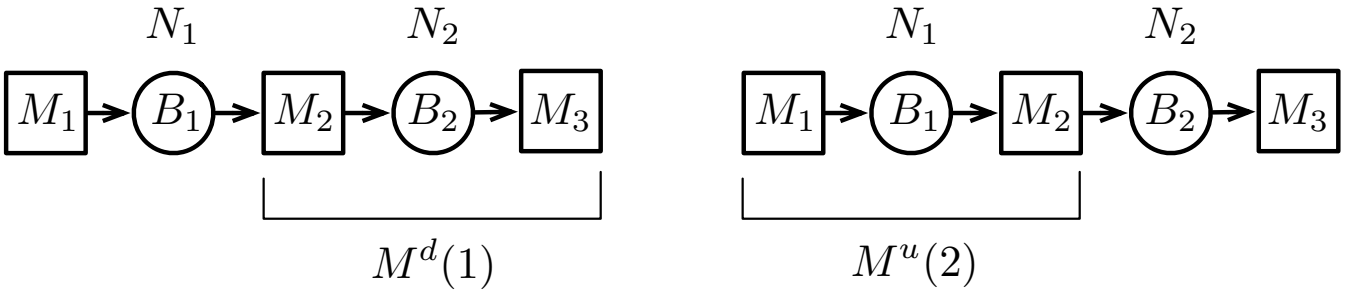
- Consider a three-machine line.
- Break up the line at each buffer into a single machine and a two-machine one-buffer line and compare the production rates of each.



- For the split at B_1, consider the *average inventory* in B_1 as the *size* of B_2 varies from 0 to ∞.
- For the split at B_2, consider the *average inventory* in B_2 as the *size* of B_1 varies from 0 to ∞.

Long Lines — Behavior of $P(\mathbf{N})$ and $\bar{n}_i(\mathbf{N})$

Effect of one buffer's size on the average inventory in others



Definitions:

- P_1 is the isolated production rate of Machine 1.
- $P_{\{2,3\}}(N_2)$ is the production rate of the line formed by M_2, B_2, M_3.
- P_3 is the isolated production rate of Machine 3.
- $P_{\{1,2\}}(N_1)$ is the production rate of the line formed by M_1, B_1, M_2.

- *We consider $\bar{n}_1$ when $N_2 = 0$ or ∞.*
- *We consider $\bar{n}_2$ when $N_2 = 0$ or ∞.*

Long Lines — Behavior of $P(\mathbf{N})$ and $\bar{n}_i(\mathbf{N})$

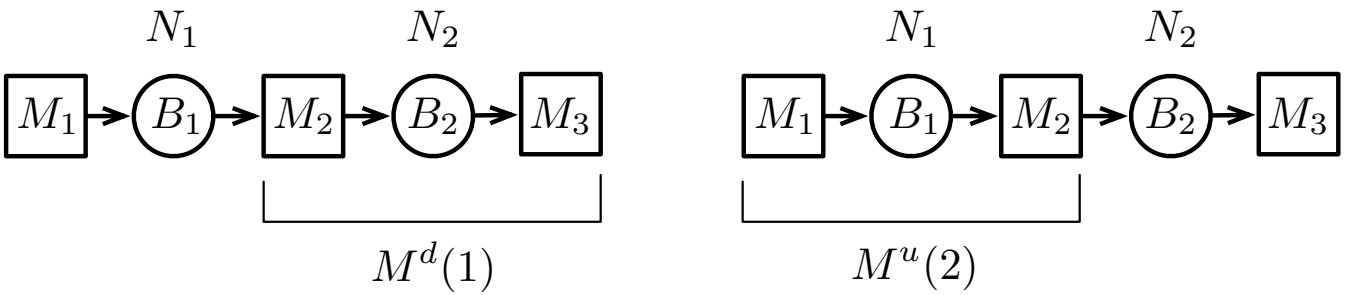
Effect of one buffer's size on the average inventory in others

There are five possible cases. Two cases are interesting.

	Properties
Type 1	$P_3 \geq P_{\{1,2\}}(\infty)$ and $P_1 \geq P_{\{2,3\}}(\infty)$
Type 2	$P_3 \geq P_{\{1,2\}}(\infty)$ and $P_{\{2,3\}}(0) < P_1 < P_{\{2,3\}}(\infty)$
Type 3	$P_3 \geq P_{\{1,2\}}(\infty)$ and $P_1 \leq P_{\{2,3\}}(0)$
Type 4	$P_{\{1,2\}}(0) < P_3 < P_{\{1,2\}}(\infty)$ and $P_1 \geq P_{\{2,3\}}(\infty)$
Type 5	$P_3 \leq P_{\{1,2\}}(0)$ and $P_1 \geq P_{\{2,3\}}(\infty)$

Long Lines — Behavior of $P(\mathbf{N})$ and $\bar{n}_i(\mathbf{N})$

Effect of one buffer's size on the average inventory in others



	Properties
Type 2	$P_3 \geq P_{\{1,2\}}(\infty)$ and $\star$ $P_{\{2,3\}}(0) < P_1 < P_{\{2,3\}}(\infty)$

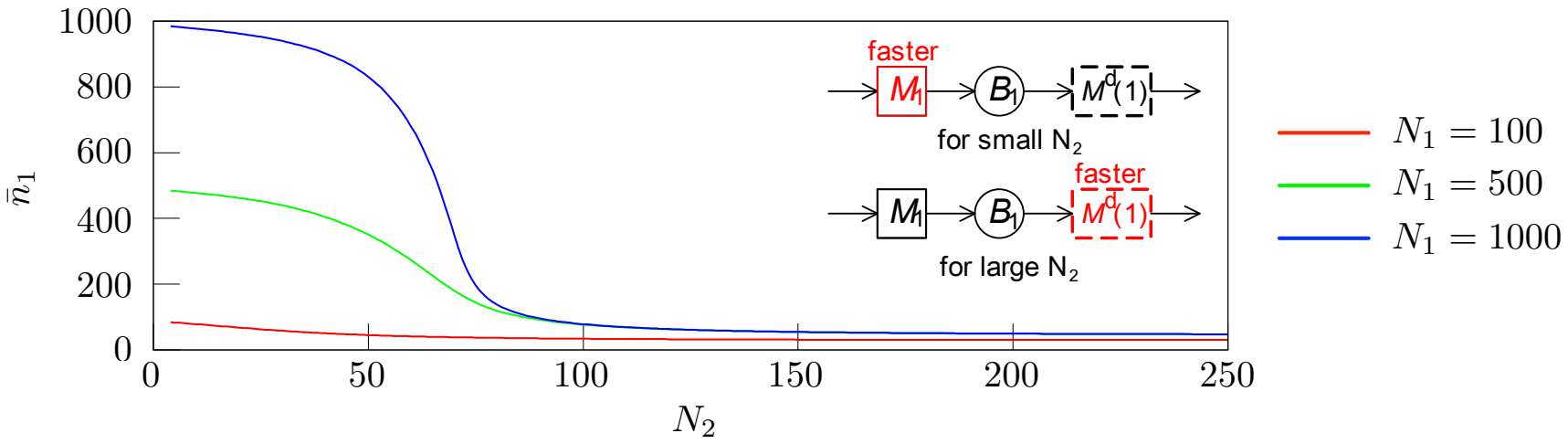
- If N_2 increases, $\bar{n}_1$ decreases.
- $\star$ means that
  - when N_2 is small, $M^d(1)$ is a bottleneck.
  - when N_2 is large, M_1 is a bottleneck.

Long Lines — Behavior of $P(\mathbf{N})$ and $\bar{n}_i(\mathbf{N})$

Effect of one buffer's size on the average inventory in others

An example of $\bar{n}_1(N_2)$ in Type 2

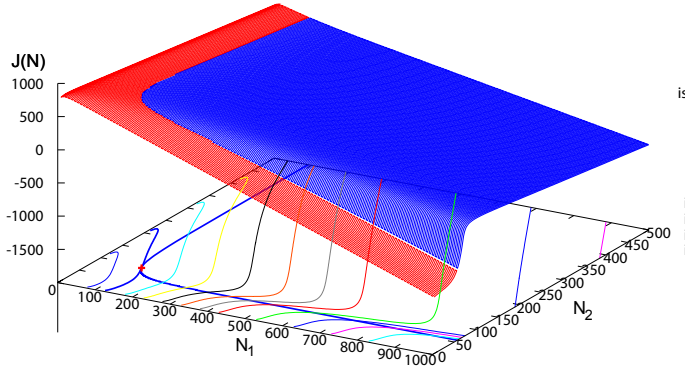
- $r_1 = .8$, $p_1 = .096$, $r_2 = .1$, $p_2 = .01$, $r_3 = .1$, and $p_3 = .01$.
- These parameters satisfy $P_3 \geq P_{\{1,2\}}(\infty)$ and $P_{\{2,3\}}(0) < P_1 < P_{\{2,3\}}(\infty)$.

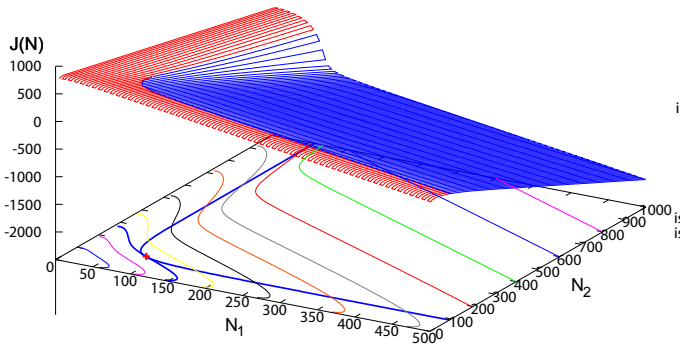


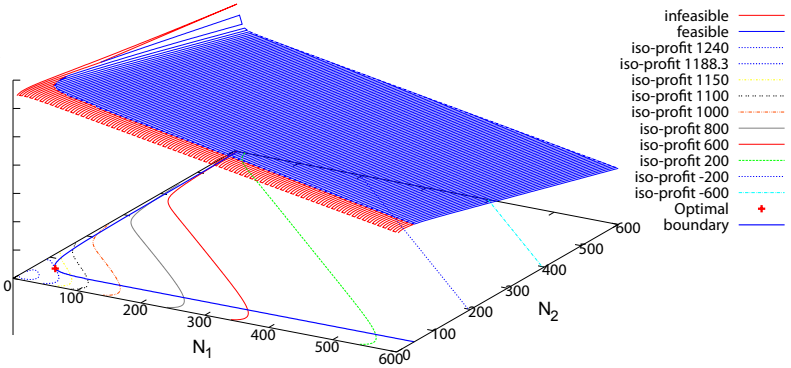
Long Lines — Behavior of $P(\mathbf{N})$ and $\bar{n}_i(\mathbf{N})$

Effect of one buffer's size on the average inventory in others

Consider profit $J(N_1, N_2) = 1000P(N_1, N_2) - N_1 - N_2 - \bar{n}_1 - \bar{n}_2$.





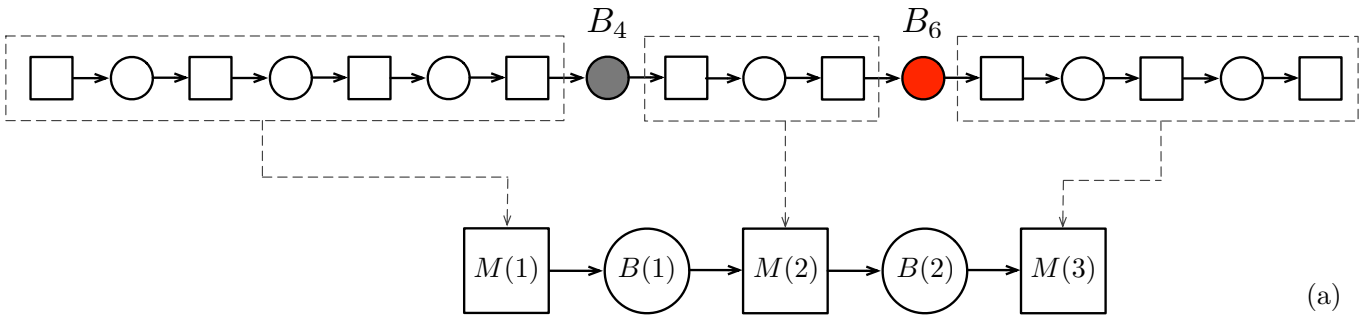


In spite of the non-convexity of the profit, we ran 5000 randomly-generated examples and observed that they all had a single local maximum.

Long Lines — Behavior of $P(\mathbf{N})$ and $\bar{n}_i(\mathbf{N})$

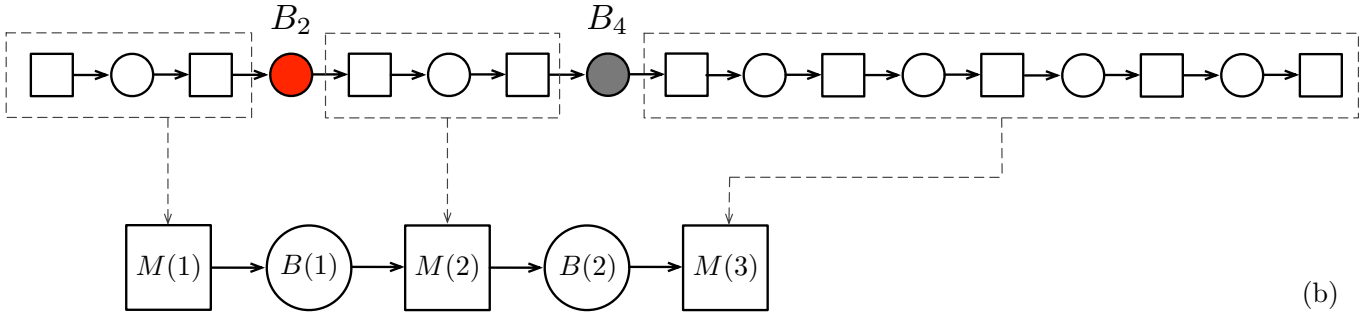
Effect of one buffer's size on the average inventory in others

Longer lines: Effect on $\bar{n}_4$ of varying N_2 or N_6.



(a)

$P(1) = .8990$
$P(2) = .9122$
$P(3) = .9097$
$P_{(2),(3)}(0) = .8614$
Type 2

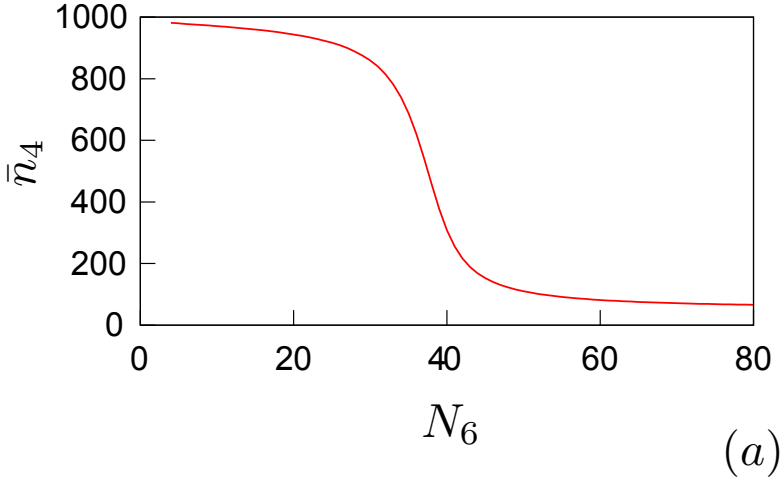


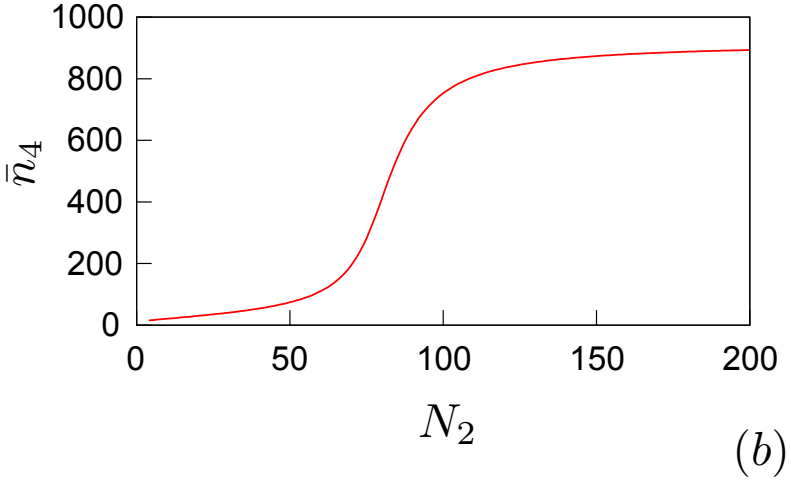
(b)

$P(1) = .9092$
$P(2) = .9092$
$P(3) = .9014$
$P_{(2),(3)}(0) = .8535$
Type 4

Long Lines — Behavior of $P(\mathbf{N})$ and $\bar{n}_i(\mathbf{N})$

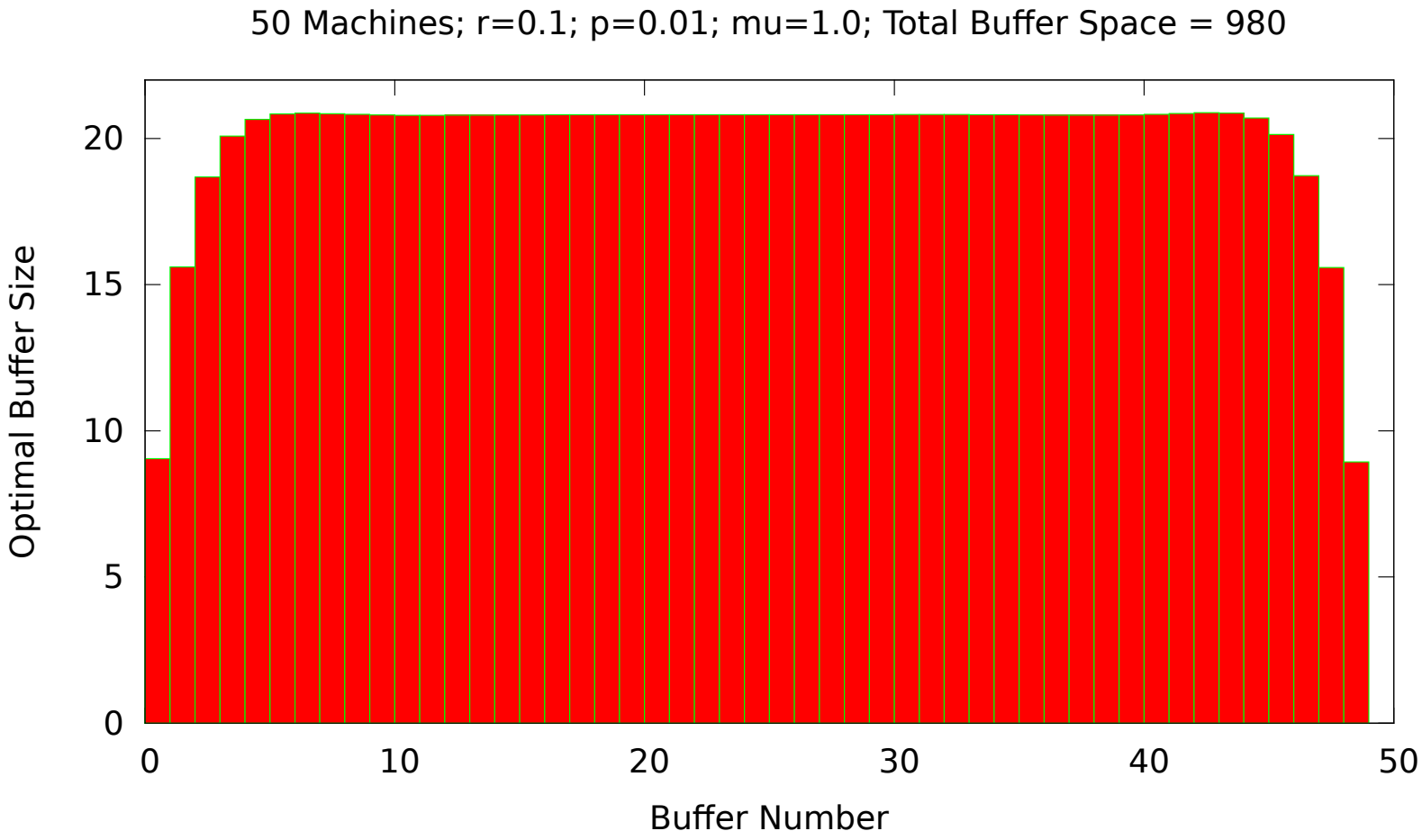
Effect of one buffer's size on the average inventory in others





- In this example, $\bar{n}_4$ is affected differently as N_2 and N_6 vary.
- The effect of each buffer's size on each other buffer's average inventory in a long line can be treated as one of the five types.
- $\bar{n}_i(N_j)$ can be *non-convex* and *non-concave.*

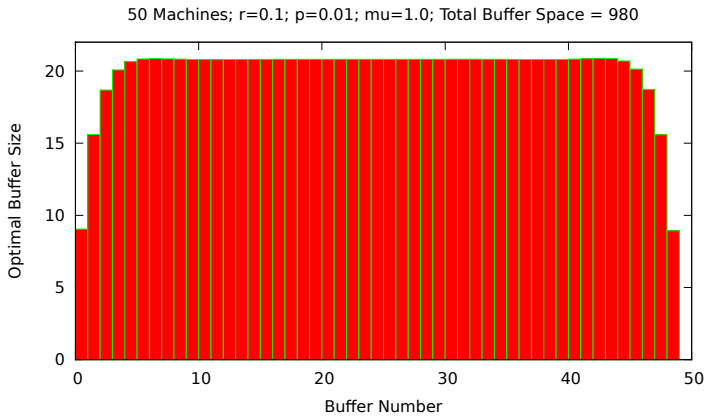
Long Lines — Optimization



Allocation of buffer space that maximizes production rate.

P= .79; Total average inventory= 469

Long Lines — Optimization

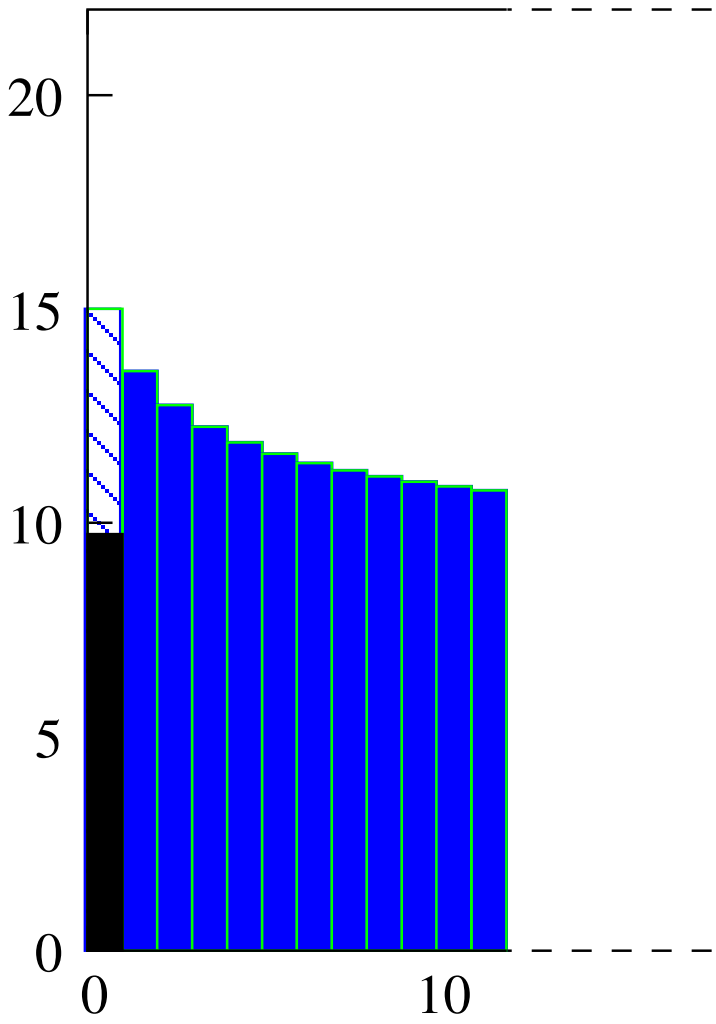


- Buffers are smaller at the ends of the line because no disruptions are considered that originate outside of the line.
- Larger buffers in the rest of the line are all about the same size because they are affected by local disruptions more than remote disruptions.
- Similar to the "bowl phenomenon" of Hillier and Boling.

Long Lines — Optimization

- *BUT,* there seems to be a paradox.
- The distribution of inventory in a line with the same machines has more inventory at the upstream end and less at the downstream end.
- *Doesn't that mean there should be more space at the upstream end and less at the downstream end?*

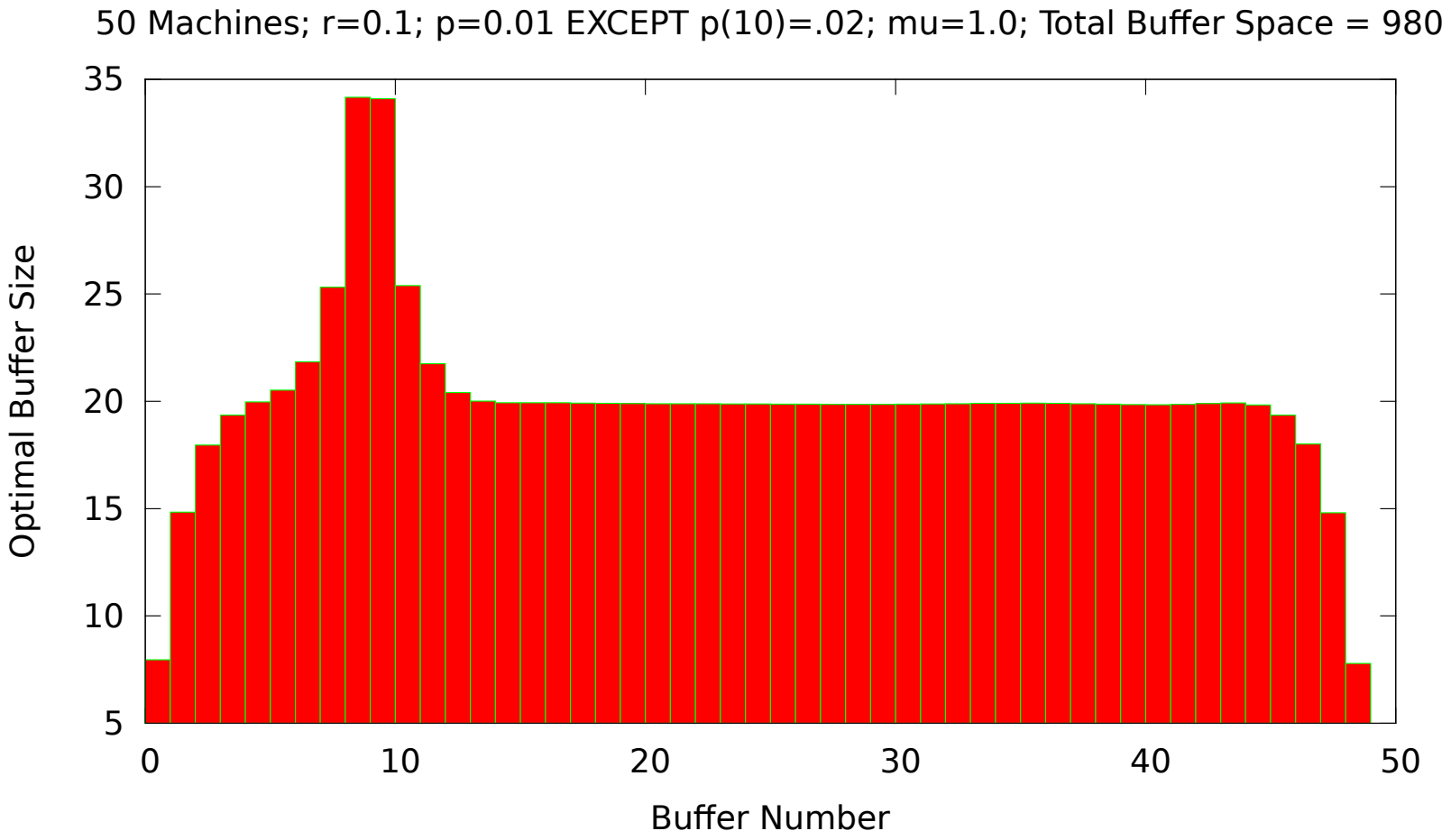
Long Lines — Optimization



- No! The level of buffer B_1 rarely gets below 10.
- *Thought experiment:* Suppose the first buffer was Last-In-First-Out. Then the bottom 10 parts would stay there for a very long time.
- Therefore we would get almost the same production rate if we replaced the bottom 10 parts with *bricks.*

Long Lines — Optimization

Effect of a bottleneck



Long Lines — Optimization

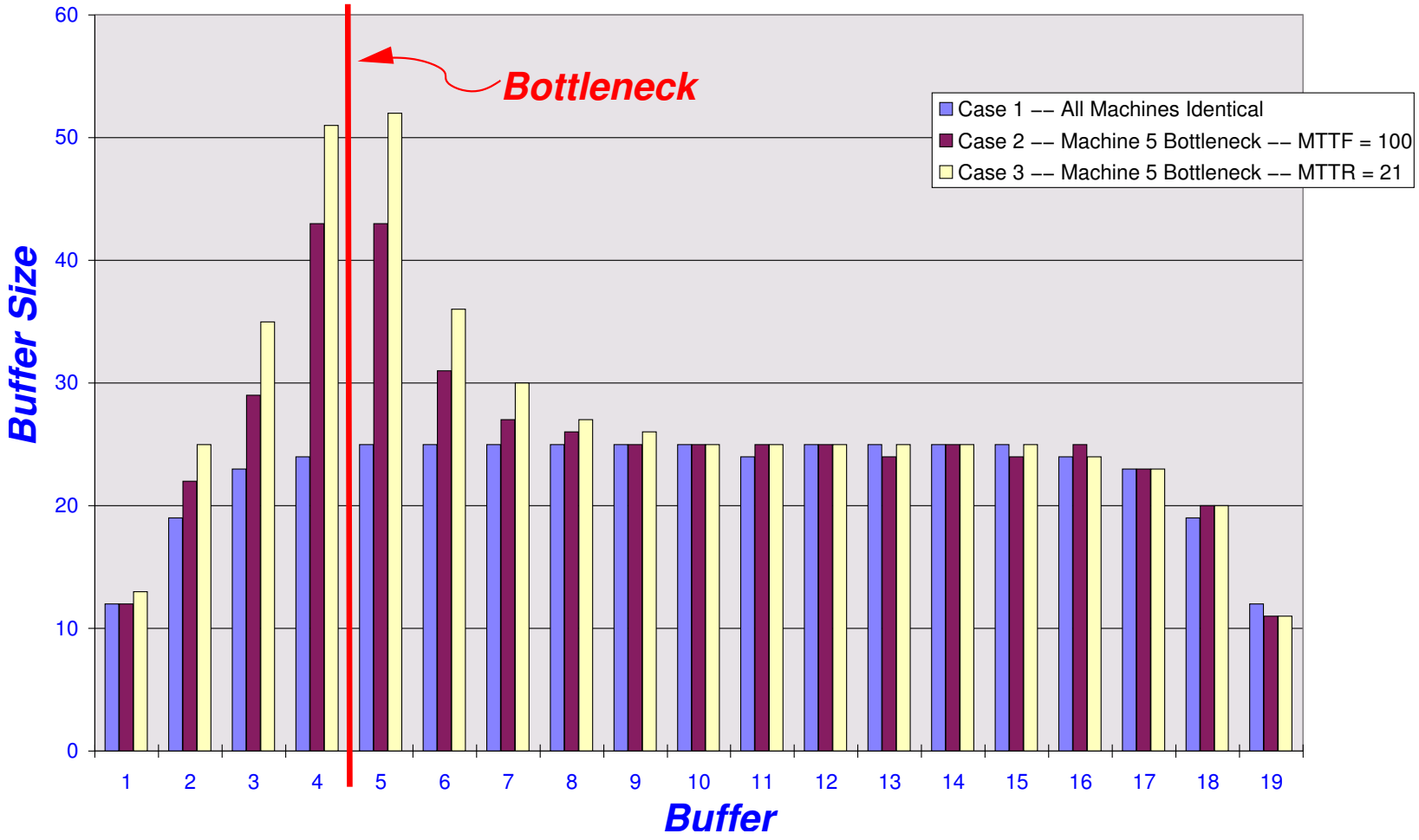
Problem: pick buffer sizes so that $P \geq .88$ to minimize total buffer space.

Three cases:

- *Case 1* MTTF= 200 minutes and MTTR = 10.5 minutes for all machines ($P = .95$ parts per minute).
- *Case 2* Like Case 1 except Machine 5. For Machine 5, MTTF = 100 and MTTR = 10.5 minutes ($P = .905$ parts per minute).
- *Case 3* Like Case 1 except Machine 5. For Machine 5, MTTF = 200 and MTTR = 21 minutes ($P = .905$ parts per minute).

Long Lines — Optimization

Effect of a bottleneck



Other Phenomena

- Reversibility/equivalence
- Buffers as low-pass filters.

Conclusions

- Buffers absorb variability. Buffers are needed most where
  - variability is greatest (largest MTTR); or
  - vulnerability to variability is greatest (a bottleneck).
- Two-machine lines can tell us a lot about the behavior of larger systems.

