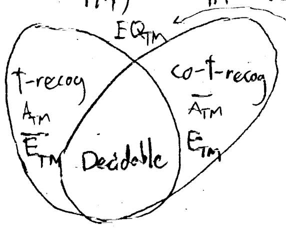
Review;

- Undecidability (explicit proof for A_{TM}, proof for E_{TM}, $HALT_{TM}$ etc via reductions)



Also, $\overline{EQ}_{TM}$

Decidable = T-recog & co-T-recog

- Mapping reducibility, ($A \leq_m B$)

Q] $L = \{ \langle M, w \rangle \mid$ M is a TM that on input w, tries to move off the end of the input tape (at some point) $\}$

Show L is undecidable

Proof] Want to reduce A_{TM} to L. Construct machine A deciding A_{TM}

A = "on input $\langle M, w \rangle$

1) Construct M', a TM which (on the empty string) simulates M on w, and moves off left iff M accepts.
2) Run decider for L on $\langle M', \varepsilon \rangle$

M' = "If input is ε,

1) Mark the first tape cell, move right, write out w.
2) Simulate M on w,

Never move left passed mark, unless
If M accepts, move left off the end.

$A \leq_M B$ means there is a computable function f st $w \in A \iff f(w) \in B \ \forall w$

Must give function f to show that $A \leq_M B$

Example $A_{TM} \leq L$, using function $f(\langle M, w \rangle) = \langle M', \epsilon \rangle$

Some Basic Facts about Mapping Reducibility:

- If $A \leq B \iff \bar{A} \leq \bar{B}$. why? use same f.
- If $A \leq B$, B decidable $\Rightarrow$ A decidable
  (A undecidable $\Rightarrow$ B undecidable)
- If $A \leq B$, B recognizable $\Rightarrow$ A recognizable
  (A unrecog $\Rightarrow$ B unrecog)
- If $A \leq \bar{B}$, B decidable $\Rightarrow$ A is decidable (since $\bar{B}$ also decidable)
- If $A \leq \bar{B}$, B recognizable $\Rightarrow$ A is co-recognizable (since $\bar{A} \leq B$, so $\bar{A}$ recognizable)

Caveats:

- $A \leq B \not\Rightarrow B \leq A$
- $A \leq B \not\Rightarrow A \leq \bar{B}$

Q] Remember $A_{TM} \leq_M \bar{E}_{TM}$ from class.

Show that $A_{TM} \not\leq_M E_{TM}$.

Proof] Suppose $A_{TM} \leq E_{TM}$

Then $\bar{A}_{TM} \leq \bar{E}_{TM}$.

But since $\bar{E}_{TM}$ is recognizable (why?), this would imply $\bar{A}_{TM}$ is recognizable.

Contradiction $\Rightarrow\Leftarrow$