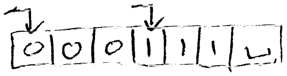
Today:
Comp History Method
Recursion Thm
Review

Define a 2-headed DFA (2HDFA) to be a TM with two read-only heads, both of which can only move right or stay put (not necessarily in sync). Moreover, there is only one "␣" symbol to mark the end of the input.

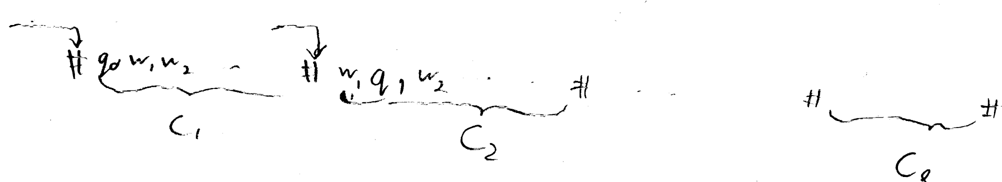


A 2HDFA can, for example, recognize $\{0^n 1^n \mid n \geq 0\}$
But probably cannot recognize $\{w w^R \mid w \in \Sigma^*\}$

Q) Show $E_{2HDFA} = \{\langle D \rangle \mid D \text{ is a 2HDFA and } L(D) = \phi\}$ is undecidable

Proof) Comp History method,
For particular M and w, show how to construct a 2HDFA $D_{M,w}$ which only accepts valid, accepting comp histories for M on w
Then, given a decider for E_{2HDFA}, we decide A_{TM} as follows
" On input $\langle M, w \rangle$
1. Construct $\langle D_{M,w} \rangle$
2. Run decider for E_{2HDFA} on $\langle D_{M,w} \rangle$, accept if decider rejects
reject if decider accepts "

How does $D_{M,w}$ work?



1) First read head verifies C_1 is correct starting config of M on w
2) Two heads move in lock-step to verify each C_i and C_{i+1} correctly line up according to M's transition function
3) First head checks that q_{accept} is encountered. If so, and all configs seen so far are valid, accept.

Q | Define the "Busy Beaver" function $f(K)$ =
to be the maximum length string printed by a K-state machine on a blank input
which halts on a blank input.
Show f is not computable.

Proof #1 | Suppose it were computable. Then define machine

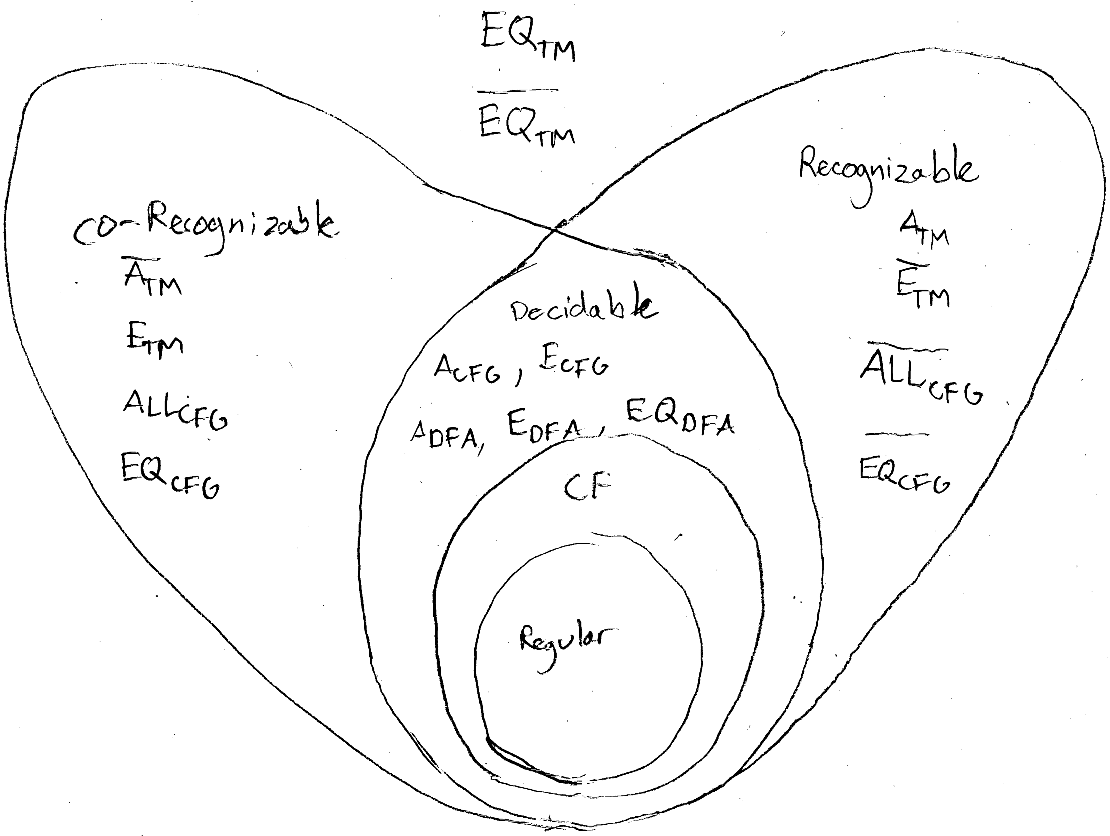
G = "on blank input

1) Get own description $\langle G \rangle$, let K = # states in $\langle G \rangle$
2) Compute $f(K)$
3) Print $f(K)+1$ symbols on the tape"

$\Rightarrow \Leftarrow$

Alternative solution : Show how to decide the halting problem if we can compute the Busy Beaver function.

10/21/05 ③


EQ_{TM}
$\overline{EQ_{TM}}$
co-Recognizable
$\overline{A_{TM}}$
E_{TM}
ALL_{CFG}
EQ_{CFG}
Recognizable
A_{TM}
$\overline{E_{TM}}$
$\overline{ALL_{CFG}}$
$\overline{EQ_{CFG}}$
Decidable
A_{CFG}, E_{CFG}
A_{DFA}, E_{DFA}, EQ_{DFA}
CF
Regular