

# The Compton Effect

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We measured the angular dependence of the energies of 661.6 keV photons scattered from free electrons and verified the Compton scattering formula. We determined the rest energy of the electron to be  $m_e c^2 = 507.9 \pm 20.6 \text{ keV}$ , which is 1% less than the accepted value  $m_e c^2 = 511 \text{ keV}$ . The total scattering cross-section per electron was determined to be  $\sigma_{total} = (2.4483 \pm .04893) \times 10^{-25} \text{ cm}^2$ . The classically determined cross section is  $6.662 \times 10^{-25} \text{ cm}^2$  and the cross-section determined by the relativistic and quantum-mechanical Klein-Nishina formula is  $2.4483 \times 10^{-25} \text{ cm}^2$ . The excellent agreement of the experimentally determined cross section and the cross-section predicted by the Klein-Nishina formula confirms that a quantum mechanical description of the Compton effect is necessary.

## INTRODUCTION

In 1920 A.H. Compton investigated the scattering of monochromatic x-rays from various materials and observed that after scattering, the frequency of the x-rays decreased. This frequency shift cannot be explained by classical electro-magnetic theory since frequency is a property of the incoming electro-magnetic field and cannot be altered by scattering, which produces only a change in direction of the electro-magnetic field. It was also observed that the cross-section for scattering depends on the frequency of the incident photon, which is in contradiction with the classically derived Thompson cross-section which is independent of the frequency of the incident photon. To understand these two experimental results, a relativistic and quantum-mechanical description of Compton scattering is necessary.

## THEORY

The phenomenon of Compton scattering may be analyzed as an elastic collision of a photon with a free electron using relativistic kinematics (as shown in figure 1). Since the energy of the photons (661.6 keV) is much greater than the binding energy of electrons (the most tightly bound electrons have a binding energy less than 1 keV), the electrons which scatter the photons may be considered free electrons. Because energy and momentum must be conserved in an elastic collision, we obtain the formula for the energy of the scattered photon,  $E'_\gamma$  as a function of scattering angle  $\theta$ : 
$$E'_\gamma = \frac{E_\gamma}{1 + \frac{E_\gamma}{m_e c^2} (1 - \cos\theta)},$$
 where  $E_\gamma$  is the energy of the incident photon and  $m_e c^2$  is the rest energy an electron. This result predicts that the scattered photon will have energy  $E = h\nu'$ , which is less than the energy of the incident photon ( $E = h\nu$ ), and that the frequency shift will depend on the angle of scattering.

Scattered radiation which is not shifted in frequency

is observed in addition to the frequency shifted gamma rays. The unshifted rays are due to scattering from electrons that remain bound in the atom. In this situation the recoiling system is the entire atom rather than a single electron ( $m_A \approx 2000 \times m_e$ ), which produces a negligible wavelength shift.

The cross-section for scattering is the effective area provided by the target for collisions. The cross section,  $\sigma$ , for scattering from a single particle is defined as:  $\sigma = \frac{\text{scattered flux}}{\text{incident flux per unit area}}$ , which has dimensions of area and can be visualized as the area of the scattering center projected on a plane normal to the incoming beam of particles.

The classically derived Thompson cross-section is  $\sigma_T = \frac{8\pi r_0^2}{3} = 6.6622 \times 10^{-25} \text{ cm}^2$  ( $r_0$  is the "classical electron radius"). The Thompson cross section is independent of the frequency of the incident photon, which contradicts experimental observations. The true cross section is described by the relativistic and quantum mechanical Klein-Nishina formula. (Heiter, p. 221) The ratio between the classically determined Thompson cross-section and the Klein-Nishina formula for different values of  $\gamma$  is illustrated in figure 2. The most important aspect of the Klein-Nishina formula is that the size of the cross-section depends on the energy of the incident photon.

## Compton Scattering Experiments

The experimental setup for the Compton scattering experiment is illustrated in figure 3. The target scintillation detector is irradiated by a beam of 661.6 keV photons emitted by  $^{137}\text{Cs}$ . When a photon scatters off a free electron in the target counter, the resulting recoil electron may lose all its energy through coulomb interactions with the scintillation material, producing a pulse with amplitude proportional to the energy of the recoil electron. If the scattered photon emerges from the target scintillator without further interaction and it passes through the scatter detector, it will have a large probabil-

ity of depositing all its energy in the scintillator through photoelectric interactions or a sequence of Compton scatterings. The counter signals are amplified and the energies of the scattered photon and recoil electron are each registered using a coincidence technique on a multichannel analyzer (MCA) in a PC.

The counter signals were calibrated using three gamma-ray sources with known photo-peaks:  $^{137}\text{Cs}$  (661.6keV),  $^{22}\text{Na}$  (511keV), and  $^{133}\text{Ba}$  (356 and 81 keV) as shown in figure 4.

The experimental setup for the cross-section measurement is the same as in the previous experiment except the scatter counter is removed and the target MCA and the coincidence circuit are not used. The scatter counter is positioned directly across from the Howitzer ( $\theta = 0$ ) and the counting rates registered by the MCA in the 661.6 keV peak is determined when no absorber and three different thicknesses of plastic are placed in front of the exit hole of the Howitzer.

### The Compton Scattering Formula

The energies of the scattered photon and recoil electron for seven scattering angles ( $0, \pi/6, \pi/4, \pi/3, \pi/2, 2\pi/3, 5\pi/6$ ), were determined by fitting Gaussians to the peaks registered on the MCA. In figure 5 the experimentally determined energies of the scattered photon and recoil electron are plotted for each of the seven angles. For the scattered photon energies, the theoretical prediction is within the  $2\sigma$  error bars for all angles. The experimentally determined energies of the recoil electron do not as closely match the theoretical predictions. For the majority of the angles measured the experimentally determined value was lower than the value predicted by the Compton formula by approximately 30 keV. This discrepancy is due to K-shell excitation. Rather than losing all its energy through coulomb interactions, the recoil electron may excite a K shell electron in an atom of the scintillation material. When the excited electron falls back to its ground state, it emits an x-ray. Because our scintillator is transparent to x-rays, this energy escapes without being registered on the MCA. The energy needed to excite the K-shell for sodium iodide (the scintillator used in this experiment) is approximately 30 keV, which is exactly the energy discrepancy observed between the experimental results and the theoretical predictions of the Compton scattering formula.

A manipulation of the Compton formula yields:  $\frac{1}{E'_\gamma} = \frac{1}{m_e c^2}(1 - \cos(\theta)) + \frac{1}{E_\gamma}$ . This formula demonstrates that there is a linear relationship between  $1/E'_\gamma$  and  $(1 - \cos(\theta))$ , as depicted in figure 6. The reciprocal of the slope of this line is the rest energy of an electron in keV. The rest energy of an electron was determined to

be  $m_e c^2 = 507.9 \pm 20.6 \text{keV}$ , which is less than 1% less than the accepted value  $m_e c^2 = 511 \text{keV}$ .

### Determination of the Total Scattering Cross-Section

The attenuation of a collimated beam of particles by interactions in a slab of material of thickness  $x$  (cm) is described by the formula  $\frac{I(x)}{I_0} = \exp(-\mu x)$ , where  $I_0$  is the initial intensity,  $I(x)$  is the intensity after interaction, and  $\mu$  is the linear attenuation coefficient ( $\text{cm}^{-1}$ ). The ratio  $\frac{I(x)}{I_0}$  for each of the three different thicknesses of plastic was determined by dividing the counts per second (weighted average of five measurements) by the counts per second with no plastic block. Fitting a decaying exponential to the data (see figure 7), yields  $\mu = .0800 \pm .0016 \text{cm}^{-1}$ . Because the attenuation is due almost entirely to Compton scattering for 661.6 keV photons, the total scattering cross-section per electron is related to the linear attenuation coefficient  $\mu$  by the equation  $\sigma_{total} = \frac{\mu}{n_e}$ , where  $n_e$  is the number of electrons per  $\text{cm}^3$  in the material. The total number of electrons was determined by the equation  $n_e = \frac{\rho N_0 Z}{A}$  where  $\rho$  is the density of the material ( $\text{g}/\text{cm}^3$ ),  $Z$  is the atomic number,  $A$  is the atomic weight, and  $N_0$  is Avogadro's number. The number of electrons per unit volume in the plastic scintillator was determined to be  $n_e = 3.266 \times 10^{23} \text{cm}^{-3}$ . The total cross-section per electron was determined to be  $\sigma_{total} = (2.4483 \pm .04893) \times 10^{-25} \text{cm}^2$ . The classically determined cross section is  $6.662 \times 10^{-25} \text{cm}^2$  and the cross-section determined by the relativistic and quantum-mechanical Klein-Nishina formula is  $2.4483 \times 10^{-25} \text{cm}^2$ . Note the excellent agreement between the experimentally determined cross-section and the Klein-Nishina formula.

## CONCLUSIONS

The experimentally determined energies of the scattered photon very closely matched the theoretical predictions of the Compton formula. However, the experimentally determined energies of the recoil electron were consistently about 30 keV less than the energy predicted by the Compton formula. We attribute this discrepancy to K-shell excitation. We determined the rest energy of the electron to be  $m_e c^2 = 507.9 \pm 20.6 \text{keV}$ , which is 1% less than the accepted value  $m_e c^2 = 511 \text{keV}$ . The total scattering cross-section per electron was determined to be  $\sigma_{total} = (2.4483 \pm .04893) \times 10^{-25} \text{cm}^2$ . The classically determined cross section is  $6.662 \times 10^{-25} \text{cm}^2$  and the cross-section determined by the relativistic and quantum-mechanical Klein-Nishina formula is  $2.4483 \times 10^{-25} \text{cm}^2$ , which exactly matches our experimentally determined cross section. Our experimental verification of Compton scattering formula and the Klein-Nishina formula demon-

strate that relativity and quantum mechanics are necessary to describe the phenomenon of Compton scattering.



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- [3] Melissinos, Adrian C., “Experiments In Modern Physics”, pp. 123-126, New York, Academic Press [1966]
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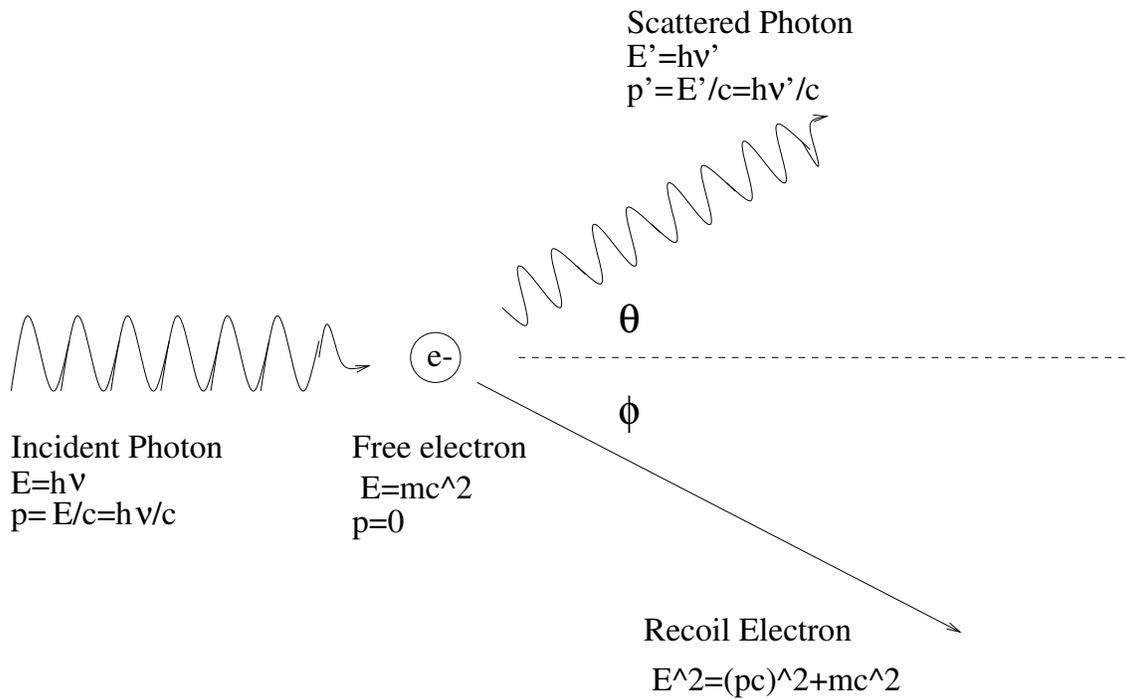


FIG. 1: Compton Scattering: A Relativistic Collision

Apply conservation of energy and momentum to find relationship between energy of incident photon  $E_\gamma$ , energy of scattered photon  $E'_\gamma$ , and energy of recoil electron  $E_e$ . Energy:  $E_\gamma + m_e c^2 = E'_\gamma + \sqrt{(pc)^2 + (m_e c^2)^2}$   $m_e c^2 =$  rest energy of an electron = 511 keV

Momentum: x direction  $\frac{E_\gamma}{c} = \frac{E'_\gamma \cos(\theta)}{c} + p \cos(\phi)$

Momentum: y direction  $0 = \frac{E'_\gamma \sin(\theta)}{c} + p \sin(\phi)$

$p =$  momentum of recoil electron  $\theta$  angle of scattered photon with respect to incident photon  $\phi$  angle of recoil with respect to incident photon

Conservation of energy and momentum yields  $E'_\gamma$ , as a function of scattering angle,  $\theta$ :

$$E'_\gamma = \frac{E_\gamma}{1 + \frac{E_\gamma}{m_e c^2} (1 - \cos\theta)}$$

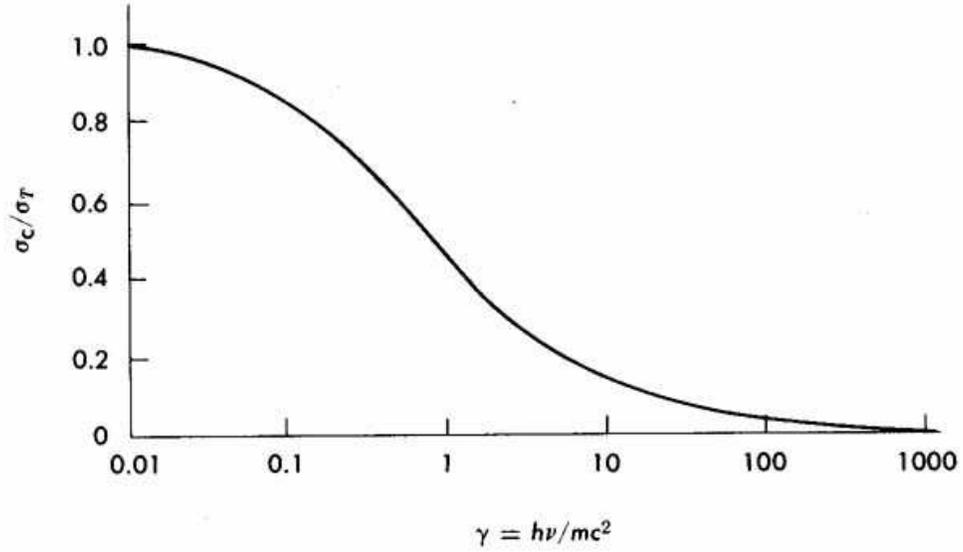


FIG. 2: This figure depicts the ratio of the classically determined Thomson cross-section per electron (Melissinos, p 167):  $\sigma_T = \frac{8\pi r_0^2}{3} = 6.6622 \times 10^{-25} \text{ cm}^2$  to the Klein-Nishina formula  $\sigma_K = \sigma_T \times \frac{3}{4} \frac{1+\gamma}{\gamma^3} \left[ \frac{2\gamma(1+\gamma)}{1+2\gamma} - \ln(1+2\gamma) \right] + \frac{1}{2\gamma} \ln(1+2\gamma) - \frac{1+3\gamma}{(1+2\gamma)^2}$ , where  $\gamma = \frac{h\nu}{m_e c^2}$ , where  $\nu$  is the frequency of the incident photon. The most important aspect of the Klein-Nishina formula is that the size of the cross-section depends on the energy of the incident photon. Note that for 662 keV photons, the ration between the Thomson cross-section and the Klein-Nishina formula is approximately .4.

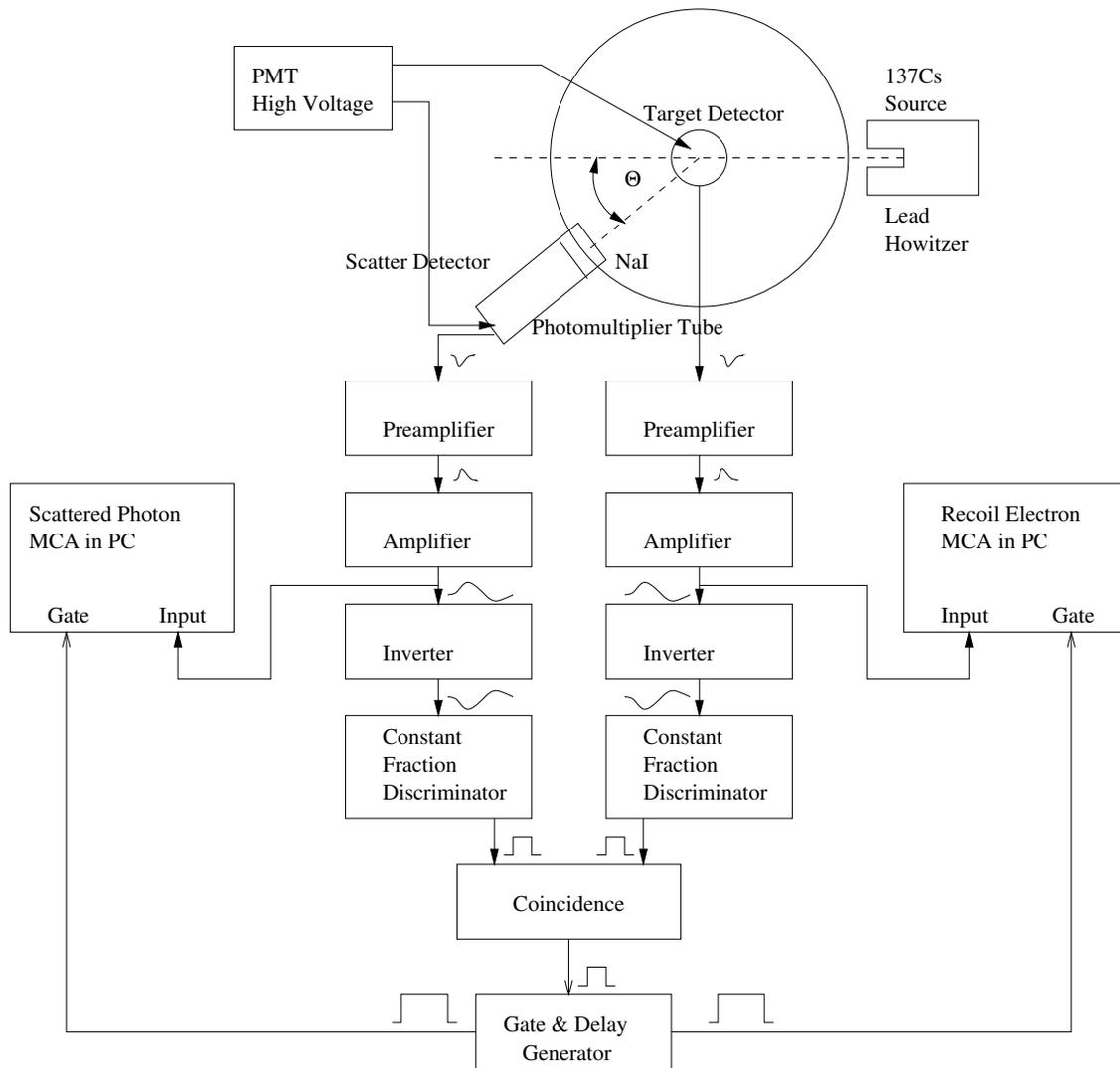


FIG. 3: The experimental setup for the Compton scattering experiment. The counter signal registered from the target and scatter counters amplified and each fed into a separate multichannel analyzer (MCA) in a PC. However, because we are interested in determining only the scintillation pulses due to the Compton effect in the target detector in which the scattered electron passes through the scatter detector, a logic pulse created by feeding each signal into the coincidence circuit is used for gating on the MCA. When the two signals are coincident within approximately  $1\mu\text{s}$ , the gating of the MCA is turned on and the target MCA acquires the energies of the recoil electron and the scatter MCA acquires the energies of the scattered photon.

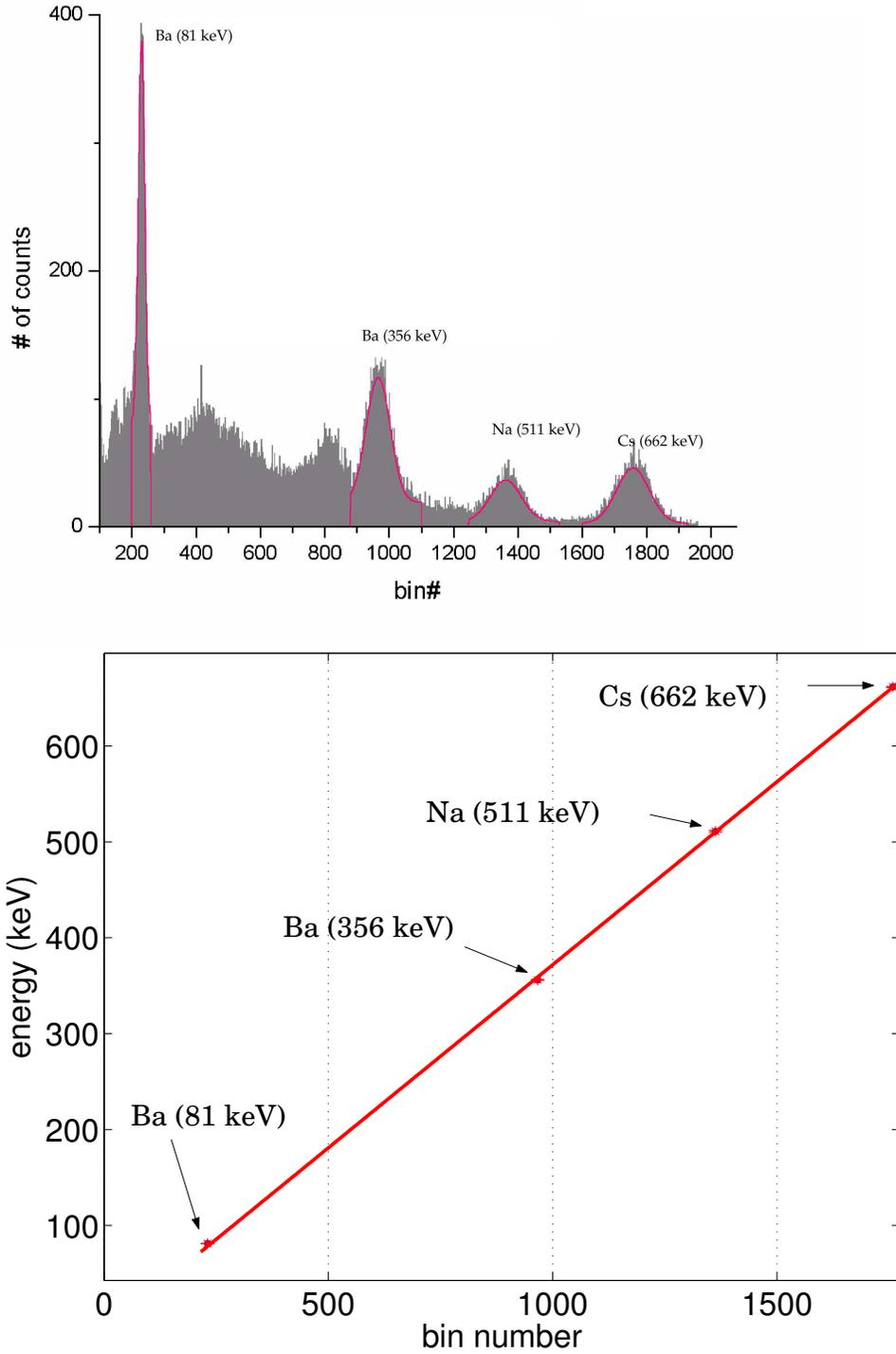


FIG. 4: This figure depicts the calibration of the signal from the scatter counter. The counter signals were calibrated using three gamma-ray sources with known photo-peaks: Cs (661.6 keV), Na (511 keV), and Ba (356 and 81 keV). The relationship between MCA bins ( $b$ ) and energy ( $E$ ) is very close to linear. A linear fit to the data yields:  $E = an + b$ ,  $a = .382 \pm .061$  and  $b = -9.96 \pm 8.13$ .

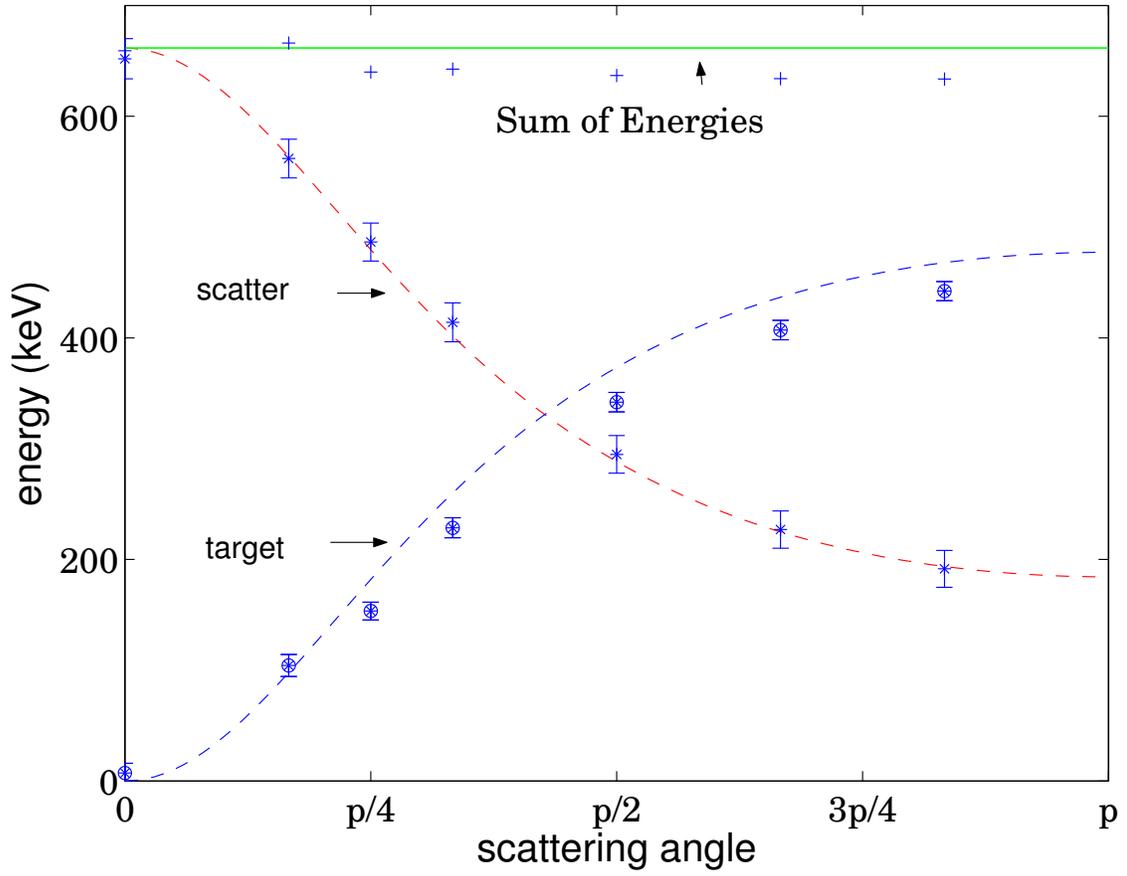


FIG. 5: The energies of the scattered photon and recoil electron for seven scattering angles  $(0, \pi/6, \pi/4, \pi/3, \pi/2, 2\pi/3, 5\pi/6)$ . The dashed curves show the theoretical predictions of the energies of the scattered photon ( $E'_\gamma$ ) and recoil electron ( $E_e$ ) in terms of the scattering angle  $\theta$  for an initial photon energy  $E_\gamma = 661.6 \text{ keV}$ .  $E'_\gamma = \frac{E_\gamma}{1 + \frac{E_\gamma}{m_e c^2} (1 - \cos\theta)}$  and  $E_e = E_\gamma - E'_\gamma$ . The green line indicates the energy of the incident photon, 661.6 keV, and the +'s show the sum of the energies of the scattered photon and recoil electron, demonstrating the conservation of energy.

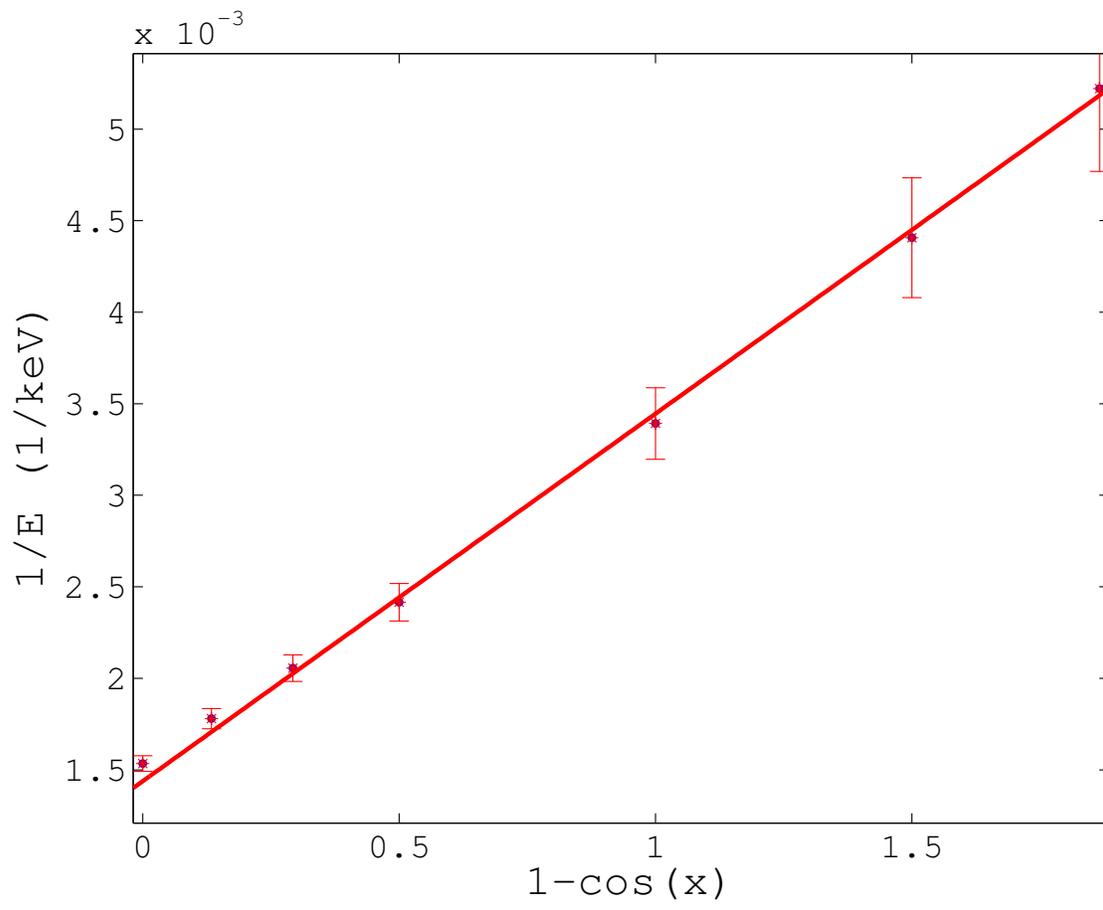


FIG. 6: The Determination of the Rest Energy of an Electron:

$$\frac{1}{E_\gamma} = \frac{1}{m_e c^2} (1 - \cos(\theta)) + \frac{1}{E_\gamma}$$

Apply Linear Fit:

$$m_e c^2 = 507.9 \pm 20.6 \text{ keV}$$

$$E_\gamma = 674.3 \pm 37.7$$

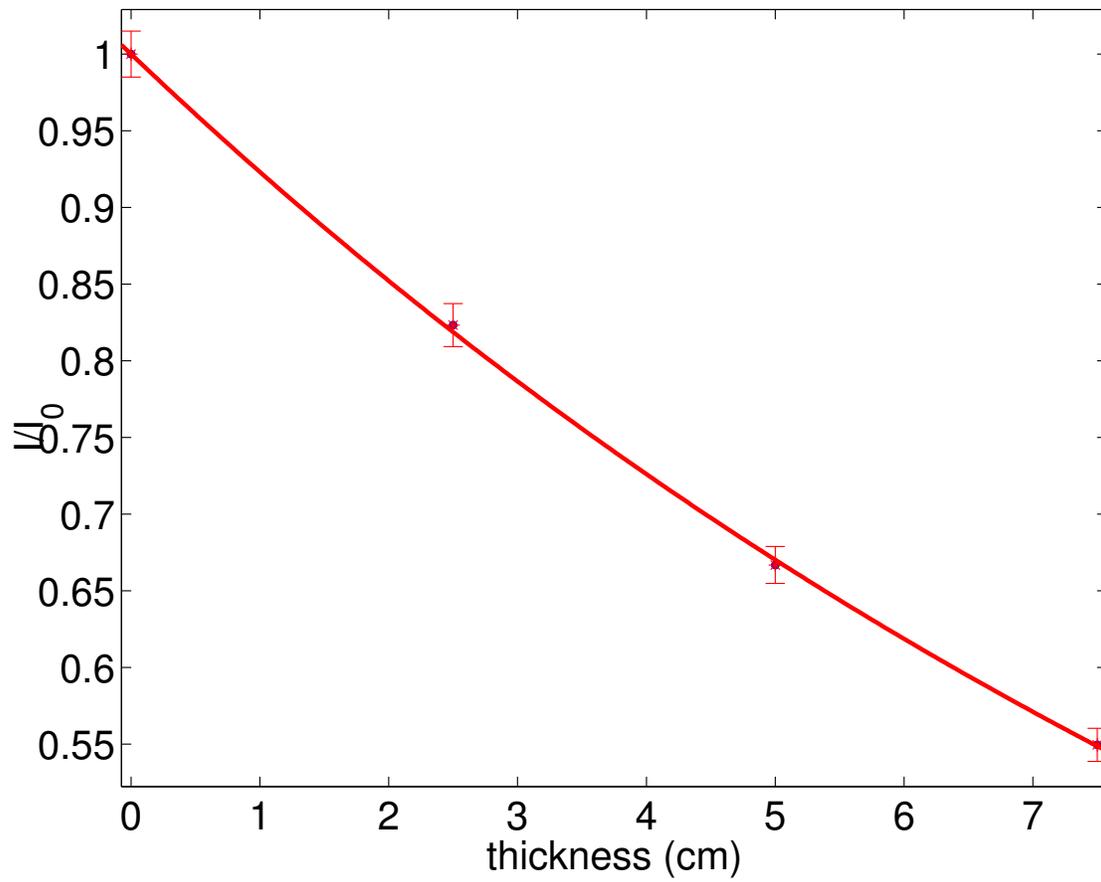


FIG. 7: Determination of the attenuation coefficient of plastic scintillator:

The attenuation of a collimated beam of particles by interactions in a slab of material of thickness  $x$ :

$\frac{I(x)}{I_0} = \exp(-\mu x)$ , where  $I_0$  is the initial intensity,  $I(x)$  is the intensity after interaction, and  $\mu$  is the linear attenuation coefficient ( $cm^{-1}$ ). Fitting a decaying exponential to the data yields  $\mu = .0800 \pm .0016 cm^{-1}$ .