Optimal Control of the DISC Engine using Hierarchical and Quantized Control

Presentation overview

- Overview of the DISC Engine
- Modeling the DISC Engine as a Switched System
- Control Objectives
- Reducing the complexity of controlling switched systems
- Application to the DISC engine
- Concluding remarks

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Overview of the Direct Injection Stratified Charge (DISC) Engine

DISC Engine uses fuel injection to operate in two combustion regimes

- Homogenous mode
- Stratified mode

Homogenous: similar to PFI engine characteristics

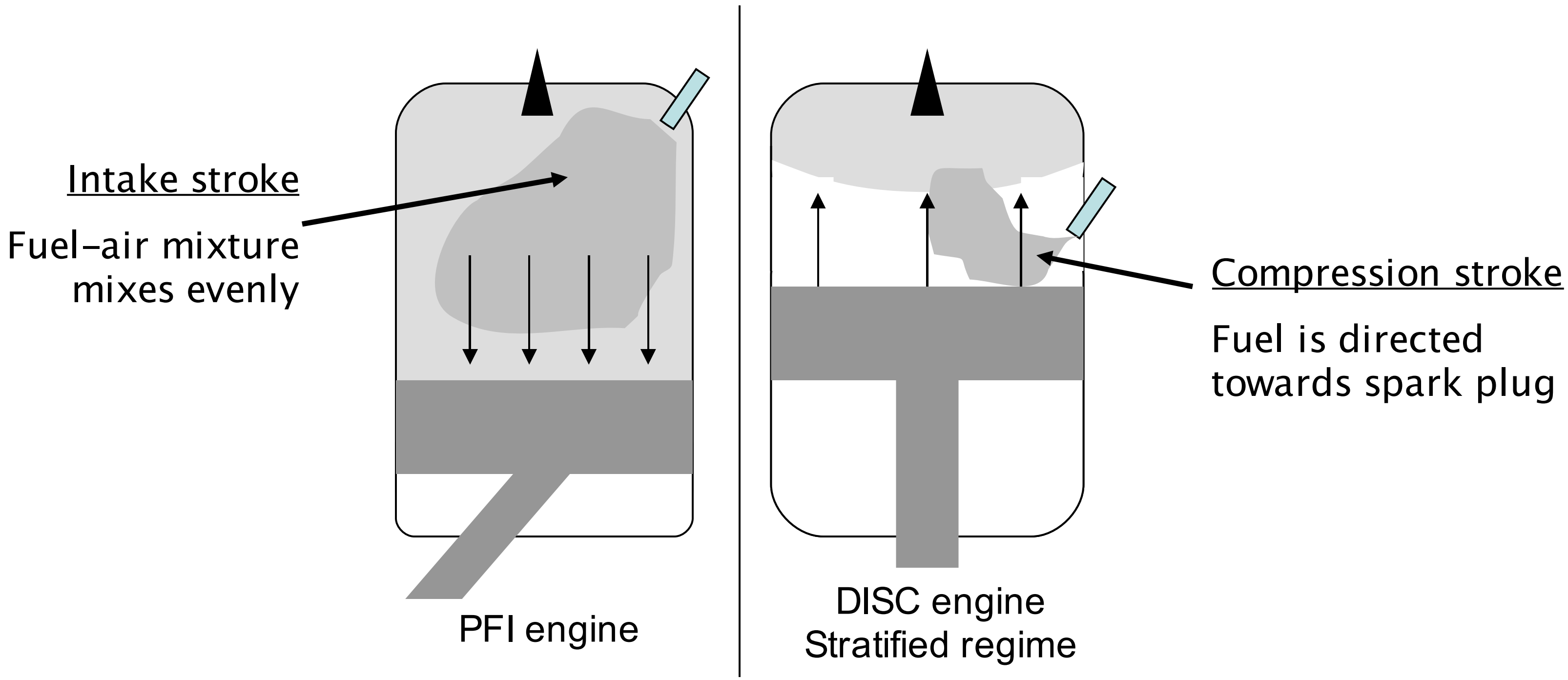
- 14.67:1 air-to-fuel ratio (AFR)
- Used to generate “high” torques for fast acceleration and maintaining high speeds

Stratified: high fuel-efficiency mode

- ~ 30:1 AFR
- Cannot generate high torques – best for maintaining slower speeds

Detailed Overview of the Stratified Regime

- High AFR in PFI engines results in poorer combustion
- Stratified regime overcomes this by concentrating air-fuel mixture around spark plug



- High temperatures and high-AFR results in excessive NOx emissions
  - NOx catalysts lose effectiveness over time
  - Need to purge catalysts periodically by switching to homogenous mode

Modeling the DISC Engine as a Switched System

Control inputs

- Mass flow rate of air: W_{th}
- Fueling rate: W_f
- Spark timing: δ
- Combustion regime: ρ

Output parameters

- Intake manifold pressure (state): p
- AFR: λ
- Brake torque: τ
- Combustion regime: ρ

Linear, Discrete-time Model

$$p(n+1) = a_\rho p(n) + b_\rho W_{th}(n)$$

$$\begin{bmatrix} \lambda(n) \\ \tau(n) \end{bmatrix} = C_\rho p(n) + D_\rho \begin{bmatrix} W_f(n) \\ \delta(n) \end{bmatrix}$$

All input and output parameters are hard-constrained

Each operating mode is a constrained system

DISC Engine Control

Given an initial state and initial combustion mode

$$\left(p_0, \rho_0\right)$$

and an output reference and reference mode

$$\left(\begin{bmatrix} p_{ref} \\ \lambda_{ref} \\ \tau_{ref} \end{bmatrix}, \rho_{ref}\right)$$

choose $W_{th}(n), W_f(n), \delta(n), \rho(n)$

such that the following is minimized

$$\sum_n \left\| \begin{bmatrix} p(n) - p_{ref} \\ \lambda(n) - \lambda_{ref} \\ \tau(n) - \tau_{ref} \end{bmatrix} \right\|_Q^2 \quad \text{subj. to} \quad \rho(n) \underset{n\to\infty}{\longrightarrow} \rho_{ref}$$

Optimal Control of Switched Systems

A controllable switched system is of the form

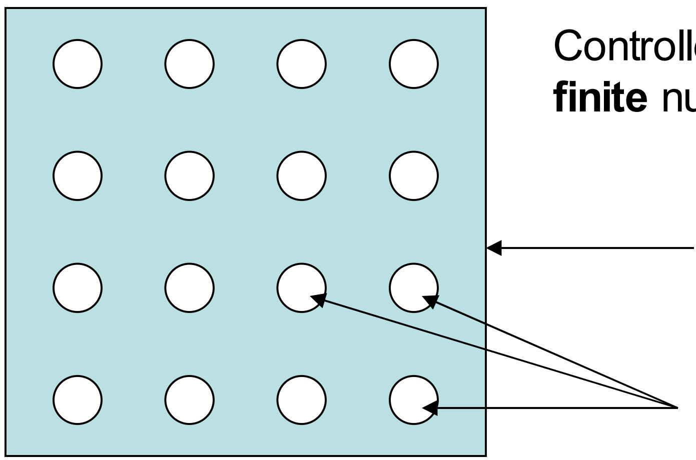
$$\dot{x} = A_\rho x + B_\rho u \qquad \rho \in \{1,2,\ldots,M\}$$
$$y = C_\rho x + D_\rho u$$

- Optimal control of such systems is generally NP-Complete
- Strive for easier-to-compute sub-optimal control
- Can reduce complexity be restricting switching to a chosen set of *Switching States*
- Consider constrained and unconstrained systems separately

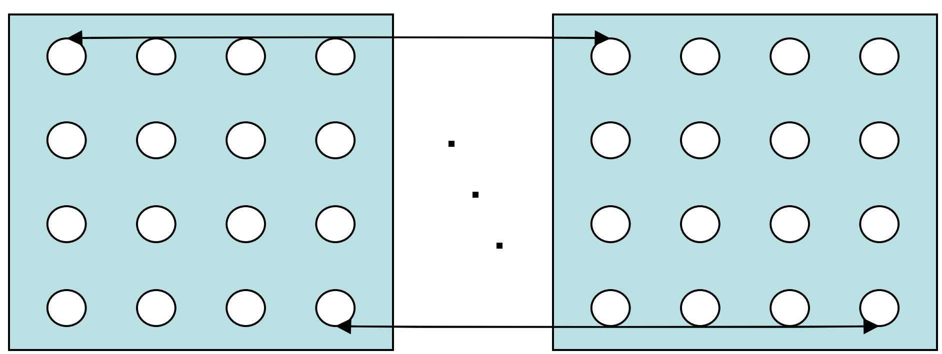
Optimal Control of Constrained Switched Systems: Reducing Decisions

For constrained systems, state-space may be assumed to be bounded

Controller only changes modes when state is within one of a **finite** number of *Switching States*

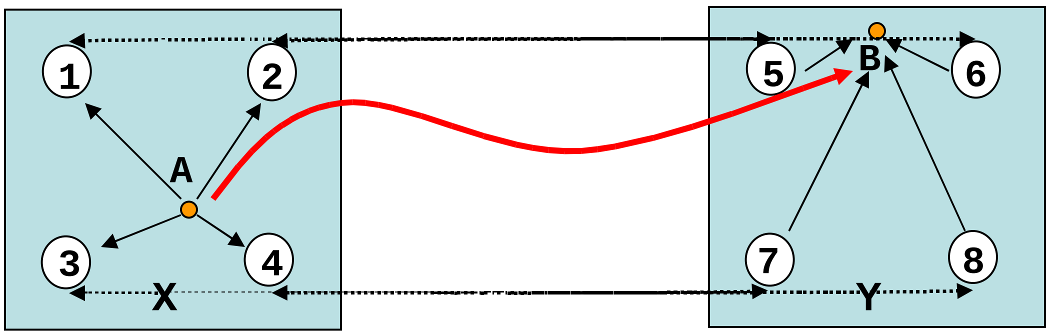


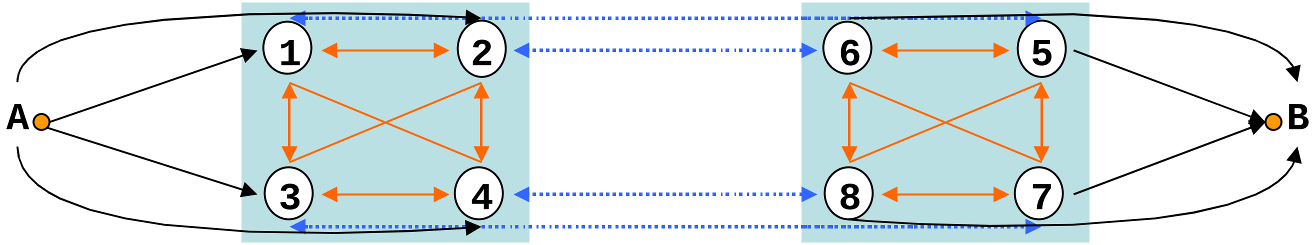
There is a 1-1 correspondence between the switching regions of two spaces



Optimal Control of Constrained Switched Systems: Switching Graph

Moving from a state *A* in a mode *X* to a state *B* in a mode *Y*:



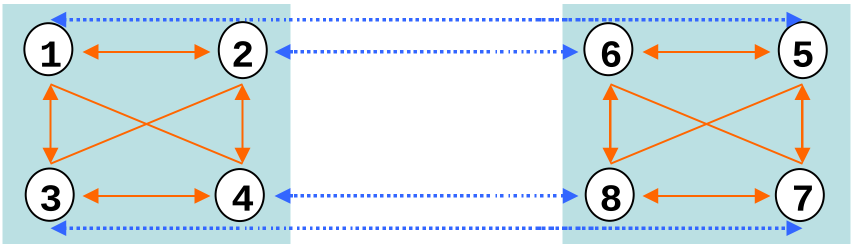


Nodes represent switching states

Edges represent the cost of transitioning between nodes

Since *A* is the source, all edges exit *A*. Similiarly, all edges enter *B*.

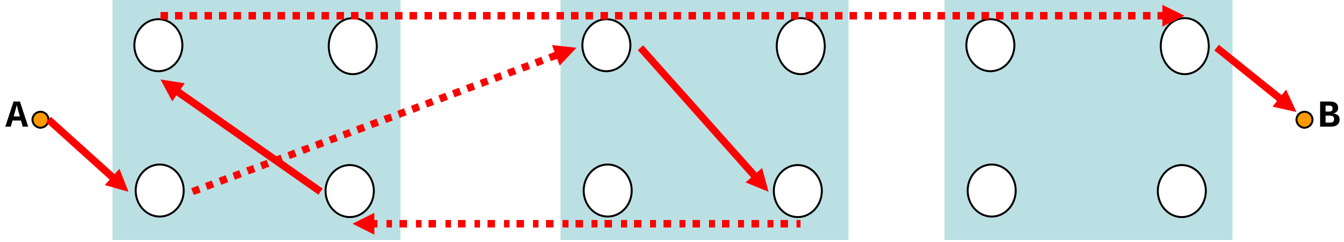
Robust Hybrid Switching Graph (RHSG)



- Edges and nodes are always fixed
- Edge weights vary

For a given source *A* and destination *B*

2. Compute the edge weights
3. Quickly find the "shortest" path (path of least cost)
4. Track the open-loop path using closed-loop controllers in each mode



Computing the Edge Weights of the RHSG: Subsystem Controllers

Take any *switching path*: $\left((x_0, \rho_0), (x_1, \rho_1), \ldots (x_{N-1}, \rho_{ref}), (x_{ref}, \rho_{ref}) \right)$

For a cost function of the form:

$$J = \sum_{n=0}^{\infty} L_{\rho(n)}\left(y(n) - y_{ref}, u\right)$$

It splits apart:

$$J = \sum_{n=0}^{n_1} L_{\rho_0}\left(y(n) - y_{ref}, u\right) + \sum_{n=n_1}^{n_2} L_{\rho_1}\left(y(n) - y_{ref}, u\right) + \cdots + \sum_{n=n_{N-1}}^{\infty} L_{\rho_{ref}}\left(y(n) - y_{ref}, u\right)$$

subject to

$$x(0) = x_0 \quad x(n_1) = x_1 \quad \cdots \quad x(n_{N-1}) = x_{N-1} \quad x(\infty) = x_{ref}$$

and the switching times are unknown (maybe optimized)

For each operating mode ρ, need a controller that minimizes

$$J = \sum_{n=0}^{N} L_{\rho}\left(y(n) - y_{ref}, u\right) \qquad \text{subj. to} \quad x(0) = x_0 \quad x(N) = x_N$$

over all admissible u and N.

Subsystem Controllers are Independent of the Switched Control

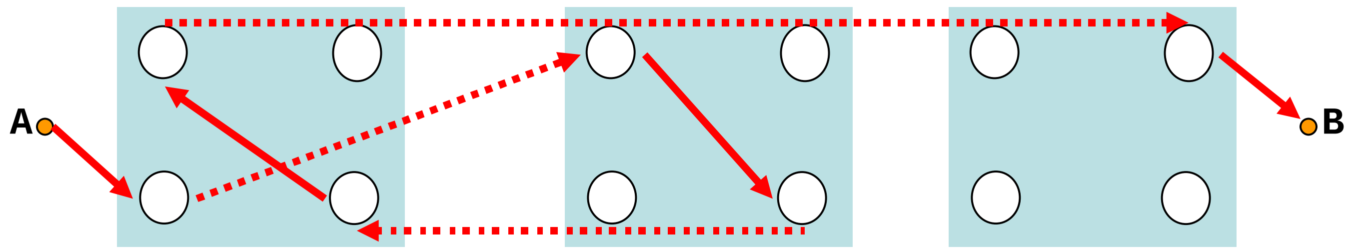
RHSG: Overall Algorithm

For a fixed reference output y_{ref} cooresponding to a target (x_{ref}, ρ_{ref})

- Compute the optimal cost between all pairs of switching states
- Compute the optimal cost from initial state to all switching states in the same mode
- Compute the best switching path
- Track the switching path using closed-loop controllers

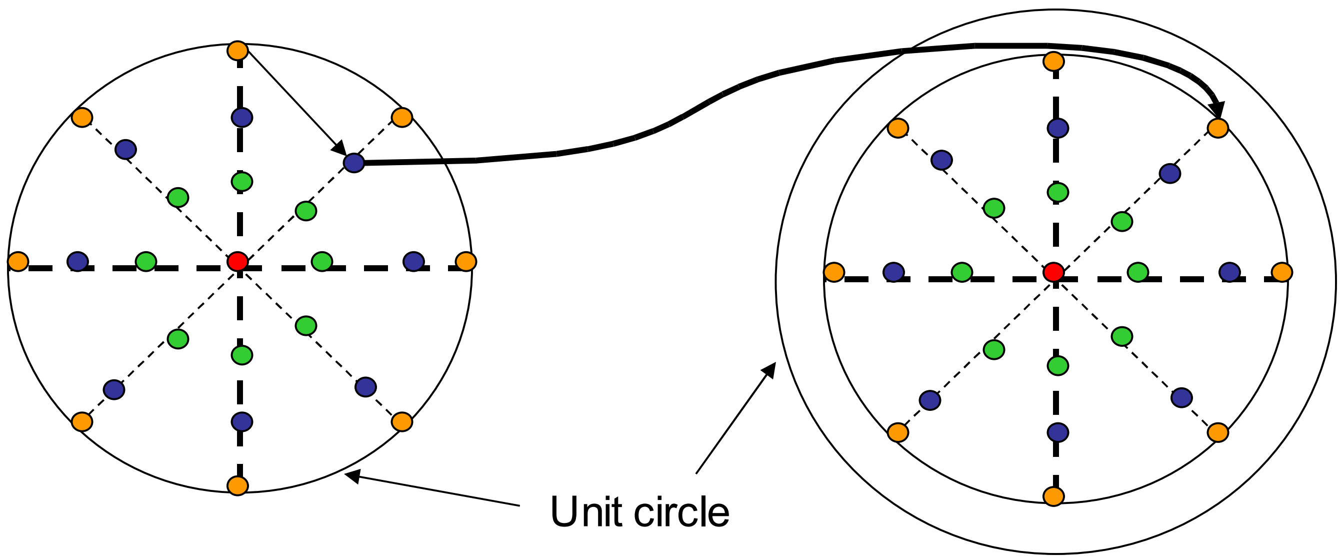
For fast processes

- Quantize the (bounded) state and output reference spaces -- small shifts not important
- At each point of the quantization, store the switching state to track



Unconstrained Systems: Dynamic RHSG (DRHSG) for LQ Problems

- Finite number of switching states not sufficient
- Leverage homogeneity of subsystems
- Force homogeneity of the switching: pose same problem at all scales



From x : Optimal cost = $J(x)$, Optimal trajectory = $x(t)$, Input $= u(t)$

From λx : Optimal cost $= \lambda^2 J(x)$, Optimal trajectory $= \lambda x(t)$, Input $= \lambda u(t)$

Use a Linear Program to get optimal costs and switching state to track

Subsystem controllers are independent of switching control

RHSG Applied to the DISC Engine: Subsystem Controllers

Controlling the Intake-Manifold Pressure (state)

$$p(n+1) = a_\rho p(n) + b_\rho W_{th}(n)$$

$$W_{th,\min} \le W_{th} \le W_{th,\max}$$

Apply a simple bang-bang control to track switching states & final state

Controlling the AFR and Torque (outputs)

$$\begin{bmatrix} \lambda(n) \\ \tau(n) \end{bmatrix} = C_\rho p(n) + D_\rho \begin{bmatrix} W_f(n) \\ \delta(n) \end{bmatrix}$$

$$W_{f,\min} \le W_f \le W_{f,\max} \quad ; \quad 0 \le \delta \le \delta_{mbt} = \beta\lambda + \eta$$

$$\lambda_{\min} \le \lambda \le \lambda_{\max} \quad ; \quad \tau_{\min} \le \tau \le \tau_{\max}$$

Use Quadratic Programming to minimize error in Torque and AFR during tracking

(MP-QP for fast evaluation)

Torque and AFR Subsystem Controller

- The optimal input is computed by finding optimal torque and AFR:

$$F\left(p(n),\begin{bmatrix}\lambda_{ref}\\ \tau_{ref}\end{bmatrix},\rho\right)=\min\quad\left\|\begin{bmatrix}\hat{\tau}(n)-\tau_{ref}\\ \hat{\lambda}(n)-\lambda_{ref}\end{bmatrix}\right\|_Q^2$$

$$\text{subj. to previous fixed and affine constraints}$$

$$\text{where}\quad\begin{bmatrix}\hat{W}_f(n)\\ \hat{\delta}(n)\end{bmatrix}=(D(\rho))^{-1}\left(\begin{bmatrix}\hat{\lambda}(n)\\ \hat{\tau}(n)\end{bmatrix}-C(\rho)p(n)\right)$$

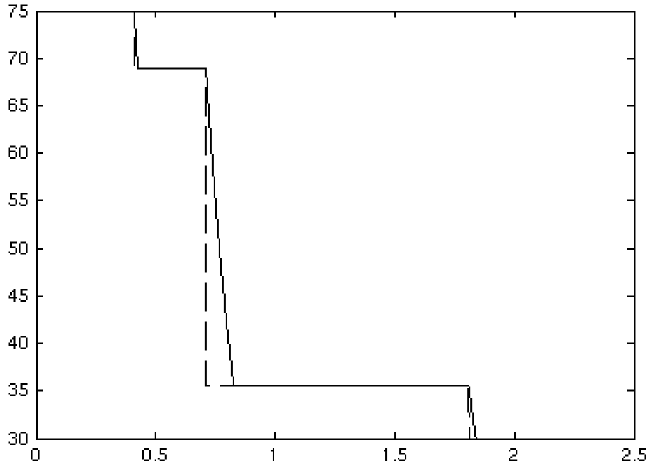
- Add an integrator with anti-windup for closed-loop:

1. $e \leftarrow (1-\alpha)\left(y^{ref}-y(n)\right)+e_I(n-1)$
2. $\tilde{y}^{ref}=y^{ref}+e$
3. $y' = F\left(p(n),\tilde{y}^{ref},\rho\right)$
4. $e_I(n) \leftarrow y'-y^{ref}$
5. $\hat{u}_{23}=(D(\rho))^{-1}\left(\hat{y}'-C(\rho)\hat{x}(n)\right)$

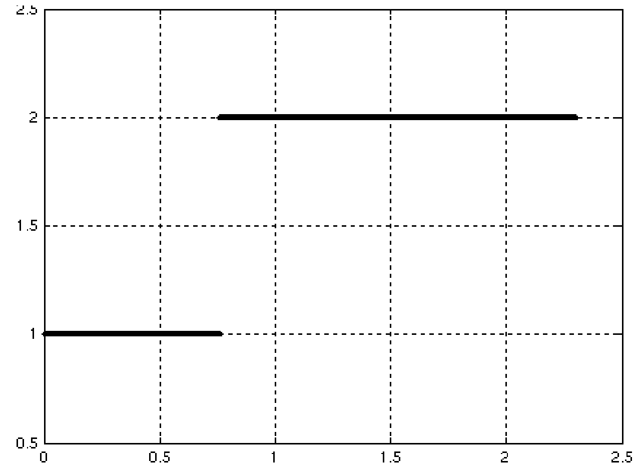
Can prove that this algorithm will stabilize the torque and AFR to the “optimal” admissable outputs.

Can add another level to handle multiple linearizations of the nonlinear system.

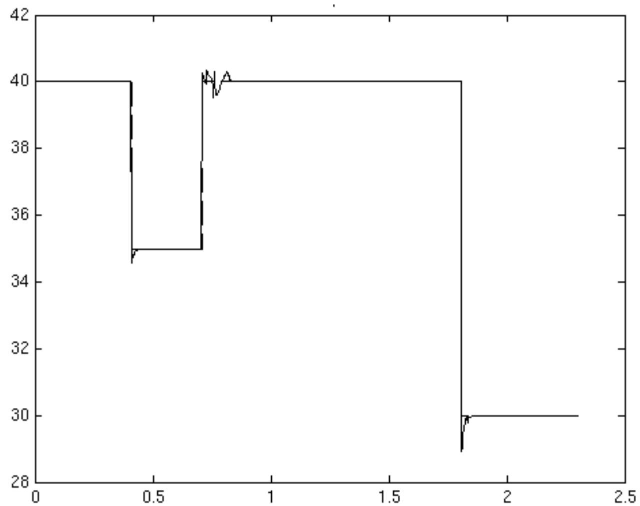
Simulation Results



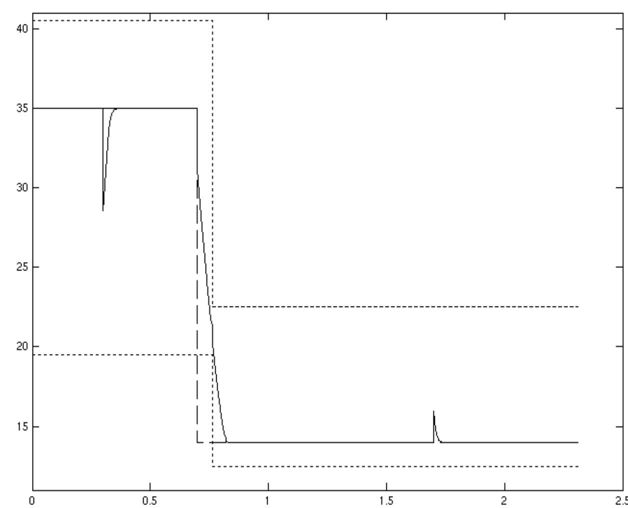
Intake-Manifold Pressure



Combustion Regime



Torque



Air-to-Fuel Ratio

Conclusions and Future Work

- Switched system control is, in general, very difficult (NP-complete)
- Reducing complexity by applying switching states separated the continuous and discrete portions of control
- Polynomial-complexity
- Guarantees convergence to the target