Notes available at http://web.mit.edu/matulef/www/18.404

9/30/05 ①

Review:

- L is Decidable - TM M always halts in accept or reject state (e.g. A_{NFA}, EQ_{DFA}, A_{CFG}, E_{CFG}) ...
- L is Recognizable - TM M enters accept state on strings in L, loops or enters reject state on other strings (Example: A_{TM})
- Equivalence of TM models. In particular, multitape & nondeterministic TM recognize and decide the same class of languages as normal TMs
- Recognizable $\Longleftrightarrow$ Enumerable by "enumerator"

no input, starts with blank tape, has a printer to output strings

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Q) Let $E_{TM} = \{\langle M\rangle \mid M$ is a TM which accepts no strings (i.e. $L(M) = \emptyset$)$\}$

Is E_{TM} decidable? No, but we'll prove this next week.

Is $\bar{E}_{TM}$ Recognizable? Yes.

Proof) Let $s_1, s_2, \ldots$ be a list of all strings in Σ^*

On input "$\langle M\rangle$"

1. If $\langle M\rangle$ is not a TM description, accept.
2. For $i = 1, 2, 3 \ldots$

WRONG → a) Simulate $\langle M\rangle$ on string s_i ← WRONG. This step might not end.

b) If $\langle M\rangle$ accepts, accept

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correction:

2. For $i = 1, 2, 3 \ldots$

a) Simulate $\langle M\rangle$ on $s_1, s_2, \ldots s_i$ for i steps each,

b) If $\langle M\rangle$ accepts, accept

Note similarity to "Recognizable $\Longrightarrow$ Enumarable" proof from class

If TM M recognizes some language, can create enumerator E which prints out that language

E = " For $i = 1, 2, \ldots$

a) simulate M on $s_1, s_2, \ldots s_i$ for i steps each

b) If M accepts a string, print it out

Aside: proof is also similar to "counting" the rationals

	1	2	3	4
1	1/1	1/2	1/3	
2	2/1	2/2	2/3	
3	3/1	3/2	3/3	

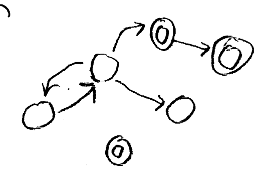
Q: $A = \{ \langle R \rangle \mid R$ is a reg exp and $L(R)$ is prefix-free $\}$

does not contain strings x and y where x is a prefix of y.

Show A is decidable.

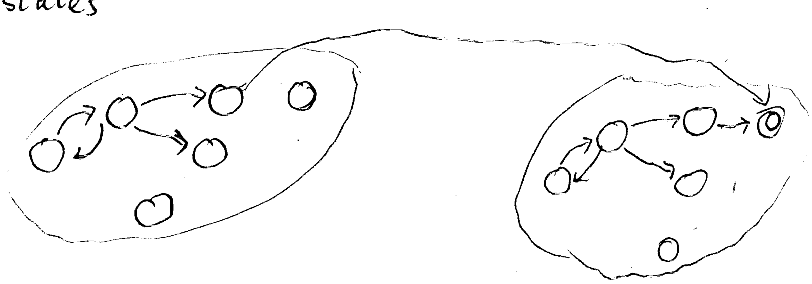
Proof (1) | On input $\langle R \rangle$

1. Convert $\langle R \rangle$ to a DFA
2. Search for accept states reachable from q_0
3. From each of ~~the~~ those states, do a new search to see if any accept state is reachable
   If so, accept, otherwise reject



Proof (2) |

Construct machine M' recognizing only those strings which pass through two accept states



Then decide if $L(M') = \phi$

Q| Let $BAL_{DFA} = \{\langle M\rangle \mid$ M is a DFA that accepts some string with an equal # of 0's and 1's $\}$

Show BAL_{DFA} is decidable.

Proof| "On input $\langle M\rangle$

1) Construct PDA P which recognizes strings with equal # of 0's and 1's
2) Construct PDA P' which recognizes $L(P) \cap L(M)$
3) Decide if $L(P')$ is empty or not. If so, reject. otherwise, accept

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Review of language classes so far: