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A Lower Bound to Transmission Power for Multicast in Underwater Networks using Network Coding

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Abstract

The purpose of this paper is to show that convexity of the underwater acoustic channel and that near-convexity of an approximate closed-form model for that channel holds, in order to use network optimization techniques. We obtain a lower bound on transmission power using subgraph selection to establish minimum-cost multicast connections in underwater acoustic networks with network coding. We solve this problem for a range of transmission distances that are of interest for practical systems and exploiting physical properties of the underwater acoustic channel. Since the complete model for the underwater channel is complex, an approximate model is used for numerical computations. We illustrate results numerically determining the lower bound on transmission power for different random two-dimensional deployment scenarios and unicast rates. Also, we quantify the performance gap of two practical network layer schemes with respect to the lower bound.

1. INTRODUCTION

With the advances in acoustic communication technology, the interest in study and experimental deployment of underwater networks has been growing [1]. The underwater acoustic channel is characterized by a path loss that depends on both the transmission distance and the signal frequency. Thus, not only the power consumption, but also the useful bandwidth depend on the transmission distance [1]. Also, underwater networks may be constrained in terms of energy supply, especially for fixed, battery-powered sensors [1].

Routing and network coding schemes have previously been considered for underwater networks, with network coding showing better performance than routing [2] [3]. Existing results compare different network schemes, but their performance is not known with respect to a theoretical optimum in terms of transmission power.

Reference [4] presents the complete model that relates transmission power, transmission band, distance, and link capacity for an underwater acoustic link, taking into account physical models of acoustic propagation loss and colored Gaussian ambient noise. This reference shows that the transmission power can be thought as a function of the capacity C and link distance l, i.e. $P(l, C)$. We show that the complete model of $P(l, C)$ is convex. However, the complete model is a complex one. As an alternative, reference [4] presented a closed-form approximate model, in which the power is given in the form $10^{\frac{a_2(C)}{10}} l^{a_1(C)}$. We show the operating conditions under which the approximate model is convex.

We obtain a lower bound on transmission power using subgraph selection [5] to establish minimum-cost multicast connections with network coding. We neglect interference between nodes, making the cost function separable. It is shown that the no-interference assumption in the underwater scenario is justified for low multicast rates, and randomly placed nodes with distances under 10 km between each other. As cost function for each link, we use the model for transmission power, which constitutes the minimum transmission power required to achieve a desired data rate.

We illustrate results numerically, computing the lower bound on transmission power for different random two-dimensional deployment scenarios and unicast rates. Also, we quantify the performance gap with respect to the lower bound of two network layer schemes: routing with link-by-link acknowledgements and net-

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work coding with implicit acknowledgements [3].

2. CHANNEL MODEL

An underwater acoustic channel is characterized by a path loss that depends on both distance l and signal frequency f. A common empirical model used for the attenuation is Thorp's formula [1] which captures the dependence on the frequency, the distance and a spreading factor k. The spreading factor describes the geometry of propagation, e.g. $k = 1$, $k = 2$ and $k = 1.5$ correspond to cylindrical, spherical, and practical spreading, respectively. The noise in an acoustic channel can be modeled through four basic sources: turbulence, thermal noise, shipping, and waves. Thus, the noise depends on the frequency, and on two additional factors: the shipping activity s and the wind speed w in m/s [1].

The complete model for a colored Gaussian underwater link was presented in [1] where power was allocated through waterfilling. This model assumes the absence of multipath and channel fading. The capacity of a point-to-point link is

$$C = \int_{B(l,C)} \log_2\left(\tfrac{K(l,C)}{A(l,f)N(f)}\right) df \qquad (1)$$

where $A(l, f)$ is the path loss, $N(f)$ is the power spectral density (psd) of the noise, $B(l,C)$ is the optimum band of operation and $K(l,C)$ is a constant. The band $B(l,C)$ is a union of non-overlapping intervals, $B(l,C) = \cup_i[f^i_{ini}(l,C), f^i_{end}(l,C)]$, where each non-overlapping interval i has the lower-end frequency $f^i_{ini}(l,C)$ and the higher-end frequency $f^i_{end}(l,C)$ associated with it.

The transmission power associated with a particular choice of (l,C) is given by

$$P(l,C) = \int_{B(l,C)} S(l,C,f)df \qquad (2)$$

where $S(l,C,f) = K(l,C) - A(l,f)N(f), f \in B(l,C)$ is the psd of the signal.

The relationship among capacity, transmission band, transmission power, and distance is quite complicated to be computationally manageable, especially if the objective is to assess network optimization. Thus, an approximate model presented in [4] will be used instead of the complete model. In this model, the power $P(l,C)$ constitutes the transmission power required in an underwater link to achieve some capacity C at some distance l. This model for transmission power is used as the cost function for the optimization problem, where it represents the minimum transmission power needed

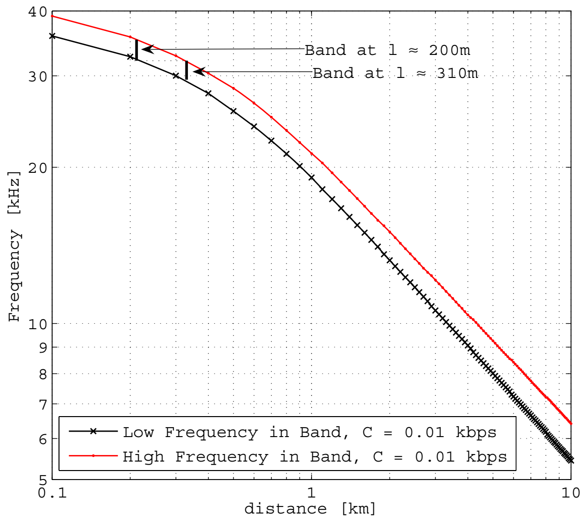

Figure 1: High and low band edge frequency of the transmission band for $C = 0.01$ kbps, $k = 1.5$, $s = 0.5$, and $w = 0$ m/s

Table 1: a_1 and a_2 approximation parameter values for $P(l,C)$, with $l \in [0, 10]$ km, $C \in [0, 2]$ kbps, $k = 1.5$, $s = 0.5$ and $w = 0$ m/s.

α_1	α_2	α_3	β_1	β_2	β_3
-0.0023518	0.015649	2.1329	0.014798	1.0148	74.175

to achieve some data rate C, for a link distance l, and has the form of

$$P(l,C) = 10^{\frac{a_2(C)}{10}} l^{a_1(C)} \qquad (3)$$
$$a_2(C) = \beta_3 + \beta_2 10\log_{10} C + \beta_1(10\log_{10}(C+1))^2$$
$$a_1(C) = \alpha_3 + \alpha_2 C + \alpha_1 C^2$$

An important characteristic of the underwater acoustic channel is that the optimal transmission band of a link depends strongly on the distance [4]. In fact, if the capacity requirement for a link is low, e.g. less than 2 kbps, and the transmission range is below 10 km, the transmission bandwidth will be low, and its optimal location in the spectrum changes dramatically with the link distance. Figure 1 shows that if two links with the same $C = 0.01$ kbps are established, one with $l \approx 200$ m and the other with $l \approx 310$ m, the optimal transmission bands for these links will not overlap; thus, they do not interfere with one another. This suggests that if an underwater network is established where nodes are at different distances from one another, each node has a limited range of transmission, and the data rate requirement is very low, then the no-interference assumption between transmissions of the various links is justified.

3. CONVEXITY OF MODELS

Transmission power in the exact model is convex with respect to C, using l as a parameter. This result is stated in Lemma 1.

Lemma 1 The complete model of $P(l, C)$ is a convex, increasing function with respect to C, $\forall C > 0$ and $l > 0$, if $A(l, f)N(f) > 0$ and $B(l, C) = \cup_i[f^i_{ini}(l, C), f^i_{end}(l, C)]$, with $f^i_{ini}(l, C) < f^i_{end}(l, C) < \infty, \forall i$ and $f^i_{ini}(l, C), f^i_{end}(l, C) \notin [f^j_{ini}(l, C), f^j_{end}(l, C)], \forall i \neq j$, i.e. a union of non-overlapping finite bands.

Proof: Using the Leibniz Integral rule, taking the first and second derivatives of $P(l, C)$ and C with respect to C, and substituting appropriately,

$$\frac{\partial P(l,C)}{\partial C} = \ln(2)K(l, C) \quad (4)$$

$$\frac{\partial^2 P(l,C)}{\partial C^2} = \left(\frac{\partial K(l,C)}{\partial C}\right)^2 \sum_i (f^i_{end}(l, C) - f^i_{ini}(l, C)) \quad (5)$$

where we used the fact that the i bands are non-overlapping, the fact that $K(l, C) = A(l, f_{end}(l, C))N(f_{end}(l, C))$ and $K(l, C) = A(l, f_{ini}(l, C))N(f_{ini}(l, C))$. The proof is completed considering that $K(l, C) > 0$ for any $l > 0$ and $C > 0$, $f^i_{end}(l, C) - f^i_{ini}(l, C) > 0, \forall C > 0, l > 0$. $\square$

Since the exact model is complicated from a computation view point, we determine the values of the parameter l for which the approximate model of $P(l, C)$ presented in the equation (3) is convex and increasing with respect to C.

Lemma 2 The approximate model for $P(l, C)$ is a convex and increasing function if

$$\ln(l) > -\frac{\ln(10)}{10}\frac{\dot{a}_2(C)}{\dot{a}_1(C)} + \max[0, -\frac{\ddot{a}_1(C)}{2\dot{a}_1(C)^2} + \sqrt{\frac{\ln(10)}{10}\left(\frac{\dot{a}_2(C)\ddot{a}_1(C)}{\dot{a}_1(C)^3} - \frac{\ddot{a}_2(C)}{\dot{a}_1(C)^2}\right) + \frac{\ddot{a}_1(C)^2}{4\dot{a}_1(C)^4}}] \quad (6)$$

Proof: Since the α and β parameters come from fitting the data, the only variable left to analyze is the distance l. Note that ensuring that the approximate model of $P(l, C)$ is increasing and convex translates into the following inequalities:

$$\ln(l)\frac{\partial a_1(C)}{\partial C} + \frac{\ln(10)}{10}\frac{\partial a_2(C)}{\partial C} > 0 \quad (7)$$

$$\ln^2(l)\left(\frac{\partial a_1(C)}{\partial C}\right)^2 + \frac{\ln(10)}{10}\frac{\partial^2 a_2(C)}{\partial C^2} + \left(\frac{\ln(10)}{10}\frac{\partial a_2(C)}{\partial C}\right)^2 + \ln(l)\left(2\frac{\ln(10)}{10}\frac{\partial a_1(C)}{\partial C}\frac{\partial a_2(C)}{\partial C} + \frac{\partial^2 a_1(C)}{\partial C^2}\right) \geq 0 \quad (8)$$

Since these constraints are also functions of C, the range of values of this parameter should be considered. For $C < 2$ kbps, $\alpha_1 < 0$, $\alpha_2 > 0$, $2\alpha_1 C + \alpha_2 > 0$, $\beta_1 > 0$ and $\beta_2 > 0$. Thus, for the choices of $a_1(C)$ and $a_2(C)$, the first and second derivatives of these functions with respect to C are $\dot{a}_1(C) > 0$, $\ddot{a}_1(C) < 0$, $\dot{a}_2(C) > 0$ $\ddot{a}_2(C) < 0$. Using these conditions, (7) and (8) can be simplified to the form presented in the lemma. $\square$

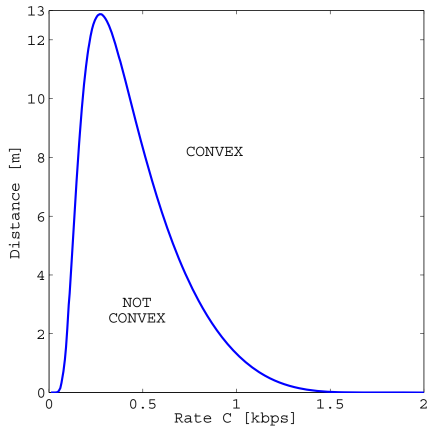

Figure 2: Region of (l, C) where the approximate model for $k = 1.5$, $s = 0.5$ and $w = 0$ m/s is convex with respect to C. The model is convex $\forall(l, C)$ over the curve. Note that for $l > 13$ m the model is convex $\forall C$ of interest.

The values in Table 1 guarantee that the term in square root in (6) is positive. Figure 2 illustrates the region of (l, C) values for which the approximate model is convex or not using the parameters in Table 1. Any pair (l, C) yields a convex function if it is above the curve. Hence, a sufficient condition for convexity for C <2 kbps is having a distance between the nodes $d >$ 13 m.

4. LOWER BOUND TO TRANSMISSION POWER IN UNDERWATER NETWORKS

The problem of achieving minimum-energy multicast using network coding in a wireless network has been studied previously [5]. An underwater acoustic network can be represented through a directed hypergraph $H = (\aleph, A)$ where $\aleph$ is the set of nodes and A is the set of hyperarcs [5]. A hypergraph is a generalization of a graph, where there are hyperarcs instead of arcs. A hyperarc is a pair (i, J), where i, the start node, is an element of $\aleph$,and J is the set of end nodes is a nonempty subset of $\aleph$. Each hyperarc (i, J) represents a broadcast link from node i to nodes in the nonempty set J. Let us denote by z_{iJ} the rate at which coded packets are injected into hyperarc (i, J). Since the cost function is separable, the optimization problem can be expressed as

$$\min \sum_{(i,J)\in A} P(l_{iJ}, z_{iJ})$$
$$\text{subject to } z \in Z$$
$$z_{iJ} \geq \sum_{j\in J} x^{(t)}_{iJj}, \forall (i,J) \in A, t \in T$$
$$\sum_{\{J|(i,J)\in A\}} \sum_{j\in J} x^{(t)}_{iJj} - \sum_{\{j|(j,I)\in A\}, i\in I} x^{(t)}_{jIi} = \delta^{(t)}_i$$
$$x^{(t)}_{iJj} \geq 0, \forall (i,J) \in A, j \in J, t \in T \quad (9)$$

with

$$\delta^{(t)}_i = \begin{cases} R & \text{if } i = s, \\ -R & \text{if } i = t, \\ 0 & \text{otherwise} \end{cases} \quad (10)$$

where T is a non-empty set of sink terminals, a source s, and a multicast rate R. $x^{(t)}_{iJj}$ represents the flow associated with terminal t, sent through hyperarc (i,J) and received by node $j \in J$. A simplification of this problem can be made under the assumption that transmissions are omnidirectional, and considering that, if a node transmits over a certain range, all nodes in that range will be able to receive the information. Also, according to the model of $P(l,z)$, any z value can be achieved as long as enough power is used. Then, under the assumption that power consumption is not a constraint for any hyperarc, it is possible to drop the constraint set Z of the optimization problem. The cost function for each particular hyperarc corresponds to a link transmission power $P(l_{iJ}, z_{iJ})$ in order to obtain the minimum transmission power required to achieve a data rate of z_{iJ}, where l_{iJ} represents the distance from i to the farthest node $j \in J$.

In the underwater scenario this formulation is used to establish a lower bound on the transmission power required to achieve a multicast rate R. Assuming no interference for transmissions in different hyperarcs yields a separable cost function. Note that if interference was taken into account, the power to reach the desired data rate would increase. Although the problem for minimum-cost multicast is well known for wireless radio networks, the cost function presented here is different because it represents the minimum transmission power for an hyperarc transmitting at a data rate Z, given by the power needed to transmit at capacity $C = Z$, without assumption on technology or a specific transmission band which is usually the case for wireless radio networks. Thus, we provide a lower bound valid for any acoustic underwater network for the case of Gaussian noise.

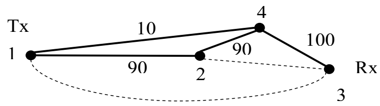

Figure 3: Selected subgraph for scheme (1) with values of z_{iJ} to provide a unicast rate of 100. Dashed lines represent unused transmission ranges, e.g. used hyperarcs with node 1 as the starting point are 1{2},1{2, 4}, but not 1{2, 3, 4}

5. NUMERICAL RESULTS

We present numerical results for unicast transmissions, which is a particular case of the multicast problem, to 1) illustrate characteristics of the lower bound to transmission power under different data rates and node density scenarios, and 2) compare the lower bound to implementations of a routing with link-by-link acknowledgements scheme and a network coding with implicit acknowledgements scheme [3] using an ALOHA-like MAC layer. For these schemes, nodes are assumed to have perfect knowledge of the network, i.e. location of the remaining nodes, allowing them to set their transmission power and bandwidth to ensure a fixed SNR level for link transmissions. The transmitter and receiver are chosen randomly. Let us explain in more detail each of the schemes.

1)Network coding in rateless fashion with implicit ACK: For a concatenated relay network as in Figure 4, the path between a source and sink node is fixed, and includes all relay nodes. If a node b is closer than node a to the collecting node, a is said to be upstream with respect to b, and b is said to be downstream with respect to a. This problem was studied in [3]. We use a similar scheme as in [3] with the following modifications needed for a two-dimensional scenario. First, subgraph selection [5] with linear and separable cost functions are used to determine the active hyperarcs in the network and the rate transmitted through each hyperarc. The cost function of each hyperarc is computed based on the approximate formulas for transmission power, bandwidth and capacity for a fixed SNR level [1]. The weight of each hyperarc is given by $P(l, SNR)/B(l, SNR)$. For optimal modulation, i.e. Gaussian signaling, the weight for each hyperarc is $P(l, SNR)/C(l, SNR)$, where $C(l, SNR)$ is the function of capacity related to the pair (l, SNR). We assume that the coding is over very large number of data packets. Second, once the subgraph has been selected, if several hyperarcs share the same transmitting node, this node will randomly choose the hyperarc to use. This choice is performed based on the fraction of data rate the optimization problem assigned to each of the hyperarcs. Let us consider the network in Figure 3 as an example, where dashed lines represent unused hyperarcs. If node 1 transmits to both node 2 and 4, but the optimization states that a rate of 90 units should be sent through hyperarc z_{12}, while 10 units through hyperarc $z_{12,4}$, then when node 1 transmits it will do so 90 % of the time to reach node 2 only, and 10 % of time using enough power to reach nodes 2 and 4. Finally, to determine which nodes are upstream and downstream of each node in the subgraph we follow the following heuristic. If the subgraph is a single path, the choice is clear. If there are multiple paths, we start by ordering the nodes starting at the transmitter and looking at the nodes directly connected to it in the optimal subgraph. These nodes are ordered as follows: the node associated with the link with higher data rate from the transmitter is considered to be directly downstream from the source node, the node with the second highest data rate is considered to be downstream with respect to the previous one, and so on. In Figure 3, 2 is directly downstream of 1, and 4 is downstream of 2. Once all nodes connected to the transmitter (let us call this set of nodes S) are ordered, we proceed to order the nodes connected to S by a similar procedure as described before. In the example, $S = \{2, 4\}$ and the nodes connected to it are $\{3, 4\}$. If a node connected to one of the nodes in S has already been ordered, as node 4 in the example, the link is discarded keeping the previous order of the nodes. We update S with the nodes that were connected to S and not previously in it, until we reach the receiver. For the network example in Figure 3 the ordering is 1,2,4,3.

2)Routing using link-by-link acknowledgement: This scheme was explained in [3] for nodes in a concatenated relay network. In order to use the same scheme in the two-dimensional scenario, we compute the shortest path between sink and source nodes before starting data transmission. The weight of each link is computed based on the approximate formulas for transmission power and bandwidth for a fixed SNR level in the same fashion as the cost function per link of scheme (1). In terms of the physical layer, schemes (1) and (2) use both PSK modulation and Gaussian signaling assuming that the encoding is over a large number of bits. We deal with the SNR requirement using the approximate model in [1]. When PSK modulation is used, the probability of packet error due to noise over the link from node i to j is obtained from the probability of bit error by $1 - (1 - P_{\text{bit}})^n$, where n (1000 in the simulations) is the number of bits in the packet, and P_{bit} is computed using the standard PSK bit error probability. Nodes farther away from the transmitter

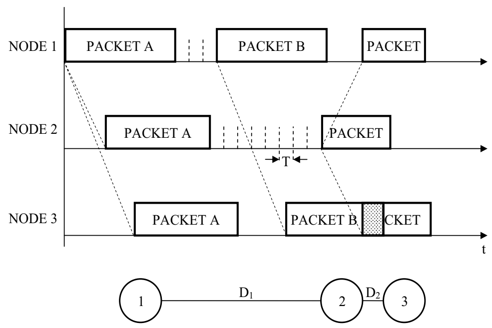

Figure 4: Medium Access Protocol for the comparison schemes

have some probability of receiving the packet correctly. For Gaussian signaling, the probability of packet error due to noise is considered to be zero for all nodes in range, and 1 for all nodes further away. In terms of the MAC layer, schemes (1) and (2) use an ALOHA MAC layer that transmits a fixed number of bits per data packet (n) and uses the optimal transmission band for an SNR requirement per link. Thus, the duration of the transmitted packet depends on the transmission distance [1]. Every node has a probability to access the medium every T units of time following a Bernoulli process. Transmission delay is considered using a typical value of sound speed (1500 m/s). Figure 4 shows an example of using this MAC layer for three nodes with $D_1 >> D_2$. In this example, when node 1 transmits a packet to node 2, this packet also reaches node 3. Note that the duration of the packet transmitted from node 1 to node 2 (Packet A) is large compared to the packet transmitted from node 2 to node 3 because of the relation of distance to bandwidth/capacity for a fixed SNR value mentioned above [1]. Figure 4 shows the case when the data packet transmitted from node 2 to node 3 suffers a collision at node 3 with a new packet transmitted from node 1 to node 2. We consider that a collision at any receiver causes a loss of all packets involved in the collision for that receiver.

We present the comparison of the lower bound to the implementations of a routing (scheme 2) and a network coding scheme (scheme 1) performed for a fixed area of deployment with increasing number of nodes. Figure 5 presents the average transmission power for the various schemes considered transmitting at approximately 1 kbps with nodes deployed randomly in a 1x1 km^2 square. These results are compared to the lower bound of transmission power for $R = 1$ kbps, computed as an average over a large number of deployments. This figure shows optimal signaling (Gaussian signalling) and a PSK modulation, which illustrates that close to 6 dB in the gap between a PSK mod-

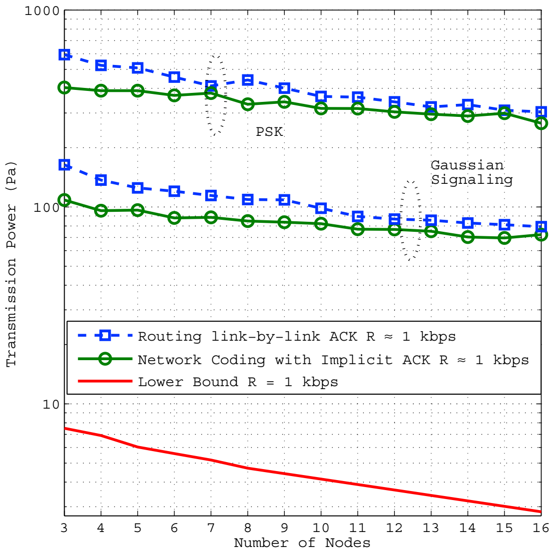

Figure 5: Average transmission power of networks with nodes deployed randomly in a 1x1 km^2 square. Network schemes operate at SNR = 10 dB with data rate $R \approx 1$ kbps and the lower bound for transmission power is presented for $R = 1$ kbps

ulation and the lower bound is due to the choice of the modulation. Notice that the gap between the average transmission power for Gaussian signaling and the lower bound of $R = 1$ kbps in Figure 5 is about 11 dB for scheme (1) and 13 dB for scheme (2). Also, it shows that this gap is maintained for a small number of deployed nodes, and increases for larger values. This is explained by an increased collision probability and, in the network coding scheme, the need of explicit ACK to avoid excessive transmissions. Some part of the gap is related to the MAC protocol used. Another is related to the 10 dB SNR requirement, which is used for practical implementations. For higher SNR's, we expect the average power of the schemes to scale in dB by the same amount increased by the SNR, e.g. 20 dB SNR would correspond to a curve 10 dB above the current result, because packet error and collision probabilities are similar for high SNR. For lower SNR's, the relationship is more complex. Packet error and collision probabilities will be larger due to SNR, increased packet durations, and a larger number of retransmissions.

This figure also shows that there is an effect of diminishing returns in the lower bound. With respect to deployments of 3 nodes, note that to reduce transmission power by 20 % or 40%, it requires 2 and 5 nodes, respectively, to be added to the network. Thus, it becomes harder to reduce transmission power by 20% by adding nodes.

6. CONCLUSIONS

A formulation to obtain a lower bound for the transmission power for the multicast scenario using network coding in underwater acoustic networks is presented. In this formulation, interference between nodes is not considered. Subgraph selection [5] is used to establish minimum-cost multicast connections in order to compute the minimum power consumption. The no-interference assumption in the underwater scenario is justified for low multicast rates, and randomly placed nodes with distances under 10 km between each other. From a network optimization view point, this assumption makes the cost function separable. Our lower bound has no assumption on technology or a specific transmission band, and is thus valid for any acoustic underwater network for the case of Gaussian noise.

Convexity of the complete model is proven. Since the complete model that relates transmission power, capacity, distance and transmission band is complicated, an approximate model presented in [4] is used for the computations. We present conditions that ensure convexity of the approximate model.

Finally, this formulation can be used to determine the gap of different medium access protocols and network schemes to the minimum transmission power for some multicast rate in underwater networks. An example with a routing and a network coding scheme is presented in the numerical section.

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