

Delay Improvements from Multiple Wavelengths in an Optical Folded Bus

by

Cécile Le Cocq

Submitted to the Department of Electrical Engineering and Computer
Science

in partial fulfillment of the requirements for the degree of

Masters of Engineering in Electrical Engineering and Computer
Science

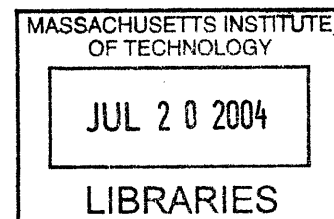
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Abstract

In this thesis, we compared the delay of a single wavelength local area optical network and a multiple wavelength local area optical network. We used an optical folded bus as the architecture for the LAN and analyzed the system's delay for both time division multiplexing and a reservations. The multiple wavelength system is assumed to have a very limited number of wavelengths and therefore does not apply a typical wavelength division multiplexing. The analysis of the delay involves mostly queueing analysis, with extensive use of Laplace transforms and Z-transforms. The purpose of this comparison is to decide whether the improvements in delay are worth the extra costs of increasing the number of wavelengths in optical LANs.

Thesis Supervisor: Muriel Médard
Title: Associate Professor

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Chapter 1

Introduction

1.1 Background and Significance

There are three main goals when designing a network: low delays, high throughput and robustness. A well designed network must reflect the area it is spanning and the type of data it is transmitting. Current technology applies a mix of optical networks and electrical networks. This is a direct result of the advantages and disadvantages of optical networks. Optical fiber started replacing copper across country in the 1980s. Optical fiber is cheap to manufacture in large quantities. However, routing and switching data in the optical field is difficult. It is still expensive and inefficient. Therefore data is converted into electronic form for all the routing and switching. Current work is being done to lower the costs of optical routers so that data may remain in the optical field and keep the rates higher by avoiding the electronic bottleneck. Optical switching is very difficult to do as it is much harder to delay an optical signal than an electronic one.

Results from advancements in technology have led to faster and higher capacity networks. However a network can only be efficient if two major issues are resolved. The first is designing the architecture of the network. This includes node features, topologies and routing. In this thesis we will be focusing on one type of architecture commonly used, the folded bus. The second issue is designing an effective medium access control (MAC). This is an algorithm which avoids interference from other nodes

when data is sent. The ideal MAC for a certain networks depends on the data. The data going through the network is usually unpredictable. We can assume that some information is known, such as the average rate of the data flow through a network, and in certain cases we also know a general pattern. For example, the data might have a constant flow, a bursty flow or a periodic burst pattern. Any such knowledge should be taken into account when deciding on the appropriate MAC. We will later discuss two different MACs and when they are applied to the network.

1.2 Local, Metropolitan and Wide Area Networks

It is very difficult to analyze how data is transmitted across the world. Therefore networks are usually analyzed in subdivisions. One subsection is the wide area network (WAN). This subsection deals with transmitting data from one city hub, to another perhaps across the country or world. For example, algorithms that route data from San Francisco to Boston only deal with the WAN and do not worry about where the data goes once it reaches the city. Next is the metropolitan area network (MAN), which involves a large backbone connecting a single city or large campus together. Last is the local area network (LAN), which involves the section of the network connecting a small groups of users to a main backbone network. Many LANs will branch off of one MAN, and many MANs will branch off of one WAN, as in Figure 1-1. LANs cover a smaller number of users, for example from one single office building or dorm. With multiple LANs attached to each MAN, minimizing cost and complexity is a higher priority for a constructing a LAN verses a MAN. On the other hand, the number of users depending on a single MAN is larger than for one LAN, and therefore high data rate and reliability is a higher priority for MANs. The issues for a WAN are more focused on the costs and delays existing with sending data over great distances.

Two different methods to route the data on a network are circuit switching and packet switching. With circuit switching, each session is allocated a fixed fraction of the capacity on each link along its path. The path for each session is fixed. With

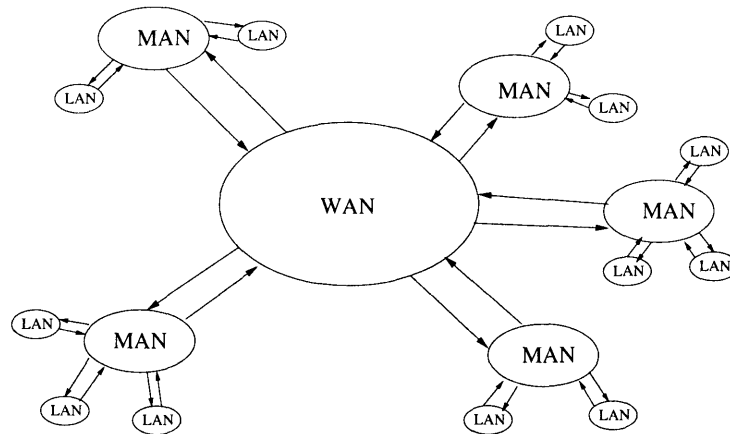


Figure 1-1: Network Hierarchy

packet switching, the data is broken up into packets. A header, which contains information on the destination of the packet, is attached to each packet. The route is chosen on a packet by packet basis and therefore the packets may arrive in a different order from when they were sent. When the data is already broken into packets, circuit switching can be emulated. This is known as virtual circuit packet switching. With virtual circuit packet switching, all packets from a session follow the same path. The advantages of virtual circuit packet switching over packet switching are that the packet order remains intact throughout the route, the required header is smaller, and routes are only computed once per session. The disadvantages occur when the data is bursty. Portions of the network capacity are reserved for sessions which contain long pauses while other busy sessions cannot take advantage of this unused capacity. When routing data on a LAN, it involves few users which may have very bursty data, whereas on a WAN there are so many users that it has an even flow despite the bursty data of individual users. Therefore, routing schemes for LANs use packet switching whereas WANs use virtual circuit packet switching. For the MAN, having a medium number of users, there exist schemes which use both methods.

1.2.1 Architectures for LANs

With different requirements, it is not surprising that LANs and MANs are built on different architectures and use different protocols and schemes. LANs favor cheaper and simpler architectures than MANs, such as some types of buses and rings. Buses link users together in a linear fashion. A single bus sends data in one direction. The first node at the head of the bus is known as the head node whereas the last node is known as the end node. To allow nodes to send data in both directions, a second bus can be made allowing the flow of the data in the other direction. This is known as a dual bus. Another solution is to have the bus fold over at the end of the line of nodes, and go back through the same nodes in the opposing direction (from end node to head node). This is the architecture we will be focusing on and is known as a folded bus. It will be explained in detail in the next chapter. Three types of buses are shown in Figure 1-2. There are specific optical bus schemes which exist all of which use packet switching, such as distributed queue dual bus (DQDB) [1, 5, 11, 12, 18, 22, 23], cyclic reservation multiple access (CRMA) [17, 22, 23, 24], helical LAN (HLAN) [3, 13, 22, 23, 26] and optical reservation multiple access (ORMA) [16, 22, 23]. The main purpose of these schemes are to allow the nodes fair access to the bus. One could imagine a greedy style algorithm where a node transmits queued data onto the bus as soon as it is empty. This type of method has good throughput, however the delays in the queues are not balanced between the nodes. Nodes at the end of the bus are forced to have longer delays while nodes at the head of the bus have virtually no delays. DQDB is defined by IEEE 802.6, is a standard for MANs and also used in LANs. It is a scheme which implements a dual bus where every node receives and transmits on both unidirectional slotted buses. DQDB has two control fields for the access protocol: the busy bit, which indicates whether or not the slot is being used and the request field to indicate whether a node has data in its queue. Reservations for one bus are made on the other bus, and a counter counts these reservations to keep track of the next free slot. With these two control fields, data is sent on a first-in first-out order approximately independent of its path. CRMA, introduced after

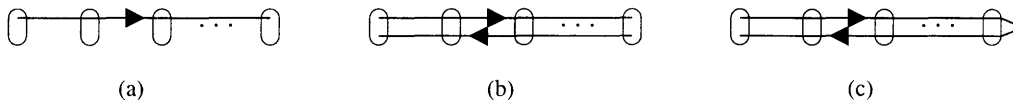


Figure 1-2: Example of buses: a) single bus; b) dual bus; c) folded bus

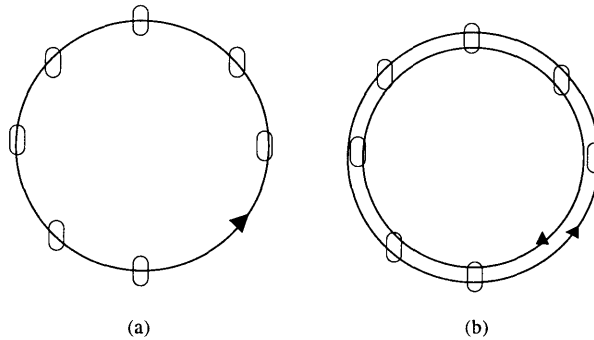


Figure 1-3: Example of rings: a) unidirectional ring; b) bidirectional ring

DQDB, is a scheme for both LANs and MANs and uses a folded bus. In CRMA, the nodes access the bus according to cycles of slots. At the beginning of each cycle, the nodes reserve slots. Then the headend allocates the cycle to be sufficiently long to satisfy these reservations. HLAN uses time division multiplexing, explained in the next chapter, on either a helical bus or a folded bus. Like CRMA, ORMA uses a folded bus architecture. ORMA uses three data channels: the data channel for sending and receiving messages, the reference channel and the select channel, these two being used to make the reservations.

A ring can be thought of as a bus with the added link from the end node to the head node. All the nodes are connected together and even with a unidirectional bus running through all the nodes, any pair of node can communicate with the closed loop. To help robustness, one can add a second ring, through all the nodes, in the other direction. Figure 1-3 shows the two different types of rings.

Stars have a central hub connected to each node of the network, as shown in Figure 1-4 (a). When one node sends data, the data goes to the hub, and then is directed from the hub to the correct node. A significant feature for a star is that all data goes from the sender node to the receiver node almost directly, passing through

only the hub on the way, as opposed to the ring and bus architecture where data passes through many nodes on the way to the receiver. Schemes for these usually involve a passive optical broadcast star [14, 15, 22, 23], and implement WDM (as explained in Section 1.3).

1.2.2 Architectures for MANs

MANs being larger scale networks may require more complicated architectures, such as meshes. This is not always the case as they do use bus and ring layouts. Fiber distributed data interface (FDDI, IEEE 802.3) [22, 23, 27, 28], synchronous optical network (SONET) [25], synchronous digital hierarchy (SDH) [25], and resilient packet ring (RPR, IEEE 802.17) [2, 22] are all schemes which implement bidirectional rings. FDDI, the first to be implemented, operates at 100 Mbps. It uses a token which rotates around the ring. The node with the token is the only node that may send data, therefore eliminating collisions. FDDI works either as a packet switching or a virtual circuit packet switching scheme as MANs can benefit from both styles as discussed earlier. Because of the high cost of hardware components in DQDB and the lower throughput of FDDI, these schemes were superseded by SONET, SDH and perhaps RPR in the near future. SONET is currently implemented in the US and SDH is very similar and implemented in Japan and Europe, while RPR is currently being reviewed to be passed as a new standard. Both SONET/SDH and RPR use dual optical ring. A large difference in RPR is that it uses packet switching. This packet-based scheme is better suited to some MANs with bursty data than SONET. To span the large number of nodes in the network, multiple rings can be created to cover the network. The individual rings share nodes with each other to allow traffic to pass through the whole network [19].

New shapes are sometimes implemented for the larger networks such as meshes. A mesh, as in Figure 1-4 (b) is a more complicated architecture. Unlike the bus, ring or star, each of the nodes may have multiple paths arriving or leaving the node. This helps with robustness as there are often multiple paths connecting two nodes. If a link or even a node is down, the other nodes are capable of rerouting data around

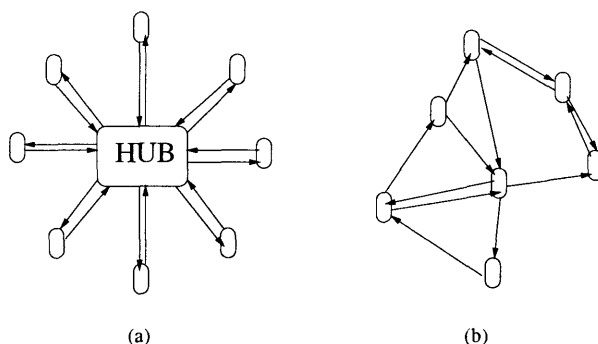


Figure 1-4: Example of: a) star; b) mesh

the failed component. However, this added flexibility adds complexity. Each node is now similar to a mini hub in the star network, it must be able to route data through to the correct path. The schemes are more complex and harder to analyze since the shape of the graph is not as simple.

1.3 Wavelength Division Multiplexing

One of the main goals in the field of optical networks is to increase the rate of data transmitted. One way to achieve this is to increase the number of wavelengths used to transmit data. Transmitting in multiple wavelengths is known as Wavelength Division Multiplexing, or WDM [25]. In a single optic fiber, many users may send data simultaneously by each using a different wavelength. WDM is used in current metropolitan area networks (MANs) with a range of 50 up to 160 different wavelengths. These wavelengths are separated by as little as 0.4 nm (50 GHz).

1.3.1 Components needed for a WDM system

WDM requires different devices to be used when building the network compared to the simpler single wavelength networks. First, with each wavelength requiring its own laser, many very well tuned lasers are required to generate the specific wavelengths. Second, amplifiers are required to amplify data on each wavelength without distorting it. Third, switches and routers need to differentiate between the different wavelengths.

Last, there are multiplexers, de-multiplexers, add/drop multiplexers that are now used to join, separate, add and drop wavelengths from the system. Another issue with WDM is the added complexity in routing. For example, if two different paths share a link and data is being sent on both of these paths in the same wavelength, then without preventive measures, there will be a collision when the data reaches the shared link. One solution is to have switches which allow data from one path to switch wavelengths for that link. Another solution is to delay the data from one path while the conflicting data passes that link. Both these solutions have drawbacks such as cost, delay and complexity.

The components required to make a multiple wavelength network function exist. These are found in some current WDM systems, however research is still being done on all components to lower the costs.

Vertical Cavity Surface Emitting Lasers

The vertical cavity surface emitting laser (VCSEL) [10] is a laser diode which may revolutionize fiber optic communications by bringing to the field a cheaper and more efficient laser than the traditional edge emitting diodes. The older diodes emit coherent light parallel to the boundaries between the semiconductor layers, while VCSELs emit their coherent energy perpendicular to the boundaries between the layers. VCSELs are currently constructed to emit energy at 850 nanometers (nm) and 1300 nm. These wavelengths are in the infrared portion of the light spectrum. VCSELs are manufactured with the materials including gallium arsenide (GaAs), aluminum gallium arsenide (AlGaAs) and indium gallium arsenide nitride (InGaAsN). There are several advantages to VCSELs which make these lasers so promising. They are cheaper to manufacture, easier to test and more efficient than edge emitting diodes. They require less electrical current to function and produce a more circular beam. There are two improvements currently being worked on with VCSELs which will enable them to be used in LANs. The first is constructing a VCSELs which emits 1550 nm. 1550 nm is a wavelength commonly found in optical networks. This is due to the fact that it is the wavelength at which the popular Erbium Doped Amplifier

(EDFA) operates. The second is the work being done on composite resonator VCSELs (CRVCL). CRVCLs have the potential ability of multiple wavelength lasing. Currently, there are BiVCSELs [6] which can emit light in two different wavelengths.

The combined costs of the new or more complicated components causes the WDM networks to be magnitudes more expensive than single wavelength systems. The added cost is justified when the increase in data rate is valuable for the network. MANs, which are large but few in number, benefit from the increase data rate of WDM and the cost is acceptable. While, for the smaller and numerous LANs, the increase in price for having a faster network is not justified, since the network only supports a few users.

However, with the VCSEL technology flourishing, improvements in optical networks are forthcoming. With the relatively low cost in manufacturing of VCSELs, as compared to other current lasers, we can expect LANs to be greatly effected. With VCSELs, LANs may be able to afford more than one wavelength. Instead of having only single wavelength systems, they can implement a simple version of WDM with a couple of wavelengths. However, the architecture of LANs will remain the same as before. Analyzing delays and throughput of WDM on LAN architectures, such as a folded bus has now become relevant. The first step is to construct a LAN using the BiVCSEL, therefore taking advantage of two wavelength rather than one. The BiVCSEL would increase the cost of the system, but by much less than past two wavelengths systems, and would increase the throuput up to twice that of the single wavelength system.

Chapter 2

Modelling and Design

2.1 The Problem Statement

In the past, the focus for optical networks was on making single wavelength devices for LANs or very large multiple wavelength devices for MANs. Recently however, there are improvements being made on small multiple wavelength devices such as the VCSEL introduced in Section 1.3.1. These devices support a relatively small number of wavelengths (two at the present, perhaps four in the near future). Because VCSELs are simple and cost efficient to manufacture, building a network with few wavelengths has a new advantage: affordability. We are interested in figuring out how using a small number of wavelengths in a LAN would compare to the single wavelength system and whether the increase in efficiency will be worth the increase in cost. We will take the first step by analyzing a common architecture used for a LAN, the folded bus.

2.2 Folded Bus

Different network architectures are used to relay information to multiple nodes. One of the simpler topologies is the folded bus. The way a folded bus works is that there is a single unidirectional bus that goes through all the inputs of each of the nodes. After the last node, the bus folds around and goes through all the outputs of the

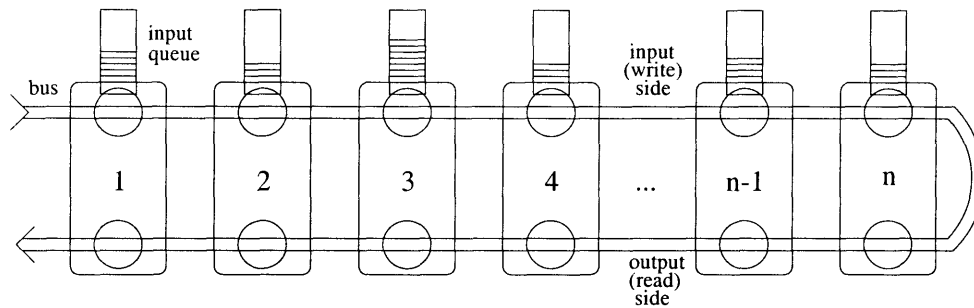


Figure 2-1: Example of a folded bus

nodes. The data therefore passes the outputs of the nodes in the opposite order that it passed their inputs. In this layout all the nodes can send data to any of the other nodes using the same bus.

As in Figure 2-1, we name all the nodes 1 through n , in the order of that the bus passes through the inputs. If node i wants to send a packet to node j , node i writes the packet on the bus and the packet must pass the inputs of nodes $i + 1$ through n and then pass the outputs of nodes n through $j + 1$ until it reaches node j .

A folded bus is a simple option for LANs and certain MANs as commented on in section 1.2.1 and 1.2.2. One characteristic of this architecture which makes it simple is that there are no routing decisions to be made. Every node on the bus can connect to any other node on the bus by exactly one path. On the other hand, on a bidirectional ring, one node may send data to another specific node on two different paths, clockwise and counter clockwise. On a star network there is only one path that the data can take to connect two nodes, through the hub. However the star adds some complexity with the hub which is a more complicated node. The hub must route the incoming data to the correct node and therefore serves as a type of switch. Meshes being the more complicated structure incorporates multiple paths between nodes and multiple nodes needing the capability to route data.

Many routing schemes use either the folded bus or the dual bus as explained in Chapter 1. One of the the most common and extensively analyzed is DQDB which is now standardized as the IEEE 802.6 metropolitan area network. HLAN is designed primarily for a helical bus can also be implemented for the folded bus [13]. CRMA and

ORMA both use a folded bus. CRMA is a single wavelength system while ORMA has three wavelengths, though only one is used to transfer data while the other two reserve the bus for the nodes. Folded bus schemes can also be used with tree architectures. A tree can be viewed as a single array where each unit on the array is actually an array itself. Therefore when connecting a tree, one can use a type of folded bus. The bus spans the tree in a depth first search (DFS) order and loops back at all the leaves of the trees. With all these routing schemes currently using folded buses or close relatives of it, we chose the folded bus as the architecture to analyze.

When using the folded bus in an optical network, we have a few limitations. Two nodes cannot write onto the bus at the same location in the same wavelength, otherwise one of the packets will be dropped. Also one output cannot read more than one packet at a time. Therefore, doubling the number of wavelengths of a system does not necessarily double the throughput.

To avoid collisions there exist different MACs (media access control) such as TDM (time division multiplexing), or a reservation system. Combinations of the two are also possible. These methods reserve locations on the bus for specific types of packets when the bus is free, and prevent collisions. They must also guarantee fairness between different node's access to the bus. If a node was to write on the bus whenever the bus was empty, the head nodes would have little to no wait while the nodes at the end would all be backlogged, forced to wait for earlier nodes to send their data. Each method reserves the bus differently and has its advantages and disadvantages. Using TDM, reservation, or combinations of those, will prevent collisions but also force data to wait at the input buffers. The data is delayed from the moment it arrives at the input node until it is in the correct reservation slot. We can calculate the expected waiting time for an incoming packet to the system. Minimizing the expected wait, while keeping the system simple and fair will give us the best result.

Comparing the single and multiple wavelength performance will be based on the expected wait for their packets. Since different MACs are used today, we will evaluate the expected wait for each of these using both methods TDM and reservation. Once these are evaluated, we will be able to decide for each MAC whether the increase in

efficiency from using multiple wavelengths is worth the higher cost.

2.3 Modelling the Folded Bus as an M/M/1 system

There has been a very thorough amount of research done on queueing analysis. The expected wait for a packet in an M/M/1 system has already been calculated. An M/M/1 system is defined to be a system with memoryless arrivals (Poisson), memoryless service times (exponential) and one server. We can relate the folded bus to an M/M/1 system. We will think of the bus as the server, since we only have one bus, we relate it to a system with one server. Therefore the space a packet takes on the bus will be equivalent to the service time of the M/M/1 system. With a constant bus rate and packet sizes being exponentially distributed, the time the packet spends on the bus, or the service time, is exponentially distributed. The arrivals to the nodes in the folded bus will be approximated as a Poisson arrival. Because there are many nodes sharing the same bus, say n nodes, it is equivalent to an M/M/1 system with $n^2 - n$ users. Each user represents a path a packet may take.

When reserving the bus, we reserve it for each user. The scheduled reservations prevent collisions from occurring on the bus and inform the nodes when to write on the bus and when to read from it. In a general M/M/1 system, we would reserve time slots. However in a folded bus, sending packets in the same location on the bus causes collisions rather than sending at the same time. Therefore when referring to making reservations, we are now reserving locations on the bus rather than time slots.

For analyzing a folded bus with multiple wavelengths, we will approximate the system to an M/M/1 system using Frequency Division Multiplexing (FDM). We treat different wavelengths as different frequencies (this is not hard to do since $\lambda = \frac{c}{f}$ and is inversely proportional to the frequency of the the light wave). There are some differences however, in FDM one usually assigns one frequency per user, therefore eliminating collisions. In our model we have a limited number of wavelengths available,

much fewer than the number of nodes on the folded bus. Therefore each wavelength is shared by multiple users.

2.4 Stability of the Folded Bus

In this thesis we will be focusing on how to analyze the waiting time of packets on the folded bus. However, before the analysis begins, it is important to understand whether the bus remains stable using the scheduling we will analyze. A non stable bus would have a nonzero possibility of having a packet with infinite waiting time. Assuming that the arrival rate of the packets is manageable, we know that the scheduling of the bus will not affect its stability. The different schedules we will be analyzing will only affect the order the packets are processed in. Problems in stability would only arise if we introduced a feedback loop, such as colliding packets that are reentered in the queue. A feedback system would affect the packet arrival rate and we would need to prove that our system remains stable. However, we will not be introducing any feedback into our schedules, and so we may proceed to analyze time-division multiplexing and reservation systems content that the bus will remain stable.

Chapter 3

Time Division Multiplexing

3.1 Description of Time Division Multiplexing

The optical folded bus we are studying is a multiplexed system. This is because the nodes on the bus must share the bus' resources to send packets. Even with multiple wavelengths, we are analyzing a system with limited wavelength where each user is allotted less than a full wavelength. Therefore there must be a controlled way of allocating the bus between the nodes. We can multiplex a system in two ways. The first is a set division of resources, independent of the current state of the server, while the second is takes into account feedback from the queues at the user. Time-division multiplexing (TDM) is an example of the first case, while reservation, discussed in Chapter 4, is an example of the second.

In TDM, the bus is divided into time slots. A time slot is a section of the bus where only one user may write onto a wavelength on the bus, thereby preventing any collisions. A user may not write onto the bus until its time slot is ready. The different user's slots cycle through in order. In between two slots of a single user, all other users must have exactly one slot. The size of the time slots are proportional to the average amount of data a user has to send. When the users have equal average arrival rates and packet size, then all time slots are the same size. Figure 3-1 shows an example of a bus split up into time slots using TDM.

TDM is applied in HLAN, a MAC used on helical and folded buses, as discussed

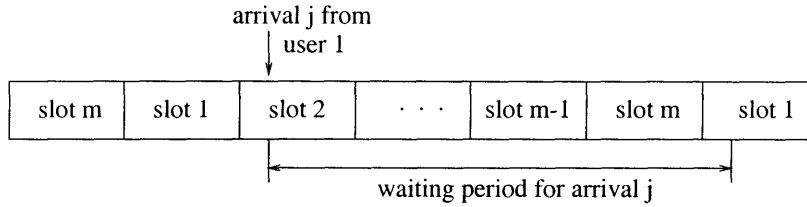


Figure 3-1: Example of System using TDM

in Section 1.2.1. TDM is a good choice for it's simplicity to construct and analyze, however some efficiency maybe lost when comparing to multiplexing methods which take into account feedback from the users. This is especially evident when the arrivals of the packets are bursty. For example, in TDM a user with a very full queue, right after a burst of arrivals, must wait until its slot is up before writing onto the bus, even though the slots currently on the bus might be for users with empty queues. The portions of the bus which are empty while some users have packets in queue, can be better utilized with a different procedure.

3.2 Analyzing TDM using M/M/1

The expected time for a packet for an M/M/1 system with n users using TDM has already been analyzed. λ represents the average arrival rate for each user, and μ is the average service time for one packet. To calculate the total time T , we treat the system like n different M/M/1 systems, one for each user, each with λ packet arrival rate and μ/n service time. The reason we can use this trick, is if we assume each of the user receives on average equal traffic, then each user can only use $1/n^{th}$ of the server's time. Therefore it is equivalent to having one user with packets that are n times the size. Since the waiting time of one packet for a M/M/1 system is:

$$T_{M/M/1} = \frac{1}{\mu - \lambda}. \quad (3.1)$$

We can derive the expected time for one packet to go through a TDM system with n users to be:

$$T_{TDM} = \frac{1}{\mu/n - \lambda}. \quad (3.2)$$

3.3 Analysis of Waiting time for Single Wavelength System Using TDM

Next we analyze the expected wait for packets on the folded bus with a single wavelength. This is the simplest system. Since there is just one wavelength allowed on the bus, we will compare this one wavelength to a single M/M/1 system. Instead of reserving server time for each user with the time slots, we will now reserve space on the bus for an specific packet path, or an input node. Using TDM on the folded bus will prevent two nodes from writing on the same location on the bus and causing a collision. We are interested in the average time a packet takes from arrival to the system and until it is written onto the bus. We do not compare the time a packet arrives and leaves the system, since once the packet is written onto the bus, there is a determined propagation delay.

3.3.1 Equal Average Arrival Rate and Packet Size

It is always best to look at the simplest example first. We will first use one generalized average packet size, and arrival rate. Then we will later use specific arrival rates and packet sizes for the different paths. The expected arrival rate for packets to a user is defined as λ , the average rate of writing a packet onto the bus is μ . μ is linearly related to the average package size ($\mu = \text{average packet size} \cdot \text{bus rate}$). There are n nodes on the folded bus, and therefore $n^2 - n$ different paths a packet can take (we will assume a packet does not want to ride the bus just to get back to where it started). T is the average time a packet will take to be written onto the bus. Each user in the system corresponds to a path on the bus. Therefore in one slot, all the packets will have the same input and output nodes. We can use the equation 3.2, as

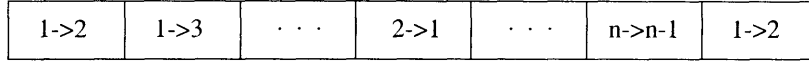


Figure 3-2: Equal sized slots by path using TDM on single wavelength bus

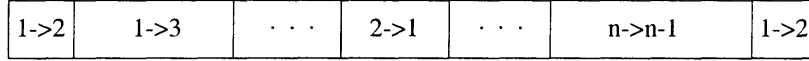


Figure 3-3: Different sized slots by path using TDM on single wavelength bus

we can directly relate this system to TDM with $n^2 - n$ users. An example is given in Figure 3-2. Therefore we come up with the equation:

$$T = \frac{n^2 - n}{\mu - (n^2 - n)\lambda}. \quad (3.3)$$

3.3.2 Different Average Arrival Rate and Packet Size

The next step is to calculate T when λ and μ are different for each path. This leads to a system where one path is more popular and has more packets, and another path may have on average larger packets. Therefore we specify λ_{ij} and μ_{ij} to be the λ and μ corresponding to the packets going from node i to node j . Once we make this distinction we must remember that the size of the slots should be proportioned correctly and no longer all equal. First we separate the slots by the path of the packets as we see in Figure 3-3. Therefore, slot ij is reserved for packets going from node i to node j . Paths that have a high arrival rate should have larger reservation slots, however paths with larger packets also need larger slots. To proportion the slots fairly they will be proportional to:

$$\rho_{ij} = \lambda_{ij} / \mu_{ij}. \quad (3.4)$$

Therefore the corresponding ratio term is:

$$\alpha_{ij} = \frac{\rho_{ij}}{\sum_k \sum_{l \neq k} \rho_{lk}}. \quad (3.5)$$

wavelength									
1	1->2	1->3	...	1->n-4	1->n-3	1->n-2	1->n-1	1->n	2->1
2	2->3	2->4	...	2->n-3	2->n-2	2->n-1	2->n	3->1	3->2
3	3->4	3->5	...	3->n-2	3->n-1	3->n	4->1	4->2	4->3
4	4->5	4->6	...	4->n-1	4->n	5->1	5->2	5->3	5->4
5	5->6	5->7	...	5->n	6->1	6->2	6->3	6->4	6->5

Figure 3-4: Equal sized slots by path using TDM on bus with 5 wavelengths

Now that we have α_{ij} defined, we can treat this system as one M/M/1 system for each path where the corresponding λ is λ_{ij} . The new μ is $\alpha_{ij} \cdot \mu_{ij}$ because each path is only allotted α_{ij} of the bus and its packets have an average service time of μ_{ij} . We can use the M/M/1 equation for T to come up with:

$$T_{ij} = \frac{1}{\alpha_{ij} \cdot \mu_{ij} - \lambda_{ij}}. \quad (3.6)$$

3.4 Analysis of Waiting time for Multiple Wavelength System Using TDM

When we start to analyze the folded bus system with multiple wavelengths, there are new complications to deal with. Now, more than one packet is allowed on the bus at once on different wavelengths. Different nodes may therefore write onto the bus in the same location but on different wavelengths. The two restrictions are that a node may only write onto the bus in one wavelength at a time, and a node can only read from the bus one packet in one wavelength at a time. We will denote the number of wavelengths in the system as w ($1 < w < n$).

3.4.1 Equal Average Arrival Rate and Packet Size

When we assume equal average arrival rates and packet sizes for all paths, the bus is separated into even slots. We will first divide the bus by paths, as shown in

wavelength					
1	1->2	1->3	...	n->n-1	1->2
2	3->4	3->5	...	3->2	3->4
⋮	⋮	⋮	⋮	⋮	⋮
y	5->6	5->7	...	5->4	5->6

Figure 3-5: Different sized slots by path using TDM on bus with y wavelengths

Figure 3-4. Scheduling the slots so there is no overlap in the input or output node of slots occurring at the same time on the different wavelengths is not intuitive. In Section 3.5, we discuss scheduling the different slots. We find in this section that we are guaranteed to be able to schedule the slots with no conflict, therefore we will analyze the waiting time with out specifying a specific schedule. Once you have the slots organized so that there are no collisions, you have $n^2 - n$ slots spread out onto w wavelengths. We can relate this system to w different one wavelength systems each with $(n^2 - n)/w$ slots working simultaneously. Therefore we come up with a new equation for T :

$$T = \frac{n^2 - n}{w\mu - (n^2 - n)\lambda}. \quad (3.7)$$

3.4.2 Different Average Arrival Rate and Packet Size

When we keep different average arrival rates and packet sizes for each path, the transitions to a different slot will no longer line up on all the wavelengths. In Section 3.5, we discuss how to schedule such slots.

Once again we will start off with separating the slots by the path of the packet. Figure 3-5 shows an example of this type of system. The proportion for the slot of packets going $i \rightarrow j$ is α_{ij} and the arrival rate for that path is λ_{ij} . These are the same α_{ij} , (equation 3.5) and λ_{ij} as we had with the one wavelength system. This time however since the are w wavelengths, the slots have actually $w \cdot \alpha_{ij}$ of the bus' time.

Therefore we come to the equation for the new T_{ij} :

$$T_{ij} = \frac{1}{w \cdot \alpha_{ij} \cdot \mu_{ij} - \lambda_{ij}}. \quad (3.8)$$

3.5 Scheduling slots for multiple wavelengths

Scheduling the slots for a multiple wavelength system can be a little confusing. For a system with equal rates for all packet paths there exists a simple schedule. If we number the wavelengths 1 through w , and have each wavelength k start with path $k \rightarrow k + 1$. Then have them cycle in order. The order is simple: $1 \rightarrow 2, 1 \rightarrow 3, \dots 1 \rightarrow n, 2 \rightarrow 1, 2 \rightarrow 3, \dots n \rightarrow n - 1$. Part of this schedule is shown in Figure 3-4. We can see that the inputs and outputs are always a string of w adjacent nodes (if you consider 1 and n adjacent), and therefore there will never be a collision.

With different rates this simple schedule does not work. In fact, it is not obvious whether an optimal schedule is even possible for all systems. To solve this problem it is very useful to treat the folded bus as a switch. When a slot is on the bus it is like a connection from the input node to the output node. We are only allowed w connections at a time since that is how many slots the bus can handle at one time. For efficiency, we would like to have w connections always on and not have the bus idling. The switch can be represented by a matrix with the input nodes on the rows and the output nodes on the columns. The numbers in the matrix represent the proportion of

bus each slot required. Therefore, the matrix resembles:

$$\begin{matrix} & \begin{matrix} 1 & 2 & \dots & n \end{matrix} \\ \begin{matrix} 1 \\ 2 \\ \vdots \\ n \end{matrix} & \begin{pmatrix} 0 & \alpha_{12} & \dots & \alpha_{1n} \\ \alpha_{21} & 0 & \dots & \alpha_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \alpha_{n1} & \alpha_{n2} & \dots & 0 \end{pmatrix} \end{matrix}$$

We then break up this matrix into state matrices. A state matrix represents a configuration of the switch. This state matrix has all zeros except for the w paths connected during that configuration, which each have a one. The state matrix is then multiplied by the proportion that the switch will stay in this configuration. Summing up all the state matrices returns the original matrix representing the switch.

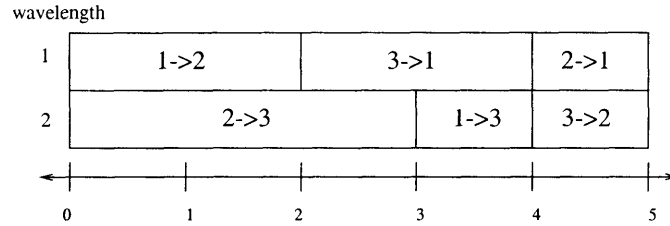


Figure 3-6: Different sized slots by groups using TDM on bus with y wavelengths

For example, if we have a bus with 2 wavelengths and 3 nodes, with the following proportions, they could be separated in the following matrices:

$$R = \begin{pmatrix} 0 & .2 & .1 \\ .1 & 0 & .3 \\ .2 & .1 & 0 \end{pmatrix}$$

$$= .2 \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix} + .1 \begin{pmatrix} 0 & 0 & 0 \\ .0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix} + .1 \begin{pmatrix} 0 & 0 & .1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix} + .1 \begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$$

When drawing this system as a bus with slots, we can see how the matrix decomposition results in Figure 3-6.

There have been many studies done on scheduling a switch and how to achieve this type of decomposition. Birkhoff and von Neumann separately came up with decomposition algorithms which take any doubly substochastic matrix and decompose them into these state matrices [7, 8, 9, 21]. This general algorithm can be used for this case and helps show that a decomposition where the bus is never idle is always possible.

Chapter 4

Reservation

4.1 Description of Reservation

Organizing transmissions from several users into a multiplexed system requires some form of scheduling. The scheduling must maintain fairness between the different users, high throughput and low packet delays. As explained in Chapter 3, time division multiplexing accomplishes this. In TDM, scheduling is independent of the current state of the user's queue and slots for a user are reserved regardless of whether or not the user actually has any packets to send. A different type of scheduling can be arranged so that only users with packets to send are reserved the bus. This type of scheduling is known as reservation.

With the scheduling now dependent on the state of the users, a controller is required. The controller must obtain the status of the different users and then schedule slots on the bus accordingly. Each user has specific time slots dedicated to it. The controller must have a mean of communicating with the users to not only learn their status but also report back to each user when its slot is reserved. There exist two different ways for this communication to take place, the first is for the controller to poll the users and the second, which we will be mostly focusing on, is for the users to request slots from the controller. Polling is one way for the controller to communicate with the users: the controller periodically takes a poll and asks the users their status [4]. There exist different algorithms to decide how and when the controller polls

the users. The first goal for these algorithms is to minimize the delay and a secondary goal is to maintain fairness between the different users. For example, the controller may poll the users by having part of the bus used to send a periodical update of each user's status to the controller. The second method is for the users to decide at what point they need a reservation slot and request one from the controller. The state at which the user requests a reservation is dependent on the size of the queue. Unlike polling this requires an open communication line from each user to the controller at any time. This is because requests will be made with unpredictable timing.

As previously discussed in Chapter 1, there have already been MACs applied onto a folded bus, or related bus architectures, which use reservations. DQDB, CRMA and ORMA are all examples of these.

4.2 Modelled Reservation System on the Folded Bus

There is some flexibility in designing the way the reservations are made. For the model we elected, when the number of packets in a certain user's queue reaches a threshold, the user will request a slot from the controller. The controller must then organize the different requested slots. In the single wavelength system, the controller can set up the slots to be first come first serve. In other words, when a reservation request by a certain user reaches the controller, it enters a queue, and when there are no reservations ahead of it in the queue, the controller reserves a slot on the bus for that user.

For the multiple wavelength system, we can make multiple reservations at once. Therefore if we have w wavelengths on the bus, w reservations can be scheduled at once, one for each wavelength. We come across the problem that it can no longer be first come first serve for the reservations. This is due to the fact that conflicting reservations might be requested sequentially, yet they cannot be given at the same time regardless of the fact that there may be wavelengths free for reservation. Con-

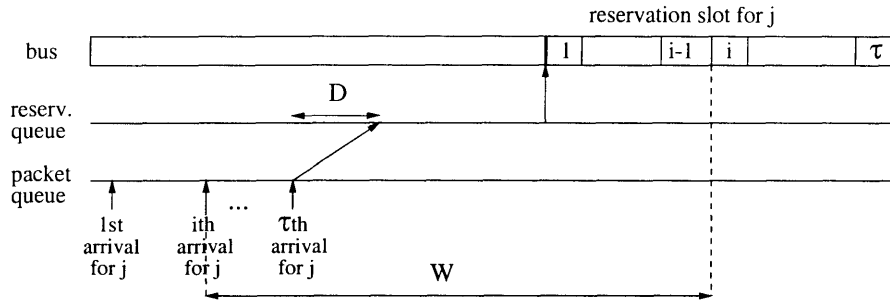


Figure 4-1: Waiting time of i^{th} packet for user n using reservations for every τ packets

flicting reservations refer to reservations for packets with either the same input node or the same output node. To resolve this conflict, one of the conflicting reservations must be pushed back by the scheduler, even if it means allowing wavelengths to go unused.

Reservation can perform better than TDM, especially for irregular packet arrivals. This is because the bus is never reserved for a user with no packets. The only time lost is from making the reservation by communicating to the controller. With bursty data, there are long pauses where a user has no packets and then an unpredictable sudden surge of multiple packets. Since the burst was unpredictable, any set time slot for the user such as in TDM would be unlikely to come soon after the burst and therefore cause a long wait in the queue of the user. However the more constant the arrivals, the better TDM performs. This is because reservation starts to look more like TDM with the even flow. The difference then is that TDM has the reservations done automatically, while in the reservation system, each reservation needs to be made individually. This means time is lost communicating to the controller.

4.3 Analysis of Waiting Time Using Reservations

The expected waiting time of a packet for the chosen reservation scheme is shown in Figure 4-1. We shall be using the variables described in Table 4.1. We will be analyzing the packet delay of a reservation scheme in a similar fashion to the way M/G/1 systems are analyzed in [20]. In analyzing M/G/1, we first define an equation

variable	description
T	waiting time for a packet in queue
λ	average arrival rate for each user
μ	average service rate for one packet
ρ	λ/μ
τ	threshold number of packets for reserving (constant)
D	delay from communicating with controller (constant)
q_n	number of packets in queue after n^{th} reservation slot
v_n	number of arrivals during the n^{th} reservation slot
m	number of users in system
n	number of nodes on the bus
w	number of wavelengths

Table 4.1: Description of Variables Used in Calculating the Expected Wait with Reservations

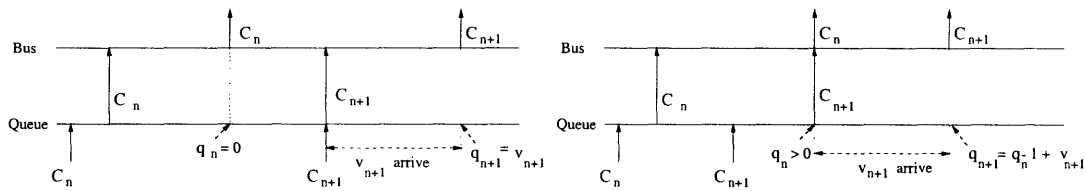


Figure 4-2: q_n for an M/G/1 system

describing q_n , the number in the queue after the service of the n^{th} packet as shown in Figure 4-2, then we find the z-transform of q and its expectation:

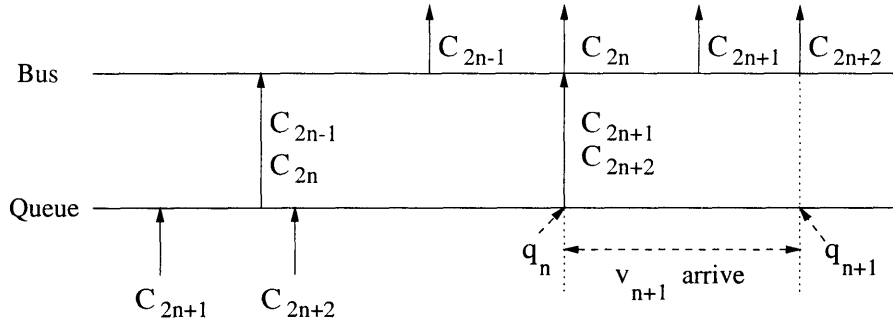


Figure 4-3: Case where $q_n \geq 2$

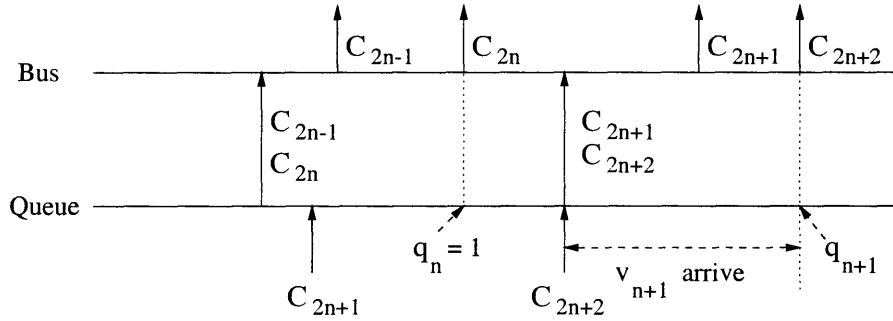


Figure 4-4: Case where $q_n = 1$

$$\Delta_k = \begin{cases} 1 & \text{if } k \geq 1, \\ 0 & \text{if } k = 0, \end{cases}$$

B^* = the Laplace transform of the service time of a packet

$$q_{n+1} = q_n - \Delta_{q_n} + v_{n+1} \quad (4.1)$$

$$V(z) = B^*(\lambda - \lambda z) \quad (4.2)$$

$$Q(z) = V(z) \frac{(1-\rho)(1-z)}{V(z)-z} \text{ Pollaczek-Khinchin} \quad (4.3)$$

$$\bar{q} = \rho + \frac{\overline{v^2} - \bar{v}}{2(1-\rho)} \quad (4.4)$$

4.3.1 Single Wavelength System

Simplifying to one user with $\tau = 2$

We first simplify the problem by looking at a system with only one user to send information. We also simplify the problem by setting τ to equal 2. That is, the user

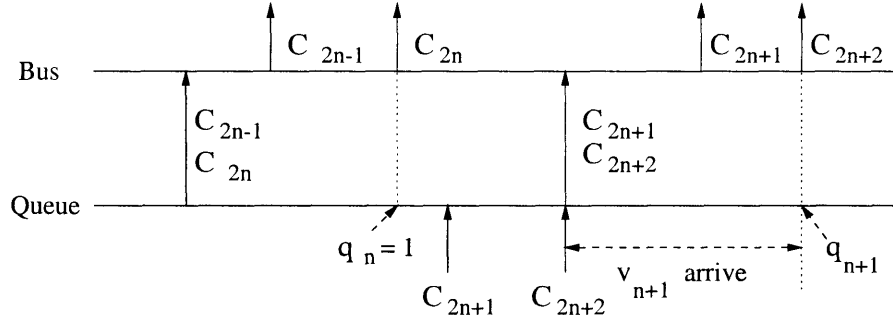


Figure 4-5: Case where $q_n = 0$

only sends packets to the bus two at a time. If there is only one packet in the queue, then this packet waits until another packet arrives. When the two packets are sent to the bus, they are back to back with no breaks in between. The space used on a bus by the pair of packets is described as a reservation slot. The delay from communicating with the controller is a constant which we will add in at the end. Therefore, the calculations initially deal with a system with no delay.

First we analyze the expected number of packets in the queue.

- q_n = number of packets left in the queue
immediately following the n^{th} reservation slot.
- v_n = number of packets arriving
during the n^{th} reservation slot.

We can derive a relationship between q_{n+1} , q_n and v_{n+1} . As shown in Figures 4-3, 4-4 and 4-5:

$$q_{n+1} = \begin{cases} q_n - 2 + v_{n+1} & \text{if } q_n \geq 2, \\ v_{n+1} & \text{if } q_n \leq 1. \end{cases} \quad (4.5)$$

Introducing the function:

$$\Delta_{\tau,k} = \begin{cases} \tau & \text{if } k \geq 2, \\ k & \text{if } k \leq 1, \end{cases} \quad (4.6)$$

we have

$$q_{n+1} = q_n - \Delta_{2,q_n} + v_{n+1} \quad (4.7)$$

Taking the expectation we obtain

$$E[q_{n+1}] = E[q_n] - E[\Delta_{2,q_n}] + E[v_{n+1}]. \quad (4.8)$$

As $n \Rightarrow \infty$, $q_n \Rightarrow \tilde{q}$, and equation 4.8 becomes:

$$E[\tilde{q}] = E[\tilde{q}] - E[\Delta_{2,\tilde{q}}] + E[\tilde{v}] \quad (4.9)$$

$$E[\tilde{v}] = E[\Delta_{2,\tilde{q}}] \quad (4.10)$$

$$\begin{aligned} &= \sum_{k=0}^{\infty} \Delta_{2,k} \cdot P(\tilde{q} = k) \\ &= P(\tilde{q} = 1) + \sum_{k=0}^{\infty} 2 \cdot P(\tilde{q} = k) - 2P(\tilde{q} = 0) - 2P(\tilde{q} = 1) \\ &= 2 - 2p_0 - p_1 = \bar{v}. \end{aligned} \quad (4.11)$$

We set $p_k = P(\tilde{q} = k)$, $\bar{q} = E[\tilde{q}]$ and $\bar{v} = E[\tilde{v}]$. This derivation has given us useful information in the relationship among p_0, p_1 and \bar{v} :

$$p_1 = 2 - 2p_0 - \bar{v}. \quad (4.12)$$

However we still need to find \bar{q} and so we take the expectation of the square of equation 4.7:

$$\begin{aligned} E[\tilde{q}^2] &= E[(\tilde{q} - \Delta_{2,\tilde{q}} + \tilde{v})^2] \\ \bar{q}^2 &= \bar{q}^2 + E[\Delta_{2,\tilde{q}}^2] + \bar{v}^2 - 2E[\tilde{q}\Delta_{2,\tilde{q}}] - 2\bar{v}E[\Delta_{2,\tilde{q}}] + 2\bar{q}\bar{v}. \end{aligned} \quad (4.13)$$

We can solve for $E[\Delta_{2,\bar{q}}^2]$ and $E[\tilde{q}\Delta_{2,\bar{q}}]$ in the following way:

$$\begin{aligned}
E[\Delta_{2,\bar{q}}^2] &= p_1 + \sum_{k=2}^{\infty} 4p_k \\
&= 4 \sum_{k=0}^{\infty} p_k - 3p_1 - 4p_0 \\
&= 4 - 3p_1 - 4p_0
\end{aligned} \tag{4.14}$$

$$\begin{aligned}
E[\tilde{q}\Delta_{2,\bar{q}}] &= \sum_{k=0}^{\infty} k\Delta_{2,k}p_k \\
&= p_1 + \sum_{k=2}^{\infty} 2kp_k \\
&= 2 \sum_{k=0}^{\infty} kp_k - p_1 \\
&= 2\bar{q} - p_1.
\end{aligned} \tag{4.15}$$

We can find \bar{v} and $\overline{v^2}$ in terms of $\rho = \lambda/\mu$ where λ is the packet arrival rate and $1/\mu$ is the average service time. From M/G/1 systems [20], we know:

$$\bar{v} = \lambda \cdot \bar{x} \tag{4.16}$$

$$\overline{v^2} = \bar{v} + \lambda^2 \overline{x^2} = \lambda(\bar{x} + \lambda \overline{x^2}). \tag{4.17}$$

x represents the length of time over which arrivals are included in \bar{v} . Therefore in our case $x = x_1 + x_2$ where x_1 and x_2 are independent exponentially distributed variables with mean $1/\mu$. This leaves us with $\bar{v} = 2\rho$ and $\overline{v^2} = 2\rho + 6\rho^2$. Putting equations

4.13, 4.14, 4.15, 4.16 and 4.17 together we can simplify to:

$$\begin{aligned}
\bar{q}(4 - 2\bar{v}) &= 4 - 3p_1 - 4p_0 + \bar{v}^2 + 2p_1 - 2\bar{v}^2 \\
\bar{q} &= \frac{4 - 4p_0 - p_1 + \bar{v}^2 - 2\bar{v}^2}{2(2 - \bar{v})} \\
&= \frac{2 - 2p_0 + \bar{v}^2 + \bar{v} - 2\bar{v}^2}{2(2 - \bar{v})} \\
&= \frac{1 - p_0 + 2\rho - \rho^2}{2(1 - \rho)}. \tag{4.18}
\end{aligned}$$

We need another equation to solve for p_0 . To do this we will take the z-transform of \tilde{q} . We label this transform $Q(z)$.

$$\begin{aligned}
Q(z) &= E[z^{\tilde{q}}] = E[z^{\tilde{q} - \Delta_{2,\tilde{q}} + \tilde{v}}] \\
&= E[z^{\tilde{v}}]E[z^{\tilde{q} - \Delta_{2,\tilde{q}}}] = V(z)E[z^{\tilde{q} - \Delta_{2,\tilde{q}}}], \tag{4.19}
\end{aligned}$$

moreover

$$\begin{aligned}
E[z^{\tilde{q} - \Delta_{2,\tilde{q}}}] &= \sum_{k=0}^{\infty} z^{k - \Delta_{2,k}} p_k \\
&= p_0 + p_1 + \sum_{k=2}^{\infty} z^{k-2} p_k \\
&= p_0 + p_1 + \frac{1}{z^2} [Q(z) - p_0 - p_1 z], \tag{4.20}
\end{aligned}$$

thus

$$\begin{aligned}
Q(z) &= \frac{V(z)}{z^2} [Q(z) + (z^2 - 1)p_0 + (z^2 - z)p_1] \\
&= V(z) \frac{(1 - z^2)p_0 + (z - z^2)p_1}{V(z) - z^2} \\
&= V(z) \frac{(1 - 2z + z^2)p_0 + (z - z^2)(2 - 2\rho)}{V(z) - z^2}. \tag{4.21}
\end{aligned}$$

We can find $V(z)$ in the following fashion, we know from [20] that $V(z) = B^*(\lambda - \lambda z)$ where $B^*(s)$ is the Laplace transform of the time over which arrivals are counted for v_n . Therefore in our case this interval is the sum of two exponentially distributed

variables. Thus $B^*(s) = \left(\frac{\mu}{\mu+s}\right)^2$ and $V(z) = \left(\frac{\mu}{\mu+\lambda-\lambda z}\right)^2$. Now we can rewrite $Q(z)$ in the following matter:

$$Q(z) = \mu^2 \frac{(1 - 2z + z^2)p_0 + (z - z^2)(2 - 2\rho)}{\mu^2 - z^2(\mu + \lambda - \lambda z)^2} \quad (4.22)$$

To eliminate p_0 we equate roots of the denominator of $Q(z)$ with that of the numerator for $|z| < 1$ using analyticity of $Q(z)$. The four zeros of the denominator are:

$$\begin{aligned} z_1 &= 1 \\ z_2 &= 1/\rho \\ z_3 &= \frac{\mu + \lambda + \sqrt{\mu^2 + \lambda^2 + 6\mu\lambda}}{2\lambda} \\ z_4 &= \frac{\mu + \lambda - \sqrt{\mu^2 + \lambda^2 + 6\mu\lambda}}{2\lambda} \end{aligned}$$

z_4 is the only root which has an absolute value less than 1. Therefore we can continue with:

$$\begin{aligned} (1 - 2z_4 + z_4^2)p_0 + (z_4 - z_4^2)(2 - 2\rho) &= 0 \\ p_0 &= \frac{2z_4(1 - \rho)}{z_4 - 1} \end{aligned} \quad (4.23)$$

Now that we have \bar{q} in known terms, we can find \bar{T} or the average waiting time of a packet. We can accomplish this by adding the constant delay D , the time lost in communicating with the controller, to the result we find using Little's result. Little's result states $\bar{T} = \frac{\bar{q}}{\lambda}$. Therefore:

$$\bar{T} = D + \frac{\bar{q}}{\lambda} \quad (4.24)$$

Multiple users with general τ

For the case with m users in the system, we will start with a very similar equation to equation 4.7:

$$q_{m,n+1} = q_{m,n} - \Delta_{\tau,q_{m,n}} + v_{m,n+1} \quad (4.25)$$

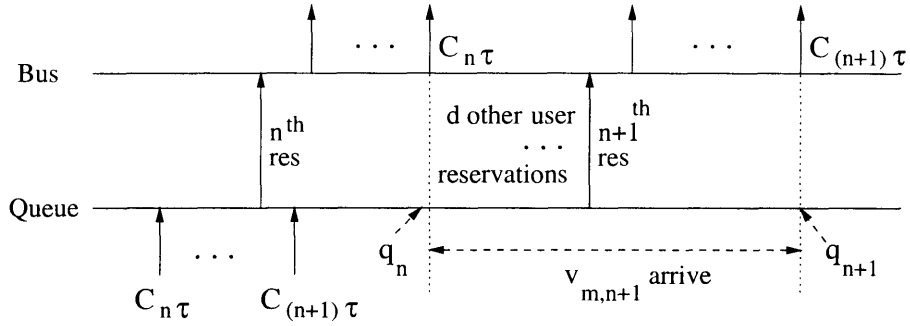


Figure 4-6: Multi-user system: queue for user j

variable	description
d	number of reservation slots from other users between n^{th} and $(n+1)^{\text{th}}$ reservations of user j
c	service time for $(d+1)\tau$ packets
r_i	number of reservations requested by user i between arrival of $(n\tau)^{\text{th}}$ and $((n+1)\tau)^{\text{th}}$ packets of user j
t	time between arrival of $(n\tau)^{\text{th}}$ and $((n+1)\tau)^{\text{th}}$ packets of user j
a	number of packet arrivals for one user between arrival of $(n\tau)^{\text{th}}$ and $((n+1)\tau)^{\text{th}}$ packets of user j

Table 4.2: Description of Intermediate Variables Used in Calculating $V_m(z)$

We assume there is a v_m which is independent of q which satisfies the system. We first define v_m and then go through some of the steps to find \bar{q}_m .

By looking at Figure 4-6, we can see that v_m must now include the number of arrivals during reservation slots from other users. We introduce a few intermediate variables to help describing $V_m(z)$ as described in Table 4.2.

We assume that in between the end of a n^{th} reservation slot and the start of a new reservation $(n+1)^{\text{th}}$ slot by user j , there were d reservation slots from other users, where d is a random variable. From Figure 4-6 and equation 4.41 we must define $v_{m,n+1}$ as the number of packet arrivals for one user during the service of $(d+1)\tau$

packets, or time c . We have the following equations:

$$V_m(z) = C(\lambda - \lambda z) \quad (4.26)$$

$$C(s) = (z \cdot D(z))|_{z=X(s)\tau} = D\left(\frac{\mu}{\mu+s}\right) \cdot \left(\frac{\mu}{\mu+s}\right)^\tau \quad (4.27)$$

$$d = \sum_{i \neq j} r_i \quad (4.28)$$

$$D(z) = R(z)^{m-1} \quad (4.29)$$

$$\begin{aligned} R(z) &= E[z^r] = E[z^{a/\tau}] = E[E[z^{a/\tau}|t]] = E[E[z^a|t]^{1/\tau}] \\ &= E[e^{\frac{\lambda t(z-1)}{\tau}}] = T(s)|_{s=-\frac{\lambda(z-1)}{\tau}} = \left(\frac{\lambda}{\lambda - \frac{\lambda(z-1)}{\tau}}\right)^\tau \\ &= \left(\frac{\tau}{\tau+1-z}\right)^\tau \end{aligned} \quad (4.30)$$

$$C(s) = \left(\frac{\tau}{\tau+1 - \left(\frac{\mu}{\mu+s}\right)^\tau}\right)^{\tau(m-1)} \left(\frac{\mu}{\mu+s}\right)^\tau \quad (4.31)$$

$$V_m(z) = \left(\frac{\tau}{\tau+1 - \left(\frac{\mu}{\mu+\lambda-\lambda z}\right)^\tau}\right)^{\tau(m-1)} \left(\frac{\mu}{\mu+\lambda-\lambda z}\right)^\tau. \quad (4.32)$$

Now that we have $V_m(z)$ we can find $\overline{v_m}$ and $\overline{v_m^2}$:

$$\overline{v_m} = \dot{V}_m(z)|_{z=1} \quad (4.33)$$

$$= \tau \rho m \quad (4.34)$$

$$\begin{aligned} \overline{v_m^2} &= \ddot{V}_m(z)|_{z=1} + \dot{V}_m(z)|_{z=1} \\ &= \tau \rho m + \rho^2 \tau [(\tau+1) + (m-1)(\tau(m-1) + 2 + 3\tau)] \end{aligned} \quad (4.35)$$

Now that we have obtained the necessary information for V_m , we may solve for $\overline{q_m}$ and T .

As $n \rightarrow \infty$ the expectation of equation 4.25 we obtain:

$$E[v_{\tilde{m}}] = E[\Delta_{\tau, \tilde{q}_m}] \quad (4.36)$$

$$\begin{aligned} &= \sum_{k=0}^{\infty} \Delta_{\tau, k} P(\tilde{q}_m = k) \\ &= \sum_{k=0}^{\tau-1} k p_k + \sum_{k=\tau}^{\infty} \tau p_k \end{aligned}$$

$$\overline{v_m} = \tau - \sum_{k=0}^{\tau-1} (\tau - k) p_k. \quad (4.37)$$

This is similar to the equation 4.11 and gives a relationship among $\overline{v_m}$ ($E[v_{\tilde{m}}]$) and p_k ($P(\tilde{q}_m = k)$) for $k \leq \tau - 1$. To find $\overline{q_m}$ we square equation 4.25 and take it's expectation as $n \rightarrow \infty$, which gives us:

$$\begin{aligned} E[\tilde{q}_m^2] &= E[\tilde{q}_m^2] + E[\Delta_{\tau, \tilde{q}_m}^2] + E[v_{\tilde{m}}^2] \\ &\quad + 2E[\tilde{q}_m v_{\tilde{m}}] - 2E[\Delta_{\tau, \tilde{q}_m} \tilde{q}_m] - 2E[\Delta_{\tau, \tilde{q}_m} v_{\tilde{m}}]. \end{aligned} \quad (4.38)$$

To solve for $E[\Delta_{\tau, \tilde{q}_m}^2]$ and $E[\Delta_{\tau, \tilde{q}_m} \tilde{q}_m]$, we proceed as follows:

$$\begin{aligned} E[\Delta_{\tau, \tilde{q}_m}^2] &= \sum_{k=0}^{\infty} \Delta_{\tau, k}^2 p_k \\ &= \sum_{k=0}^{\tau-1} k^2 p_k + \sum_{k=\tau}^{\infty} \tau^2 p_k \\ &= \tau^2 - \sum_{k=0}^{\tau-1} (\tau^2 - k^2) p_k, \end{aligned} \quad (4.39)$$

$$\begin{aligned} E[\Delta_{\tau, \tilde{q}_m} \tilde{q}_m] &= \sum_{k=0}^{\infty} k \Delta_{\tau, k} p_k \\ &= \sum_{k=0}^{\tau-1} k^2 p_k + \sum_{k=\tau}^{\infty} k \tau p_k \\ &= \tau \overline{q_m} - \sum_{k=0}^{\tau-1} (k \tau - k^2) p_k. \end{aligned} \quad (4.40)$$

We then use equation 4.36, 4.39, 4.40, and the independence of \tilde{v}_m with \tilde{q}_m to rewrite 4.38 as:

$$\overline{q}_m = \frac{\tau^2 + \overline{v}_m^2 - 2\overline{v}_m^2 - \sum_{k=0}^{\tau-1} (\tau - k)^2 p_k}{2(\tau - \overline{v}_m)}. \quad (4.41)$$

We see a clear connection between 4.41 and the regular M/G/1 P-K formula as shown in equation 4.3. To find p_k for $0 \leq k \leq \tau - 1$, we take the Z-transform of q .

$$\begin{aligned} Q_m(z) &= E[z^{\tilde{q}_m - \Delta\tau, \tilde{q}_m + \tilde{v}_m}] \\ &= V_m(z) E[z^{\tilde{q}_m - \Delta\tau, \tilde{q}_m}] \\ &= V_m(z) \left(\sum_{k=0}^{\tau-1} p_k + \sum_{k=\tau}^{\infty} z^{k-\tau} p_k \right) \\ &= V_m(z) \left(\sum_{k=0}^{\tau-1} p_k + \frac{Q_m(z) - \sum_{k=0}^{\tau-1} z^k p_k}{z^\tau} \right) \\ &= \frac{V_m(z)}{z^\tau} (Q_m(z) + \sum_{k=0}^{\tau-1} (z^\tau - z^k) p_k) \\ &= \frac{V_m(z) (\sum_{k=0}^{\tau-1} (z^k - z^\tau) p_k)}{V_m(z) - z^\tau} \end{aligned} \quad (4.42)$$

In the same fashion that we solved for p_0 and p_1 with the single user, $\tau = 2$, we find the zeros of the denominator of $Q_m(z)$. As we know our system is stable from section 2.4 and $Q_m(z) = E[z^{\tilde{q}_m}]$, $Q_m(z)$ should be less than infinity for any $|z| < 1$. Therefore we take the $\tau - 1$ zeros of the denominator (z_i for $0 \leq i \leq \tau - 1$) which have an absolute value less than one and set them as zeros for the numerator. Along with the equation 4.37, this gives us the matrix system:

$$\begin{bmatrix} (1 - z_1^\tau) & (z_1 - z_1^\tau) & \dots & (z_1^{\tau-1} - z_1^\tau) \\ (1 - z_2^\tau) & (z_2 - z_2^\tau) & \dots & (z_2^{\tau-1} - z_2^\tau) \\ \vdots & \vdots & \ddots & \vdots \\ (1 - z_{\tau-1}^\tau) & (z_{\tau-1} - z_{\tau-1}^\tau) & \dots & (z_{\tau-1}^{\tau-1} - z_{\tau-1}^\tau) \\ \tau & (\tau - 1) & \dots & (\tau - (\tau - 1)) \end{bmatrix} \begin{bmatrix} p_0 \\ p_1 \\ \vdots \\ p_{\tau-1} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ \tau - \overline{v}_m \end{bmatrix} \quad (4.43)$$

p_i , where $0 \leq i \leq \tau - 1$, is now solvable, and we have all terms of equation 4.41 in terms of the set variables: τ, m, μ and λ .

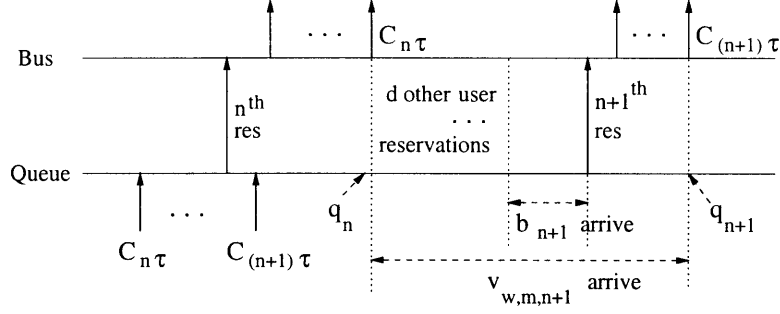


Figure 4-7: Packet delay caused by conflict from other wavelength

Since there are m users, there are on average $m\bar{q}_m$ packets in the system at once. By Little's Theorem [4, 20] a system with no extra delay would have a packet waiting time of:

$$T_{m,\text{no delay}} = \frac{m\bar{q}_m}{\lambda}. \quad (4.44)$$

With the extra delay of communicating with the controller, we have a total waiting time of:

$$T_m = D + \frac{m\bar{q}_m}{\lambda}. \quad (4.45)$$

The folded bus acts as a switch between the nodes, therefore each path possible on the bus is a different user. For a bus with n nodes, there are $n^2 - n$ paths. Therefore the average waiting time of a packet in a single wavelength folded bus using reservation is:

$$T_{n^2-n} = D + \frac{(n^2 - n)\bar{q}_{n^2-n}}{\lambda}. \quad (4.46)$$

4.3.2 Multiple Wavelength System

A system with w wavelengths and m users is very similar to w independent single wavelength systems, each with m/w users. However there is one major difference, two or more wavelengths may not have reservation slots overlapping that conflict. Conflicting reservation slots include slots for packets that have the same departure and or arrival node. To prevent any overlap in the departure node, we have nodes always writing onto the same wavelength. For example, the nodes are divided into w even groups and the nodes of the first group always reserve slots on the first

wavelength, the second group reserves on the second wavelength and so on and so on. However, there can still be overlap in the arrival node. The solution for this system is when the controller is trying to reserve a slot, it will wait until all conflicting reservations on other wavelengths are done before allowing the new slot.

Therefore the $v_{w,m}$ now depends on the number of wavelengths, the number of users and includes the packet arrivals which occur during the delay caused by conflicting reservations as shown in Figure 4-7. If we name the number of arrivals during the conflict b , then we have:

$$v_{w,m} = v_{m/w} + b \quad (4.47)$$

$$V_{w,m}(z) = V_{m/w}(z)B(z) \quad (4.48)$$

$$\overline{v_{w,m}} = \overline{v_{m/w}} + \overline{b} \quad (4.49)$$

$$\overline{v_{w,m}^2} = \overline{v_{m/w}^2} + \overline{b^2} + 2\overline{v_{m/w}b}. \quad (4.50)$$

Once we define b , we can use the same equations as the one wavelength system replacing v_m with $v_{w,m}$. We will first break up b into smaller pieces. h_i , $0 \leq i \leq w-1$, is the number of arrivals caused by the i^{th} wavelength to conflict. For example, if a reservation is made and exactly one wavelength has a conflict, then $b = h_1$ for all $h_i = 0, i \neq j$. In general:

$$h_i = \text{the number of arrivals due to the } i^{th} \text{ conflicting slot} \quad (4.51)$$

$$b = \sum_{i=1}^{w-1} h_i \quad (4.52)$$

$$\overline{b} = \sum_{i=1}^{w-1} \overline{h_i} \quad (4.53)$$

$$\overline{b^2} = \sum_{i=1}^{w-1} \overline{h_i^2} + \sum_{i=1}^{w-1} \sum_{j \neq i}^{w-1} \overline{h_i h_j} \quad (4.54)$$

We introduce the variable:

$$pc_i = P(\text{at least } i \text{ wavelengths conflict}).$$

For $i > 1$, we know that the entire reservation will be included in the wait. The transform of the number of arrivals in an entire reservation slot is $V_{1,1}(z) = \left(\frac{\mu}{\mu + \lambda + \lambda z}\right)^\tau$. Therefore in calculating $H_i(z)$ for $i > 1$, we have:

$$\begin{aligned} H_i(z) &= V_{1,1}(z^{pc_i}) \text{ for } i > 1 \\ &= \left(\frac{\mu}{\mu + \lambda + \lambda z^{pc_i}}\right)^\tau. \end{aligned} \quad (4.55)$$

We can calculate $\overline{h_i}$ and $\overline{h_i^2}$ using $H_i(z)$:

$$\overline{h_i} = \dot{H}_i(1) = pc_i \tau \rho, \quad (4.56)$$

$$\overline{h_i^2} = \ddot{H}_i(1) + \dot{H}_i(1) = (pc_i)^2 \tau (\rho + (\tau + 1) \rho^2). \quad (4.57)$$

With $i = 1$, it is a different equation. This is because the reservation can come in anytime during the first conflicting slot. This is called calculating the residual of a function. When calculating the residual of a function f with a Laplace transform $F(s)$, the Laplace transform of the residual is: $F_r(s) = \frac{1-F(s)}{sf}$. Therefore using this form we have:

$$\begin{aligned} H_1(z) &= \frac{1 - V_{1,1}(z^{pc_1})}{(\lambda - \lambda z^{pc_1}) pc_1 v_{1,1}} \\ &= \frac{1 - \left(\frac{\mu}{\mu + \lambda + \lambda z^{pc_1}}\right)^\tau}{(\lambda - \lambda z^{pc_1}) pc_1 \tau \rho}. \end{aligned} \quad (4.58)$$

Having solved for the z-transform, we can now find the first and second moments of h_1 :

$$\overline{h_1} = pc_1 \rho (\tau + 1) / 2 \quad (4.59)$$

$$\overline{h_1^2} = pc_1^2 \rho (\lambda (\tau + 2) / 3 + (\tau + 1) / 2). \quad (4.60)$$

With all the h_i 's solved for, we can now solve for b , which is what we needed to describe $V_{w,m}(z)$, $\overline{v_{w,m}}$ and $\overline{v_{w,m}^2}$. These values can be used in the matrix system 4.43

to find the corresponding p_i . Next, we use equation 4.41 and 4.45 to solve for $\overline{q_{w,m}}$ and $T_{w,m}$.

For the specific case of the folded bus, each packet path is a user, therefore:

$$m = n^2 - n$$

We can specify pc_i to be the probability that there are at least i wavelengths conflicting with the reservation slot. We specified that the conflict would come from the output node. The number of different possible conflicting slots on a wavelength which does not have that output node as one of its input nodes is $\frac{n}{w}$, while if it is an input node on the wavelength, there are $\frac{n}{w} - 1$ conflicting slots possibly reserved on that wavelength. The probability that i of these might occur at the same time is:

$$\begin{aligned}
pc_i &= P(i \text{ wavelengths busy}) \cdot P(i \text{ wavelengths conflict} | i \text{ wavelengths busy}) \\
&= \left(\frac{\lambda(n^2 - n)}{\mu} \right)^i \frac{w}{n^2 - n} \\
&\quad \cdot \left[\frac{n(n-w)}{w^2(n-1)^i} \binom{w-1}{i} \right. \\
&\quad \left. + \frac{n^2(w-1)}{w^2} \left(\binom{w-2}{i} \frac{1}{(n-1)^i} + \binom{w-2}{i-1} \left(\frac{n-w}{(n^2-n)(n-1)^{i-1}} \right) \right) \right] \\
&= \frac{(\rho n)^i \binom{w-1}{i} (n-i-1)}{w^i (n-1)} \tag{4.61}
\end{aligned}$$

Using this pc_i , we can solve for $T_{n^2-n,w}$ to be:

$$T_{n^2-n,w} = D + \frac{(n^2 - n)\overline{q_{n^2-n,w}}}{\lambda}. \tag{4.62}$$

We now have a systematic way of solving for the average waiting time of system. We define a v , solve for its z-transform and then its first and second moments. We solve for necessary p_i 's and with these components are able to find the average queue size and packet waiting time.

Chapter 5

Discussion and Simulations

In this chapter, we simulate the results from Chapters 3 and 4 and discuss their significance. We begin by analyzing the effect of varying τ on the expected number of packets in the a user's queue. We then graph the changes in the size of the queue caused by varying ρ_T , the total traffic on the bus. Finally, we examine the relationship between the number of wavelengths on the bus and the expected packet delay for a bus using TDM and reservation.

5.1 Effects from varying τ

τ is the number of packets sent in one reservation slot. Varying the value of τ effects the efficiency of the bus. The average queue size increases with τ regardless of the traffic on the bus. For a higher τ , the bus must wait for more packets before sending them, causing the average queue size to rise. From Little's Law, we know that the average expected delay is linearly dependent on the average queue size. Therefore, raising τ increases the average packet delay. In Figure 5-1, we have two different graphs showing high and low traffic scenarios for a single wavelength bus. The high traffic has a $\rho = .9$ while the low traffic has a value of $\rho = .5$. We use these two values of ρ throughout this chapter so that we may compare the behavior of a very busy bus ($\rho = .9$) to a bus with almost half the traffic ($\rho = .5$). TDM is not dependent on τ , however we are interested in showing where reservation and TDM intersect since

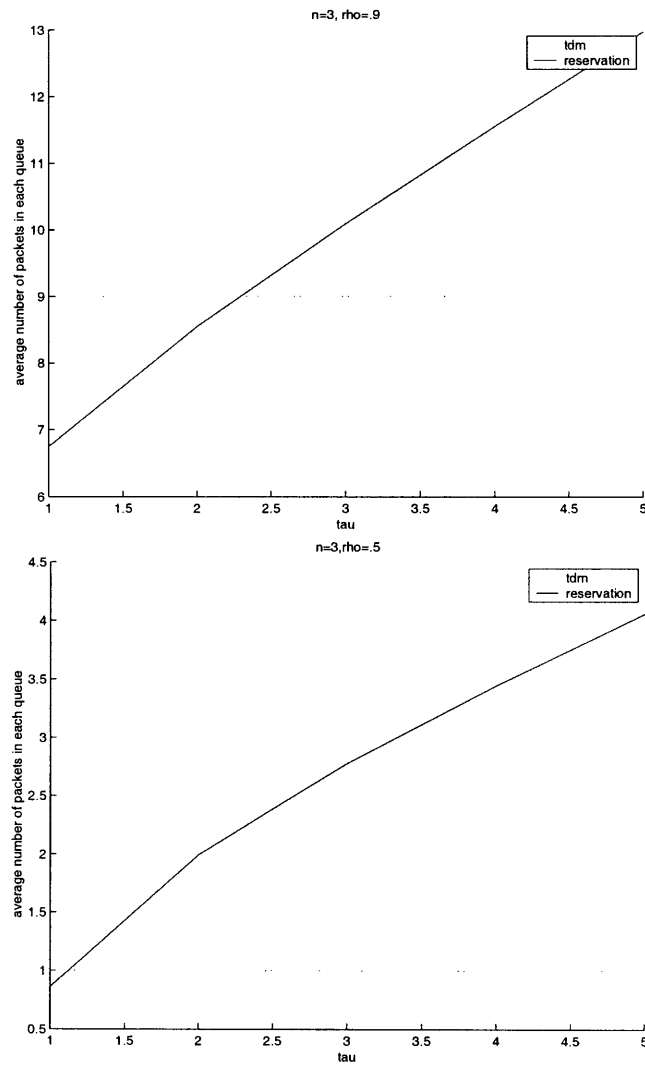


Figure 5-1: Varying τ on a single wavelength bus

this will identify the range of values in τ for which reservation is preferable to TDM. This point will be discussed in more detail later in Section 5.3. We can see that as the traffic increases, this intersection occurs at a larger τ . In Figure 5-1, the size of the queue when traffic is low stays on average under τ , while with high traffic the average size of the queue is larger than τ . This indicates that with low traffic, the bus can send packets as soon as the τ packets arrive, whereas with high traffic the reservation slots are backed up.

5.2 Effects from varying the traffic load

For the remainder of the graphs, we set $\tau = 3$. We want to keep τ small to allow reservation to perform comparatively to TDM. In Figure 5-2, there are three graphs which compare the average queue size to the number of wavelengths on the bus. For this comparison, we used two different values of ρ_T (the arrival rate for the entire bus divided by the service rate), 0.5 (for low traffic) and 0.9 (for high traffic). We chose these two values because we are limited to $\rho_T < 1$ when it is a single wavelength bus and we want a large enough difference in the values to allow for significant comparisons. With $\rho_T = 0.5$ and 0.9, we can compare two buses, one with nearly twice the traffic of the other, and see significant differences in their behavior. For reservation, we also set the number of nodes on the bus to two different values, 6 and 24, to determine if the size of the bus has a noticeable effect on the queue sizes. For TDM, the average number of packets in each queue is independent of the number of nodes on the bus. We postpone the comparison between TDM and reservation until Section 5.3, while here we focus on the effects of ρ_T on the bus. As we see in Figure 5-2, the number of wavelengths in the system does diminish the average queue size; however, the improvement is less impressive for a bus with low traffic. As the number of wavelengths increases, the total traffic on the bus remains constant and the traffic on each wavelength diminishes accordingly. Since there are fewer backed-up packets in a low traffic bus, the advantages of the extra wavelengths are less pronounced.

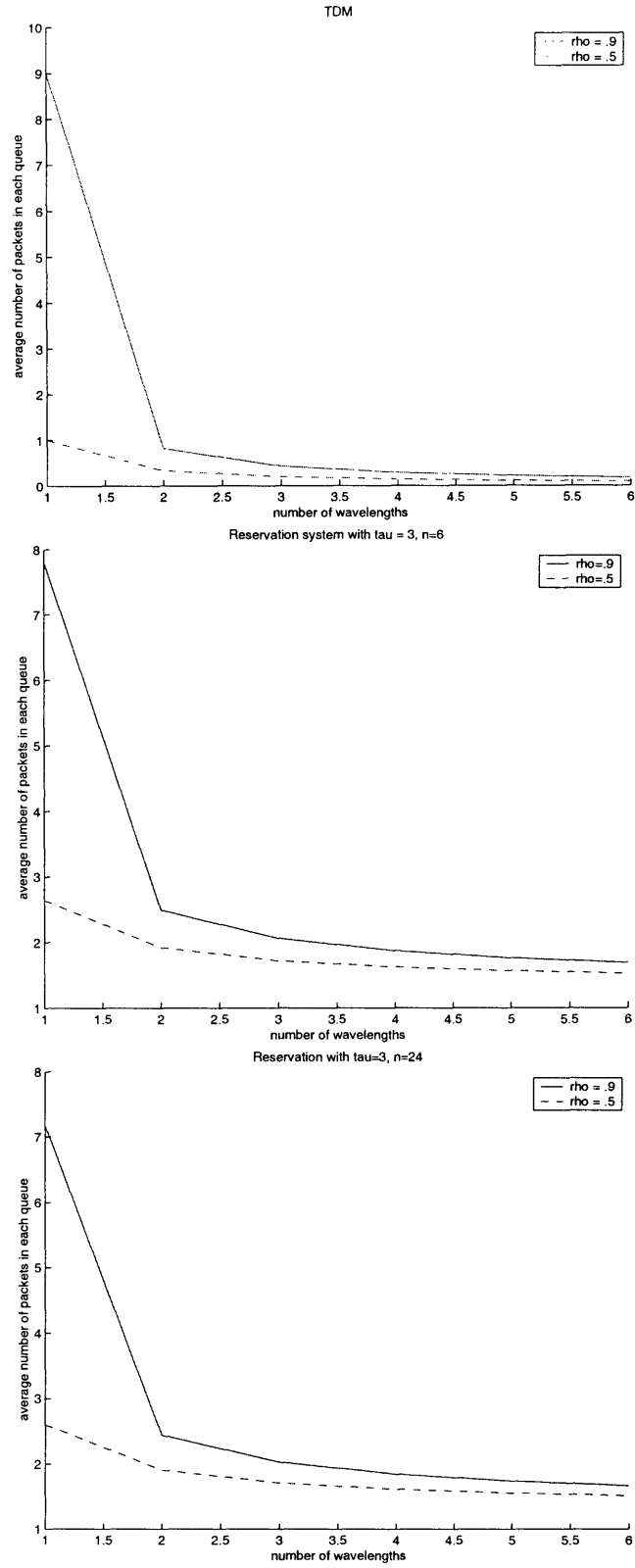


Figure 5-2: The average queue size vs. number of wavelengths

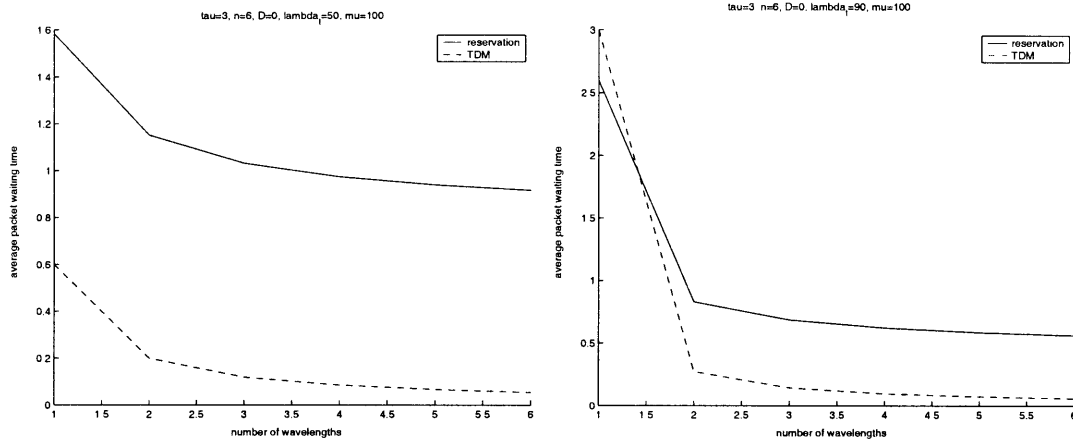


Figure 5-3: The average packet waiting time for a bus with 6 nodes vs. number of wavelengths

5.3 Comparing TDM and reservation

In Figures 5-3 and 5-4, we have the average packet waiting time for a bus with 6 and 24 nodes respectively versus the number of wavelengths on the bus. Each figure contains two graphs, the first with low traffic and the second with high traffic. As μ is set to 100, the buses where $\lambda_t = 50$ and $\lambda_t = 90$ are equivalent to the buses with $\rho_t = .5$ and $\rho_t = .9$ respectively. The four graphs include TDM and reservation, allowing for comparison between the two methods.

TDM is preferable to reservation for buses with less traffic. Lower traffic can result from setting λ_t (the total arrival rate to the bus) to 50 rather than 90, or by increasing the number of wavelengths. This goes against what we would initially suppose with TDM verses reservation. For a high traffic bus, the slots in TDM are nearly full and little space on the bus is wasted; therefore, one might suppose reservation would not be as efficient. However, since the simulated data corresponds to a reservation system with no communication delay, it is logical that reservation will better TDM. With no communication delay, reservation slots will be backed up and sequential similar to TDM. However, the difference arises when users with the largest queues can reserve more than one slot before reservations from all users have passed. This allows reservation to be more efficient than TDM, and supports the above figures for

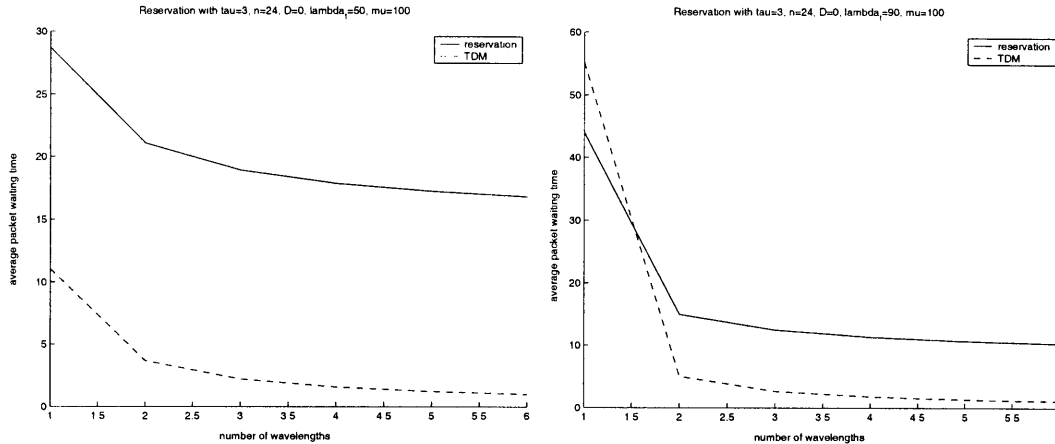


Figure 5-4: The average packet waiting time for a bus with 24 nodes vs. number of wavelengths

the high traffic case.

In a low traffic bus, TDM forces packets to wait through empty slots while reservation reserves the slot only when needed. Our initial hypothesis was that reservation should have lower packet delays than TDM for a low traffic bus. However, the graphs once again go against our initial hypothesis. Figure 5-2 helps us explain this discrepancy. As mentioned in Section 5.2, we see that for reservation with low traffic, the average queue size remains below τ , from which we deduced that the reservation slots were not backed up at the controller. Since the average queue size of a bus with TDM and low traffic remains between 0 and 1, its time slots remain mostly empty. However, for reservation the slots must contain τ packets from the same user. Since the arrival rate remains constant, the queue spends on average the same amount of time with 0, 1, 2 up to τ packets, for an average queue size of at least $\tau/2$. With low traffic, the average queue of TDM remains below $\tau/2$ and therefore has a lower delay, explaining the discrepancy with our initial hypothesis. In Figure 5-2 where τ is set to 3, the average queue diminishes to 0 with more wavelengths for TDM and $1.5 = \tau/2$ for reservation.

In Figure 5-5, we compare the waiting time between TDM and a transposed version of reservation. We can use Figure 5-5 to compare the slopes of the graphs from Figure 5-4. We notice that the slopes are very similar. Both resemble an exponentially

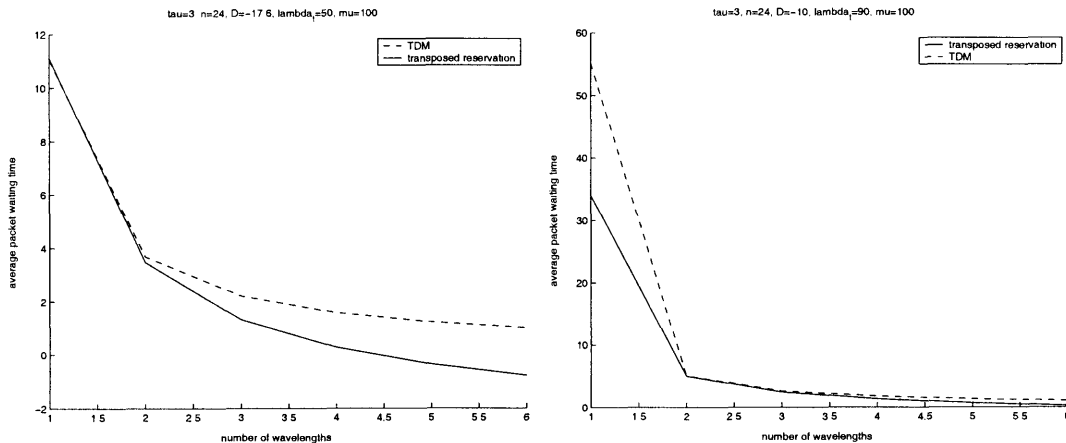


Figure 5-5: Transposed waiting time for a bus with 24 nodes vs. number of wavelengths

decaying slope, although reservation has a steeper slope as the number of wavelengths increases. This shows that the increase in wavelengths helps reservation reach its asymptote faster than TDM. However because of the offset of $\tau/2$ in the queue size, the asymptote is $\tau/2 \cdot \lambda_T$ for reservation rather than 0 for TDM.

5.4 Guidelines for designing an optical folded bus network

In this chapter, we have shown both TDM and reservation offer significant performance improvements with added wavelengths. We find TDM to be the superior method for low traffic buses whereas reservation is better for high traffic buses and improves with lower values of τ . When designing a network, we must weigh the performance improvements against the budget available to us in order to determine how many wavelengths to use. Hopefully the quantitative results of this chapter can simplify this process for the network designer.

As a network designer, our first task is to characterize correctly the demands that will be placed on the bus. The parameters we need are the following: the number of nodes the bus is spanning (n), the average size of packets being sent ($1/\mu$), the

average packet arrival rate (λ), the budget available, and the required performance (maximum packet delay). Once these values have been identified, we decide on the minimum possible value of τ . The lower the value of τ , the faster components must work. However, faster components raise the cost of the total network and therefore our budget will ultimately determine τ . Next, we decide on the maximum number of wavelengths used on our network. Again we are faced with a tradeoff as the components required for a multiple wavelength system (e.g. VCSELs, wavelength detectors) are more expensive and the higher the number of wavelengths the higher the expense. Therefore, our budget will also set the maximum number of wavelengths possible.

Our ultimate objective is to choose the number of wavelengths so as to minimize the expected packet delay subject to our budget constraint. In order to do this we can use the parameters above to graph the packet delay versus the number of wavelengths for both TDM and reservation (e.g. Figure 5-4 and 5-3). Next, we add a third axis to the existing plot where we calculate the estimated cost for each configuration on the plot. We then block off the regions of the plot which are out of the scope of our interests, such as delays above our maximum delay allowed and costs beyond our budget, and choose the configuration that yields the best delay versus cost.

We can see from our results in Section 5.3 that TDM performs better than reservation with low traffic and reservation performs better than TDM for high traffic. Reservation fails to perform as well in low traffic because, as reasoned earlier, of the delay of waiting for τ packets to accumulate in the queue. Therefore, reservation has lower delays than TDM when the queues are overloaded. We can imagine reservation might prevail for a network where the packet arrivals are not accurately portrayed by the Poisson distribution, but rather with a bursty pattern. Reservation performs efficiently when packets are likely to arrive to a user in a large group as there would be little waiting for τ packets to accumulate even if the average arrival rate was low. TDM is most appropriate when the packet arrivals are even. With knowledge on the type of data our bus is servicing, we have more information with which to choose our MAC. The equations developed in the last two chapters are only valid if we assume

the packet arrival rates to be Poisson, but still provide a useful estimate for packet arrivals resembling Poisson. The more the traffic heavy and bursty, the stronger we lean towards using reservation, whereas the more the traffic is light and even, the more TDM is the appropriate choice.

Chapter 6

Conclusion

The main purpose of this thesis was to give a systematic way of evaluating whether increasing the number of wavelengths for certain LANS using VCSELs would be worth the extra cost and complexity. In this thesis, we focused on one architecture and two different MACs which are commonly used. We analyzed the packet waiting time for both protocols using two different techniques. We then simulated both TDM and reservation, allowing us to compare the two and characterize the potential improvements offered by multiple wavelengths.

We picked the folded bus for its simplicity and its popularity as a LAN architecture. We analyzed the folded bus for TDM and reservation as they are common and fundamental protocols. Most scheduling techniques used in LANs will resemble either TDM or some type of reservation system. We can now build on this model to analyze other MACs (e.g. DQDB, HLAN, CRMA, ORMA) and other architectures (e.g. dual buses, rings). Since our reservation analysis is based on the M/G/1 queue, it should be powerful and flexible enough to extend to more complicated MACs. For a different system, we can calculate a new v as we did in `refeqn:v` and follow similar steps to 4.43 to solve for any p_i 's required. These lead to a new T ; however, the expression for T in our multiple wavelength reservation analysis is already quite complicated, so with more intricate MACs, we can imagine this analysis may no longer be manageable. Yet, we may still use it to analyze simplifications of the model. A first model to consider would be a combination of TDM and reservation on the folded

bus. As we learned in Section 5.3, TDM is the more efficient of the two when the traffic is low, whereas reservation has the lower packet waiting time for higher traffic. By combining the two we can therefore utilize the advantages of both methods. The protocol would work as such: TDM is the default MAC until one of the queues becomes overloaded, then a reservation interrupts TDM to alleviate the heavy queue after which point TDM resumes. Analyzing the delays from such a combination will build on the initial analysis done in this thesis.

Our model of the folded bus is a simplified model which does not take into account all the characteristics of a real bus. For example, we assume the controller delay in reservation is a constant for all nodes on the bus. Realistically, the controller will be physically farther from some nodes and delay those packets more. Also the analysis for reservation assumes the average packet size and arrival rate are equal for all nodes. This assumption is easily removed as we may calculate the reservation delay with a different λ_i and μ_i as we did with TDM. To analyze the performance of the folded bus fully, we would like to calculate the delay from the time the packet arrives to the time it leaves the bus, whereas we only analyze the delay up to the time packets are written onto the bus. This calculation includes issues formerly neglected, such as the physical length of the bus and delays at the output node. The length of the path used on the bus delays a packet accordingly. A packet which travels between two nodes near the fold of the bus has a shorter delay than a packet travelling between two nodes near the head of the bus. Development of more robust models which utilize these extra parameters is a viable avenue for future research.

We hope further work will be continued in the same direction as this thesis. There are many LANs that might benefit from using multiple wavelengths with the VCSEL technology.

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