What Drives Hedge Fund Returns? Models of Flows, Autocorrelation, Optimal Size, Limits to Arbitrage and Fund Failures

by

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Bachelor of Science, Chemical Engineering
Massachusetts Institute of Technology, 1998

Submitted to the Alfred P. Sloan School of Management
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ABSTRACT

Hedge funds provide an opportunity for investing with few government regulations and high potential returns. Since 1980 this has lead to a dramatic 25% annual increase in the number of hedge funds, with nearly $700 billion managed by hedge funds in 2003. However, high risks associated with hedge fund strategies, competition and limited arbitrage opportunities contributed to an annual attrition rate of 7.10%. In this thesis, models were developed and tested that describe the characteristics of fund returns, fund flows, optimal size and hedge fund life cycles. The TASS hedge fund database provided by the Tremont Company was used for analysis. In Essay One, it was found that hedge fund returns are highly serially correlated compared to the returns of more traditional investment vehicles such as mutual funds. Several sources of such high serial correlation were explored and the research illustrated that the most likely explanation of this derived from asset illiquidity and smoothing of returns. Illiquid securities are not actively traded and market prices are not always available for them. In the case of smoothing, brokers or managers have the flexibility to report partial returns. Consequently, for portfolios of illiquid or smoothed securities, reported returns will tend to be smoother than true economic returns, thereby understating volatility and increasing risk-adjusted performance measures such as the Sharpe ratio. An econometric model of illiquidity exposure was further proposed and estimators for the smoothing profile as well as a smoothing-adjusted Sharpe ratio were developed. Estimated smoothing coefficients were found to vary considerably across hedge-fund style categories and may be a useful proxy for quantifying illiquidity exposure. In Essay Two, the life cycles of hedge funds were analyzed. The findings show that in general, investors chasing individual fund performance decrease the probability of an individual hedge fund liquidating. However, when investors pursue a category of hedge funds that has performed well, the probability of hedge funds liquidating within that category increases because of growing competition among hedge funds; and in such environment, marginal funds are more likely to be liquidated than funds that deliver superior risk-adjusted returns. In the Essay, a model was proposed for calculating an optimal asset size by balancing out the effects of past returns, fund flows, market impact, competition and favorable category positioning.

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With profound gratitude for all they have done, I dedicate this thesis to my Mom and Dad and my future husband Woody.

To Mom, Dad, and Woody

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An Econometric Model of Serial Correlation and Illiquidity In Hedge Fund Returns

Mila Getmansky, Andrew W. Lo, and Igor Makarov

Abstract

The returns to hedge funds and other alternative investments are often highly serially correlated, in sharp contrast to the returns of more traditional investment vehicles such as long-only equity portfolios and mutual funds. In this paper, we explore several sources of such serial correlation and show that the most likely explanation is illiquidity exposure, i.e., investments in securities that are not actively traded and for which market prices are not always readily available. For portfolios of illiquid securities, reported returns will tend to be smoother than true economic returns, which will understate volatility and increase risk-adjusted performance measures such as the Sharpe ratio. We propose an econometric model of illiquidity exposure and develop estimators for the smoothing profile as well as a smoothing-adjusted Sharpe ratio. For a sample of 908 hedge funds drawn from the TASS database, we show that our estimated smoothing coefficients vary considerably across hedge-fund style categories and may be a useful proxy for quantifying illiquidity exposure.

Contents

1 Introduction

One of the fastest growing sectors of the financial services industry is the hedge-fund or "alternative investments" sector. Long the province of foundations, family offices, and high-net-worth investors, hedge funds are now attracting major institutional investors such as large state and corporate pension funds and university endowments, and efforts are underway to make hedge-fund investments available to individual investors through more traditional mutual-fund investment vehicles. One of the main reasons for such interest is the performance characteristics of hedge funds—often known as "high-octane" investments, many hedge funds have yielded double-digit returns to their investors and, in some cases, in a fashion that seems uncorrelated with general market swings and with relatively low volatility. Most hedge funds accomplish this by maintaining both long and short positions in securities—hence the term "hedge" fund—which, in principle, gives investors an opportunity to profit from both positive and negative information while, at the same time, providing some degree of "market neutrality" because of the simultaneous long and short positions.

However, several recent empirical studies have challenged these characterizations of hedge-fund returns, arguing that the standard methods of assessing their risks and rewards may be misleading. For example, Asness, Krail and Liew (2001) show in some cases where hedge funds purport to be market neutral, i.e., funds with relatively small market betas, including both contemporaneous and lagged market returns as regressors and summing the coefficients yields significantly higher market exposure. Moreover, in deriving statistical estimators for Sharpe ratios of a sample of mutual and hedge funds, Lo (2002) shows that the correct method for computing annual Sharpe ratios based on monthly means and standard deviations can yield point estimates that differ from the naive Sharpe ratio estimator by as much as 70%.

These empirical properties may have potentially significant implications for assessing the risks and expected returns of hedge-fund investments, and can be traced to a single common source: significant serial correlation in their returns.

This may come as some surprise because serial correlation is often (though incorrectly) associated with market inefficiencies, implying a violation of the Random Walk Hypothesis and the presence of predictability in returns. This seems inconsistent with the popular belief that the hedge-fund industry attracts the best and the brightest fund managers in the financial services sector. In particular, if a fund manager's returns are predictable, the implication is that the manager's investment policy is not optimal; if his returns next month can be reliably forecasted to be positive, he should increase his positions this month to take advantage of this forecast, and vice versa for the opposite forecast. By taking advantage of such predictability the fund manager will eventually eliminate it, along the lines of Samuelson's (1965) original "proof that properly anticipated prices fluctuate randomly". Given the outsize financial incentives of hedge-fund managers to produce profitable investment strategies, the existence of significant unexploited sources of predictability seems unlikely.

In this paper, we argue that in most cases, serial correlation in hedge-fund returns is not due to unexploited profit opportunities, but is more likely the result of illiquid securities that are contained in the fund, i.e., securities that are not actively traded and for which market prices are not always readily available. In such cases, the reported returns of funds containing illiquid securities will appear to be smoother than "true" economic returns—returns that fully reflect all available market information concerning those securities—and this, in turn, will impart a downward bias on the estimated return variance and yield positive serial return correlation. The prospect of spurious serial correlation and biased sample moments in reported returns is not new. Such effects have been derived and empirically documented extensively in the literature on "nonsynchronous trading", which refers to security prices recorded at different times but which are erroneously treated as if they were recorded simultaneously.[1] However, this literature has focused exclusively on equity market-microstructure

[1] For example, the daily prices of financial securities quoted in the *Wall Street Journal* are usually "closing" prices, prices at which the last transaction in each of those securities occurred on the previous business day. If the last transaction in security A occurs at 2:00pm and the last transaction in security B occurs at 4:00pm, then included in B's closing price is information not available when A's closing price was set. This can create spurious serial correlation in asset returns since economy-wide shocks will be reflected first in the prices of the most frequently traded securities, with less frequently traded stocks responding with a lag. Even when there is no statistical relation between securities A and B, their reported returns will appear to be serially correlated and cross-correlated simply because we have mistakenly assumed that they are measured simultaneously. One of the first to recognize the potential impact of nonsynchronous price quotes was Fisher (1966). Since then more explicit models of non-trading have been developed by Atchison, Butler, and Simonds (1987), Dimson (1979), Cohen, Hawawini, et al. (1983a,b), Shanken (1987), Cohen, Maier, et al. (1978, 1979, 1986), Kadlec and Patterson (1999), Lo and MacKinlay (1988, 1990), and Scholes and Williams (1977). See Campbell, Lo, and MacKinlay (1997, Chapter 3) for a more detailed review of this

effects as the sources of nonsynchronicity—closing prices that are set at different times, or prices that are "stale"—where the temporal displacement is on the order of minutes, hours, or, in extreme cases, several days.[2] In the context of hedge funds, we argue in this paper that serial correlation is the outcome of illiquidity exposure, and while nonsynchronous trading may be one symptom or by-product of illiquidity, it is not the only aspect of illiquidity that affects hedge-fund returns. Even if prices were sampled synchronously, they may still yield highly serially correlated returns if the securities are not actively traded.[3] Therefore, although our formal econometric model of illiquidity is similar to those in the nonsynchronous trading literature, the motivation is considerably broader—linear extrapolation of prices for thinly traded securities, the use of smoothed broker-dealer quotes, trading restrictions arising from control positions and other regulatory requirements, and, in some cases, deliberate performance-smoothing behavior—and the corresponding interpretations of the parameter estimates must be modified accordingly.

Regardless of the particular mechanism by which hedge-fund returns are smoothed and serial correlation is induced, the common theme and underlying driver is illiquidity exposure, and although we argue that the sources of serial correlation are spurious for most hedge funds, nevertheless, the economic impact of serial correlation can be quite real. For example, spurious serial correlation yields misleading performance statistics such as volatility, Sharpe ratio, correlation, and market beta estimates, statistics commonly used by investors to determine whether or not they will invest in a fund, how much capital to allocate to a fund, what kinds of risk exposures they are bearing, and when to redeem their investments. Moreover, spurious serial correlation can lead to wealth transfers between new, existing, and departing investors, in much the same way that using stale prices for individual securities to compute mutual-fund net-asset-values can lead to wealth transfers between buy-and-hold investors and day-traders (see, for example, Boudoukh et al., 2002).

In this paper, we develop an explicit econometric model of smoothed returns and derive

literature.

[2]For such application, Lo and MacKinlay (1988, 1990) and Kadlec and Patterson (1999) show that nonsynchronous trading cannot explain all of the serial correlation in weekly returns of equal- and value-weighted portfolios of US equities during the past three decades.

[3]In fact, for most hedge funds, returns computed on a monthly basis, hence the pricing or "mark-to-market" of a fund's securities typically occurs synchronously on the last day of the month.

its implications for common performance statistics such as the mean, standard deviation, and Sharpe ratio. We find that the induced serial correlation and impact on the Sharpe ratio can be quite significant even for mild forms of smoothing. We estimate the model using historical hedge-fund returns from the TASS Database, and show how to infer the true risk exposures of a smoothed fund for a given smoothing profile. Our empirical findings are quite intuitive: funds with the highest serial correlation tend to be the more illiquid funds, e.g., emerging market debt, fixed income arbitrage, etc., and after correcting for the effects of smoothed returns, some of the most successful types of funds tend to have considerably less attractive performance characteristics.

Before describing our econometric model of smoothed returns, we provide a brief literature review in Section 2 and then consider other potential sources of serial correlation in hedge-fund returns in Section 3. We show that these other alternatives—time-varying expected returns, time-varying leverage, and incentive fees with high-water marks—are unlikely to be able to generate the magnitudes of serial correlation observed in the data. We develop a model of smoothed returns in Section 4 and derive its implications for serial correlation in observed returns, and we propose several methods for estimating the smoothing profile and smoothing-adjusted Sharpe ratios in Section 5. We apply these methods to a dataset of 909 hedge funds spanning the period from November 1977 to January 2001 and summarize our findings in Section 6, and conclude in Section 7.

2 Literature Review

Thanks to the availability of hedge-fund returns data from sources such as AltVest, Hedge Fund Research (HFR), Managed Account Reports (MAR), and TASS, a number of empirical studies of hedge funds have been published recently. For example, Ackermann, McEnally, and Ravenscraft (1999), Agarwal and Naik (2000b, 2000c), Edwards and Caglayan (2001), Fung and Hsieh (1999, 2000, 2001), Kao (2002), and Liang (1999, 2000, 2001) provide comprehensive empirical studies of historical hedge-fund performance using various hedge-fund databases. Agarwal and Naik (2000a), Brown and Goetzmann (2001), Brown, Goetzmann, and Ibbotson (1999), Brown, Goetzmann, and Park (1997, 2000, 2001), Fung and

Hsieh (1997a, 1997b), and Lochoff (2002) present more detailed performance attribution and "style" analysis for hedge funds. None of these empirical studies focus directly on the serial correlation in hedge-fund returns or the sources of such correlation.

However, several authors have examined the persistence of hedge-fund performance over various time intervals, and such persistence may be indirectly linked to serial correlation, e.g., persistence in performance usually implies positively autocorrelated returns. Agarwal and Naik (2000c) examine the persistence of hedge-fund performance over quarterly, half-yearly, and yearly intervals by examining the series of wins and losses for two, three, and more consecutive time periods. Using net-of-fee returns, they find that persistence is highest at the quarterly horizon and decreases when moving to the yearly horizon. The authors also find that performance persistence, whenever present, is unrelated to the type of a hedge fund strategy. Brown, Goetzmann, Ibbotson, and Ross (1992) show that survivorship gives rise to biases in the first and second moments and cross-moments of returns, and apparent persistence in performance where there is dispersion of risk among the population of managers. However, using annual returns of both defunct and currently operating offshore hedge funds between 1989 and 1995, Brown, Goetzmann, and Ibbotson (1999) find virtually no evidence of performance persistence in raw returns or risk-adjusted returns, even after breaking funds down according to their returns-based style classifications. None of these studies considers illiquidity and smoothed returns as a source of serial correlation in hedge-fund returns.

The findings by Asness, Krail, and Liew (2001)—that lagged market returns are often significant explanatory variables for the returns of supposedly market-neutral hedge funds—is closely related to serial correlation and smoothed returns, as we shall demonstrate in Section 4. In particular, we show that even simple models of smoothed returns can explain both serial correlation in hedge-fund returns and correlation between hedge-fund returns and lagged index returns, and our empirically estimated smoothing profiles imply lagged beta coefficients that are consistent with the lagged beta estimates reported in Asness, Krail, and Liew (2001). Their framework is derived from the nonsynchronous trading literature, specifically the estimators for market beta for infrequently traded securities proposed by Dimson (1977), Scholes and Williams (1977), and Schwert (1977) (see footnote 1 for additional references to this literature). A similar set of issues affects real-estate prices and price indexes, and Ross and Zisler (1991), Gyourko and Keim (1992), Fisher, Geltner, and Webb (1994), and Fisher et al. (2003), have proposed various econometric estimators that have much in common with those in the nonsynchronous trading literature.

An economic implication of nonsynchronous trading that is closely related to the hedge-fund context is the impact of stale prices on the computation of daily net-asset-values (NAVs) of certain open-end mutual funds, e.g., Bhargava, Bose and Dubofsky (1998), Chalmers, Edelen, and Kadlec (2001), Goetzmann, Ivkovic, and Rouwenhorst (2001), Boudoukh et al. (2002), Greene and Hodges (2002), and Zitzewitz (2002). In these studies, serially correlated mutual fund returns are traced to nonsynchronous trading effects in the prices of the securities contained in the funds, and although the correlation is spurious, it has real effects in the form of wealth transfers from a fund's buy-and-hold shareholders to those engaged in opportunistic buying and selling of shares based on forecasts of the fund's daily NAVs. Although few hedge funds compute daily NAVs or provide daily liquidity, the predictability in some hedge-fund return series far exceeds levels found among mutual funds, hence the magnitude of wealth transfers attributable to hedge-fund NAV-timing may still be significant.

With respect to the deliberate smoothing of performance by managers, a recent study of closed-end funds by Chandar and Bricker (2002) concludes that managers seem to use accounting discretion in valuing restricted securities so as to optimize fund returns with respect to a passive benchmark. Because mutual funds are highly regulated entities that are required to disclose considerably more information about their holdings than hedge funds, Chandar and Bricker (2002) were able to perform a detailed analysis of the periodic adjustments—both discretionary and non-discretionary—that fund managers made to the valuation of their restricted securities. Their findings suggest that performance smoothing may be even more relevant in the hedge-fund industry which is not nearly as transparent, and that econometric models of smoothed returns may be an important tool for detecting such behavior and unraveling its effects on true economic returns.

3 Other Sources of Serial Correlation

Before turning to our econometric model of smoothed returns in Section 4, we first consider four other potential sources of serial correlation in asset returns: (1) market inefficiencies; (2) time-varying expected returns; (3) time-varying leverage; and (4) incentive fees with high water marks.

Perhaps the most common explanation (at least among industry professionals and certain academics) for the presence of serial correlation in asset returns is a violation of the Efficient Markets Hypothesis, one of the central pillars of modern finance theory. As with so many of the ideas of modern economics, the origins of the Efficient Markets Hypothesis can be traced back to Paul Samuelson (1965), whose contribution is neatly summarized by the title of his article: "Proof that Properly Anticipated Prices Fluctuate Randomly". In an informationally efficient market, price changes must be unforecastable if they are properly anticipated, i.e., if they fully incorporate the expectations and information of all market participants. Fama (1970) operationalizes this hypothesis, which he summarizes in the well-known epithet "prices fully reflect all available information", by placing structure on various information sets available to market participants. This concept of informational efficiency has a wonderfully counter-intuitive and seemingly contradictory flavor to it: the more efficient the market, the more random the sequence of price changes generated by such a market, and the most efficient market of all is one in which price changes are completely random and unpredictable. This, of course, is not an accident of Nature but is the direct result of many active participants attempting to profit from their information. Unable to curtail their greed, an army of investors aggressively pounce on even the smallest informational advantages at their disposal, and in doing so, they incorporate their information into market prices and quickly eliminate the profit opportunities that gave rise to their aggression. If this occurs instantaneously, which it must in an idealized world of "frictionless" markets and costless trading, then prices must always fully reflect all available information and no profits can be garnered from information-based trading (because such profits have already been captured).

In the context of hedge-fund returns, one interpretation of the presence of serial correlation is that the hedge-fund manager is not taking full advantage of the information or "alpha" contained in his strategy. For example, if a manager's returns are highly positively autocorrelated, then it should be possible for him to improve his performance by exploiting this fact—in months where his performance is good, he should increase his bets in anticipation of continued good performance (due to positive serial correlation), and in months where his performance is poor, he should reduce his bets accordingly. The reverse argument can be made for the case of negative serial correlation. By taking advantage of serial correlation of either sign in his returns, the hedge-fund manager will eventually eliminate it along the lines of Samuelson (1965), i.e., properly anticipated hedge-fund returns should fluctuate randomly. And if this self-correcting mechanism of the Efficient Markets Hypothesis is at work among any group of investors in the financial community, it surely must be at work among hedge-fund managers, which consists of a highly trained, highly motivated, and highly competitive group of sophisticated investment professionals.

Of course, the natural counter-argument to this somewhat naive application of the Efficient Markets Hypothesis is that hedge-fund managers cannot fully exploit such serial correlation because of transactions costs and liquidity constraints. But once again, this leads to the main thesis of this paper: serial correlation is a proxy for illiquidity and smoothed returns.

There are, however, at least three additional explanations for the presence of serial correlation. One of the central tenets of modern financial economics is the necessity of some trade-off between risk and expected return, hence serial correlation may not be exploitable in the sense that an attempt to take advantage of predictabilities in fund returns might be offset by corresponding changes in risk, leaving the fund manager indifferent at the margin between his current investment policy and other alternatives. Specifically, LeRoy (1973), Rubinstein (1976), and Lucas (1978) have demonstrated conclusively that serial correlation in asset returns need not be the result of market inefficiencies, but may be the result of time-varying expected returns, which is perfectly consistent with the Efficient Markets Hypothesis.[4] If an investment strategy's required expected return varies through time—because

[4] Grossman (1976) and Grossman and Stiglitz (1980) go even further. They argue that perfectly informationally efficient markets are an *impossibility*, for if markets are perfectly efficient, the return to gathering information is nil, in which case there would be little reason to trade and markets would eventually collapse. Alternatively, the degree of market *inefficiency* determines the effort investors are willing to expend

of changes in its risk exposures, for example—then serial correlation may be induced in realized returns without implying any violation of market efficiency (see Figure 1). We examine this possibility more formally in Section 3.1.

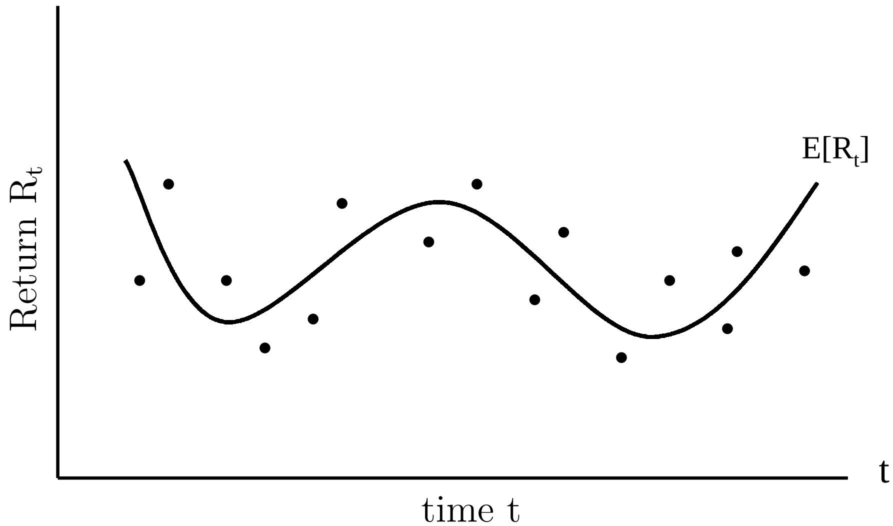

Figure 1: Time-varying expected returns can induce serial correlation in asset returns.

Another possible source of serial correlation in hedge-fund returns is time-varying leverage. If managers change the degree to which they leverage their investment strategies, and if these changes occur in response to lagged market conditions, this is tantamount to time-varying expected returns. We consider this case in Section 3.2.

Finally, we investigate one more potential explanation for serial correlation: the compensation structure of the typical hedge fund. Because most hedge funds charge an incentive fee coupled with a "high water mark" that must be surpassed before incentive fees are paid, this path dependence in the computation for net-of-fee returns may induce serial correlation. We develop a formal model of this phenomenon in Section 3.3.

The analysis of Sections 3.1–3.3 show that time-varying expected returns, time-varying leverage, and incentive fees with high water marks can all generate serial correlation in

to gather and trade on information, hence a non-degenerate market equilibrium will arise only when there are sufficient profit opportunities, i.e., inefficiencies, to compensate investors for the costs of trading and information-gathering. The profits earned by these attentive investors may be viewed as economic rents that accrue to those willing to engage in such activities. Who are the providers of these rents? Black (1986) gives a provocative answer: noise traders, individuals who trade on what they think is information but is in fact merely noise.

hedge-fund returns, but none of these effects can plausibly generate serial correlation to the degree observed in the data, e.g., 30% to 50% for monthly returns. Therefore, illiquidity and smoothed returns are more likely sources of serial correlation in hedge-fund returns.

3.1 Time-Varying Expected Returns

Let R_t denote a hedge fund's return in month t, and suppose that its dynamics are given by the following time-series process:

$$R_t \;=\; \mu_1\, I_t \;+\; \mu_0\,(1-I_t) \;+\; \epsilon_t \tag{1}$$

where ϵ_t is assumed to be independently and identically distributed (IID) with mean 0 and variance σ_ϵ^2, and I_t is a two-state Markov process with transition matrix:

$$P \;\equiv\; \begin{array}{c} \\ I_t\!=\!1 \\ I_t\!=\!0 \end{array} \!\!\begin{array}{c} \begin{array}{cc} I_{t+1}\!=\!1 & I_{t+1}\!=\!0 \end{array} \\ \left(\begin{array}{cc} p & 1-p \\ 1-q & q \end{array}\right) \end{array} \tag{2}$$

and μ_0 and μ_1 are the equilibrium expected returns of fund i in states 0 and 1, respectively. This is a particularly simple model of time-varying expected returns in which we abstract from the underlying structure of the economy that gives rise to (1), but focus instead on the serial correlation induced by the Markov regime-switching process (2).[5] In particular, observe that

$$P^k \;=\; \frac{1}{2-p-q}\begin{pmatrix} 1-q & 1-p \\ 1-q & 1-p \end{pmatrix} + \frac{(p+q-1)^k}{2-p-q}\begin{pmatrix} 1-p & -(1-p) \\ -(1-q) & 1-q \end{pmatrix} \tag{3}$$

[5]For examples of dynamic general equilibrium models that yield a Markov-switching process for asset prices, and econometric methods to estimate such processes, see Cecchetti and Mark (1990), Goodwin (1993), Hamilton (1989, 1990, 1996), Kandel and Stambaugh (1991), and Turner, Startz, and Nelson (1989).

assuming that $|p+q-1| < 1$, hence the steady-state probabilities and moments for the regime-switching process I_t are:

$$P^{\infty} = \begin{pmatrix} \pi_1 \\ \\ \pi_0 \end{pmatrix} = \begin{pmatrix} \frac{1-q}{2-p-q} \\ \\ \frac{1-p}{2-p-q} \end{pmatrix} \tag{4}$$

$$\mathrm{E}[I_t] = \frac{1-q}{2-p-q} \tag{5}$$

$$\mathrm{Var}[I_t] = \frac{(1-p)(1-q)}{(2-p-q)^2} \tag{6}$$

These, in turn, imply the following moments for R_t:

$$\mathrm{E}[R_t] = \mu_1 \frac{1-q}{2-p-q} + \mu_0 \frac{1-p}{2-p-q} \tag{7}$$

$$\mathrm{Var}[R_t] = (\mu_1 - \mu_0)^2 \frac{(1-p)(1-q)}{(2-p-q)^2} + \sigma^2_{\epsilon_i} \tag{8}$$

$$\rho_k \equiv \mathrm{Corr}[R_{t-k}, R_t] = \frac{(p+q-1)^k}{1+\sigma^2_\epsilon / \left[(\mu_1-\mu_0)^2 \frac{(1-p)(1-q)}{(2-p-q)^2}\right]} \tag{9}$$

By calibrating the parameters μ_1, μ_0, p, q, and σ^2_ϵ to empirically plausible values, we can compute the serial correlation induced by time-varying expected returns using (9).

Observe from (9) that the serial correlation of returns depends on the squared difference of expected returns, $(\mu_1-\mu_0)^2$, not on the particular values in either regime. Moreover, the absolute magnitudes of the autocorrelation coefficients ρ_k are monotonically increasing in $(\mu_1-\mu_0)^2$—the larger the difference in expected returns between the two states, the more serial correlation is induced. Therefore, we begin our calibration exercise by considering an extreme case where $|\mu_1-\mu_0|$ is 5% per month, or 60% per year, which yields rather dramatic shifts in regimes. To complete the calibration exercise, we fix the unconditional variance of returns at a particular value, say $(20\%)^2/12$ (which is comparable to the volatility of the S&P 500 over the past 30 years), vary p and q, and solve for the values of σ^2_ϵ that are consistent with the values of p, q, $(\mu_1-\mu_0)^2$, and the unconditional variance of returns.

The top panel of Table 1 reports the first-order autocorrelation coefficients for various values of p and q under these assumptions, and we see that even in this most extreme case, the largest absolute magnitude of serial correlation is only 15%. The second panel of Table 1 shows that when the unconditional variance of returns is increased from 20% to 50% per year, the correlations decline in magnitude with the largest absolute correlation of 2.4%. And the bottom panel illustrates the kind of extreme parameter values needed to obtain autocorrelations that are empirically relevant for hedge-fund returns—a difference in expected returns of 20% per month or 240% per year, and probabilities p and q that are either both 80% or higher, or both 20% or lower. Given the implausibility of these parameter values, we conclude that time-varying expected returns—at least of this form—may not be the most likely explanation for serial correlation in hedge-fund returns.

3.2 Time-Varying Leverage

Another possible source of serial correlation in hedge-fund returns is time-varying leverage. Since leverage directly affects the expected return of any investment strategy, this can be considered a special case of the time-varying expected returns model of Section 3.1. Specifically, if L_t denotes a hedge fund's leverage ratio, then the actual return R_t^o of the fund at date t is given by:

$$R_t^o \quad = \quad L_t \, R_t \tag{10}$$

where R_t is the fund's unlevered return.[6] For example if a fund's unlevered strategy yields a 2% return in a given month, but 50% of the funds are borrowed from various counterparties at fixed borrowing rates, the return to the fund's investors is approximately 4%,[7] hence the leverage ratio is 2.

The specific mechanisms by which a hedge fund determines its leverage can be quite complex and often depend on a number of factors including market volatility, credit risk,

[6] For simplicity, and with little loss in generality, we have ignored the borrowing costs associated with leverage in our specification (10). Although including such costs will obviously reduce the net return, the serial correlation properties will be largely unaffected because the time variation in borrowing rates is not significant relative to R_t and L_t.

[7] Less the borrowing rate, of course, which we assume is 0 for simplicity.

ρ_1 (%)		q (%)								
		10	20	30	40	50	60	70	80	90
$\lvert\mu_1 - \mu_0\rvert = 5\%$, $\text{Var}[R_t] = (20\%)^2/12$										
	10	−15.0	−13.1	−11.1	−9.0	−6.9	−4.8	−2.8	−1.1	0.0
	20	−13.1	−11.3	−9.3	−7.3	−5.3	−3.3	−1.5	0.0	0.7
	30	−11.1	−9.3	−7.5	−5.6	−3.6	−1.7	0.0	1.3	1.6
	40	−9.0	−7.3	−5.6	−3.8	−1.9	0.0	1.7	2.8	2.8
p (%)	50	−6.9	−5.3	−3.6	−1.9	0.0	1.9	3.5	4.6	4.2
	60	−4.8	−3.3	−1.7	0.0	1.9	3.8	5.5	6.7	6.0
	70	−2.8	−1.5	0.0	1.7	3.5	5.5	7.5	9.0	8.4
	80	−1.1	0.0	1.3	2.8	4.6	6.7	9.0	11.3	11.7
	90	0.0	0.7	1.6	2.8	4.2	6.0	8.4	11.7	15.0
$\lvert\mu_1 - \mu_0\rvert = 5\%$, $\text{Var}[R_t] = (50\%)^2/12$										
	10	−2.4	−2.1	−1.8	−1.4	−1.1	−0.8	−0.5	−0.2	0.0
	20	−2.1	−1.8	−1.5	−1.2	−0.9	−0.5	−0.2	0.0	0.1
	30	−1.8	−1.5	−1.2	−0.9	−0.6	−0.3	0.0	0.2	0.3
	40	−1.4	−1.2	−0.9	−0.6	−0.3	0.0	0.3	0.5	0.4
p (%)	50	−1.1	−0.9	−0.6	−0.3	0.0	0.3	0.6	0.7	0.7
	60	−0.8	−0.5	−0.3	0.0	0.3	0.6	0.9	1.1	1.0
	70	−0.5	−0.2	0.0	0.3	0.6	0.9	1.2	1.4	1.4
	80	−0.2	0.0	0.2	0.5	0.7	1.1	1.4	1.8	1.9
	90	0.0	0.1	0.3	0.4	0.7	1.0	1.4	1.9	2.4
$\lvert\mu_1 - \mu_0\rvert = 20\%$, $\text{Var}[R_t] = (50\%)^2/12$										
	10	−38.4	−33.5	−28.4	−23.0	−17.6	−12.3	−7.2	−2.9	0.0
	20	−33.5	−28.8	−23.9	−18.8	−13.6	−8.5	−3.8	0.0	1.9
	30	−28.4	−23.9	−19.2	−14.3	−9.3	−4.4	0.0	3.3	4.2
	40	−23.0	−18.8	−14.3	−9.6	−4.8	0.0	4.3	7.2	7.1
p (%)	50	−17.6	−13.6	−9.3	−4.8	0.0	4.7	9.0	11.8	10.7
	60	−12.3	−8.5	−4.4	0.0	4.7	9.6	14.1	17.1	15.4
	70	−7.2	−3.8	0.0	4.3	9.0	14.1	19.2	23.0	21.6
	80	−2.9	0.0	3.3	7.2	11.8	17.1	23.0	28.8	29.9
	90	0.0	1.9	4.2	7.1	10.7	15.4	21.6	29.9	38.4

Table 1: First-order autocorrelation coefficients of returns from a two-state Markov model of time-varying expected returns, $R_t = \mu_1 I_t + \mu_0(1 - I_t) + \epsilon_t$, where $p \equiv \text{Prob}(I_{t+1} = 1 | I_t = 1)$, $q \equiv \text{Prob}(I_{t+1} = 0 | I_t = 0)$, μ_1 and μ_0 are the monthly expected returns in states 1 and 0, respectively, and $\epsilon_t \sim \mathcal{N}(0, \sigma_\epsilon^2)$ and σ_ϵ^2 is calibrated to fix the unconditional variance $\text{Var}[R_t]$ of returns at a prespecified level.

and various constraints imposed by investors, regulatory bodies, banks, brokers, and other counterparties. But the basic motivation for typical leverage dynamics is the well-known trade-off between risk and expected return: by increasing its leverage ratio, a hedge fund boosts its expected returns proportionally, but also increases its return volatility and, eventually, its credit risk or risk of default. Therefore, counterparties providing credit facilities for hedge funds will impose some ceiling on the degree of leverage they are willing to provide. More importantly, as market prices move against a hedge fund's portfolio, thereby reducing the value of the fund's collateral and increasing its leverage ratio, or as markets become more volatile and the fund's risk exposure increases significantly, creditors (and, in some cases, securities regulations) will require the fund to either post additional collateral or liquidate a portion of its portfolio to bring the leverage ratio back down to an acceptable level. As a result, the leverage ratio of a typical hedge fund varies through time in a specific manner, usually as a function of market prices and market volatility. Therefore we propose a simple data-dependent mechanism through which a hedge fund determines its ideal leverage ratio.

Denote by R_t the return of a fund in the absence of any leverage, and to focus squarely on the ability of leverage to generate serial correlation, let R_t be IID through time, hence:

$$R_t \;\;=\;\; \mu \;+\; \epsilon_t \quad , \quad \epsilon_t \text{ IID } \mathcal{N}(0, \sigma_\epsilon^2) \tag{11}$$

where we have assumed that ϵ_t is normally distributed only for expositional convenience.[8] Given (10), the k-th order autocorrelation coefficient of leveraged returns R_t^o is:

$$\begin{aligned}\rho_k \;\;=\;\; \frac{1}{\text{Var}[R_t^o]} \Bigg[& \mu^2 \, \text{Cov}[L_t, L_{t+k}] + \mu \, \text{Cov}[L_t, L_{t+k}\epsilon_{t+k}] \; + \\ & \mu \, \text{Cov}[L_{t+k}, L_t\epsilon_t] \; + \text{Cov}[L_t\epsilon_t, L_{t+k}\epsilon_{t+k}] \Bigg] \, . \end{aligned} \tag{12}$$

Now suppose that the leverage process L_t is independently distributed through time and also independent of ϵ_{t+k} for all k. Then (12) implies that $\rho_k = 0$ for all $k \neq 0$, hence time-varying

[8]Other distributions can easily be used instead of the normal in the Monte Carlo simulation experiment described below.

leverage of this sort will not induce any serial correlation in returns R_t^o.

However, as discussed above, leverage is typically a function of market conditions, which can induce serial dependence in L_t and dependence between L_{t+k} and ϵ_t for $k \geq 0$, yielding serially correlated observed returns R_t^o.

To see how, we propose a simple but realistic mechanism by which a hedge fund might manage its leverage ratio. Suppose that, as part of its enterprise-wide risk management protocol, a fund has adopted a policy of limiting the 95% Value-at-Risk of its portfolio to no worse than δ—for example, if $\delta = -10\%$, this policy requires managing the portfolio so that the probability of a loss greater than or equal to 10% of the fund's assets is at most 5%. If we assume that the only control variable available to the manager is the leverage ratio L_t and that unleveraged returns R_t are given by (11), this implies the following constraint on leverage:

$$
\begin{aligned}
\text{Prob}(R_t^o \leq \delta) &\leq 5\% \quad , \quad \delta \leq 0 \\
\text{Prob}(L_t R_t \leq \delta) &\leq 5\% \\
\text{Prob}\left(\frac{R_t - \mu}{\sigma} \leq \frac{\delta / L_t - \mu}{\sigma} \right) &\leq 5\% \\
\Phi\left(\frac{\delta / L_t - \mu}{\sigma} \right) &\leq 5\% \qquad (13) \\
\delta / L_t &\leq \sigma \Phi^{-1}(5\%) \qquad (14) \\
\Rightarrow \quad L_t &\leq \frac{\delta}{\sigma \Phi^{-1}(5\%)} \qquad (15)
\end{aligned}
$$

where, following common industry practice, we have set $\mu = 0$ in (13) to arrive at (14).[9] Now in implementing the constraint (15), the manager must estimate the portfolio volatility σ, which is typically estimated using some rolling window of historical data, hence the manager's estimate is likely to be time-varying but persistent to some degree. This persistence, and

[9]Setting the expected return of a portfolio equal to 0 for purposes of risk management is often motivated by a desire to be conservative. Most portfolios will tend to have positive expected return, hence setting μ equal to 0 will generally yield larger values for VaR. However, for actively managed portfolios that contain both long and short positions, Lo (2002) shows that the practice of setting expected returns equal to 0 need not be conservative, but in some cases, can yield severely downward-biased estimates of VaR.

the dependence of the volatility estimate on past returns, will both induce serial correlation in observed returns R_t^o. Specifically, let:

$$\hat{\sigma}_t^2 \equiv \frac{1}{n}\sum_{k=1}^{n}(R_{t-k}-\hat{\mu})^2 \quad , \quad \hat{\mu}_t \equiv \frac{1}{n}\sum_{k=1}^{n}R_{t-k} \tag{16}$$

$$L_t = \frac{\delta}{\hat{\sigma}_t \Phi^{-1}(5\%)} \tag{17}$$

where we have assumed that the manager sets his leverage ratio L_t to the maximum allowable level subject to the VaR constraint (15).

To derive the impact of this heuristic risk management policy on the serial correlation of observed returns, we perform a Monte Carlo simulation experiment where we simulate a time series of 100,000 returns $\{R_t\}$ and implement the leverage policy (17) to obtain a time series of observed returns $\{R_t^o\}$, from which we compute its autocorrelation coefficients $\{\rho_k\}$. Given the large sample size, our estimate should yield an excellent approximation to the population values of the autocorrelation coefficients. This procedure is performed for the following combinations of parameter values:

$$\begin{aligned} n &= 3, 6, 9, 12, 24, 36, 48, 60 \\ 12\,\mu &= 5\% \\ \sqrt{12}\,\sigma &= 10\%, 20\%, 50\% \\ \delta &= -25\% \end{aligned}$$

and the results are summarized in Table 2. Note that the autocorrelation of observed returns (12) is homogeneous of degree 0 in δ, hence we need only simulate our return process for one value of δ without loss of generality as far as ρ_k is concerned. Of course, the mean and standard of observed returns and leverage will be affected by our choice of δ, but because these variables are homogeneous of degree 1, we can obtain results for any arbitrary δ simply by rescaling our results for $\delta = -25\%$.

n	Return R_t^o		Leverage L_t		Return R_t^o			Leverage L_t		
	12 Mean (%)	$\sqrt{12}$ SD (%)	Mean	SD	ρ_1 (%)	ρ_2 (%)	ρ_3 (%)	ρ_1 (%)	ρ_2 (%)	ρ_3 (%)
			$12\,\mu = 5\%$,	$\sqrt{12}\,\sigma = 10\%$,	$\delta = -25\%$					
3	50.53	191.76	9.52	15.14	0.7	0.3	0.4	17.5	2.9	0.0
6	29.71	62.61	5.73	2.45	0.1	0.4	0.3	70.6	48.5	32.1
12	24.34	51.07	4.96	1.19	0.1	0.4	−0.3	88.9	78.6	68.8
24	24.29	47.27	4.66	0.71	0.3	0.1	−0.2	95.0	90.0	85.1
36	21.46	46.20	4.57	0.57	−0.2	0.0	0.1	96.9	93.9	90.9
48	22.67	45.61	4.54	0.46	0.3	−0.5	0.3	97.6	95.3	92.9
60	22.22	45.38	4.51	0.43	−0.2	0.0	0.2	98.2	96.5	94.7
			$12\,\mu = 5\%$,	$\sqrt{12}\,\sigma = 20\%$,	$\delta = -25\%$					
3	26.13	183.78	4.80	8.02	0.0	−0.1	0.0	13.4	1.9	−0.6
6	14.26	62.55	2.87	1.19	0.2	0.1	0.4	70.7	48.6	32.0
12	12.95	50.99	2.48	0.59	0.2	−0.1	0.1	89.1	79.0	69.4
24	11.58	47.22	2.33	0.36	0.2	0.0	0.1	95.2	90.4	85.8
36	11.23	46.14	2.29	0.28	−0.1	0.3	−0.3	97.0	94.0	90.9
48	11.00	45.63	2.27	0.24	0.2	−0.5	−0.1	97.8	95.5	93.3
60	12.18	45.37	2.26	0.21	0.1	−0.1	0.4	98.3	96.5	94.8
			$12\,\mu = 5\%$,	$\sqrt{12}\,\sigma = 50\%$,	$\delta = -25\%$					
3	9.68	186.59	1.93	3.42	−1.1	0.0	−0.5	14.7	1.8	−0.1
6	6.25	62.43	1.16	0.48	−0.2	0.3	−0.2	70.9	49.4	32.9
12	5.90	50.94	0.99	0.23	−0.1	0.1	0.0	89.0	78.6	69.0
24	5.30	47.29	0.93	0.15	0.2	0.3	0.4	95.2	90.5	85.7
36	5.59	46.14	0.92	0.12	−0.1	0.3	−0.2	97.0	94.1	91.1
48	4.07	45.64	0.91	0.10	−0.4	−0.6	0.1	97.8	95.7	93.5
60	5.11	45.34	0.90	0.08	0.4	0.3	−0.3	98.2	96.5	94.7

Table 2: Monte Carlo simulation results for time-varying leverage model with a VaR constraint. Each row corresponds to a separate and independent simulation of 100,000 observations of independently and identically distributed $\mathcal{N}(\mu, \sigma^2)$ returns R_t which are multiplied by a time-varying leverage factor L_t to generated observed returns $R_t^o \equiv L_t R_t$.

For a VaR constraint of -25% and an annual standard deviation of unlevered returns of 10%, the mean leverage ratio ranges from 9.52 when $n = 3$ to 4.51 when $n = 60$. For small n, there is considerably more sampling variation in the estimated standard deviation of returns, hence the leverage ratio—which is proportional to the reciprocal of $\hat{\sigma}_t$—takes on more extreme values as well and has a higher expectation in this case.

As n increases, the volatility estimator becomes more stable over time since each month's estimator has more data in common with the previous month's estimator, leading to more persistence in L_t as expected. For example, when $n=3$, the average first-order autocorrelation coefficient of L_t is 43.2%, but increases to 98.2% when $n=60$. However, even with such extreme levels of persistence in L_t, the autocorrelation induced in observed returns R_t^o is still only -0.2%. In fact, the largest absolute return-autocorrelation reported in Table 2 is only 0.7%, despite the fact that leverage ratios are sometimes nearly perfectly autocorrelated from month to month. This suggests that time-varying leverage, at least of the form described by the VaR constraint (15), cannot fully account for the magnitudes of serial correlation in historical hedge-fund returns.

3.3 Incentive Fees with High-Water Marks

Yet another source of serial correlation in hedge-fund returns is an aspect of the fee structure that is commonly used in the hedge-fund industry: an incentive fee—typically 20% of excess returns above a benchmark—which is subject to a "high-water mark", meaning that incentive fees are paid only if the cumulative returns of the fund are "above water", i.e., if they exceed the *cumulative* return of the benchmark since inception.[10] This type of nonlinearity can induce serial correlation in net-of-fee returns because of the path dependence inherent in the definition of the high-water mark—when the fund is "below water" the incentive fee is not charged, but over time, as the fund's cumulative performance rises "above water", the incentive fee is reinstated and the net-of-fee returns is reduced accordingly.

Specifically, denote by F_t the incentive fee paid to the manager in period t and for

[10] For more detailed analyses of high water marks and other incentive-fee arrangements in the context of delegated portfolio management, see Bhattacharya and Pfleiderer (1985), Brown, Goetzmann, and Liang (2002), Carpenter (2000), Carpenter, Dybvig, and Farnsworth (2001), Elton and Gruber (2002), and Goetzmann, Ingersoll, and Ross (1997).

simplicity, set the benchmark to 0. Then:

$$F_t \equiv \text{Max}[\,0\,,\ \gamma(X_{t-1}+R_t)\,]\ ,\quad \gamma>0 \tag{18a}$$

$$X_t \equiv \text{Min}[\,0\,,\ X_{t-1}+R_t\,] \tag{18b}$$

where X_t is a state variable that is non-zero only when the manager is "under water", in which case it measures the cumulative losses that must be recovered before an incentive fee is paid. The net-of-fee returns R_t^o are then given by:

$$R_t^o = R_t - F_t = (1-\gamma)R_t + \gamma(X_t - X_{t-1}) \tag{19}$$

which is clearly serially correlated due to the presence of the lagged state variable X_{t-1}.[11]

Because the high-water mark variable X_t is a nonlinear recursive function of X_{t-1} and R_t, its statistical properties are quite complex and difficult to derive in closed form. Therefore, we perform a Monte Carlo simulation experiment in which we simulate a time series of returns $\{R_t\}$ of length $T=100{,}000$ where R_t is given by (11), compute the net-of-fee returns $\{R_t^o\}$, and estimate the first-order autocorrelation coefficient ρ_1. We follow this procedure for each of the combinations of the following parameter values:

$$\begin{aligned} 12\,\mu &= 5\%, 10\%, 15\%, \ldots, 50\% \\ \sqrt{12}\,\sigma &= 10\%, 20\%, \ldots, 50\% \\ \gamma &= 20\%\ . \end{aligned}$$

Table 3 summarizes the results of the simulations which show that although incentive fees

[11] This is a simplified model of how a typical hedge fund's incentive fee is structured. In particular, (18) ignores the fact that incentive fees are usually paid on an annual or quarterly basis whereas high-water marks are tracked on a monthly basis. Using the more realistic fee cycle did not have significant impact on our simulation results, hence we use (18) for expositional simplicity. Also, some funds do pay their employees and partners monthly incentive compensation, in which case (18) is the exact specification of their fee structure.

with high-water marks do induce some serial correlation in net-of-fee returns, they are generally quite small in absolute value. For example, the largest absolute value of all the entries in Table 3 is only 4.4%. Moreover, all of the averages are negative, a result of the fact that all of the serial correlation in R_t^o is due to the first difference of X_t in (19). This implies that incentive fees with high-water marks are even less likely to be able to explain the large positive serial correlation in historical hedge-fund returns.

ρ_1 (%)		$12\,\mu$ (%) 5	10	15	20	25	30	35	40	45	50
$\sigma \times \sqrt{12}$ (%)	10	−1.4	−2.5	−3.2	−3.4	−3.4	−3.2	−2.9	−2.4	−2.0	−1.5
	20	−1.6	−2.3	−2.9	−3.4	−3.8	−4.1	−4.3	−4.4	−4.4	−4.3
	30	−0.6	−1.1	−1.6	−2.1	−2.4	−2.8	−3.0	−3.3	−3.5	−3.6
	40	−0.2	−0.7	−1.1	−1.4	−1.8	−2.1	−2.3	−2.6	−2.8	−3.0
	50	0.0	−0.3	−0.6	−0.9	−1.2	−1.5	−1.7	−1.9	−2.1	−2.3

Table 3: First-order autocorrelation coefficients for Monte Carlo simulation of net-of-fee returns under an incentive fee with a high-water mark. Each entry corresponds to a separate and independent simulation of 100,000 observations of independently and identically distributed $\mathcal{N}(\mu, \sigma^2)$ returns R_t, from which a 20% incentive fee $F_t \equiv \text{Max}[0, 0.2\times(X_{t-1}+R_t)]$ is subtracted each period to yield net-of-fee returns $R_t^o \equiv R_t - F_t$, where $X_t \equiv \text{Min}[0, X_{t-1}+R_t]$ is a state variable that is non-zero only when the fund is "under water", in which case it measures the cumulative losses that must be recovered before an incentive fee is paid.

4 An Econometric Model of Smoothed Returns

Having shown in Section 3 that other possible sources of serial correlation in hedge-fund returns are hard-pressed to yield empirically plausible levels of autocorrelation, we now turn to the main focus of this study: illiquidity and smoothed returns. Although illiquidity and smoothed returns are two distinct phenomena, it is important to consider them in tandem because one facilitates the other—for actively traded securities, both theory and empirical evidence suggest that in the absence of transactions costs and other market frictions, returns are unlikely to be very smooth.

As we argued in Section 1, nonsynchronous trading is a plausible source of serial correlation in hedge-fund returns. In contrast to the studies by Lo and MacKinlay (1988, 1990) and Kadlec and Patterson (1999) in which they conclude that it is difficult to generate serial correlations in weekly US equity portfolio returns much greater than 10% to 15% through nonsynchronous trading effects alone, we argue that in the context of hedge funds, significantly higher levels of serial correlation can be explained by the combination of illiquidity and smoothed returns, of which nonsynchronous trading is a special case. To see why, note that the empirical analysis in the nonsynchronous-trading literature is devoted exclusively to exchange-traded equity returns, not hedge-fund returns, hence their conclusions may not be relevant in our context. For example, Lo and MacKinlay (1990) argue that securities would have to go without trading for several days on average to induce serial correlations of 30%, and they dismiss such nontrading intervals as unrealistic for most exchange-traded US equity issues. However, such nontrading intervals are considerably more realistic for the types of securities held by many hedge funds, e.g., emerging-market debt, real estate, restricted securities, control positions in publicly traded companies, asset-backed securities, and other exotic OTC derivatives. Therefore, nonsynchronous trading of this magnitude is likely to be an explanation for the serial correlation observed in hedge-fund returns.

But even when prices are synchronously measured—as they are for many funds that mark their portfolios to market at the end of the month to strike a net-asset-value at which investors can buy into or cash out of the fund—there are several other channels by which illiquidity exposure can induce serial correlation in the reported returns of hedge funds. Apart from the nonsynchronous-trading effect, naive methods for determining the fair market value or "marks" for illiquid securities can yield serially correlated returns. For example, one approach to valuing illiquid securities is to extrapolate linearly from the most recent transaction price (which, in the case of emerging-market debt, might be several months ago), which yields a price path that is a straight line, or at best a series of straight lines. Returns computed from such marks will be smoother, exhibiting lower volatility and higher serial correlation than true economic returns, i.e., returns computed from mark-to-market prices where the market is sufficiently active to allow all available information to be impounded in the price of the security. Of course, for securities that are more easily traded and with deeper markets, mark-to-market prices are more readily available, extrapolated marks are not necessary, and serial correlation is therefore less of an issue. But for securities that are thinly traded, or not traded at all for extended periods of time, marking them to market is often an expensive and time-consuming procedure that cannot easily be performed frequently. Therefore, we argue in this paper that serial correlation may serve as a proxy for a fund's liquidity exposure.

Even if a hedge-fund manager does not make use of any form of linear extrapolation to mark the securities in his portfolio, he may still be subject to smoothed returns if he obtains marks from broker-dealers that engage in such extrapolation. For example, consider the case of a conscientious hedge-fund manager attempting to obtain the most accurate mark for his portfolio at month end by getting bid/offer quotes from three independent broker-dealers for every security in his portfolio, and then marking each security at the average of the three quote midpoints. By averaging the quote midpoints, the manager is inadvertently downward-biasing price volatility, and if any of the broker-dealers employ linear extrapolation in formulating their quotes (and many do, through sheer necessity because they have little else to go on for the most illiquid securities), or if they fail to update their quotes because of light volume, serial correlation will also be induced in reported returns.

Finally, a more prosaic channel by which serial correlation may arise in the reported returns of hedge funds is through "performance smoothing", the unsavory practice of reporting only part of the gains in months when a fund has positive returns so as to partially offset potential future losses and thereby reduce volatility and improve risk-adjusted performance measures such as the Sharpe ratio. For funds containing liquid securities that can be easily marked to market, performance smoothing is more difficult and, as a result, less of a concern. Indeed, it is only for portfolios of illiquid securities that managers and brokers have any discretion in marking their positions. Such practices are generally prohibited by various securities laws and accounting principles, and great care must be exercised in interpreting smoothed returns as deliberate attempts to manipulate performance statistics. After all, as we have discussed above, there are many other sources of serial correlation in the presence of illiquidity, none of which is motivated by deceit. Nevertheless, managers do have certain degrees of freedom in valuing illiquid securities—for example, discretionary accruals for unregistered private placements and venture capital investments—and Chandar and Bricker (2002) conclude that managers of certain closed-end mutual funds do use accounting discretion to manage fund returns around a passive benchmark. Therefore, the possibility of deliberate performance smoothing in the less regulated hedge-fund industry must be kept in mind in interpreting our empirical analysis of smoothed returns.

To quantify the impact of all of these possible sources of serial correlation, denote by R_t the true economic return of a hedge fund in period t, and let R_t satisfy the following linear single-factor model:

$$R_t = \mu + \beta\Lambda_t + \epsilon_t \ , \quad \mathrm{E}[\Lambda_t] = \mathrm{E}[\epsilon_t] = 0 \ , \quad \epsilon_t \ , \ \Lambda_t \sim \text{IID} \tag{20a}$$

$$\mathrm{Var}[R_t] \equiv \sigma^2 \ . \tag{20b}$$

True returns represent the flow of information that would determine the equilibrium value of the fund's securities in a frictionless market. However, true economic returns are not observed. Instead, R_t^o denotes the reported or observed return in period t, and let

$$R_t^o = \theta_0 R_t + \theta_1 R_{t-1} + \cdots + \theta_k R_{t-k} \tag{21}$$

$$\theta_j \in [0,1] \ , \quad j = 0, \ldots, k \tag{22}$$

$$1 = \theta_0 + \theta_1 + \cdots + \theta_k \tag{23}$$

which is a weighted average of the fund's true returns over the most recent $k+1$ periods, including the current period.

This averaging process captures the essence of smoothed returns in several respects. From the perspective of illiquidity-driven smoothing, (21) is consistent with several models in the nonsynchronous trading literature. For example, Cohen, Maier et al. (1986, Chapter 6.1) propose a similar weighted-average model for observed returns.[12] Alternatively, (21) can be

[12]In particular, their specification for observed returns is:

$$r_{j,t}^o = \sum_{l=0}^{N} (\gamma_{j,t-l,l} r_{j,t-l} + \theta_{j,t-l})$$

where $r_{j,t-l}$ is the true but unobserved return for security j in period $t-l$, the coefficients $\{\gamma_{j,t-l,l}\}$ are assumed to sum to 1, and $\theta_{j,t-l}$ are random variables meant to capture "bid/ask bounce". The authors motivate their specification of nonsynchronous trading in the following way (p. 116): "Alternatively stated, the $\gamma_{j,t,0}, \gamma_{j,t,1}, \ldots, \gamma_{j,t,N}$ comprise a delay distribution that shows how the true return generated in period t impacts on the returns actually observed during t and the next N periods". In other words, the essential viewed as the outcome of marking portfolios to simple linear extrapolations of acquisition prices when market prices are unavailable, or "mark-to-model" returns where the pricing model is slowly varying through time. And of course, (21) also captures the intentional smoothing of performance.

The constraint (23) that the weights sum to 1 implies that the information driving the fund's performance in period t will eventually be fully reflected in observed returns, but this process could take up to $k+1$ periods from the time the information is generated.[13] This is a sensible restriction in the current context of hedge funds for several reasons. Even the most illiquid securities will trade eventually, and when that occurs, all of the cumulative information affecting that security will be fully impounded into its transaction price. Therefore the parameter k should be selected to match the kind of illiquidity of the fund—a fund comprised mostly of exchange-traded US equities would require a much lower value of k than a private equity fund. Alternatively, in the case of intentional smoothing of performance, the necessity of periodic external audits of fund performance imposes a finite limit on the extent to which deliberate smoothing can persist.[14]

4.1 Implications For Performance Statistics

Given the smoothing mechanism outlined above, we have the following implications for the statistical properties of observed returns:

---

feature of nonsynchronous trading is the fact that information generated at date t may not be fully impounded into prices until several periods later.

[13] In Lo and MacKinlay's (1990) model of nonsynchronous trading, they propose a stochastic non-trading horizon so that observed returns are an infinite-order moving average of past true returns, where the coefficients are stochastic. In that framework, the waiting time for information to become fully impounded into future returns may be arbitrarily long (but with increasingly remote probability).

[14] In fact, if a fund allows investors to invest and withdraw capital only at pre-specified intervals, imposing lock-ups in between, and external audits are conducted at these same pre-specified intervals, then it may be argued that performance smoothing is irrelevant. For example, no investor should be disadvantaged by investing in a fund that offers annual liquidity and engages in annual external audits with which the fund's net-asset-value is determined by a disinterested third party for purposes of redemptions and new investments. However, there are at least two additional concerns that remain—historical track records are still affected by smoothed returns, and estimates of a fund's liquidity exposure are also affected—both of which are important factors in the typical hedge-fund investor's overall investment process. Moreover, given the apparently unscrupulous role that the auditors at Arthur Andersen played in the Enron affair, there is the further concern of whether third-party auditors are truly objective and free of all conflicts of interest.

Proposition 1 *Under (21)–(23), the statistical properties of observed returns are characterized by:*

$$
\begin{aligned}
\mathrm{E}[R_t^o] &= \mu \qquad (24)\\
\mathrm{Var}[R_t^o] &= c_\sigma^2\,\sigma^2 \;\leq\; \sigma^2 \qquad (25)
\end{aligned}
$$

$$
\mathrm{SR}^o \equiv \frac{\mathrm{E}[R_t^o]}{\sqrt{\mathrm{Var}[R_t^o]}} = c_s\,\mathrm{SR} \;\geq\; \mathrm{SR} \equiv \frac{\mathrm{E}[R_t]}{\sqrt{\mathrm{Var}[R_t]}} \qquad (26)
$$

$$
\beta_m^o \equiv \frac{\mathrm{Cov}[R_t^o, \Lambda_{t-m}]}{\mathrm{Var}[\Lambda_{t-m}]} = \begin{cases} c_{\beta,m}\,\beta & \text{if } 0 \leq m \leq k \\ 0 & \text{if } m > k \end{cases} \qquad (27)
$$

$$
\mathrm{Cov}[R_t^o, R_{t-m}^o] = \begin{cases} \left(\sum_{j=0}^{k-m} \theta_j\theta_{j+m}\right)\sigma^2 & \text{if } 0 \leq m \leq k \\ 0 & \text{if } m > k \end{cases} \qquad (28)
$$

$$
\mathrm{Corr}[R_t^o, R_{t-m}^o] = \frac{\mathrm{Cov}[R_t^o, R_{t-m}^o]}{\mathrm{Var}[R_t^o]} = \begin{cases} \frac{\sum_{j=0}^{k-m}\theta_j\theta_{j+m}}{\sum_{j=0}^{k}\theta_j^2} & \text{if } 0 \leq m \leq k \\ 0 & \text{if } m > k \end{cases} \qquad (29)
$$

where:

$$
\begin{aligned}
c_\mu &\equiv \theta_0 + \theta_1 + \cdots + \theta_k \qquad (30)\\
c_\sigma^2 &\equiv \theta_0^2 + \theta_1^2 + \cdots + \theta_k^2 \qquad (31)\\
c_s &\equiv 1/\sqrt{\theta_0^2 + \cdots + \theta_k^2} \qquad (32)\\
c_{\beta,m} &\equiv \theta_m \qquad (33)
\end{aligned}
$$

Proposition 1 shows that smoothed returns of the form (21)–(23) do not affect the expected value of R_t^o but reduce its variance, hence boosting the Sharpe ratio of observed returns by a factor of c_s. From (27), we see that smoothing also affects β_0^o, the contemporaneous market beta of observed returns, biasing it towards 0 or "market neutrality", and induces correlation between current observed returns and lagged market returns up to lag k. This provides a formal interpretation of the empirical analysis of Asness, Krail, and Liew (2001) in which many hedge funds were found to have significant lagged market exposure despite relatively low contemporaneous market betas.

Smoothed returns also exhibit positive serial correlation up to order k according to (29), and the magnitude of the effect is determined by the pattern of weights $\{\theta_j\}$. If, for example, the weights are disproportionately centered on a small number of lags, relatively little serial correlation will be induced. However, if the weights are evenly distributed among many lags, this will result in higher serial correlation. A useful summary statistic for measuring the concentration of weights is

$$\xi \equiv \sum_{j=0}^{k} \theta_j^2 \in [0,1] \tag{34}$$

which is simply the denominator of (29). This measure is well known in the industrial organization literature as the *Herfindahl index*, a measure of the concentration of firms in a given industry where θ_j represents the market share of firm j. Because $\theta_j \in [0,1]$, ξ is also confined to the unit interval, and is minimized when all the θ_j's are identical, which implies a value of $1/(k+1)$ for ξ, and is maximized when one coefficient is 1 and the rest are 0, in which case $\xi = 1$. In the context of smoothed returns, a lower value of ξ implies more smoothing, and the upper bound of 1 implies no smoothing, hence we shall refer to ξ as a "smoothing index".

In the special case of equal weights, $\theta_j = 1/(k+1)$ for $j=0,\ldots,k$, the serial correlation of observed returns takes on a particularly simple form:

$$\mathrm{Corr}[R_t^o, R_{t-m}^o] = 1 - \frac{m}{k+1} \quad , \quad 1 \le m \le k \tag{35}$$

which declines linearly in the lag m. This can yield substantial correlations even when k is small—for example, if $k = 2$ so that smoothing takes place only over a current quarter (i.e. this month and the previous two months), the first-order autocorrelation of monthly observed returns is 66.7%.

To develop a sense for just how much observed returns can differ from true returns under the smoothed-return mechanism (21)–(23), denote by $\Delta(T)$ the difference between the cumulative observed and true returns over T holding periods, where we assume that $T > k$:

$$
\begin{aligned}
\Delta(T) &\equiv (R_1^o + R_2^o + \cdots + R_T^o) \; - \; (R_1 + R_2 + \cdots + R_T) && (36) \\
&= \sum_{j=0}^{k-1} (R_{-j} - R_{T-j})(1 - \sum_{i=0}^{j} \theta_i) && (37)
\end{aligned}
$$

Then we have:

Proposition 2 *Under (21)–(23) and for $T > k$,*

$$
\begin{aligned}
\mathrm{E}[\Delta(T)] &= 0 && (38) \\
\mathrm{Var}[\Delta(T)] &= 2\sigma^2 \sum_{j=0}^{k-1} \left(1 - \sum_{l=0}^{j} \theta_l\right)^2 \; = \; 2\sigma^2 \zeta && (39) \\
\zeta &\equiv \sum_{j=0}^{k-1} \left(1 - \sum_{l=0}^{j} \theta_l\right)^2 \; \leq \; k && (40)
\end{aligned}
$$

Proposition 2 shows that the cumulative difference between observed and true returns has 0 expected value, and its variance is bounded above by $2k\sigma^2$.

4.2 Examples of Smoothing Profiles

To develop further intuition for the impact of smoothed returns on observed returns, we consider the following three specific sets of weights $\{\theta_j\}$ or "smoothing profiles":[15]

$$\theta_j = \frac{1}{k+1} \qquad \text{(Straightline)} \tag{41}$$

$$\theta_j = \frac{k+1-j}{(k+1)(k+2)/2} \qquad \text{(Sum-of-Years)} \tag{42}$$

$$\theta_j = \frac{\delta^j(1-\delta)}{1-\delta^{k+1}} \ , \quad \delta \in (0,1) \qquad \text{(Geometric)}\ . \tag{43}$$

The straightline profile weights each return equally. In contrast, the sum-of-years and geometric profiles weight the current return the most heavily, and then has monotonically declining weights for lagged returns, with the sum-of-years weights declining linearly and the geometric weights declining more rapidly (see Figure 2).

More detailed information about the three smoothing profiles is contained in Table 4. The first panel reports the smoothing coefficients $\{\theta_j\}$, constants $c_{\beta,0}$, c_σ, c_s, ζ, and the first three autocorrelations of observed returns for the straightline profile for $k = 0, 1, \ldots, 5$. Consider the case where $k = 2$. Despite the relatively short smoothing period of three months, the effects are dramatic: smoothing reduces the market beta by 67%, increases the Sharpe ratio by 73%, and induces first- and second-order serial correlation of 67% and 33%, respectively, in observed returns. Moreover, the variance of the cumulative discrepancy between observed and true returns, $2\sigma^2\zeta$, is only slightly larger than the variance of monthly true returns σ^2, suggesting that it may be difficult to detect this type of smoothed returns even over time.

As k increases, the effects become more pronounced—for $k=5$, the market beta is reduced by 83%, the Sharpe ratio is increased by 145%, and first three autocorrelation coefficients are 83%, 67%, and 50%, respectively. However, in this extreme case, the variance of the discrepancy between true and observed returns is approximately three times the monthly variance of true returns, in which case it may be easier to identify smoothing from realized

[15] Students of accounting will recognize these profiles as commonly used methods for computing depreciation. The motivation for these depreciation schedules is not entirely without relevance in the smoothed-return context.

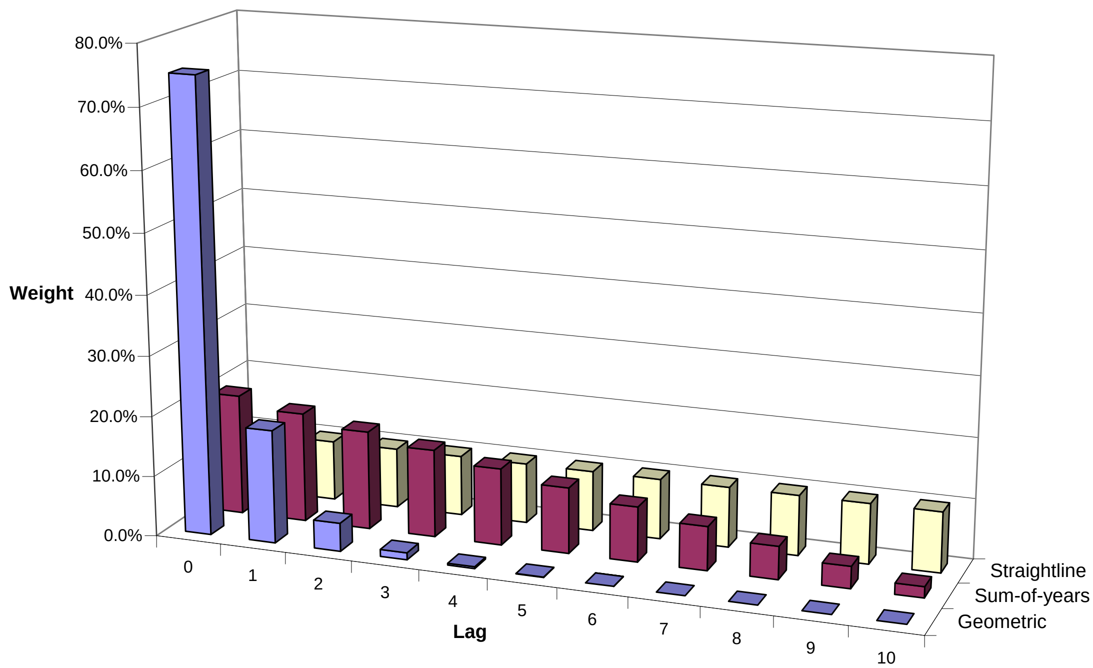

Figure 2: Straightline, sum-of-years, and geometric smoothing profiles for $k = 10$.

returns.

The sum-of-years profile is similar to, although somewhat less extreme than, the straightline profile for the same values of k because more weight is being placed on the current return. For example, even in the extreme case of $k=5$, the sum-of-years profile reduces the market beta by 71%, increases the Sharpe ratio by 120%, induces autocorrelations of 77%, 55%, and 35%, respectively, in the first three lags, and has a discrepancy variance that is approximately 1.6 times the monthly variance of true returns.

The last two panels of Table 4 contain results for the geometric smoothing profile for two values of δ, 0.25 and 0.50. For $\delta=0.25$, the geometric profile places more weight on the current return than the other two smoothing profiles for all values of k, hence the effects tend to be less dramatic. Even in the extreme case of $k=5$, 75% of current true returns are incorporated into observed returns, the market beta is reduced by only 25%, the Sharpe ratio is increased by only 29%, the first three autocorrelations are 25%, 6%, and 1% respectively, and the discrepancy variance is approximately 13% of the monthly variance of true returns. As δ increases, less weight is placed on the current observation and the effects on performance statistics become more significant. When $\delta=0.50$ and $k=5$, geometric smoothing reduces the market beta by 49%, increases the Sharpe ratio by 71%, induces autocorrelations of 50%, 25%, and 12%, respectively, for the first three lags, and yields a discrepancy variance that is approximately 63% of the monthly variance of true returns.

The three smoothing profiles have very different values for ζ in (40):

$$\zeta = \frac{k(2k+1)}{6(k+1)} \tag{44}$$

$$\zeta = \frac{k(3k^2+6k+1)}{15(k+1)(k+2)} \tag{45}$$

$$\zeta = \frac{\delta^2(-1+\delta^k(2+2\delta+\delta^k(-1-2\delta+k(\delta^2-1))))}{(\delta^2-1)(\delta^{k+1}-1)^2} \tag{46}$$

with the straightline and sum-of-years profiles implying variances for $\Delta(T)$ that grow approximately linearly in k, and the geometric profile implying a variance for $\Delta(T)$ that asymptotes to a finite limit (see Figure 3).

k	θ_0 (%)	θ_1 (%)	θ_2 (%)	θ_3 (%)	θ_4 (%)	θ_5 (%)	c_β	c_σ	c_s	ρ_1^o (%)	ρ_2^o (%)	ρ_3^o (%)	ρ_4^o (%)	ρ_5^o (%)	ζ (%)
Straightline Smoothing															
0	100.0	—	—	—	—	—	1.00	1.00	1.00	0.0	0.0	0.0	0.0	0.0	—
1	50.0	50.0	—	—	—	—	0.50	0.71	1.41	50.0	0.0	0.0	0.0	0.0	25.0
2	33.3	33.3	33.3	—	—	—	0.33	0.58	1.73	66.7	33.3	0.0	0.0	0.0	55.6
3	25.0	25.0	25.0	25.0	—	—	0.25	0.50	2.00	75.0	50.0	25.0	0.0	0.0	87.5
4	20.0	20.0	20.0	20.0	20.0	—	0.20	0.45	2.24	80.0	60.0	40.0	20.0	0.0	120.0
5	16.7	16.7	16.7	16.7	16.7	16.7	0.17	0.41	2.45	83.3	66.7	50.0	33.3	16.7	152.8
Sum-of-Years Smoothing															
0	100.0	—	—	—	—	—	1.00	1.00	1.00	0.0	0.0	0.0	0.0	0.0	—
1	66.7	33.3	—	—	—	—	0.67	0.75	1.34	40.0	0.0	0.0	0.0	0.0	11.1
2	50.0	33.3	16.7	—	—	—	0.50	0.62	1.60	57.1	21.4	0.0	0.0	0.0	27.8
3	40.0	30.0	20.0	10.0	—	—	0.40	0.55	1.83	66.7	36.7	13.3	0.0	0.0	46.0
4	33.3	26.7	20.0	13.3	6.7	—	0.33	0.49	2.02	72.7	47.3	25.5	9.1	0.0	64.9
5	28.6	23.8	19.0	14.3	9.5	4.8	0.29	0.45	2.20	76.9	54.9	35.2	18.7	6.6	84.1
Geometric Smoothing ($\delta = 0.25$)															
0	100.0	—	—	—	—	—	1.00	1.00	1.00	0.0	0.0	0.0	0.0	0.0	—
1	80.0	20.0	—	—	—	—	0.80	0.82	1.21	23.5	0.0	0.0	0.0	0.0	4.0
2	76.2	19.0	4.8	—	—	—	0.76	0.79	1.27	24.9	5.9	0.0	0.0	0.0	5.9
3	75.3	18.8	4.7	1.2	—	—	0.75	0.78	1.29	25.0	6.2	1.5	0.0	0.0	6.5
4	75.1	18.8	4.7	1.2	0.3	—	0.75	0.78	1.29	25.0	6.2	1.6	0.4	0.0	6.6
5	75.0	18.8	4.7	1.2	0.3	0.1	0.75	0.77	1.29	25.0	6.2	1.6	0.4	0.1	6.7
Geometric Smoothing ($\delta = 0.50$)															
0	100.0	—	—	—	—	—	1.00	1.00	1.00	0.0	0.0	0.0	0.0	0.0	—
1	66.7	33.3	—	—	—	—	0.67	0.75	1.34	40.0	0.0	0.0	0.0	0.0	11.1
2	57.1	28.6	14.3	—	—	—	0.57	0.65	1.53	47.6	19.0	0.0	0.0	0.0	20.4
3	53.3	26.7	13.3	6.7	—	—	0.53	0.61	1.63	49.4	23.5	9.4	0.0	0.0	26.2
4	51.6	25.8	12.9	6.5	3.2	—	0.52	0.60	1.68	49.9	24.6	11.7	4.7	0.0	29.6
5	50.8	25.4	12.7	6.3	3.2	1.6	0.51	0.59	1.71	50.0	24.9	12.3	5.9	2.3	31.4

Table 4: Implications of three different smoothing profiles for observed betas, standard deviations, Sharpe ratios, and serial correlation coefficients for a fund with IID true returns. Straightline smoothing is given by $\theta_j = 1/(k+1)$; sum-of-years smoothing is given by $\theta_j = (k+1-j)/[(k+1)(k+2)/2]$; geometric smooothing is given by $\theta_j = \delta^j(1-\delta)/(1-\delta^{k+1})$. c_β, c_σ, and c_s denote multipliers associated with the beta, standard deviation, and Sharpe ratio of observed returns, respectively, ρ_j^o denotes the j-th autocorrelation coefficient of observed returns, and ζ is proportional to the variance of the discrepancy between true and observed multi-period returns.

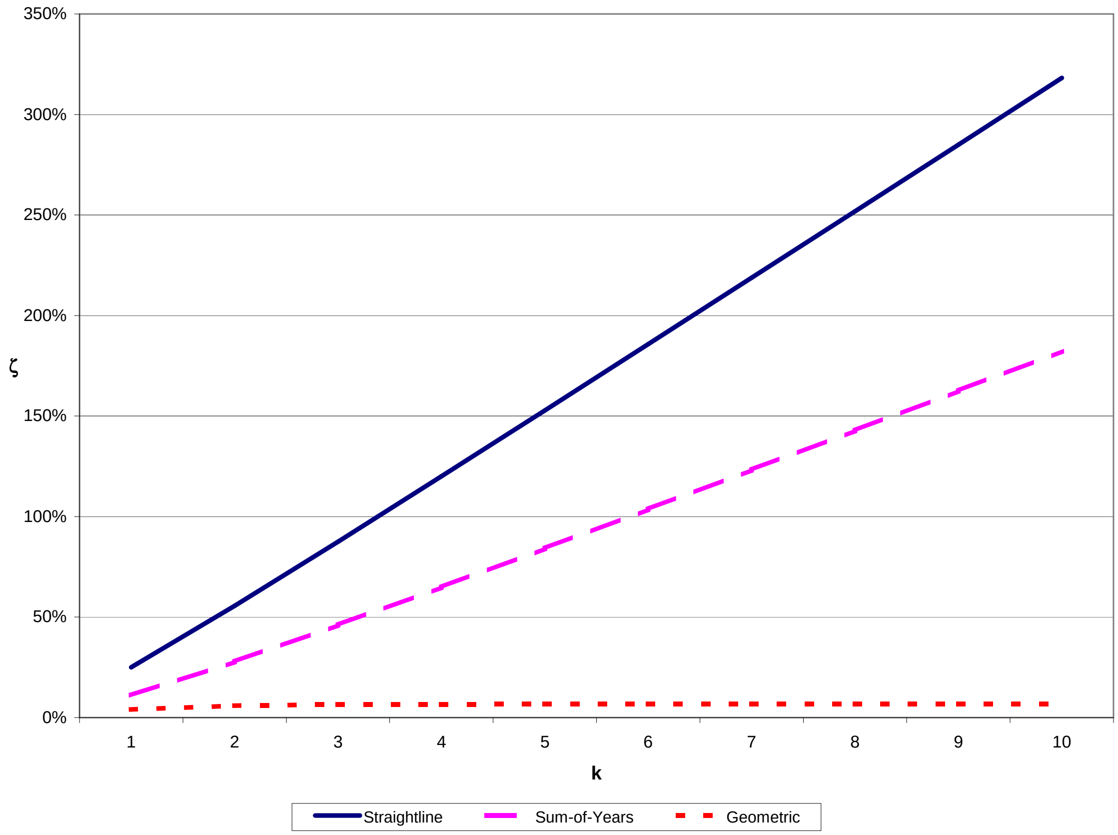

Figure 3: Straightline, sum-of-years, and geometric smoothing profiles for $k = 10$.

The results in Table 4 and Figure 3 show that a rich set of biases can be generated by even simple smoothing profiles, and even the most casual empirical observation suggests that smoothed returns may be an important source of serial correlation in hedge-fund returns. To address this issue directly, we propose methods for estimating the smoothing profile in Section 5 and apply these methods to the data in Section 6.

5 Estimation of Smoothing Profiles and Sharpe Ratios

Although the smoothing profiles described in Section 4.2 can all be easily estimated from the sample moments of fund returns, e.g., means, variances, and autocorrelations, we wish to be able to estimate more general forms of smoothing. Therefore, in this section we propose two estimation procedures—maximum likelihood and linear regression—that place fewer restrictions on a fund's smoothing profile than the three examples in Section 4.2. In Section 5.1 we review the steps for maximum likelihood estimation of an MA(k) process, slightly modified to accommodate our context and constraints, and in Section 5.2 we consider a simpler alternative based on linear regression under the assumption that true returns are generated by the linear single-factor model (20). We propose several specification checks to evaluate the robustness of our smoothing model in Section 5.3, and in Section 5.4 we show how to adjust Sharpe ratios to take smoothed returns into account.

5.1 Maximum Likelihood Estimation

Given the specification of the smoothing process in (21)–(23), we can estimate the smoothing profile using maximum likelihood estimation in a fashion similar to the estimation of standard moving-average time series models (see, for example, Brockwell and Davis, 1991, Chapter 8). We begin by defining the de-meaned observed returns process X_t:

$$X_t \;=\; R_t^o \,-\, \mu \tag{47}$$

and observing that (21)–(23) implies the following properties for X_t:

$$X_t = \theta_0\eta_t + \theta_1\eta_{t-1} + \cdots + \theta_k\eta_{t-k} \tag{48}$$

$$1 = \theta_0 + \theta_1 + \cdots + \theta_k \tag{49}$$

$$\eta_k \sim \mathcal{N}(0, \sigma_\eta^2) \tag{50}$$

where, for purposes of estimation, we have added the parametric assumption (50) that η_k is normally distributed. From (48), it is apparent that X_t is a moving-average process of order k, or an "MA(k)". For a given set of observations $\mathbf{X} \equiv [\ X_1 \ \cdots \ X_T\ ]'$, the likelihood function is well known to be:

$$\mathcal{L}(\boldsymbol{\theta}, \sigma_\eta) = (2\pi)^{-T/2}(\det\mathbf{\Gamma})^{-1/2}\exp(-\tfrac{1}{2}\mathbf{X}'\mathbf{\Gamma}^{-1}\mathbf{X}) \ , \quad \mathbf{\Gamma} \equiv \mathrm{E}[\mathbf{X}\mathbf{X}'] \tag{51}$$

where $\boldsymbol{\theta} \equiv [\ \theta_0 \ \cdots \ \theta_k\ ]'$ and the covariance matrix $\mathbf{\Gamma}$ is a function of the parameters $\boldsymbol{\theta}$ and σ_η. It can be shown that for any constant κ,

$$\mathcal{L}(\kappa\boldsymbol{\theta}, \sigma_\eta/\kappa) = \mathcal{L}(\boldsymbol{\theta}, \sigma_\eta) \ , \tag{52}$$

therefore, an additional identification condition is required. The most common identification condition imposed in the time-series literature is the normalization $\theta_0 \equiv 1$. However, in our context, we impose the condition (49) that the MA coefficients sum to 1—an economic restriction that smoothing takes place over only the most recent $k+1$ periods—and this is sufficient to identify the parameters $\boldsymbol{\theta}$ and σ_η. The likelihood function (51) may be then evaluated and maximized via the "innovations algorithm" of Brockwell and Davis (1991, Chapter 8.3),[16] and the properties of the estimator are given by:

Proposition 3 *Under the specification (48)–(50), X_t is invertible on the set $\{\,\boldsymbol{\theta} : \theta_0 + \theta_1 + \theta_2 = 1\,,\, \theta_1 < 1/2\,,\, \theta_1 < 1 - 2\theta_2\,\}$, and the maximum likelihood estimator $\hat{\boldsymbol{\theta}}$ satisfies the following properties:*

$$1 \;=\; \hat{\theta}_0 \;+\; \hat{\theta}_1 \;+\; \hat{\theta}_2 \tag{56}$$

$$\sqrt{T}\left(\begin{bmatrix} \hat{\theta}_1 \\ \hat{\theta}_2 \end{bmatrix} - \begin{bmatrix} \theta_1 \\ \theta_2 \end{bmatrix} \right) \;\overset{a}{\sim}\; \mathcal{N}(\,0\,,\, \mathbf{V}_\theta\,) \tag{57}$$

$$\mathbf{V}_\theta \;=\;$$

$$\begin{bmatrix} -(-1+\theta_1)(-1+2\theta_1)(-1+\theta_1+2\theta_2) & -\theta_2(-1+2\theta_1)(-1+\theta_1+2\theta_2) \\ -\theta_2(-1+2\theta_1)(-1+\theta_1+2\theta_2) & (-1+\theta_1-2(-1+\theta_2)\theta_2)(-1+\theta_1+2\theta_2) \end{bmatrix}. \tag{58}$$

[16] Specifically, let $\hat{\mathbf{X}} = [\;\hat{X}_1 \;\cdots\; \hat{X}_T\;]'$ where $\hat{X}_1 = 0$ and $\hat{X}_j = \mathrm{E}[X_j | X_1, \ldots, X_{j-1}]$, $j \geq 2$. Let $r_t = \mathrm{E}[(X_{t+1} - \hat{X}_{t+1})^2]/\sigma^2_\eta$. Brockwell and Davis (1991) show that (51) can be rewritten as:

$$\mathcal{L}(\boldsymbol{\theta}, \sigma^2_\eta) \;=\; (2\pi\sigma^2_\eta)^{-T/2}(r_0 \cdots r_{T-1})^{-1/2} \exp\left[-\tfrac{1}{2}\sigma^2_\eta \sum_{t=1}^{T} (X_t - \hat{X}_t)^2 / r_{t-1} \right] \tag{53}$$

where the one-step-ahead predictors $\hat{X}_t$ and their normalized mean-squared errors r_{t-1}, $t = 1, \ldots, T$ are calculated recursively according to the formulas given in Brockwell and Davis (1991, Proposition 5.2.2). Taking the derivative of (53) with respect to σ^2_η, see that the maximum likelihood estimator $\hat{\sigma}^2_\eta$ is given by:

$$\hat{\sigma}^2_\eta \;=\; S(\boldsymbol{\theta}) \;=\; T^{-1} \sum_{t=1}^{T} (X_t - \hat{X}_t)^2 / r_{t-1} \tag{54}$$

hence we can "concentrate" the likelihood function by substituting (54) into (53) to obtain:

$$\mathcal{L}_o(\boldsymbol{\theta}) \;=\; \log S(\boldsymbol{\theta}) \;+\; T^{-1} \sum_{t=1}^{T} \log r_{t-1} \tag{55}$$

which can be minimized in $\boldsymbol{\theta}$ subject to the constraint (49) using standard numerical optimization packages (we use Matlab's Optimization Toolbox in our empirical analysis). Maximum likelihood estimates obtained in this fashion need not yield an invertible MA(k) process, but it is well known that any non-invertible process can always be transformed into an invertible one simply by adjusting the parameters σ^2_η and $\boldsymbol{\theta}$. To address this identification problem, we impose the additional restriction that the estimated MA(k) process be invertible.

By applying the above procedure to observed de-meaned returns, we may obtain estimates of the smoothing profile $\hat{\boldsymbol{\theta}}$ for each fund.[17] Because of the scaling property (52) of the MA(k) likelihood function, a simple procedure for obtaining estimates of our smoothing model with the normalization (49) is to transform estimates $(\check{\boldsymbol{\theta}}, \check{\sigma})$ from standard MA(k) estimation packages such as SAS or RATS by dividing each $\check{\theta}_i$ by $1+\check{\theta}_1+\cdots+\check{\theta}_k$ and multiplying $\check{\sigma}$ by the same factor. The likelihood function remains unchanged but the transformed smoothing coefficients will now satisfy (49).

5.2 Linear Regression Analysis

Although we proposed a linear single-factor model (20) in Section 4 for true returns so as to derive the implications of smoothed returns for the market beta of observed returns, the maximum likelihood procedure outlined in Section 5.1 is designed to estimate the more general specification of IID Gaussian returns, regardless of any factor structure. However, if we are willing to impose (20), a simpler method for estimating the smoothing profile is available. By substituting (20) into (21), we can re-express observed returns as:

$$R_t^o = \mu + \beta\left(\theta_0\Lambda_t + \theta_1\Lambda_{t-1} + \cdots + \theta_k\Lambda_{t-k}\right) + u_t \tag{59}$$

$$u_t = \theta_0\epsilon_t + \theta_1\epsilon_{t-1} + \cdots + \theta_k\epsilon_{t-k} \, . \tag{60}$$

Suppose we estimate the following linear regression of observed returns on contemporaneous and lagged market returns:

$$R_t^o = \mu + \gamma_0\Lambda_t + \gamma_1\Lambda_{t-1} + \cdots + \gamma_k\Lambda_{t-k} + u_t \tag{61}$$

[17] Recall from Proposition 1 that the smoothing process (21)–(23) does not affect the expected return, i.e., the sample mean of observed returns is a consistent estimator of the true expected return. Therefore, we may use $R_t^o - \hat{\mu}$ in place of X_t in the estimation process without altering any of the asymptotic properties of the maximum likelihood estimator.

as in Asness, Krail and Liew (2001). Using the normalization (23) from our smoothing model, we can obtain estimators for β and $\{\theta_j\}$ readily:

$$\hat{\beta} \;=\; \hat{\gamma}_0 + \hat{\gamma}_1 + \cdots + \hat{\gamma}_k \quad , \quad \hat{\theta}_j \;=\; \hat{\gamma}_j/\hat{\beta} \, . \tag{62}$$

Moreover, a specification check for (59)–(60) can be performed by testing the following set of equalities:

$$\beta \;=\; \frac{\gamma_0}{\theta_0} \;=\; \frac{\gamma_1}{\theta_1} \;=\; \cdots \;=\; \frac{\gamma_k}{\theta_k} \, . \tag{63}$$

Because of serial correlation in u_t, ordinary least squares estimates (62) will not be efficient and the usual standard errors are incorrect, but the estimates are still consistent and may be a useful first approximation for identifying smoothing in hedge-fund returns.[18]

There is yet another variation of the linear single-factor model that may help to disentangle the effects of illiquidity from return smoothing.[19] Suppose that a fund's true economic returns R_t satisfies:

$$R_t \;=\; \mu + \beta\Lambda_t + \epsilon_t \;\; , \;\; \epsilon_t \sim \text{IID}(0, \sigma_\epsilon^2) \tag{64}$$

but instead of assuming that the common factor Λ_t is IID as in (20), let Λ_t be serially correlated. While this alternative may seem to be a minor variation of the smoothing model (21)–(23), the difference in interpretation is significant. A serially correlated Λ_t captures the fact that a fund's returns may be autocorrelated because of an illiquid common factor, even in the absence of any smoothing process such as (21)–(23). Of course, this still begs the question of what the ultimate source of serial correlation in the common factor might be, but by combining (64) with the smoothing process (21)–(23), it may be possible to distinguish

[18]To obtain efficient estimates of the smoothing coefficients, a procedure like the maximum likelihood estimator of Section 5.1 must be used.

[19]We thank the referee for encouraging us to explore this alternative.

between "systematic" versus "idiosyncratic" smoothing, the former attributable to the asset class and the latter resulting from fund-specific characteristics.

To see why the combination of (64) and (21)–(23) may have different implications for observed returns, suppose for the moment that there is no smoothing, i.e., $\theta_0 = 1$ and $\theta_k = 0$ for $k > 0$ in (21)–(23). Then observed returns are simply given by:

$$R_t^o \;=\; \mu \;+\; \beta\Lambda_t \;+\; \epsilon_t \;\;,\;\; \epsilon_t \sim \text{IID}(0, \sigma_\epsilon^2) \tag{65}$$

where R_t^o is now serially correlated solely through Λ_t. This specification implies that the ratios of observed-return autocovariances will be identical across all funds with the same common factor:

$$\frac{\text{Cov}[R_t^o, R_{t-k}^o]}{\text{Cov}[R_t^o, R_{t-l}^o]} \;=\; \frac{\beta\,\text{Cov}[\Lambda_t, \Lambda_{t-k}]}{\beta\,\text{Cov}[\Lambda_t, \Lambda_{t-l}]} \;=\; \frac{\text{Cov}[\Lambda_t, \Lambda_{t-k}]}{\text{Cov}[\Lambda_t, \Lambda_{t-l}]}\;. \tag{66}$$

Moreover, (64) implies that in the regression equation (61), the coefficients of the lagged factor returns are zero and the error term is not serially correlated.

More generally, consider the combination of a serially correlated common factor (64) and smoothed returns (21)–(23). This more general econometric model of observed returns implies that the appropriate specification of the regression equation is:

$$R_t^o \;=\; \mu \;+\; \gamma_0\Lambda_t \;+\gamma_1\Lambda_{t-1} \;+\; \cdots \;+\; \gamma_k\Lambda_{t-k} \;+\; u_t \tag{67}$$

$$u_t \;=\; \theta_0\epsilon_t \;+\theta_1\epsilon_{t-1} \;+\; \cdots \;+\; \theta_k\epsilon_{t-k} \;\;,\;\; \epsilon_t \sim \text{IID}(0, \sigma_\epsilon^2) \tag{68}$$

$$1 \;=\; \theta_0 \;+\; \theta_1 \;+\; \cdots \;+\; \theta_k\;. \tag{69}$$

To the extent that serial correlation in R_t^o can be explained mainly by the common factor, the lagged coefficient estimates of (67) will be statistically insignificant, the residuals will be serially uncorrelated, and the ratios of autocovariance coefficients will be roughly constant across funds with the same common factor. To the extent that the smoothing process (21)–

(23) is responsible for serial correlation in R_t^o, the lagged coefficient estimates of (67) will be significant, the residuals will be serially correlated, and the ratios $\hat{\gamma}_j/\hat{\theta}_j$ will be roughly the same for all $j \geq 0$ and will be a consistent estimate of the factor loading or beta of the fund's true economic returns with respect to the factor Λ_t.

Perhaps the most difficult challenge in estimating (67)–(69) is to correctly identify the common factor Λ_t. Unlike a simple market-model regression that is meant to estimate the sensitivity of a fund's returns to a broad-based market index, the ability to distinguish between the effects of systematic illiquidity and idiosyncratic return smoothing via (67) relies heavily on the correct specification of the common factor. Using a common factor in (67) that is highly serially correlated but not exactly the right factor for a given fund may yield misleading estimates for the degree of smoothing in that fund's observed returns. Therefore, the common factor Λ_t must be selected or constructed carefully to match the specific risk exposures of the fund, and the parameter estimates of (67) must be interpreted cautiously and with several specific alternative hypotheses at hand. In Section 6.4, we provide an empirical example that highlights the pitfalls and opportunities of the common factor specification (67)–(69).

5.3 Specification Checks

Although the maximum likelihood estimator proposed in Section 5.1 has some attractive properties—it is consistent and asymptotically efficient under certain regularity conditions—it may not perform well in small samples or when the underlying distribution of true returns is not normal as hypothesized.[20] Moreover, even if normality is satisfied and a sufficient sample size is available, our proposed smoothing model (21)–(23) may simply not apply to some of the funds in our sample. Therefore, it is important to have certain specification checks in mind when interpreting the empirical results.

The most obvious specification check is whether or not the maximum likelihood estimation procedure, which involves numerical optimization, converges. If not, this is one sign

[20] There is substantial evidence that financial asset returns are not normally distributed, but characterized by skewness, leptokurtosis, and other non-gaussian properties (see, for example, Lo and MacKinlay, 1999). Given the dynamic nature of hedge-fund strategies, it would be even less plausible for their returns to be normally distributed.

that our model is misspecified, either because of non-normality or because the smoothing process is inappropriate.

A second specification check is whether or not the estimated smoothing coefficients are all positive in sign (we do not impose non-negative restrictions in our estimation procedure, despite the fact that the specification does assume non-negativity). Estimated coefficients that are negative and significant may be a sign that the constraint (49) is violated, which suggests that a somewhat different smoothing model may apply.

A third specification check is to compare the smoothing-parameter estimates from the maximum likelihood approach of Section 5.1 with the linear regression approach of Section 5.2. If the linear single-factor model (20) holds, the two sets of smoothing-parameter estimates should be close. Of course, omitted factors could be a source of discrepancies between the two sets of estimates, so this specification check must be interpreted cautiously and with some auxiliary information about the economic motivation for the common factor Λ_t.

Finally, a more direct approach to testing the specification of (21)–(23) is to impose a different identification condition than (49). Suppose that the standard deviation σ_η of true returns was observable; then the smoothing parameters $\boldsymbol{\theta}$ are identified, and a simple check of the specification (21)–(23) is to see whether the estimated parameters $\hat{\boldsymbol{\theta}}$ sum to 1. Of course, σ_η is not observable, but if we had an alternative estimator $\tilde{\sigma}_\eta$ for σ_η, we can achieve identification of the MA(k) process by imposing the restriction:

$$\sigma_\eta = \tilde{\sigma}_\eta \tag{70}$$

instead of (49). If, under this normalization, the smoothing parameter estimates are significantly different, this may be a sign of misspecification.

The efficacy of this specification check depends on the quality of $\tilde{\sigma}_\eta$. We propose to construct such an estimator by exploiting the fact that the discrepancy between observed and true returns becomes "small" for multiperiod returns as the number of periods grows.

Specifically, recall from (37) that:

$$(R_1^o + R_2^o + \cdots + R_T^o) \;=\; (R_1 + R_2 + \cdots + R_T) \;+\; \sum_{j=0}^{k-1}(R_{-j} - R_{T-j})(1 - \sum_{i=0}^{j}\theta_i) \quad (71)$$

$$\frac{1}{T}\,\mathrm{Var}\left[\sum_{t=1}^{T} R_t^o\right] \;=\; \sigma_\eta^2 \;+\; \frac{\sigma_\eta^2}{T}\sum_{j=0}^{k-1}\left(1 - \sum_{i=0}^{j}\theta_i\right)\left(1 - 2\sum_{i=0}^{j}\theta_i\right) \quad (72)$$

and under the specification (21)–(23), it is easy to show that the second term on the right side of (72) vanishes as T increases without bound, hence:

$$\lim_{T\to\infty}\frac{1}{T}\,\mathrm{Var}\left[\sum_{t=1}^{T} R_t^o\right] \;=\; \sigma_\eta^2\ . \quad (73)$$

To estimate this normalized variance of multiperiod observed returns, we can apply Newey and West's (1987) estimator:

$$\tilde{\sigma}_\eta^2 \;\equiv\; \frac{1}{T}\sum_{1}^{T}(R_t^o - \hat{\mu})^2 + \frac{2}{T}\sum_{j=1}^{m}\left(1 - \frac{j}{m+1}\right)\left(\sum_{t=j+1}^{T}(R_t^o - \hat{\mu})(R_{t-j}^o - \hat{\mu})\right) \quad (74)$$

where $\hat{\mu}$ is the sample mean of $\{R_t^o\}$ and m is a truncation lag that must increase with T but at a slower rate to ensure consistency and asymptotic normality of the estimator. By imposing the identification restriction

$$\sigma_\eta \;=\; \tilde{\sigma}_\eta \quad (75)$$

in estimating the smoothing profile of observed returns, we obtain another estimator of $\boldsymbol{\theta}$ which can be compared against the first. As in the case of the normalization (49), the alternate normalization (75) can be imposed by rescaling estimates $(\check{\boldsymbol{\theta}}, \check{\sigma})$ from standard MA(k) estimation packages, in this case by dividing each $\check{\theta}_i$ by $\tilde{\sigma}_\eta/\check{\sigma}$ and multiplying $\check{\sigma}$ by the same factor.

5.4 Smoothing-Adjusted Sharpe Ratios

One of the main implications of smoothed returns is that Sharpe ratios are biased upward, in some cases substantially (see Proposition 1).[21] The mechanism by which this bias occurs is through the reduction in volatility because of the smoothing, but there is an additional bias that occurs when monthly Sharpe ratios are annualized by multiplying by $\sqrt{12}$. If monthly returns are independently and identically distributed, this is the correct procedure, but Lo (2002) shows that for non-IID returns, an alternative procedure must be used, one that accounts for serial correlation in returns in a very specific manner.[22] Specifically, denote by $R_t(q)$ the following q-period return:

$$R_t(q) \equiv R_t + R_{t-1} + \cdots + R_{t-q+1} \tag{76}$$

where we ignore the effects of compounding for computational convenience.[23] For IID returns, the variance of $R_t(q)$ is directly proportional to q, hence the Sharpe ratio satisfies the simple relation:

$$\mathrm{SR}(q) = \frac{\mathrm{E}[R_t(q)] - R_f(q)}{\sqrt{\mathrm{Var}[R_t(q)]}} = \frac{q(\mu - R_f)}{\sqrt{q}\,\sigma} = \sqrt{q}\,\mathrm{SR}\ . \tag{77}$$

[21] There are a number of other concerns regarding the use and interpretation of Sharpe ratios in the context of hedge funds. See Agarwal and Naik (2000a, 2002), Goetzmann et al. (2002), Lo (2001), Sharpe (1994), Spurgin (2001), and Weisman (2002) for examples where Sharpe ratios can be misleading indicators of the true risk-adjusted performance of hedge-fund strategies, and for alternate methods of constructing optimal portfolios of hedge funds.

[22] See also Jobson and Korkie (1981), who were perhaps the first to derive rigorous statistical properties of performance measures such as the Sharpe ratio and the Treynor measure.

[23] The exact expression is, of course:

$$R_t(q) \equiv \prod_{j=0}^{q-1}(1 + R_{t-j}) - 1\ .$$

For most (but not all) applications, (76) is an excellent approximation. Alternatively, if R_t is defined to be the continuously compounded return, i.e., $R_t \equiv \log(P_t/P_{t-1})$ where P_t is the price or net asset value at time t, then (76) is exact.

Using Hansen's (1982) GMM estimator, Lo (2002) derives the asymptotic distribution of $\widehat{\text{SR}}(q)$ as:

$$\sqrt{T}\,(\widehat{\text{SR}}(q) - \sqrt{q}\,\text{SR}) \ \stackrel{a}{\sim} \ \mathcal{N}\bigg(0 \ , \ V_{\text{IID}}(q) \bigg) \quad , \quad V_{\text{IID}}(q) \ = \ q\,V_{\text{IID}} \ = \ q\,(1+\tfrac{1}{2}\text{SR}^2)\ . \tag{78}$$

For non-IID returns, the relation between SR and SR(q) is somewhat more involved because the variance of $R_t(q)$ is not just the sum of the variances of component returns, but also includes all the covariances. Specifically, under the assumption that returns $\{R_t\}$ are stationary,

$$\text{Var}[R_t(q)] \ = \ \sum_{i=0}^{q-1}\sum_{j=0}^{q-1}\text{Cov}[R_{t-i}, R_{t-j}] \ = \ q\sigma^2 \ + \ 2\sigma^2\sum_{k=1}^{q-1}(q-k)\rho_k \tag{79}$$

where $\rho_k \equiv \text{Cov}[R_t, R_{t-k}]/\text{Var}[R_t]$. This yields the following relation between SR and SR(q):

$$\text{SR}(q) \ = \ \eta(q)\,\text{SR} \quad , \quad \eta(q) \ \equiv \ \frac{q}{\sqrt{q + 2\sum_{k=1}^{q-1}(q-k)\rho_k}}\ . \tag{80}$$

Note that (80) reduces to (77) if the autocorrelations $\{\rho_k\}$ are zero, as in the case of IID returns. However, for non-IID returns, the adjustment factor for time-aggregated Sharpe ratios is generally not $\sqrt{q}$ but a function of the first $q-1$ autocorrelations of returns, which is readily estimated from the sample autocorrelations of returns, hence:

$$\widehat{\text{SR}}(q) \ = \ \hat{\eta}(q)\,\widehat{\text{SR}} \quad , \quad \hat{\eta}(q) \ \equiv \ \frac{q}{\sqrt{q + 2\sum_{k=1}^{q-1}(q-k)\hat{\rho}_k}} \tag{81}$$

where $\hat{\rho}_k$ is the sample k-th order autocorrelation coefficient.

Lo (2002) also derives the asymptotic distribution of (81) under fairly general assumptions for the returns process (stationarity and ergodicity) using generalized method of moments.

However, in the context of hedge-fund returns, the usual asymptotic approximations may not be satisfactory because of the small sample sizes that characterize hedge-fund data—a five-year track record, which amounts to only 60 monthly observations, is considered quite a long history in this fast-paced industry. Therefore, we derive an alternate asymptotic distribution using the continuous-record asymptotics of Richardson and Stock (1989). Specifically, as the sample size T increases without bound, let q grow as well so that the ratio converges to some finite limit between 0 and 1:

$$\lim_{q,T\to\infty} q/T \quad = \quad \tau \quad \in \quad (0,1) \ . \tag{82}$$

This condition is meant to provide an asymptotic approximation that may be more accurate for small-sample situations, i.e., situations where q is a significant fraction of T. For example, in the case of a fund with a five-year track record, computing an annual Sharpe ratio with monthly data corresponds to a value of 0.20 for the ratio q/T.

Now as q increases without bound, $\mathrm{SR}(q)$ also tends to infinity, hence we must renormalize it to obtain a well-defined asymptotic sampling theory. In particular, observe that:

$$\mathrm{SR}(q) \quad = \quad \frac{\mathrm{E}[R_t(q)] - R_f(q)}{\sqrt{\mathrm{Var}[R_t(q)]}} \quad = \quad \frac{q(\mu - R_f)}{\sqrt{\mathrm{Var}[R_t(q)]}} \tag{83}$$

$$\mathrm{SR}(q)/\sqrt{q} \quad = \quad \frac{\mu - R_f}{\sqrt{\mathrm{Var}[R_t(q)]/q}} \tag{84}$$

$$\lim_{q\to\infty} \mathrm{SR}(q)/\sqrt{q} \quad = \quad \frac{\mu - R_f}{\overline{\sigma}} \tag{85}$$

where $\overline{\sigma}$ can be viewed as a kind of long-run average return standard deviation, which is generally not identical to the unconditional standard deviation σ of monthly returns except in the IID case. To estimate $\overline{\sigma}$, we can either follow Lo (2002) and use sample autocorrelations as in (81), or estimate $\overline{\sigma}$ directly accordingly to Newey and West (1987):

$$\hat{\overline{\sigma}}^2_{\mathrm{NW}} \quad \equiv \quad \frac{1}{T}\sum_{1}^{T}(R_t - \hat{\mu})^2 + \frac{2}{T}\sum_{j=1}^{m}(1 - \frac{j}{m+1})\sum_{t=j+1}^{T}(R_t - \hat{\mu})(R_{t-j} - \hat{\mu}) \tag{86}$$

where $\hat{\mu}$ is the sample mean of $\{R_t\}$. For this estimator of $\overline{\sigma}$, we have the following asymptotic result:

Proposition 4 *As m and T increase without bound so that $m/T \to \lambda \in (0,1)$, $\hat{\overline{\sigma}}^2_{\mathrm{NW}}$ converges weakly to the following functional $f(W)$ of standard Brownian motion on* $[0,1]$:[24]

$$
\begin{aligned}
f(W) \;\equiv\; & \frac{2\overline{\sigma}^2}{\lambda}\Bigg(\int_0^1 W(r)\big[W(r)-W(\min(r+\lambda,1))\big]\,dr \;- \\
& W(1)\int_0^\lambda (\lambda - r)\big(W(1-r)-W(r)\big)\,dr \;+\; \frac{\lambda(1-\frac{\lambda^2}{3})}{2}W^2(1)\Bigg) \,. \qquad (87)
\end{aligned}
$$

From (87), a straightforward computation yields the following expectations:

$$
\mathrm{E}[\hat{\overline{\sigma}}^2_{\mathrm{NW}}] \;=\; 1 \;-\; \lambda \;+\; \frac{\lambda^2}{3} \quad , \quad \mathrm{E}[1/\hat{\overline{\sigma}}_{\mathrm{NW}}] \;\approx\; \sqrt{\frac{1+\lambda}{1-\lambda+\lambda^2/3}} \qquad (88)
$$

hence we propose the following bias-corrected estimator for the Sharpe ratio for small samples:

$$
\widehat{\mathrm{SR}}(q) \;=\; \frac{\sqrt{q}(\hat{\mu}-R_f)}{\hat{\overline{\sigma}}_{\mathrm{NW}}}\sqrt{\frac{1-\lambda+\lambda^3/2}{1+\lambda}} \qquad (89)
$$

and its asymptotic distribution is given by the following proposition:

Proposition 5 *As m, q, and T increase without bound so that $m/T \to \lambda \in (0,1)$ and $q/T \to \tau \in (0,1)$, the Sharpe ratio estimator $\widehat{\mathrm{SR}}(q)$ converges weakly to the following random variable:*

$$
\widehat{\mathrm{SR}}(q) \;\Rightarrow\; \left(\frac{\mathrm{SR}(q)}{f(W)}+\frac{\sqrt{\tau}W(1)}{f(W)}\right)\sqrt{\frac{1-\lambda+\lambda^3/2}{1+\lambda}} \qquad (90)
$$

where $f(W)$ is given by (87), $\mathrm{SR}(q)$ is given by (83) and $W(\cdot)$ is standard Brownian motion

[24]See Billingsley (1968) for the definition of weak convergence and related results.

defined on $[0, 1]$.

Monte Carlo simulations show that the second term of (90) does not account for much bias when $\tau \in (0, \frac{1}{2}]$, and that (90) is an excellent approximation to the small-sample distributions of Sharpe ratios for non-IID returns.[25]

6 Empirical Analysis

For our empirical analysis, we use the TASS database of hedge funds which consists of monthly returns and accompanying information for 2,439 hedge funds (as of January 2001) from November 1977 to January 2001.[26] The database is divided into two parts: "Live" and "Graveyard" funds. Hedge funds that belong to the Live database are considered to be active as of January 1, 2001; once a hedge fund decides not to report its performance, is liquidated, restructured, or merged with other hedge funds, the fund is transferred into the Graveyard database. A hedge fund can only be listed in the Graveyard database after being listed in the Live database, but the TASS database is subject to backfill bias—when a fund decides to be included in the database, TASS adds the fund to the Live database and includes available prior performance of the fund(hedge funds do not need to meet any specific requirements to be included in the TASS database). Due to reporting delays and time lags in contacting hedge funds, some Graveyard funds can be incorrectly listed in the Live database for a period of time. However, TASS has adopted a policy of transferring funds from the Live to the Graveyard database if they do not report over a 6–8 month period.

As of January 1, 2001, the combined data set of both live and dead hedge funds contained 2,439 funds with at least one monthly net return observation. Out of these 2,439 funds, 1,512 are in the Live database and 927 are in the Graveyard database. The earliest data available for a fund in either database is November 1, 1977. The Graveyard database became active only in 1994, i.e., funds that were dropped from the Live database prior to 1994 are not included in the Graveyard database, which may yield a certain degree of survivorship bias.[27]

[25]We have tabulated the percentiles of the distribution of (90) by Monte Carlo simulation for an extensive combination of values of q, τ, and λ and would be happy to provide them to interested readers upon request.

[26]For further information about the database and TASS, see http://www.tassresearch.com.

[27] For studies attempting to quantify the degree and impact of survivorship bias, see Baquero, Horst, and

A majority of the 2,439 funds reported returns net of various fees on a monthly basis.[28] We eliminated 30 funds that reported only gross returns and/or quarterly returns (15 from each of the Live and Graveyard databases, respectively), leaving 2,409 funds in our sample. We imposed an additional filter of including only those funds with at least five years of data, leaving 651 funds in the Live database and 258 in the Graveyard database for a combined total of 909 funds. This obviously creates additional survivorship bias in our sample, but since our main objective is to estimate smoothing profiles and not to make inferences about overall performance, our filter may not be as problematic.[29]

TASS also attempts to classify funds according to one of 17 different investment styles, listed in Table 5 and described in Appendix A.4; funds that TASS are not able to categorize are assigned a category code of '0'.[30] Table 5 also reports the number of funds in each category for the Live, Graveyard, and Combined databases, and it is apparent from these figures that the representation of investment styles is not evenly distributed, but is concentrated among six categories: US Equity Hedge (162), Event Driven (109), Non-Directional/Relative Value (85), Pure Managed Futures (93), Pure Emerging Market (72), and Fund of Funds (132). Together, these six categories account for 72% of the funds in the Combined database.

To develop a sense of the dynamics of the TASS database and the impact of our minimum return-history filter, in Table 6 we report annual frequency counts of the funds in the database at the start of each year, funds entering during the year, funds exiting during the year, and funds entering and exiting within the year. The left panel contains counts for the entire TASS database, and the right panel contains counts for our sample of 909 funds with at least five years of returns. The left panel shows that despite the start date of November

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Verbeek (2002), Brown, Goetzmann, Ibbotson, and Ross (1992), Brown, Goetzmann, and Ibbotson (1999), Brown, Goetzmann, and Park (1997), Carpenter and Lynch (1999), Fung and Hsieh (1997b, 2000), Horst, Nijman, T. and M. Verbeek (2001), Hendricks, Patel, and Zeckhauser (1997), and Schneeweis and Spurgin (1996).

[28] TASS defines returns as the change in net asset value during the month (assuming the reinvestment of any distributions on the reinvestment date used by the fund) divided by the net asset value at the beginning of the month, net of management fees, incentive fees, and other fund expenses. Therefore, these reported returns should approximate the returns realized by investors. TASS also converts all foreign-currency denominated returns to US-dollar returns using the appropriate exchange rates.

[29] See the references in footnote 27.

[30] A hedge fund can have at most 2 different categories (CAT1 and CAT2) in the TASS database. For all hedge funds in the TASS database, the second category (CAT2) is always 17, 'Fund of Funds'.

Code	Category	Number of Funds In:		
		Combined	Live	Graveyard
0	Not Categorized	111	44	67
1	US Equity Hedge	162	139	23
2	European Equity Hedge	22	19	3
3	Asian Equity Hedge	5	5	0
4	Global Equity Hedge	27	24	3
5	Dedicated Shortseller	7	6	1
6	Fixed-Income Directional	13	12	1
7	Convertible Fund (Long Only)	15	12	3
8	Event Driven	109	97	12
9	Non-Directional/Relative Value	85	63	22
10	Global Macro	25	15	10
11	Global Opportunity	1	1	0
12	Natural Resources	3	1	2
13	Pure Leveraged Currency	26	15	11
14	Pure Managed Futures	93	28	65
15	Pure Emerging Market	72	54	18
16	Pure Property	1	1	0
17	Fund of Funds	132	115	17
	All	909	651	258

Table 5: Number of funds in the TASS Hedge Fund Live and Graveyard databases with at least five years of returns history during the period from November 1977 to January 2001.

1977, the database is relatively sparsely populated until the 1990's, with the largest increase in new funds in 1998 and, in the aftermath of the collapse of LTCM, the largest number of funds exiting the database in 1999 and 2000. The right panel of Table 6 illustrates the impact of our five-year filter—the number of funds is considerably smaller, and although the impact of survivorship bias can be ameliorated by the use of Live and Graveyard funds, our sample of 909 funds will not include any of the funds started in 1997 and later which is a substantial proportion of the TASS database.

The attrition rates reported in Table 6 are defined as the ratio of funds exiting in a given year to the number of existing funds at the start of the year. TASS began tracking the exits of funds starting only in 1994 hence attrition rates could not be computed in prior years. For the unfiltered sample of all funds, the average attrition rate from 1994–1999 is 9.11%, which is very similar to the 8.54% attrition rate obtained by Liang (2001) for the same period. As observed above, the attrition rate skyrocketed in 2000 in the wake of LTCM's demise. In the right panel of Table 6, we see smaller attrition rates—the average over the 1994–1999 period is only 3.81%—because of our five-year minimum return history filter; since many hedge funds fail in their first three years, our filtered sample is likely to have a much lower attrition rate by construction.

Figure 4 contains a visual depiction of the variation in sample sizes of our 909 funds. The start and end dates of the return history for each fund are connected by a vertical line and plotted in Figure 4 according to the primary category of the fund—Categories 0–7 in the top panel and Categories 8–17 in the bottom panel. It is apparent from the increasing density of the graphs as we move from the bottom to the top that the majority of funds in our sample are relatively new.

In Section 6.1 we present summary statistics for the sample of hedge funds included in our analysis. We implement the smoothing profile estimation procedures outlined in Section 5 for each of the funds, and in Section 6.2 we summarize the results of the maximum likelihood and linear regression estimation procedures for the entire sample of funds and for each style category. Section 6.3 reports the results of cross-sectional regressions of the estimated smoothing coefficients, and in Section 6.4 we consider idiosyncratic and systematic effects of illiquidity and smoothing by estimating a linear factor model with smoothing for

Year	All Funds						Funds with At Least 5 Years' History					
	Existing Funds	New Entries	New Exits	Intrayear Entry/Exit	Total Funds	Attrition Rate (%)	Existing Funds	New Entries	New Exits	Intrayear Entry/Exit	Total Funds	Attrition Rate (%)
1977	0	1	0	0	1	—	0	1	0	0	1	—
1978	1	2	0	0	3	0.0	1	2	0	0	3	0.0
1979	3	1	0	0	4	0.0	3	1	0	0	4	0.0
1980	4	1	0	0	5	0.0	4	1	0	0	5	0.0
1981	5	3	0	0	8	0.0	5	3	0	0	8	0.0
1982	8	4	0	0	12	0.0	8	4	0	0	12	0.0
1983	12	6	0	0	18	0.0	12	6	0	0	18	0.0
1984	18	14	0	0	32	0.0	18	14	0	0	32	0.0
1985	32	8	0	0	40	0.0	32	8	0	0	40	0.0
1986	40	19	0	0	59	0.0	40	19	0	0	59	0.0
1987	59	32	0	0	91	0.0	59	32	0	0	91	0.0
1988	91	28	0	0	119	0.0	91	28	0	0	119	0.0
1989	119	39	0	0	158	0.0	119	39	0	0	158	0.0
1990	158	99	0	0	257	0.0	158	93	0	0	251	0.0
1991	257	87	0	0	344	0.0	251	78	0	0	329	0.0
1992	344	156	0	0	500	0.0	329	118	0	0	447	0.0
1993	500	233	0	0	733	0.0	447	135	0	0	582	0.0
1994	733	245	28	2	950	3.8	582	153	3	0	732	0.5
1995	950	256	71	1	1135	7.5	732	142	15	0	859	2.0
1996	1135	263	127	9	1271	11.2	859	32	30	0	861	3.5
1997	1271	290	105	7	1456	8.3	861	0	14	0	847	1.6
1998	1456	255	166	10	1545	11.4	847	0	51	0	796	6.0
1999	1545	222	193	7	1574	12.5	796	0	73	0	723	9.2
2000	1574	90	421	19	1243	26.7	723	0	182	0	541	25.2

Table 6: Annual frequency counts of entries into and exits out of the TASS Hedge Fund Database from November 1977 to January 2001.

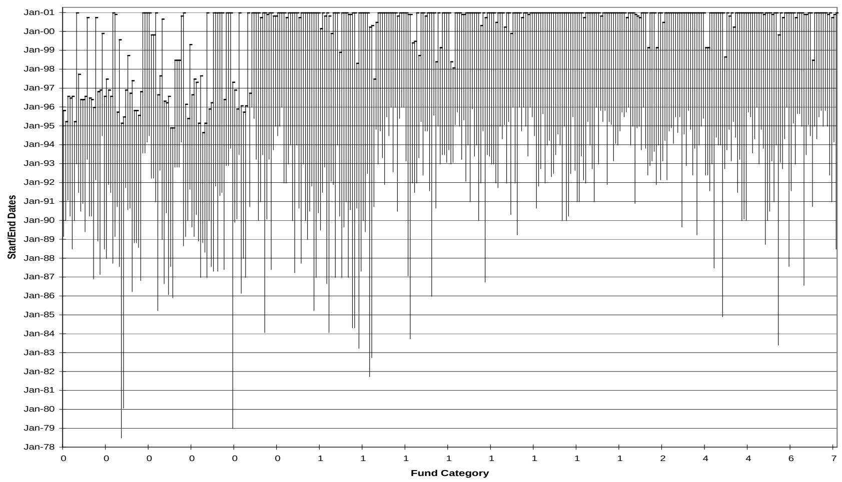
all funds in one particular style category using a common factor appropriate for that style. And in Section 6.5 we summarize the properties of smoothing-adjusted Sharpe ratios for all the funds in our sample and compare them to their unadjusted counterparts.

6.1 Summary Statistics

Table 7 contains basic summary statistics for the 909 funds in our combined extract from the TASS Live and Graveyard databases. Not surprisingly, there is a great deal of variation in mean returns and volatilities both across and within categories. For example, the 162 US Equity Hedge funds in our sample exhibited a mean return of 22.53%, but with a standard deviation of 10.80% in the cross section, and a mean volatility of 21.69% with a cross-sectional standard deviation of 11.63%. Average serial correlations also vary considerably across categories, but five categories stand out as having the highest averages:[31] Fixed Income Directional (21.6%), Convertible Fund (Long Only) (22.5%), Event Driven (20.8%), Non-Directional/Relative Value (18.2%), and Pure Emerging Market (18.8%). Given the descriptions of these categories provided by TASS (see Appendix A.4) and common wisdom about the nature of the strategies involved—these categories include some of the most illiquid securities traded—serial correlation seems to be a reasonable proxy for illiquidity and smoothed returns. Alternatively, equities and futures are among the most liquid securities in which hedge funds invest, and not surprising, the average first-order serial correlation for US Equity Hedge funds and Pure Managed Futures is 7.8% and −0.1%, respectively. In fact, all of the equity funds have average serial correlations that are much smaller than those of the top five categories. Dedicated Shortseller funds also have a low average first-order autocorrelation, 4.4%, which is consistent with the high degree of liquidity that often characterize shortsellers (since, by definition, the ability to short a security implies a certain degree of liquidity).

These summary statistics suggest that illiquidity and smoothed returns may be important attributes for hedge-fund returns which can be captured to some degree by serial correlation

[31]At 23.1% and −23.1%, respectively, Global Opportunity and Pure Property have higher first-order autocorrelation coefficients in absolute value than the other categories, but since these two categories contain only a single fund each, we set them aside in our discussions.



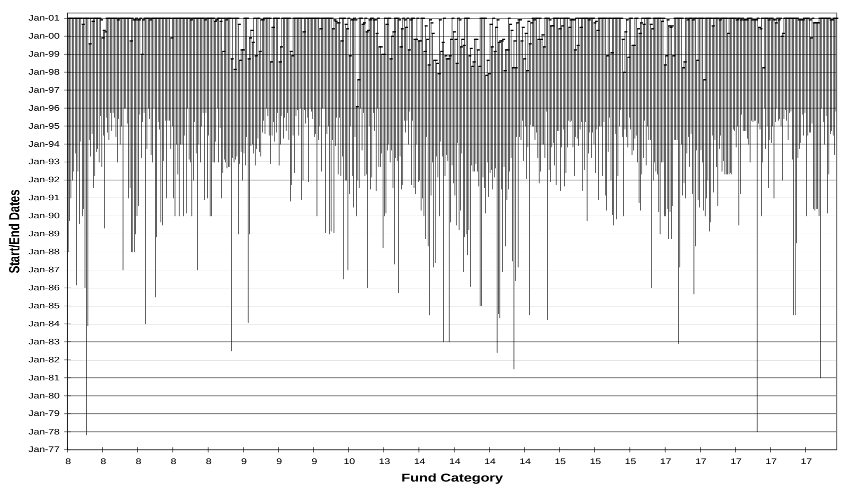


Figure 4: Length of return histories, depicted by vertical solid lines, for all funds in the TASS Hedge Fund database with at least five years of returns during the period from November 1977 to January 2001, ordered by categories 0 to 7 in the top panel and categories 8 to 17 in the bottom panel. Each fund is represented by a single solid vertical line that spans the start and end dates of the fund's return history.

and our time-series model of smoothing.

6.2 Smoothing Profile Estimates

Using the methods outlined in Section 5, we estimate the smoothing model (21)–(23) and summarize the results in Tables 8–9. Our maximum likelihood procedure—programmed in Matlab using the *Optimization Toolbox* and replicated in Stata using its MA(k) estimation routine—converged without difficulty for all but one of the 909 funds:[32] fund 1055, a Global Macro fund with returns from June 1994 to January 2001 for which the maximum likelihood estimation procedure yielded the following parameter estimates:

$$\hat{\theta}_0 \;=\; 490.47 \quad , \quad \hat{\theta}_1 \;=\; -352.63 \quad , \quad \hat{\theta}_2 \;=\; -136.83$$

which suggests that our MA(2) model is severely misspecified for this fund. Therefore, we drop this fund from our sample and for the remainder of our analysis, we focus on the smoothing profile estimates for the remaining 908 funds in our sample.[33]

Table 8 contains summary statistics for maximum likelihood estimate of the smoothing parameters $(\theta_0, \theta_1, \theta_2)$ and smoothing index ξ, Table 12 reports comparable statistics for the regression estimates of the smoothing parameters under the assumption of a linear one-factor model for true returns, and Table 9 presents the maximum likelihood estimates of the smoothing model for the 50 most illiquid funds of the 908 funds, as ranked by $\hat{\theta}_0$.

The left panel of Table 8 reports summary statistics for the maximum likelihood estimates under the normalization (49) where the smoothing coefficients sum to 1, and the right panel

[32]We also constrain our maximum likelihood estimators to yield invertible MA(2) processes, and this constraint was binding for only two funds: 1711 and 4298.

[33]The apparent source of the problem in this case is two consecutive outliers, 39.4% in December 1999 followed by −27.6% in January 2000 (these are *monthly* returns, not annualized). The effect of two outliers on the parameter estimates of the MA(2) model (21)–(23) is to pull the values of the coefficients in opposite directions so as to fit the extreme reversals. We contacted TASS to investigate these outliers and were informed that they were data errors. We also checked the remaining 908 funds in our sample for similar outliers, i.e., consecutive extreme returns of opposite sign, and found none. We also computed the maximum and minimum monthly returns for each fund in our sample, ranked the 908 funds according to these maxima and minima, and checked the parameter estimates of the top and bottom 10 funds, and none exhibited the extreme behavior of fund 1055's parameter estimates.

Category	N	Annual Mean		Annual SD		Skewness		Kurtosis		$\hat{\rho}_1(\%)$		$\hat{\rho}_2(\%)$		$\hat{\rho}_3(\%)$		p-Value(Q)	
		Mean	SD	Mean	SD	Mean	SD	Mean	SD	Mean	SD	Mean	SD	Mean	SD	Mean	SD
Not Categorized	111	10.78	6.99	15.60	11.25	0.12	1.06	3.67	4.94	5.5	17.0	0.9	14.7	0.5	11.4	36.1	30.4
US Equity Hedge	162	22.53	10.80	21.69	11.63	0.17	0.96	3.82	5.02	7.8	13.5	0.6	11.2	−4.2	11.0	33.9	28.8
European Equity Hedge	22	18.01	7.15	14.71	6.41	0.39	0.81	2.97	2.73	12.7	10.9	11.2	11.3	−2.0	8.9	21.2	23.0
Asian Equity Hedge	5	9.16	7.96	21.13	6.64	0.50	0.85	2.32	0.73	11.7	13.0	5.0	5.6	−5.4	13.4	40.2	21.1
Global Equity Hedge	27	13.71	8.41	17.41	6.61	−0.24	1.06	3.96	5.60	12.4	11.3	−0.1	9.6	−2.7	7.2	41.9	27.2
Dedicated Shortseller	7	0.33	10.26	21.55	13.80	0.49	0.20	2.28	2.40	4.4	8.4	−2.7	9.5	−6.5	6.9	31.8	27.7
Fixed-Income Directional	13	9.82	3.49	9.55	8.74	−0.94	1.29	5.37	7.32	21.6	15.7	14.1	13.2	1.9	13.6	18.1	21.0
Convertible Fund (Long Only)	15	14.95	5.33	11.03	6.76	−0.19	1.68	6.14	9.97	22.5	12.3	6.2	13.4	−2.1	12.5	17.2	25.2
Event Driven	109	15.35	6.70	9.51	9.65	−0.58	1.82	8.26	11.68	20.8	16.8	6.4	13.4	−0.1	12.3	25.7	29.6
Non-Directional/Relative Value	85	12.54	6.65	8.19	5.05	−0.76	1.71	6.44	9.50	18.2	23.5	12.2	15.4	5.0	14.5	17.1	25.0
Global Macro	25	16.75	7.94	18.92	9.53	0.53	1.03	4.66	6.82	7.2	15.0	−0.1	10.8	−0.9	8.1	33.4	22.9
Global Opportunity	1	−17.39	—	31.03	—	−0.48	—	2.80	—	23.1	—	14.1	—	0.0	—	20.8	—
Natural Resources	3	11.39	4.89	18.91	1.06	0.48	0.44	1.25	1.49	5.0	14.5	8.5	12.1	1.9	7.0	47.6	43.7
Pure Leveraged Currency	26	9.29	7.11	17.18	8.93	0.44	1.08	3.50	4.92	5.2	8.7	−6.2	9.5	−4.4	8.1	35.5	26.6
Pure Managed Futures	93	9.60	9.28	21.74	17.06	0.22	1.35	4.93	8.31	−0.1	12.9	−3.8	10.0	−3.5	10.7	35.4	29.1
Pure Emerging Market	72	9.57	10.94	26.60	14.44	−0.69	1.49	6.17	8.05	18.8	11.8	4.4	11.6	−1.4	9.0	32.6	30.6
Pure Property	1	3.96	—	9.41	—	−1.33	—	5.43	—	−23.1	—	2.3	—	7.3	—	18.9	—
Fund of Funds	132	11.04	5.16	10.07	5.63	−0.26	1.30	4.78	6.16	17.6	14.1	5.7	11.5	−0.7	9.3	26.3	27.5
All	909	13.72	9.46	16.06	12.21	−0.15	1.38	5.05	7.56	12.1	16.8	3.4	13.1	−1.2	11.2	30.1	28.7

Table 7: Means and standard deviations of basic summary statistics for 909 hedge funds in the TASS Hedge Fund Combined (Live and Graveyard) database with at least five years of returns history during the period from November 1977 to January 2001. The columns 'p-Value(Q)' contain means and standard deviations of p-values for the Box-Pierce Q-statistic for each fund using the first 6 autocorrelations of returns.

Category	N	MA(2) with Constrained Sum								MA(2) with Constrained σ_η							
		$\hat\theta_0$		$\hat\theta_1$		$\hat\theta_2$		$\hat\xi$		$\hat\theta_0$		$\hat\theta_1$		$\hat\theta_2$		$\hat\xi$	
		Mean	SD	Mean	SD	Mean	SD	Mean	SD	Mean	SD	Mean	SD	Mean	SD	Mean	SD
Not Categorized	111	1.04	0.33	0.01	0.18	−0.05	0.23	1.27	1.06	0.97	0.27	0.01	0.16	−0.04	0.19	1.08	0.60
US Equity Hedge	162	0.95	0.21	0.06	0.15	−0.01	0.15	1.00	0.54	0.98	0.20	0.07	0.14	−0.01	0.15	1.04	0.42
European Equity Hedge	22	0.82	0.15	0.08	0.09	0.11	0.11	0.72	0.28	0.90	0.17	0.09	0.10	0.13	0.12	0.88	0.32
Asian Equity Hedge	5	0.94	0.30	0.05	0.19	0.01	0.11	1.00	0.71	0.90	0.20	0.06	0.17	0.01	0.10	0.88	0.40
Global Equity Hedge	27	0.91	0.14	0.11	0.10	−0.02	0.09	0.88	0.27	0.96	0.20	0.11	0.10	−0.02	0.09	0.99	0.37
Dedicated Shortseller	7	1.03	0.20	0.03	0.12	−0.06	0.13	1.12	0.48	1.13	0.14	0.05	0.12	−0.05	0.14	1.33	0.34
Fixed-Income Directional	13	0.76	0.17	0.15	0.10	0.08	0.12	0.67	0.27	0.75	0.21	0.15	0.10	0.09	0.12	0.66	0.30
Convertible Fund (Long Only)	15	0.84	0.35	0.18	0.08	−0.02	0.35	0.98	1.36	0.89	0.27	0.20	0.10	0.00	0.24	0.97	0.58
Event Driven	109	0.81	0.17	0.15	0.13	0.04	0.11	0.74	0.28	0.84	0.24	0.16	0.12	0.04	0.11	0.82	0.44
Non-Directional/Relative Value	85	0.82	0.24	0.09	0.25	0.09	0.14	0.82	0.62	0.77	0.27	0.09	0.24	0.08	0.13	0.75	0.70
Global Macro	24	0.95	0.18	0.08	0.11	−0.03	0.15	0.99	0.40	0.97	0.23	0.08	0.10	−0.03	0.17	1.03	0.51
Global Opportunity	1	0.74	—	0.16	—	0.10	—	0.58	—	0.63	—	0.14	—	0.09	—	0.43	—
Natural Resources	3	0.91	0.19	0.02	0.11	0.07	0.09	0.87	0.34	0.85	0.04	0.03	0.10	0.08	0.10	0.74	0.04
Pure Leveraged Currency	26	1.10	0.34	0.03	0.13	−0.13	0.25	1.41	1.38	1.00	0.16	0.04	0.10	−0.10	0.16	1.07	0.37
Pure Managed Futures	93	1.13	0.32	−0.05	0.22	−0.08	0.18	1.47	1.19	1.10	0.25	−0.03	0.18	−0.07	0.16	1.33	0.63
Pure Emerging Market	72	0.83	0.16	0.15	0.09	0.02	0.13	0.76	0.31	0.79	0.17	0.14	0.08	0.02	0.13	0.70	0.30
Pure Property	1	1.23	—	−0.31	—	0.07	—	1.62	—	0.95	—	−0.24	—	0.06	—	0.96	—
Fund of Funds	132	0.85	0.21	0.13	0.13	0.03	0.13	0.81	0.50	0.85	0.23	0.13	0.12	0.03	0.13	0.83	0.45
All	908	0.92	0.26	0.08	0.17	0.00	0.17	0.98	0.76	0.91	0.25	0.08	0.16	0.01	0.15	0.95	0.53

Table 8: Means and standard deviations of maximum likelihood estimates of MA(2) smoothing process $R_t^o = \theta_0 R_t + \theta_1 R_{t-1} + \theta_2 R_{t-2}$, $\xi \equiv \theta_0^2 + \theta_1^2 + \theta_2^2$, for 908 hedge funds in the TASS combined database with at least five years of returns history during the period from November 1977 to January 2001.

reports the same statistics for the maximum likelihood estimates under the normalization (75) where the variance σ_η^2 is set equal to a nonparametric estimate $\tilde{\sigma}_\eta^2$ given by (74). A comparison of the right and left panels reveals roughly similar characteristics, indicating the general equivalence of these two normalization methods and the fact that the smoothing model (21)–(23) may be a reasonable specification for hedge-fund returns.[34]

Table 8 shows that seven categories seem to exhibit smaller average values of $\hat{\theta}_0$ than the rest—European Equity Hedge (0.82), Fixed-Income Directional (0.76), Convertible Fund (Long Only) (0.84), Event Driven (0.81), Non-Directional/Relative Value (0.82), Pure Emerging Market (0.83), and Fund of Funds (0.85).[35] Consider, in particular, the Fixed-Income Directional category, which has a mean of 0.76 for $\hat{\theta}_0$. This is, of course, the average across all 13 funds in this category, but if it were the point estimate of a given fund, it would imply that only 76% of that fund's true current monthly return would be reported, with the remaining 24% distributed over the next two months (recall the constraint that $\hat{\theta}_0 + \hat{\theta}_1 + \hat{\theta}_2 = 1$). The estimates 0.15 and 0.08 for $\hat{\theta}_1$ and $\hat{\theta}_2$ imply that on average, the current reported return also includes 15% of last month's true return and 8% of the true return two months ago.[36] These averages suggest a significant amount of smoothing and illiquidity in this category, and are approximated by the geometric smoothing model of Section 4.2 with $\delta = 0.25$ (see Table 4). Recall from Table 4 that in this case, with $k = 2$, the impact of geometric smoothing was a 24% decrease in the market beta and a 27% increase in the Sharpe ratio of observed returns. Overall, the summary statistics in Table 8 are broadly consistent with common intuition about the nature of the strategies and securities involved in these fund categories, which contain the most illiquid securities and, therefore, have the most potential for smoothed returns.

Table 9 contains the smoothing parameter estimates for the top 50 funds ranked by $\hat{\theta}_0$, which provides a more direct view of illiquidity and smoothed returns. In contrast to the

[34]However, Table 8 contains only summary statistics, not the maximum likelihood estimators of individual funds, hence differences in the two normalizations for given funds may not be apparent from this table. In particular, side-by-side comparisons of maximum likelihood estimates for an individual under these two normalizations may still be a useful specification check despite the broad similarities that these two approaches seem to exhibit in Table 8.

[35]We omit the Global Opportunity category from our discussions because it consists of only a single fund.

[36]The averages do not sum to 1 exactly because of rounding errors.

Code	Category	Period	T	Status	$\hat{\theta}_0$	SE($\hat{\theta}_0$)	$\hat{\theta}_1$	SE($\hat{\theta}_1$)	$\hat{\theta}_2$	SE($\hat{\theta}_2$)	$\hat{\xi}$
1201	Non-Directional/Relative Value	199409–200101	77	1	0.464	0.040	0.365	0.025	0.171	0.047	0.378
4346	Event Driven	199412–200011	71	1	0.471	0.041	0.335	0.029	0.195	0.048	0.371
180	Not Categorized	198906–199608	87	0	0.485	0.041	0.342	0.027	0.173	0.046	0.382
1204	Non-Directional/Relative Value	199510–200101	64	1	0.492	0.049	0.339	0.033	0.169	0.055	0.386
1584	Fund of Funds	199601–200101	61	1	0.504	0.046	0.245	0.040	0.251	0.051	0.377
518	Non-Directional/Relative Value	199312–200005	78	0	0.514	0.034	0.142	0.037	0.343	0.037	0.403
971	Non-Directional/Relative Value	199409–200012	76	1	0.515	0.038	0.176	0.038	0.309	0.041	0.392
1234	Fund of Funds	199410–200012	75	1	0.527	0.061	0.446	0.020	0.027	0.061	0.477
2185	Fixed Income Directional	199108–200101	114	1	0.532	0.040	0.265	0.032	0.202	0.043	0.395
26	Non-Directional/Relative Value	199303–200101	95	1	0.533	0.047	0.305	0.033	0.162	0.050	0.403
171	Non-Directional/Relative Value	199304–200101	94	1	0.536	0.040	0.193	0.037	0.271	0.043	0.398
1696	Fund of Funds	199501–200001	61	0	0.536	0.067	0.406	0.030	0.058	0.068	0.456
120	Non-Directional/Relative Value	198207–199810	196	0	0.546	0.032	0.238	0.026	0.215	0.034	0.402
1396	Non-Directional/Relative Value	199507–200101	67	1	0.548	0.056	0.255	0.044	0.197	0.059	0.404
2774	Non-Directional/Relative Value	199501–200006	66	0	0.553	0.058	0.262	0.044	0.185	0.061	0.409
1397	Non-Directional/Relative Value	199410–200101	76	1	0.556	0.055	0.260	0.042	0.184	0.057	0.410
57	Non-Directional/Relative Value	199210–200101	100	1	0.561	0.052	0.311	0.034	0.127	0.053	0.428
4158	Event Driven	199510–200101	64	1	0.565	0.062	0.261	0.046	0.174	0.064	0.417
1773	Fund of Funds	199506–200101	68	1	0.569	0.056	0.194	0.049	0.238	0.058	0.417
415	Not Categorized	198807–199608	98	0	0.570	0.054	0.306	0.035	0.124	0.055	0.434
1713	Fixed Income Directional	199601–200101	61	1	0.573	0.067	0.269	0.048	0.158	0.068	0.426
1576	Fund of Funds	199504–200010	67	1	0.575	0.066	0.293	0.044	0.132	0.067	0.434
1633	Event Driven	199304–199901	70	0	0.576	0.066	0.309	0.042	0.115	0.066	0.440
1883	Event Driven	199306–200101	92	1	0.578	0.053	0.236	0.041	0.187	0.054	0.424
1779	Non-Directional/Relative Value	199512–200101	62	1	0.584	0.067	0.241	0.051	0.175	0.068	0.429
3860	Fund of Funds	199511–200101	63	1	0.584	0.064	0.207	0.053	0.210	0.065	0.428
482	Non-Directional/Relative Value	199406–200101	80	1	0.584	0.060	0.253	0.044	0.163	0.060	0.432
2755	US Equity Hedge	199203–200010	104	1	0.584	0.057	0.321	0.034	0.095	0.056	0.453
1968	Fund of Funds	199402–200012	83	1	0.587	0.060	0.253	0.044	0.160	0.060	0.434
1240	Fund of Funds	199405–200101	81	1	0.592	0.055	0.168	0.049	0.240	0.056	0.436
1884	Event Driven	199301–200101	97	1	0.592	0.059	0.298	0.038	0.110	0.058	0.451
2864	Non-Directional/Relative Value	199408–200101	78	1	0.598	0.064	0.255	0.046	0.147	0.064	0.444
1	Non-Directional/Relative Value	199101–200101	121	1	0.600	0.051	0.241	0.038	0.159	0.051	0.444
412	Pure Emerging Market	199211–200101	99	1	0.600	0.061	0.318	0.036	0.082	0.059	0.468
1046	Pure Emerging Market	199406–200101	80	1	0.603	0.058	0.169	0.051	0.228	0.058	0.444
2570	Event Driven	199001–200101	133	1	0.604	0.046	0.171	0.039	0.225	0.046	0.445
945	Fund of Funds	199210–200101	100	1	0.605	0.060	0.273	0.040	0.122	0.058	0.456
1994	Non-Directional/Relative Value	199507–200101	67	1	0.606	0.069	0.217	0.053	0.177	0.068	0.446
1691	Event Driven	199310–200101	88	0	0.610	0.062	0.232	0.046	0.158	0.061	0.451
34	Non-Directional/Relative Value	199209–200101	101	1	0.612	0.052	0.143	0.047	0.245	0.052	0.455
2630	Non-Directional/Relative Value	199001–200101	133	1	0.612	0.052	0.255	0.036	0.133	0.051	0.457
2685	US Equity Hedge	199207–200101	103	1	0.613	0.060	0.271	0.040	0.116	0.058	0.463
2549	Non-Directional/Relative Value	199408–200101	78	1	0.614	0.079	0.412	0.029	−0.026	0.069	0.547
490	Event Driven	199404–199908	65	0	0.615	0.074	0.237	0.054	0.148	0.072	0.456
3099	Convertible Fund (Long Only)	199501–200101	73	1	0.615	0.060	0.121	0.057	0.263	0.060	0.463
1326	US Equity Hedge	199601–200101	61	1	0.619	0.069	0.145	0.062	0.236	0.068	0.460
1418	Fund of Funds	199301–199804	64	0	0.621	0.085	0.363	0.041	0.016	0.078	0.518
3712	Not Categorized	199304–200101	94	1	0.625	0.062	0.211	0.047	0.164	0.060	0.462
1534	Fund of Funds	199402–200101	84	1	0.625	0.068	0.246	0.048	0.129	0.065	0.468
167	Event Driven	199207–200101	103	1	0.627	0.061	0.239	0.044	0.134	0.059	0.468

Table 9: First 50 funds of ranked list of 908 hedge funds in the TASS Hedge Fund Combined (Live and Graveyard) database with at least five years of returns history during the period from November 1977 to January 2001, ranked in increasing order of the estimated smoothing parameter $\hat{\theta}_0$ of the MA(2) smoothing process $R_t^o = \theta_0 R_t + \theta_1 R_{t-1} + \theta_2 R_{t-2}$, subject to the normalization $1 = \theta_0 + \theta_1 + \theta_2$, and estimated via maximum likelihood.

averages of Table 8, the parameter estimates of θ_0 among these 50 funds range from 0.464 to 0.627, implying that only half to two-thirds of the current month's true returns are reflected in observed returns. The asymptotic standard errors are generally quite small, ranging from 0.032 to 0.085, hence the smoothing parameters seem to be estimated reasonably precisely.

The funds in Table 9 fall mainly into three categories: Non-Directional/Relative Value, Event Driven, and Fund of Funds. Together, these three categories account for 40 of the 50 funds in Table 9. A more complete summary of the distribution of smoothing parameter estimates across the different fund categories is provided in Figures 5 and 6. Figure 5 contains a graph of the smoothing coefficients $\hat{\theta}_0$ for all 908 funds by category, and Figure 6 contains a similar graph for the smoothing index $\hat{\xi}$. These figures show that although there is considerable variation within each category, nevertheless, some differences emerge between categories. For example, categories 6–9, 15, and 17 (Fixed-Income Directional, Convertible Fund (Long Only), Event Driven Non-Directional/Relative Value, Pure Emerging Market, and Fund of Funds, respectively), have clearly discernible concentrations that are lower than the other categories, suggesting more illiquid funds and more smoothed returns. On the other hand, categories 1, 4, and 14 (US Equity Hedge, Global Equity Hedge, and Pure Managed Futures, respectively) have concentrations that are at the upper end, suggesting just the opposite—more liquidity and less return-smoothing. The smoothing index estimates $\hat{\xi}$ plotted in Figure 6 show similar patterns of concentration and dispersion within and between the categories.

To develop further intuition for the smoothing model (21)–(23) and the possible interpretations of the smoothing parameter estimates, we apply the same estimation procedure to the returns of the Ibbotson stock and bond indexes, the Merrill Lynch Convertible Securities Index,[37] the CSFB/Tremont hedge-fund indexes, and two mutual funds, the highly liquid Vanguard 500 Index Fund, and the considerably less liquid American Express Extra Income Fund.[38] Table 10 contains summary statistics, market betas, contemporaneous and lagged

[37] This is described by Merrill Lynch as a "market value-weighted index that tracks the daily price only, income and total return performance of corporate convertible securities, including US domestic bonds, Eurobonds, preferred stocks and Liquid Yield Option Notes".

[38] As of January 31, 2003, the net assets of the Vanguard 500 Index Fund (ticker symbol: VFINX) and the AXP Extra Income Fund (ticker symbol: INEAX) are given by http://finance.yahoo.com/ as \$59.7 billion and \$1.5 billion, respectively, and the descriptions of the two funds are as follows:

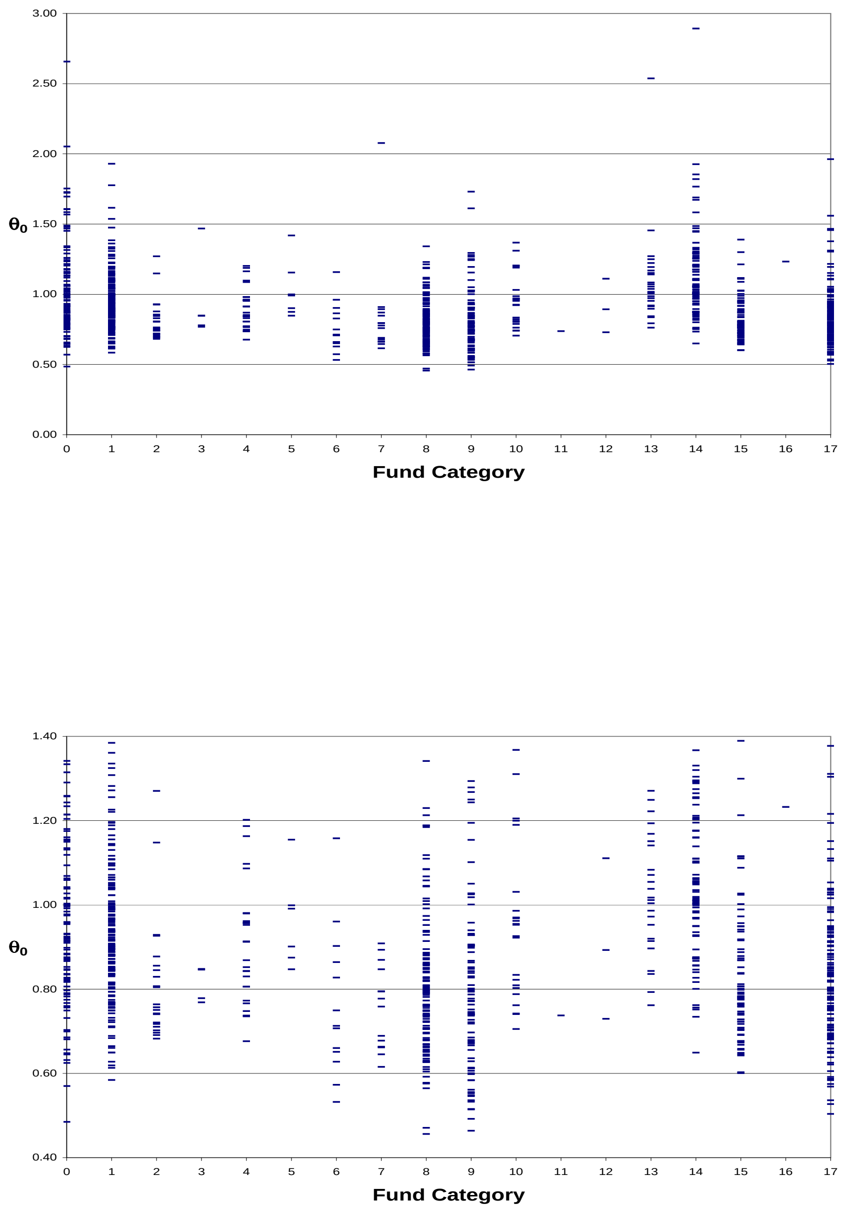


Figure 5: Estimated smoothing coefficients $\hat{\theta}_0$ for all funds in the TASS Hedge Fund database with at least five years of returns during the period from November 1977 to January 2001, ordered by categories 0 to 17. The two panels differ only in the range of the vertical axis, which is smaller for the lower panel so as to provide finer visual resolution of the distribution of estimated coefficients in the sample.

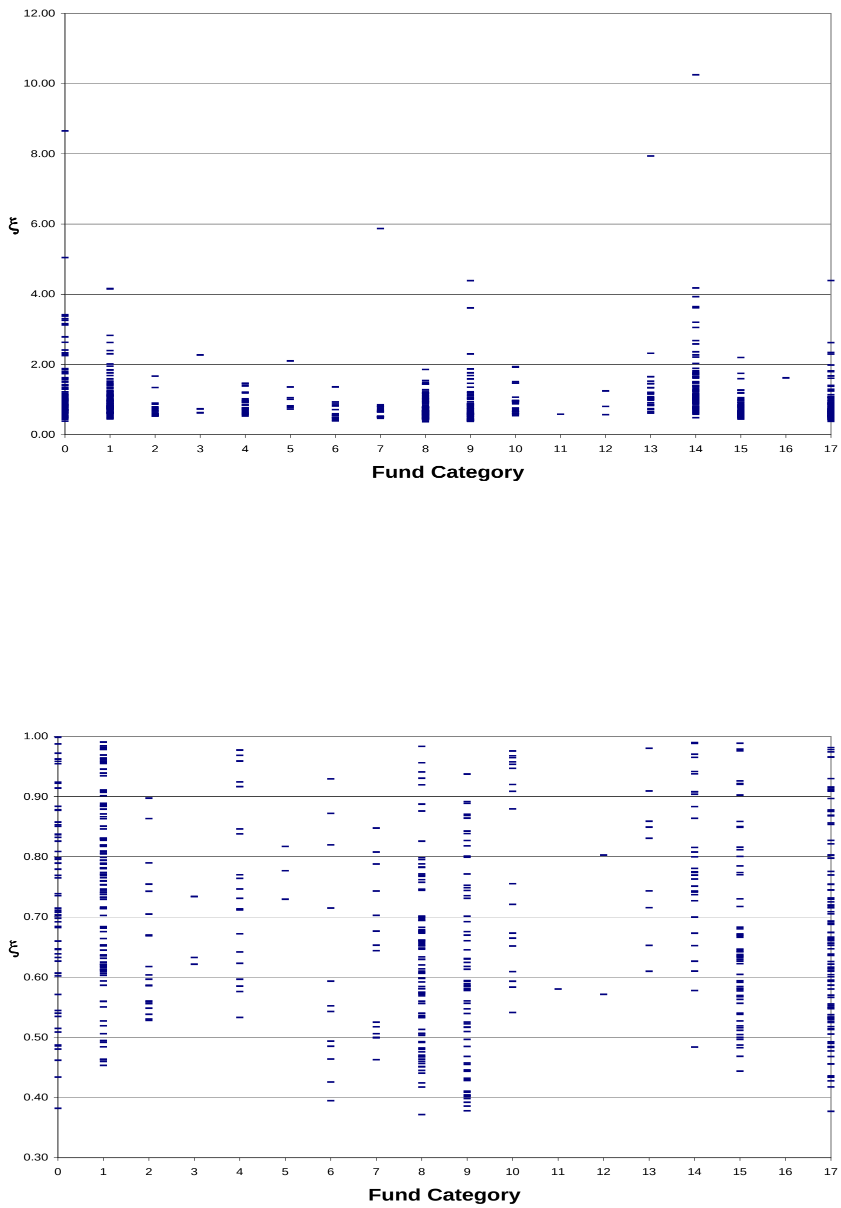


Figure 6: Estimated smoothing index $\hat{\xi}$ for all funds in the TASS Hedge Fund database with at least five years of returns during the period from November 1977 to January 2001, ordered by categories 0 to 17. The two panels differ only in the range of the vertical axis, which is smaller for the lower panel so as to provide finer visual resolution of the distribution of estimated smoothing indexes in the sample.

market betas as in Asness, Krail and Liew (2001), and smoothing-coefficient estimates for these index and mutual-fund returns.[39]

Consistent with our interpretation of $\hat{\theta}_0$ as an indicator of liquidity, the returns of the most liquid portfolios in the first panel of Table 10—the Ibbotson Large Company Index, the Vanguard 500 Index Fund (which is virtually identical to the Ibbotson Large Company Index, except for sample period and tracking error), and the Ibbotson Long-Term Government Bond Index—have smoothing parameter estimates near unity: 0.92 for the Ibbotson Large Company Index, 1.12 for the Vanguard 500 Index Fund, and 0.92 for the Ibbotson Long-Term Government Bond Index. The first-order autocorrelation coefficients and lagged market betas also confirm their lack of serial correlation; 9.8% first-order autocorrelation for the Ibbotson Large Company Index , −2.3% for the Vanguard 500 Index Fund, and 6.7% for the Ibbotson Long-Term Government Bond Index, and lagged market betas that are statistically indistinguishable from 0. However, the values of $\hat{\theta}_0$ of the less liquid portfolios are less than 1.00: 0.82 for the Ibbotson Small Company Index, 0.84 for the Ibbotson Long-Term Corporate Bond Index, 0.82 for the Merrill Lynch Convertible Securities Index, and 0.67 for the American Express Extra Income Fund, and their first-order serial correlation coefficients are 15.6%, 15.6%, 6.4% and 35.4%, respectively, which, with the exception of the Merrill Lynch Convertible Securities Index, are considerably higher than those of the more liquid portfolios.[40] Also, the lagged market betas are statistically significant at the 5% level for the Ibbotson Small Company Index (a t-statistic for $\hat{\beta}_1$: 5.41), the Ibbotson Long-Term

"The Vanguard 500 Index Fund seeks investment results that correspond with the price and yield performance of the S&P 500 Index. The fund employs a passive management strategy designed to track the performance of the S&P 500 Index, which is dominated by the stocks of large U.S. companies. It attempts to replicate the target index by investing all or substantially all of its assets in the stocks that make up the index."

"AXP Extra Income Fund seeks high current income; capital appreciation is secondary. The fund ordinarily invests in long-term high-yielding, lower-rated corporate bonds. These bonds may be issued by U.S. and foreign companies and governments. The fund may invest in other instruments such as: money market securities, convertible securities, preferred stocks, derivatives (such as futures, options and forward contracts), and common stocks."

[39] Market betas were obtained by regressing returns on a constant and the total return of the S&P 500, and contemporaneous and lagged market betas were obtained by regressing returns on a constant, the contemporaneous total return of the S&P 500, and the first two lags.

[40] However, note that the second-order autocorrelation of the Merrill Lynch Convertible Securities Index is 12.0% which is second only to the AXP Extra Income Fund in absolute magnitude, two orders of magnitude larger than the second-order autocorrelation of the Ibbotson bond indexes, and one order of magnitude larger than the Ibbotson stock indexes.

Government Bond Index (t-statistic for $\hat{\beta}_1$: -2.30), the Merrill Lynch Convertible Securities Index (t-statistic for $\hat{\beta}_1$: 3.33), and the AXP Extra Income Fund (t-statistic for $\hat{\beta}_1$: 4.64).

The results for the CSFB Hedge Fund Indexes in the second panel of Table 10 are also consistent with the empirical results in Tables 8 and 9—indexes corresponding to hedge-fund strategies involving less liquid securities tend to have lower $\hat{\theta}_0$'s. For example, the smoothing-parameter estimates $\hat{\theta}_0$ of the Convertible Arbitrage, Emerging Markets, and Fixed-Income Arbitrage Indexes are 0.49, 0.75, and 0.63, respectively, and first-order serial correlation coefficients of 56.6%, 29.4%, and 39.6%, respectively. In contrast, the smoothing-parameter estimates of the more liquid hedge-fund strategies such as Dedicated Short Bias and Managed Futures are 0.99 and 1.04, respectively, with first-order serial correlation coefficients of 7.8% and 3.2%, respectively. While these findings are generally consistent with the results in Tables 8 and 9, it should be noted that the process of aggregation can change the statistical behavior of any time series. For example, Granger (1980, 1988) observes that the aggregation of a large number of stationary autoregressive processes can yield a time series that exhibits long-term memory, characterized by serial correlation coefficients that decay very slowly (hyperbolically, as opposed to geometrically as in the case of a stationary ARMA process). Therefore, while it is true that the aggregation of a collection of illiquid funds will generally yield an index with smoothed returns,[41] the reverse need not be true—smoothed index returns need not imply that all of the funds comprising the index are illiquid. The latter inference can only be made with the benefit of additional information—essentially identification restrictions—about the statistical relations among the funds in the index, i.e., covariances and possibly other higher-order co-moments, or the existence of common factors driving fund returns.

It is interesting to note that the first lagged market beta, $\hat{\beta}_1$, for the CSFB/Tremont Indexes is statistically significant at the 5% level in only three cases (Convertible Arbitrage, Event Driven, and Managed Futures), but the second lagged beta, $\hat{\beta}_2$, is significant in five cases (the overall index, Convertible Arbitrage, Fixed Income Arbitrage, Global Macro,

[41] It is, of course, possible that the smoothing coefficients of some funds may exactly offset those of other funds so as to reduce the degree of smoothing in an aggregate index. However, such a possibility is extremely remote and pathological if each of the component funds exhibits a high degree of smoothing.

and Long/Short). Obviously, the S&P 500 Index is likely to be inappropriate for certain styles, e.g., Emerging Markets, and these somewhat inconsistent results suggest that using a lagged market-beta adjustment may not completely account for the impact of illiquidity and smoothed returns.

Overall, the patterns in Table 10 confirm our interpretation of smoothing coefficients and serial correlation as proxies for liquidity, and suggest that there may be broader applications of our model of smoothed returns to other investment strategies and asset classes.

6.3 Cross-Sectional Regressions

A more quantitative summary of the cross-sectional properties of the smoothing parameter estimates for the 908 funds is given in Table 11, which contains the results of cross-sectional regressions of the smoothing parameter $\hat{\theta}_0$ and the smoothing index $\hat{\xi}$ on a number of 0/1 indicator variables.[42] In the first two regressions, $\hat{\theta}_0$ and $\hat{\xi}$ are the dependent variables, respectively, and the regressors include a constant term, 17 indicator variables corresponding to the 17 hedge-fund categories defined by TASS (see Appendix A.4), and an indicator variable that takes on the value 1 if the fund is open and 0 if it is closed to new investors. The third and fourth regressions have the same dependent variables—$\hat{\theta}_0$ and $\hat{\xi}$, respectively—and include the same regressors as the first two regressions but also include 0/1 indicator variables that indicate whether the fund is US-based (USBASED), and whether the geographical focus of the fund is global (GF-GLB), US (GF-USA), Asia/Pacific (GF-APC), Western Europe (GF-WEU), Eastern Europe (GF-EEU), and Africa (GF-AFR).

The results of the first regression are consistent with the general intuition gleaned from Figures 5 and 6. The category indicator variables with the most negative coefficients that are statistically significant at the 5% level are European Equity Hedge (−0.212), Fixed-Income Directional (−0.262), Event Driven (−0.218), Non-Directional/Relative Value (−0.211), Pure Emerging Market (−0.195), Fund of Funds (−0.178), implying that on average, funds in these categories have smaller smoothing coefficients $\hat{\theta}_0$, i.e., less liquidity or smoother

[42] To conserve space, we report regression results only for the maximum likelihood estimates under the constraint (49). Table A.8 of the Appendix reports corresponding results for the maximum likelihood estimates under the alternate constraint (75).

Series	Period	T	Mean (%)	SD (%)	$\hat{\rho}_1$ (%)	$\hat{\rho}_2$ (%)	$\hat{\rho}_3$ (%)	Market Model			Contemporaneous and Lagged Market Model						
								$\hat{\beta}$	SE($\hat{\beta}$)	R^2 (%)	$\hat{\beta}_0$	SE($\hat{\beta}_0$)	$\hat{\beta}_1$	SE($\hat{\beta}_1$)	$\hat{\beta}_2$	SE($\hat{\beta}_2$)	R^2 (%)
Ibbotson Small Company	192601–200112	912	1.35	8.63	15.6	1.7	−10.6	1.27	0.03	66.9	1.25	0.03	0.16	0.03	0.03	0.03	68.0
Ibbotson Long-Term Government Bonds	192601–200112	912	0.46	2.22	6.7	0.3	−8.3	0.07	0.01	2.8	0.07	0.01	−0.03	0.01	−0.02	0.01	3.6
Ibbotson Long-Term Corporate Bonds	192601–200112	912	0.49	1.96	15.6	0.3	−6.0	0.08	0.01	5.2	0.08	0.01	−0.01	0.01	−0.01	0.01	5.3
Ibbotson Large Company	192601–200112	912	1.03	5.57	9.8	−3.2	−10.7	1.00	0.00	100.0	1.00	0.00	0.00	0.00	0.00	0.00	100.0
Merrill Lynch Convertibles Index	199401–200210	168	0.99	3.43	6.4	12.0	5.1	0.59	0.05	48.6	0.60	0.05	0.15	0.05	0.07	0.04	52.2
AXP Extra Income Fund (INEAX)	198401–200112	216	0.67	2.04	35.4	13.1	2.5	0.21	0.03	20.7	0.21	0.03	0.12	0.03	0.04	0.03	28.7
Vanguard 500 Index Trust (VFINX)	197609–200112	304	1.16	4.36	−2.3	−6.8	−3.2	1.00	0.00	100.0	1.00	0.00	0.00	0.00	0.00	0.00	100.0
CSFB/Tremont Indices:																	
Aggregate Hedge Fund Index	199401–200210	106	0.87	2.58	11.2	4.1	−0.4	0.31	0.05	24.9	0.32	0.05	0.06	0.05	0.16	0.05	32.1
Convertible Arbitrage	199401–200210	106	0.81	1.40	56.6	42.6	15.6	0.03	0.03	1.1	0.04	0.03	0.09	0.03	0.06	0.03	12.0
Dedicated Short Bias	199401–200210	106	0.22	5.29	7.8	−6.3	−5.0	−0.94	0.08	58.6	−0.93	0.08	−0.06	0.08	0.08	0.08	59.3
Emerging Markets	199401–200210	106	0.54	5.38	29.4	1.2	−2.1	0.62	0.11	24.0	0.63	0.11	0.19	0.11	0.03	0.12	26.2
Equity Market Neutral	199401–200210	106	0.89	0.92	29.4	18.1	8.4	0.10	0.02	21.1	0.10	0.02	0.02	0.02	0.00	0.02	22.1
Event Driven	199401–200210	106	0.83	1.81	34.8	14.7	3.8	0.23	0.04	30.2	0.23	0.03	0.11	0.03	0.04	0.03	38.2
Fixed Income Arbitrage	199401–200210	106	0.55	1.18	39.6	10.8	5.4	0.02	0.03	0.7	0.03	0.03	0.05	0.03	0.09	0.03	12.9
Global Macro	199401–200210	106	1.17	3.69	5.6	4.6	8.3	0.24	0.09	7.5	0.26	0.09	−0.01	0.09	0.23	0.09	14.1
Long/Short	199401–200210	106	0.98	3.34	15.9	5.9	−4.6	0.48	0.06	36.7	0.49	0.06	0.06	0.06	0.15	0.06	40.7
Managed Futures	199401–200210	106	0.55	3.44	3.2	−6.3	0.7	−0.12	0.08	2.5	−0.13	0.08	−0.17	0.08	0.02	0.08	7.8

Series	Period	T	$\hat{\theta}_0$	SE($\hat{\theta}_0$)	$\hat{\theta}_1$	SE($\hat{\theta}_1$)	$\hat{\theta}_2$	SE($\hat{\theta}_2$)	$\hat{\xi}$
Ibbotson Small Company	192601–200112	912	0.82	0.03	0.13	0.02	0.04	0.03	0.69
Ibbotson Long-Term Government Bonds	192601–200112	912	0.92	0.05	0.06	0.03	0.01	0.03	0.86
Ibbotson Long-Term Corporate Bonds	192601–200112	912	0.84	0.04	0.14	0.02	0.02	0.03	0.73
Ibbotson Large Company	192601–200112	912	0.92	0.05	0.09	0.03	−0.01	0.03	0.85
Merrill Lynch Convertibles Index	199401–200210	168	0.82	0.07	0.02	0.06	0.16	0.05	0.70
AXP Extra Income Fund (INEAX)	198401–200112	216	0.67	0.03	0.24	0.03	0.09	0.04	0.51
Vanguard 500 Index Trust (VFINX)	197609–200112	304	1.12	0.17	−0.03	0.07	−0.09	0.07	1.26
CSFB/Tremont Indices:									
Aggregate Hedge Fund Index	199401–200210	106	0.86	0.12	0.09	0.08	0.04	0.08	0.76
Convertible Arbitrage	199401–200210	106	0.49	0.01	0.26	0.03	0.25	0.03	0.37
Dedicated Short Bias	199401–200210	106	0.99	0.20	0.08	0.09	−0.07	0.10	0.99
Emerging Markets	199401–200210	106	0.75	0.08	0.24	0.05	0.01	0.07	0.62
Equity Market Neutral	199401–200210	106	0.71	0.06	0.18	0.05	0.12	0.06	0.54
Event Driven	199401–200210	106	0.68	0.05	0.23	0.05	0.09	0.06	0.52
Fixed Income Arbitrage	199401–200210	106	0.63	0.04	0.28	0.04	0.08	0.05	0.49
Global Macro	199401–200210	106	0.91	0.14	0.04	0.08	0.05	0.08	0.84
Long/Short	199401–200210	106	0.82	0.10	0.13	0.07	0.06	0.07	0.68
Managed Futures	199401–200210	106	1.04	0.23	0.04	0.10	−0.08	0.11	1.08

Table 10: Summary statistics and maximum likelihood estimates of MA(2) smoothing process $R_t^o = \theta_0 R_t + \theta_1 R_{t-1} + \theta_2 R_{t-2}$, $\xi \equiv \theta_0^2 + \theta_1^2 + \theta_2^2$, subject to the normalization $1 = \theta_0 + \theta_1 + \theta_2$, for the returns of various indexes and two mutual funds, the Vanguard 500 Index Trust (which tracks the S&P 500 index), and the AXP Extra Income Fund (which focuses on high current income and invests in long-term high-yielding lower-rated corporate bonds). Total returns of the S&P 500 index are used for both market models.

returns. These point estimates can be used to approximate the marginal impact that a given investment style has on the smoothing profile of the fund's monthly returns. For example, from a no-smoothing baseline of 1, conditioning on belonging to the Fixed-Income Directional category yields an expected smoothing parameter $\hat{\theta}_0$ of $1-0.262=0.738$ and an expected smoothing index of $\hat{\xi}$ of $1-0.583=0.417$, other things equal (and assuming that the remaining indicator variables in the two regression equations are 0).

In contrast, the coefficients for Dedicated Shortseller and Pure Leveraged Currency indicators—0.001 and 0.069, respectively, with t-statistics of 0.01 and 0.11, respectively—are positive and statistically insignificant at the 5% level, which is consistent with common intuition about the liquidity of these types of funds. Moreover, the coefficient for the Pure Managed Futures indicator is both positive and significant at the 5% level—0.101 with a t-statistic of 3.00—which is also consistent with the intuition that managed futures involve relatively liquid securities with well established marks that cannot easily be manipulated.

The last indicator variable included in the first two regressions takes on the value 1 if the fund is open to new investors and 0 if closed. If return-smoothing is actively being pursued, we might expect it to be more important for funds that are open since such funds are still attempting to attract new investors. This implies that the coefficient for this indicator variable should be negative—open funds should be more prone to smoothing than closed funds. Table 11 confirms this hypothesis: the estimated coefficient for OPEN is -0.040 with a t-statistic of 2.03, implying that funds open to new investors have a smoothing coefficient $\hat{\theta}_0$ that is lower by 0.040 on average than funds that are closed. An alternate interpretation is that funds that are still open to new investors are typically those with smaller assets under management, and as a result, are less likely to be able to afford costly third-party valuations of illiquid securities in their portfolios. Unfortunately, many funds in the TASS database do not report assets under management so we were unable to investigate this alternative.

The third and fourth regressions in Table 11 include additional indicator variables that capture the fund's geographical base as well as the geographical focus of its investments, and we see that being in the US has a positive marginal impact on the conditional mean of $\hat{\theta}_0$, but being US-focused in its investments has a negative marginal impact. The latter result is somewhat counterintuitive but becomes less puzzling in light of the fact that approximately

46% of the funds are US-focused, hence many of the most illiquid funds are included in this category. Apart from this indicator, the geographical aspects of our sample of funds seem to have little impact on the cross-sectional variability in smoothing parameter estimates.

With R^2's ranging from 9.0% to 17.7%, the regressions in Table 11 leave considerable cross-sectional variation unexplained, but this is no surprise given the noise inherent in the category assignments and the heterogeneity of investment styles even within each category. However, the general pattern of coefficients and t-statistics do suggest that the smoothing coefficients are capturing significant features of the cross section of hedge fund returns in our sample.

The final set of empirical estimates of the smoothing process (21)–(23) is for the linear regression model of Section 5.2, and is summarized in Table 12. Recall from Section 5.2 that the linear-regression estimates of $(\theta_0, \theta_1, \theta_2)$ are based on the assumption that true returns are given by the linear single-factor model (20) where the factor is the return on the S&P 500 index. To the extent that this assumption is a poor approximation to the true return-generating process, the corresponding smoothing parameter estimates will be flawed as well.

Table 12 reports the means and standard deviations of the estimates $(\hat{\theta}_0, \hat{\theta}_1, \hat{\theta}_2)$ and $\hat{\xi}$ for each of the categories, as well as the Durbin-Watson statistic and the regression R^2. In contrast to the maximum likelihood estimates of Table 8, the regression estimates are considerably more noisy, with cross-sectional standard deviations for the coefficients that are often an order of magnitude larger than the means, and in almost every case larger than the standard deviations of Table 8. For example, the average $\hat{\theta}_0$ for the Not Categorized category is 0.659, but the standard deviation is 8.696. The mean of $\hat{\theta}_0$ for Fixed-Income Directional funds is -1.437 and the standard deviation is 6.398. These results are not unexpected given the role that the linear single-factor model plays in the estimation process—if true returns contain additional common factors, then the linear-regression approach (62) will yield biased and inconsistent estimators for the smoothing parameters in (21)–(23).

The R^2 statistics in Table 12 yield some indication of the likelihood of omitted factors among the different categories. The highest mean R^2's are for the US Equity Hedge, Dedicated Shortseller, and Convertible Fund (Long Only) categories, with values of 26.1%, 43.0%,

Regressor	$\hat{\theta}_0$	$\hat{\xi}$	$\hat{\theta}_0$	$\hat{\xi}$
Constant	1.059	1.314	1.086	1.407
	(42.05)	(17.26)	(35.62)	(15.21)
US Equity Hedge	−0.077	−0.255	−0.076	−0.257
	(2.56)	(2.82)	(2.36)	(2.62)
European Equity Hedge	−0.212	−0.531	−0.260	−0.676
	(3.79)	(3.14)	(4.06)	(3.48)
Asian Equity Hedge	−0.086	−0.254	−0.076	−0.248
	(0.78)	(0.77)	(0.65)	(0.71)
Global Equity Hedge	−0.113	−0.366	−0.084	−0.294
	(2.19)	(2.35)	(1.55)	(1.80)
Dedicated Shortseller	0.001	−0.128	0.001	−0.130
	(0.01)	(0.45)	(0.01)	(0.46)
Fixed-Income Directional	−0.262	−0.583	−0.258	−0.575
	(3.73)	(2.75)	(3.64)	(2.67)
Convertible Fund (Long Only)	−0.180	−0.264	−0.188	−0.289
	(2.73)	(1.32)	(2.83)	(1.44)
Event Driven	−0.218	−0.514	−0.223	−0.527
	(6.69)	(5.23)	(6.57)	(5.12)
Non-Directional/Relative Value	−0.211	−0.432	−0.194	−0.399
	(6.07)	(4.12)	(5.37)	(3.64)
Global Macro	−0.075	−0.271	−0.049	−0.230
	(1.38)	(1.66)	(0.86)	(1.33)
Global Opportunity	−0.282	−0.656	−0.275	−0.647
	(1.17)	(0.9)	(1.14)	(0.89)
Natural Resources	−0.109	−0.363	−0.112	−0.372
	(0.78)	(0.86)	(0.8)	(0.87)
Pure Leveraged Currency	0.069	0.152	0.073	0.158
	(1.32)	(0.96)	(1.37)	(0.98)
Pure Managed Futures	0.101	0.210	0.116	0.244
	(3.00)	(2.06)	(3.29)	(2.29)
Pure Emerging Market	−0.195	−0.486	−0.189	−0.481
	(5.34)	(4.40)	(4.33)	(3.63)
Pure Property	0.173	0.304	0.195	0.339
	(0.72)	(0.42)	(0.80	(0.46)
Fund of Funds	−0.178	−0.433	−0.170	−0.419
	(5.66)	(4.57)	(5.28)	(4.29)
OPEN	−0.040	−0.077	−0.024	−0.041
	(2.03)	(1.30)	(1.17)	(0.65)
USBASED			0.018	0.030
			(1.00)	(0.54)
GF-GLB			−0.050	−0.139
			(2.37)	(2.19)
GF-USA			−0.053	−0.144
			(2.45)	(2.19)
GF-APC			−0.049	−0.128
			(1.59)	(1.37)
GF-WEU			0.018	0.052
			(0.61)	(0.58)
GF-EEU			−0.011	−0.045
			(0.26)	(0.35)
GF-AFR			0.016	0.054
			(0.28)	(0.31)
Sample Size	908	908	891	891
Adjusted R^2(%)	16.5	9.0	17.7	9.9

Table 11: Regressions of maximum likelihood estimated smoothing coefficient $\hat{\theta}_0$ and smoothing index $\hat{\xi}$ on indicator variables for 908 hedge funds in the TASS Hedge Fund Combined (Live and Graveyard) database with at least five years of returns history during the period from November 1977 to January 2001, where the maximum likelihood estimators of the MA coefficients $(\theta_0, \theta_1, \theta_2)$ are constrained to sum to 1. Absolute values of t-statistics are given in parentheses. The indicator variables are OPEN (1 if the fund is open, 0 otherwise); the fund categories (1 if the fund belongs to the category, 0 otherwise); USBASED (1 if the fund is based in the US, 0 otherwise); and geographical focus categories (1 if the geographical focus of the fund is in a given region, 0 otherwise, where the regions are USA, Asia Pacific, Western Europe, Eastern Europe, and Africa, respectively).

and 25.0%, respectively, which is consistent with the fact that our single factor is the S&P 500.[43] However, several categories have mean R^2's below 10%, implying relatively poor explanatory power for the single-factor model and, therefore, noisy and unreliable estimates of the smoothing process.

Overall, the results in Table 12 suggest that the linear regression approach is dominated by the maximum likelihood procedure, and that while the regression coefficients of lagged market returns may provide some insight into the net market exposure of some funds, they are considerably less useful for making inferences about illiquidity and smoothed returns.

6.4 Illiquidity Vs. Smoothing

To address the issue of systematic versus idiosyncratic effects of illiquidity and return-smoothing, we estimate the more general linear factor model of smoothing (67)–(69) with $k=3$ for the subset of Convertible Funds (Long Only), which consists of 15 funds in our sample of 908 funds. We take as our common factor Λ_t the Merrill Lynch Convertible Securities Index (see footnote 37 for a description), and estimate the linear regression equation via maximum likelihood and then renormalize the MA coefficients according to (69) and recompute the standard errors accordingly. Table 13 contains the regression coefficients as well as the smoothing coefficients, and t-statistics are reported instead of standard errors because we have specific null hypotheses to test as described in Section 5.2.

The estimates in Table 13 show that including a common factor can have a significant impact on the smoothing parameter estimates. For example, the value of $\hat{\theta}_0$ for Fund 1146 under the smoothing process (21)–(23) is 0.689, and its t-statistic under the null hypothesis that $\theta_0=1$ is -6.01. However, under the linear factor specification (67)–(69), the smoothing coefficient estimate becomes 1.172 with a t-statistic of 0.96. Nevertheless, for other funds in our sample of 15, the smoothing parameter estimates are virtually unchanged by including the contemporaneous and lagged common factors. For example, the value of $\hat{\theta}_0$ for Fund 4243 under the smoothing process (21)–(23) and the linear factor model (67)–(69) is 0.645 and 0.665, respectively, with t-statistics of -8.18 and -5.36, respectively.

[43] We have omitted the Global Opportunity category from this comparison despite its R^2 of 30.9% because it contains only a single fund.

Category	N	$\hat{\theta}_0$		$\hat{\theta}_1$		$\hat{\theta}_2$		$\hat{\xi}$		D.W.		$R^2(\%)$	
		Mean	SD	Mean	SD	Mean	SD	Mean	SD	Mean	SD	Mean	SD
Not Categorized	111	0.659	8.696	0.459	9.363	−0.118	2.260	167.540	1432.164	1.86	0.33	13.6	16.8
US Equity Hedge	162	0.695	1.502	0.094	0.415	0.211	1.220	4.427	38.503	1.80	0.28	26.1	17.5
European Equity Hedge	22	0.561	0.155	0.340	0.683	0.099	0.708	1.386	4.287	1.76	0.19	18.0	10.8
Asian Equity Hedge	5	0.405	0.252	0.323	0.179	0.272	0.088	0.425	0.086	1.82	0.24	7.4	4.5
Global Equity Hedge	27	0.846	0.400	0.057	0.331	0.097	0.364	1.115	0.913	1.65	0.22	22.1	19.8
Dedicated Shortseller	7	0.908	0.182	0.115	0.101	−0.023	0.203	0.911	0.360	1.94	0.16	43.0	14.7
Fixed-Income Directional	13	−1.437	6.398	4.385	14.552	−1.948	8.179	320.099	1151.235	1.62	0.29	13.1	9.9
Convertible Fund (Long Only)	15	0.538	0.262	0.343	0.152	0.120	0.178	0.536	0.219	1.53	0.24	25.0	15.0
Event Driven	109	0.582	0.479	0.293	0.493	0.125	0.225	0.958	3.568	1.63	0.36	18.0	11.3
Non-Directional/Relative Value	85	0.643	4.057	0.895	2.321	−0.538	4.761	45.493	244.844	1.62	0.46	12.2	9.2
Global Macro	24	0.558	0.349	0.194	0.341	0.247	0.190	0.674	0.409	1.76	0.28	13.5	8.0
Global Opportunity	1	0.619	—	0.191	—	0.191	—	0.455	—	1.55	—	30.9	—
Natural Resources	3	0.691	0.138	0.106	0.087	0.203	0.068	0.551	0.163	1.88	0.31	2.6	0.9
Pure Leveraged Currency	26	−0.080	1.376	0.360	1.066	0.720	1.987	7.363	24.457	1.89	0.16	4.4	6.0
Pure Managed Futures	93	−0.032	1.588	0.696	1.649	0.336	1.611	8.350	26.079	1.99	0.26	5.9	5.9
Pure Emerging Market	72	0.899	0.801	0.157	0.465	−0.056	0.452	1.883	6.486	1.51	0.29	22.8	10.6
Pure Property	1	1.319	—	−0.024	—	−0.295	—	1.828	—	2.31	—	15.4	—
Fund of Funds	132	0.674	1.516	−0.049	2.566	0.375	3.059	18.694	171.624	1.63	0.30	21.6	15.0
All	908	0.554	3.534	0.366	3.950	0.081	2.401	34.268	529.561	1.73	0.34	17.8	15.1

Table 12: Means and standard deviations of linear regression estimates of MA(2) smoothing process $R_t^o = \theta_0 R_t + \theta_1 R_{t-1} + \theta_2 R_{t-2}$, $\xi \equiv \theta_0^2 + \theta_1^2 + \theta_2^2$ under the assumption of a linear single-factor model for R_t where the factor is the total return of the S&P 500 Index, for 908 hedge funds in the TASS Hedge Fund Combined (Live and Graveyard) database with at least five years of returns history during the period from November 1977 to January 2001.

We see from Table 13 that the Convertible Securities Index is statistically significant at the 5% level for most, but not all, of the 15 funds, and that its first lag is significant for only four funds (818, 2245, 4204, and 4326), and its second lag is significant for only two funds (1908 and 4216). For five of these six funds, the lagged-index coefficients are positive in sign, which is consistent with the smoothing model (68)–(69) (assuming that the funds' contemporaneous factor loadings and smoothing parameters are positive). For Fund 4216, the smoothing parameter estimate $\hat{\theta}_0$ is still significantly different from 1 even after accounting for the common factor, but for Fund 4204, it is not.

It is tempting to conclude from these results that the linear factor model (67)–(69) is capable of distinguishing between systematic illiquidity and idiosyncratic return-smoothing behavior. For example, we might argue that those funds which continue to exhibit significant smoothing parameters $\hat{\theta}_0$ even after accounting for common factors must be engaged in return-smoothing behavior. However, several caveats must be kept in mind before reaching such conclusions. First, we cannot be certain that the Merrill Lynch Convertible Securities Index is the appropriate common factor for these funds, despite the TASS classification—some funds may be involved in complex trading strategies involving convertible securities while others are engaged in simpler buy-and-hold strategies.[44] Regressing a fund's returns on a highly serially correlated common factor that is not directly relevant to that fund's investment process will, nevertheless, have an effect on the smoothing-parameter estimates $\hat{\theta}_k$, and the effect may be in either direction depending on the relation between the common factor and the fund's observed returns. Second, even if a common factor can account for much of the serial correlation in a fund's observed returns, an explanation for the source of the factor's serial correlation is still required—if the fund is a buy-and-hold version of the common factor, e.g., a fund-of-funds designed to replicate the CSFB/Tremont Convertible Arbitrage Index, then it is of small comfort to investors in such a fund-of-funds that there is not much smoothing in observed returns beyond what is already present in the common

[44] In particular, of the 15 "Convertible Fund (Long Only)" funds in our sample, 12 funds involve long-only positions in convertible bonds, possibly with a limited degree of leverage, but with no equity or credit protection, and the remaining 3 funds (4204, 4145 and 4326) are convertible arbitrage funds that involve long positions in convertible bonds and short positions in the corresponding stocks. We thank Stephen Jupp of TASS for providing us with this information.

factor. And finally, no econometric model can fully capture the many qualitative and often subjective characteristics of a fund's investment process, and such information is likely to be of particular relevance in distinguishing between illiquidity and smoothed returns at the fund level.

These caveats suggest that a more comprehensive econometric analysis of hedge-fund returns may be worthwhile, with particular emphasis on constructing common factors for hedge funds with similar investment mandates and processes. By developing a better understanding for the common risk exposures that certain hedge funds share, it may be possible to differentiate between systematic and idiosyncratic illiquidity and provide investors and managers with a more refined set of tools with which to optimize their investment plans.

6.5 Smoothing-Adjusted Sharpe Ratio Estimates

For each of the 908 funds in our sample, we compute annual Sharpe ratios in three ways relative to a benchmark return of 0: the standard method ($\sqrt{12}$ times the ratio of the mean monthly return to the monthly return standard deviation), the serial-correlation-adjusted method in Lo (2002), and the small-sample method described in Section 5.4. The results are summarized in Table 14.

The largest differences between standard and smoothing-adjusted Sharpe ratios are found in the same categories that the smoothing-process estimates of Section 6.2 identified as the most illiquid: Fixed-Income Directional (20.3% higher average Sharpe ratio relative to SR^{**}), Convertible Fund (Long Only) (17.8%), Non-Directional/Relative Value (16.0%), Pure Emerging Market (16.3%), and Fund of Funds (17.8%). For two categories—Dedicated Shortseller and Managed Futures—the bias is reversed, a result of negative serial correlation in their returns. For the other categories, Table 14 shows that the smoothing-adjusted Sharpe ratios are similar in magnitude to the usual estimates. These differences across categories suggest the importance of taking illiquidity and smoothed returns into account in evaluating the performance of hedge funds.

Code	T	Period	$\hat{\mu}$ (t-stat)	$\hat{\gamma}_0$ (t-stat)	$\hat{\gamma}_1$ (t-stat)	$\hat{\gamma}_2$ (t-stat)	Maximum Likelihood Smoothing Parameter Estimates $\hat{\theta}_0$	$t(\hat{\theta}_0)$	$\hat{\theta}_1$	$t(\hat{\theta}_1)$	$\hat{\theta}_2$	$t(\hat{\theta}_2)$	$\hat{\xi}$
31	90	199307–200012	−1.159	1.171	0.036	−0.110	0.903	−0.55	−0.296	−2.39	0.393	6.42	1.06
			(−2.28)	(11.51)	(0.36)	(−0.92)	0.794	−3.24	0.083	1.27	0.123	1.94	0.65
818	72	199502–200101	−0.754	0.387	0.168	0.014	0.986	−0.10	0.139	1.31	−0.124	−0.96	1.01
			(−1.66)	(5.84)	(1.98)	(0.15)	0.759	−3.46	0.261	4.71	−0.019	−0.28	0.64
843	79	199407–200101	−0.716	0.340	0.144	0.106	0.971	−0.18	0.094	0.93	−0.065	−0.51	0.96
			(−1.86)	(4.50)	(1.61)	(0.97)	0.847	−2.02	0.159	2.34	−0.006	−0.08	0.74
1146	94	199010–199807	−1.503	1.043	0.138	−0.029	1.172	0.96	−0.044	−0.30	−0.128	−0.98	1.39
			(−7.59)	(13.15)	(1.75)	(−0.39)	0.689	−6.01	0.187	3.79	0.124	2.40	0.53
1148	73	199501–200101	−0.966	0.617	0.106	−0.009	1.125	0.51	0.312	3.16	−0.437	−1.72	1.55
			(−3.07)	(10.49)	(1.25)	(−0.11)	0.795	−2.81	0.211	3.39	−0.006	−0.08	0.68
1908	81	199405–200101	−1.191	1.325	0.121	−0.425	0.989	−0.06	0.052	0.42	−0.041	−0.37	0.98
			(−1.37)	(10.51)	(0.82)	(−2.09)	0.909	−1.12	0.140	1.93	−0.049	−0.60	0.85
2245	67	199507–200101	−0.607	0.287	0.174	0.034	0.938	−0.36	0.087	0.70	−0.025	−0.22	0.89
			(−1.27)	(3.48)	(2.37)	(0.31)	0.870	−1.53	0.173	2.34	−0.043	−0.50	0.79
2564	63	199511–200101	−0.621	0.291	0.108	0.123	0.866	−0.67	0.279	2.17	−0.145	−0.90	0.85
			(−1.10)	(3.22)	(1.39)	(1.36)	0.778	−2.86	0.303	5.45	−0.080	−1.03	0.70
3099	73	199501–200101	0.093	−0.040	−0.047	0.018	0.632	−4.10	0.104	1.71	0.264	5.49	0.48
			(0.41)	(−0.96)	(−1.32)	(0.39)	0.615	−8.31	0.121	2.37	0.263	5.68	0.46
4145	63	199511–200101	−0.487	0.368	−0.094	−0.031	2.042	4.82	0.015	0.03	−1.058	−1.08	5.29
			(−0.73)	(2.14)	(−0.61)	(−0.14)	2.077	4.41	0.161	1.16	−1.238	−5.07	5.87
4204	72	199501–200012	−0.876	0.303	0.304	0.069	0.769	−1.26	0.122	0.85	0.109	1.46	0.62
			(−2.88)	(4.52)	(6.37)	(0.95)	0.678	−5.53	0.217	4.08	0.105	1.80	0.52
4216	104	199206–200101	−0.115	0.012	0.012	0.079	0.650	−4.60	0.151	3.47	0.200	4.60	0.48
			(−0.61)	(0.56)	(0.50)	(2.09)	0.663	−7.48	0.134	2.84	0.202	4.49	0.50
4243	118	199101–200010	−0.322	0.036	0.110	0.081	0.665	−5.36	0.270	6.24	0.066	0.89	0.52
			(−1.07)	(0.59)	(1.70)	(1.15)	0.645	−8.18	0.279	7.76	0.076	1.76	0.50
4295	82	199403–200012	−0.014	0.039	−0.032	0.014	0.894	−0.61	−0.031	−0.22	0.136	1.63	0.82
			(−0.07)	(0.51)	(−0.72)	(0.21)	0.893	−1.37	0.009	0.10	0.098	1.26	0.81
4326	149	198807–200101	−0.154	0.082	0.039	0.021	0.700	−4.71	0.229	6.40	0.071	1.40	0.55
			(−2.10)	(6.02)	(2.60)	(1.24)	0.662	−8.62	0.243	7.02	0.095	2.43	0.51

Table 13: Maximum likelihood estimates of linear regression model with MA(2) errors, $R_t^o = \mu + \gamma_0\Lambda_t + \gamma_1\Lambda_{t-1} + \gamma_2\Lambda_{t-2} + u_t$, $u_t = \theta_0\epsilon_t + \theta_1\epsilon_{t-1} + \theta_2\epsilon_{t-2}$, subject to the normalization $1 = \theta_0 + \theta_1 + \theta_2$, for all 15 Convertible Funds (Long Only) among the sample of 908 hedge funds in the TASS Hedge Fund Combined (Live and Graveyard) database with at least five years of returns history during the period from November 1977 to January 2001. The factor Λ_t is taken to be the Merrill Lynch Convertible Securities Index. For comparison, maximum likelihood estimates of θ_0, θ_1, θ_2, and t-statistics from the MA(2) smoothing process $R_t^o = \theta_0 R_t + \theta_1 R_{t-1} + \theta_2 R_{t-2}$, subject to the normalization $1 = \theta_0 + \theta_1 + \theta_2$, are provided below each of the main rows. t-statistics are computed with respect to the null hypothesis that the coefficient is 0, except for $\hat{\theta}_0$, for which the null hypothesis $\theta_0 = 1$ is used, hence the t-statistic is computed as $(\hat{\theta}_0 - 1)/\mathrm{SE}(\hat{\theta}_0)$.

Category	Sharpe Ratios For Combined Sample						
	N	SR		SR*		SR**	
		Mean	SD	Mean	SD	Mean	SD
Not Categorized	111	1.12	1.09	1.06	0.87	1.00	0.82
US Equity Hedge	162	1.26	0.75	1.31	0.75	1.23	0.69
European Equity Hedge	22	1.43	0.74	1.43	0.80	1.33	0.74
Asian Equity Hedge	5	0.50	0.39	0.52	0.39	0.49	0.37
Global Equity Hedge	27	0.90	0.61	0.95	0.66	0.89	0.61
Dedicated Shortseller	7	0.28	0.59	0.32	0.64	0.30	0.61
Fixed-Income Directional	13	2.02	2.35	1.80	2.23	1.68	2.06
Convertible Fund (Long Only)	15	1.83	1.20	1.66	0.85	1.55	0.80
Event Driven	109	2.36	1.45	2.21	1.57	2.08	1.47
Non-Directional/Relative Value	85	2.20	1.86	2.03	2.39	1.89	2.22
Global Macro	24	1.08	0.67	1.14	0.73	1.07	0.70
Global Opportunity	1	−0.56	—	−0.39	—	−0.37	—
Natural Resources	3	0.60	0.25	0.56	0.23	0.52	0.21
Pure Leveraged Currency	26	0.63	0.49	0.65	0.50	0.61	0.47
Pure Managed Futures	93	0.54	0.55	0.63	0.60	0.60	0.56
Pure Emerging Market	72	0.39	0.45	0.36	0.44	0.34	0.40
Pure Property	1	0.42	—	0.45	—	0.41	—
Fund of Funds	132	1.44	1.01	1.30	0.88	1.22	0.82
All	908	1.32	1.24	1.27	1.27	1.19	1.18

Table 14: Means and standard deviations of Sharpe ratios of 908 hedge funds in the TASS Hedge Fund Combined (Live and Graveyard) database with at least five years of returns history during the period from November 1977 to January 2001. SR is the standard Sharpe ratio, SR^* is the smoothing-adjusted Sharpe ratio of Lo (2002), and SR^{**} is the smoothing-adjusted Sharpe ratio using $\hat{\sigma}_{\mathrm{NW}}$. All Sharpe ratios are computed with respect to a 0 benchmark.

7 Conclusions

Although there are several potential explanations for serial correlation in asset returns, we have argued in this paper that the serial correlation present in the returns of hedge funds is due primarily to illiquidity and smoothed returns. Using a simple econometric model in which observed returns are a finite moving-average of unobserved economic returns, we are able to generate empirically realistic levels of serial correlation for historical hedge-fund returns while, at the same time, explaining the findings of Asness, Krail and Liew (2001) regarding the significance of lagged market returns in market-model regressions for hedge funds. Although our moving-average specification is similar to some of the early models of nonsynchronous trading, our motivation is quite different and is meant to cover a broader set of factors that give rise to serial correlation and smoothed returns, even in the presence of synchronously recorded prices.

Maximum likelihood estimates of our smoothing model for the returns of 908 hedge funds in the TASS Hedge Fund database yield empirically plausible estimates of smoothing coefficients and suggest that simple time-series measures such as our smoothing index may serve as useful proxies for a hedge fund's illiquidity risk exposure. In some cases, our econometric model may also be useful for flagging possible cases of deliberate performance-smoothing behavior, although additional information will need to be gathered before any firm conclusions regarding such behavior can be made. Regardless of the sources of serial correlation, illiquidity exposure is the main implication and this has potentially important consequences for both managers and investors. Therefore, we also develop a set of tools for quantifying the degree of smoothing in the data and adjusting for smoothed returns in computing performance statistics such as means, variances, market betas, and Sharpe ratios, and derive their asymptotic distributions using continuous-record asymptotics that can better accommodate the small sample sizes of most hedge-fund datasets.

Our empirical results suggest several applications for our econometric model of illiquidity and smoothed returns. Despite the general consistency of our empirical results with common intuition regarding the levels of illiquidity among the various hedge-fund investment styles, the variation in estimated smoothing coefficients within each category indicates that there may be better ways of categorizing hedge funds. Given the importance of liquidity for the typical hedge-fund investor, it may be useful to subdivide each style category into "liquidity tranches" defined by our smoothing index. This may prove to be especially useful in identifying and avoiding the potential wealth transfers between new and existing investors that can occur from the opportunistic timing of hedge-fund investments and redemptions. Alternatively, our smoothing parameter estimates may be used to compute illiquidity exposure measures for portfolios of hedge funds or fund of funds, which may serve as the basis for a more systematic approach to managing portfolios that include alternative investments.

Although we have focused on hedge funds in this paper, our analysis may be applied to other investments and asset classes, e.g., real estate, venture capital, private equity, art and other collectibles, and other assets for which illiquidity and smoothed returns are even more problematic, and where the estimation of smoothing profiles can be particularly useful for providing investors with risk transparency. More generally, our econometric model may be applied to a number of other contexts in which there may be a gap between reported results and economic realities. For example, recent events surrounding the collapse of Enron and other cases of corporate accounting irregularities have created renewed concerns about "earnings management" in which certain corporations are alleged to have abused accounting conventions so as to smooth earnings, presumably to give the appearance of stability and consistent growth.[45] The impact of such smoothing can sometimes be "undone" using an econometric model such as ours.

There are a number of outstanding issues regarding our analysis of illiquidity and smoothed returns that warrant further study. Perhaps the most pressing issue is whether the proximate source of smoothing is inadvertent or deliberate. Our linear regression model with contemporaneous and lagged common factors may serve as the starting point for distinguishing between systematic illiquidity versus idiosyncratic smoothing behavior. However, this issue is likely to require additional information about each fund along the lines of Chandar and Bricker's (2002) study, e.g., the size of the fund, the types of the securities in which the fund invests, the accounting conventions used to mark the portfolio, the organization's compen-

[45]See Beneish (2001) and Healy and Wahlen (1999) for reviews of the extensive literature on earnings management.

sation structure, and other operational aspects of the fund. With these additional pieces of information, we may construct more relevant common factors for our linear-regression framework, or relate the cross-sectional variation in smoothing coefficients to assets under management, security type, fee structure, and other characteristics, yielding a more complete picture of the sources of smoothed returns.

It may also be fruitful to view the hedge-fund industry from a broader perspective, one that acknowledges the inherent capacity constraints of certain types of strategies as well as the time lags involved in shifting assets from one type to another. Because of the inevitable cross-sectional differences in the performance of hedge-fund styles, assets often flow in loosely coordinated fashion from one style to another, albeit under various institutional restrictions such as calendar-specific periods of liquidity, tiered redemption schedules, redemption fees, and other frictions. The interactions between asset flows and institutional rigidities—especially over time—may sometimes cause statistical side-effects that include periodicities in performance and volatility, time-varying correlations, structural breaks, and under certain conditions, serial correlation. The dynamics of the hedge-fund industry are likely to be quite different than that of more traditional investment products, hence the "ecological" framework of Niederhoffer (1998, Chapter 15), Farmer (1998), and Farmer and Lo (1999), or the system dynamics approach of Getmansky and Lo (2003) might be more conducive paradigms for addressing these issues.

Finally, from the hedge-fund investor's perspective, a natural extension of our analysis is to model illiquidity directly and quantify the illiquidity premium associated with each hedge-fund investment style, perhaps in a linear-factor framework such as Chordia, Roll and Subrahmanyam (2000) and Pastor and Stambaugh (2002). Whether such factor models can forecast liquidity crises like August 1998, and whether there are "systematic" illiquidity factors that are common to categories of hedge funds are open questions that are particularly important in the context of hedge-fund investments. We plan to address these and other related questions in our ongoing research.

A Appendix

Proofs of Propositions 3, 4, and 5 are provided in Sections A.1, A.2, and A.3, respectively. Section A.4 provides definitions of the 17 categories from TASS, and Section A.5 contains additional empirical results.

A.1 Proof of Proposition 3

The constraint (49) may be used to substitute out θ_0, hence we need only consider (θ_1, θ_2). Now it is well known that in the standard MA(2) specification where the usual identification condition is used in place of (49), i.e.,

$$X_t \;=\; \epsilon_t \;+\; a\epsilon_{t-1} \;+\; b\epsilon_{t-2} \;,$$

the asymptotic distribution of the maximum likelihood estimators $(\hat{a}, \hat{b})$ is given by:

$$\sqrt{T}\left(\begin{bmatrix} \hat{a} \\ \hat{b} \end{bmatrix} - \begin{bmatrix} a \\ b \end{bmatrix}\right) \;\overset{a}{\sim}\; \mathcal{N}(0, \mathbf{V}) \tag{A.1}$$

where

$$\mathbf{V} \;\equiv\; \begin{bmatrix} 1-b^2 & a(1-b) \\ a(1-b) & 1-b^2 \end{bmatrix} . \tag{A.2}$$

But under our normalization (49), there is a simple functional relation between $(\hat{a}, \hat{b})$ and $(\hat{\theta}_1, \hat{\theta}_2)$:

$$\hat{\theta}_1 \;=\; \frac{\hat{a}}{1+\hat{a}+\hat{b}} \;,\;\; \hat{\theta}_2 \;=\; \frac{\hat{b}}{1+\hat{a}+\hat{b}} \;. \tag{A.3}$$

Therefore, we can apply the delta method to obtain the asymptotic distribution of $(\hat{\theta}_1, \hat{\theta}_2)$ as:

$$\sqrt{T}\left(\begin{bmatrix}\hat{\theta}_1\\ \hat{\theta}_2\end{bmatrix} - \begin{bmatrix}\theta_1\\ \theta_2\end{bmatrix}\right) \overset{a}{\sim} \mathcal{N}(0, \mathbf{JVJ'}) \tag{A.4}$$

where the matrix $\mathbf{J}$ is Jacobian associated with $(A.3)$:

$$\mathbf{J} = \frac{1}{(1+a+b)^2}\begin{bmatrix}1+b & -a\\ -b & 1+a\end{bmatrix}. \tag{A.5}$$

Then we have:

$$\mathbf{JVJ'} =$$

$$\frac{1}{(1+a+b)^3}\begin{bmatrix}-(1+b)(-1+a-ab+b^2) & b(-1+a-ab+b^2)\\ b(-1+a-ab+b^2) & -(-1+b)(-1+a-ab+b^2)\end{bmatrix} \tag{A.6}$$

and solving for a and b as a function of θ_1 and θ_2 using $(A.3)$ and substituting these expressions into $A.6$ yields the desired result.

The process X_t is invertible if and only if the roots of characteristic polynomial

$$f(x) = \theta_0 x^2 + \theta_1 x + \theta_2 \tag{A.7}$$

lie inside the unit circle in the complex plane. It is easy to see that this is equivalent to the condition that the roots of

$$f(x) = f\left(\frac{z+1}{z-1}\right) = \frac{z^2 + 2(1-\theta_1-2\theta_2)z + 1 - 2\theta_1}{(z-1)^2} \tag{A.8}$$

lie in the left half-plane (Samuelson, 1941, was perhaps the first to state this result). Applying the Routh-Hurwitz necessary and sufficient conditions to (A.8) then yields the desired result. ∎

A.2 Proof of Proposition 4

Theorem 1 *[Herrndorf (1984)] If $\{\epsilon_t\}$ satisfies the following assumptions:*

(A1) $\mathrm{E}[\epsilon_t] \;=\; \mu$ *for all* t.

(A2) $\sup_t \mathrm{E}[|\epsilon_t - \mu|^\beta] \;<\; \infty$ *for some* $\beta > 2$.

(A3) $0 \;<\; \sigma_o^2 \;=\; \lim_{T\to\infty} \mathrm{E}\left[\frac{1}{T}\left(\sum_{j=1}^n (\epsilon_j - \mu\right)^2\right] \;<\; \infty$.

(A4) $\{\epsilon_t\}$ *is strong-mixing with mixing coefficients* α_k *that satisfy:*

$$\sum_{j=1}^{\infty} \alpha_j^{1-\frac{2}{\beta}} \;<\; \infty \; . \tag{A.9}$$

then as n *increases without bound,*

$$W_n(s) \;\equiv\; \frac{1}{\sigma_o\sqrt{n}} \sum_{j=1}^{[ns]} (\epsilon_j - \mu) \;\Rightarrow\; W(s) \;\;,\;\; s \in [0,1] \tag{A.10}$$

where $[ns]$ *denotes the greater integer less than or equal to* ns *and '$\Rightarrow$' denotes weak convergence.*

With these results in hand, we are ready to prove Proposition 4. Let returns R_t be given by:

$$R_t \;=\; \epsilon_t \tag{A.11}$$

where ϵ_t satisfies assumptions (A1)–(A4) of Theorem 1, and recall:

$$\hat{\mu} = \frac{1}{T}\sum_{1}^{T}\epsilon_t \tag{A.12}$$

$$\hat{\sigma}^2_{\mathrm{NW}} = \frac{1}{T}\sum_{1}^{T}(\epsilon_t - \hat{\mu})^2 + \frac{2}{T}\sum_{j=1}^{\theta}(1 - \frac{j}{\theta+1})\sum_{t=j+1}^{T}(\epsilon_t - \hat{\mu})(\epsilon_{t-j} - \hat{\mu}) \tag{A.13}$$

$$= I_1 + I_2 \qquad \text{where} \tag{A.14}$$

$$I_1 \equiv \frac{1}{T}\sum_{1}^{T}(\epsilon_t - \hat{\mu})^2 \tag{A.15}$$

$$I_2 \equiv \frac{2}{T}\sum_{j=1}^{\theta}(1 - \frac{j}{\theta+1})\sum_{t=j+1}^{T}(\epsilon_t - \hat{\mu})(\epsilon_{t-j} - \hat{\mu}) \tag{A.16}$$

Observe that

$$I_1 = \frac{1}{T}\sum_{1}^{T}(\epsilon_t - \hat{\mu})^2 = \frac{1}{T}\sum_{1}^{T}\epsilon_t^2 - \hat{\mu}^2 \xrightarrow{p} \sigma^2_\epsilon \tag{A.17}$$

where

$$\sigma^2_\epsilon \equiv \lim_{t\to\infty}\frac{1}{T}\sum_{1}^{T}\epsilon_t^2 = \mathrm{E}[\epsilon_t^2] \,. \tag{A.18}$$

Now the second term I_2 can be written as:

$$I_2\frac{T(m+1)}{2} = \sum_{j=0}^{m}(m+1-j)\sum_{t=j+1}^{T}(\epsilon_t - \hat{\mu})(\epsilon_{t-j} - \hat{\mu}) = J_1 + J_2 + J_3 \tag{A.19}$$

where

$$J_1 \equiv \sum_{j=0}^{m}(m+1-j)\sum_{t=j+1}^{T}\epsilon_t\epsilon_{t-j} \tag{A.20}$$

$$J_2 \equiv \sum_{j=0}^{m}(m+1-j)\sum_{t=j+1}^{T}\hat{\mu}(\epsilon_t+\epsilon_{t-j}) \tag{A.21}$$

$$J_3 \equiv \sum_{j=0}^{m}(m+1-j)\sum_{t=j+1}^{T}\hat{\mu}^2 \tag{A.22}$$

Consider the first term in (A.19):

$$
\begin{aligned}
J_1 &= \sum_{j=0}^{m}(m+1-j)\sum_{t=j+1}^{T}\epsilon_t\epsilon_{t-j} \;=\; m(\epsilon_1\epsilon_2+\epsilon_2\epsilon_3+\cdots+\epsilon_{T-1}\epsilon_T) \;+\\
&\qquad (m-1)(\epsilon_1\epsilon_3+\epsilon_2\epsilon_4+\cdots+\epsilon_{T-2}\epsilon_T) \;+\\
&\qquad (\epsilon_1\epsilon_{m+1}+\epsilon_2\epsilon_{m+2}+\cdots+\epsilon_{T-m}\epsilon_T) \qquad\qquad \text{(A.23)}
\end{aligned}
$$

$$
\begin{aligned}
&= \epsilon_1\big((S_2-S_1)+(S_3-S_1)+\cdots+(S_{m+1}-S_1)\big) \;+\; \cdots \;+\\
&\qquad \epsilon_{T-m}\big((S_{T-m+1}-S_{T-m})+\cdots+(S_T-S_{T-m})\big) \;+\\
&\qquad \epsilon_{T-m+1}\big((S_{T-m+2}-S_{T-m+1})+\cdots+2(S_T-S_{T-m+1})\big) \;+\; \cdots \;+\\
&\qquad \epsilon_{T-1}\big(m(S_T-S_{T-1})\big) \qquad\qquad \text{(A.24)}
\end{aligned}
$$

$$
\begin{aligned}
&= \epsilon_1(S_2+S_3+\cdots+S_{m+1}) \;-\; m\epsilon_1 S_1 \;+\; \cdots \;+\\
&\qquad \epsilon_{T-m}(S_{T-m+1}+\cdots+S_T) \;-\; m\epsilon_{T-m}S_{T-m} \;+\\
&\qquad \epsilon_{T-m+1}(S_{T-m+2}+\cdots+2S_T) \;-\; m\epsilon_{T-m+1}S_{T-m+1} \;+\; \cdots \;+\\
&\qquad \epsilon_{T-1}mS_T \;-\; m\epsilon_{T-1}S_{T-1} \qquad\qquad \text{(A.25)}
\end{aligned}
$$

$$
\begin{aligned}
&= S_2S_1 \;+\; S_3S_2 \;+\; \cdots \;+\; S_{m+1}S_m \;+\; S_{m+2}(S_{m+1}-S_1) \;+\; \cdots \;+\\
&\qquad S_T(S_{T-1}-S_{T-m-1}) \;+\; S_T(\epsilon_{T-m+1}+\cdots+m\epsilon_{T-1}) \;-\; m\sum_{t=1}^{T-1}\epsilon_t S_t \qquad \text{(A.26)}
\end{aligned}
$$

$$
\begin{aligned}
&= \sum_{t=1}^{T-1}S_t^2 \;-\; (m-1)\sum_{t=1}^{T-1}\epsilon_t S_t \;-\; \sum_{t=1}^{T-(m+1)} S_tS_{t+m+1} \;+\\
&\qquad S_T\left(mS_{T-1}-\sum_{t=T-m}^{T-1}S_t\right) \qquad\qquad \text{(A.27)}
\end{aligned}
$$

When the Functional Central Limit Theorem is applied to (A.27), we have:

$$
\begin{aligned}
J_1/T^2 \;\Rightarrow\; & \sigma^2\left(\int_0^1 W(r)(W(r)-W(\min(r+\lambda,1)))dr\right) \;-\\
& \frac{(m-1)}{2T}(\sigma^2W^2(1)+\sigma_\epsilon^2) \;+\; \frac{m}{T}\sigma^2W^2(1) \qquad\qquad \text{(A.28)}
\end{aligned}
$$

Now the second term in (A.19) can be rewritten as:

$$\begin{aligned} J_2 &= -\sum_{j=1}^{m}(m+1-j)\sum_{t=j+1}^{T}\hat{\mu}(\epsilon_t+\epsilon_{t-j}) && \text{(A.29)} \\ &= -\hat{\mu}\sum_{j=1}^{m}(m+1-j)(S_T-S_j+S_{T-j}) && \text{(A.30)} \\ &= -\frac{m(m+1)}{2}\hat{\mu}S_T \;-\; \hat{\mu}\sum_{j=1}^{m}(m+1-j)(S_{T-j}-S_j) && \text{(A.31)} \end{aligned}$$

and applying the Functional Central Limit Theorem to (A.31) yields

$$J_2/T^2 \;\Rightarrow\; -\frac{m(m+1)}{2T^2}\sigma^2W^2(1) \;-\; \sigma^2W(1)\int_0^{\lambda}(\lambda-r)(W(1-r)-W(r))dr\ . \qquad \text{(A.32)}$$

The third term of (A.19) can be rewritten in a similar manner:

$$\begin{aligned} J_3 &= \sum_{j=1}^{m}(m+1-j)\sum_{t=j+1}^{T}\hat{\mu}^2 \;=\; \hat{\mu}^2\sum_{j=1}^{m}(m+1-j)(T-j-1) && \text{(A.33)} \\ &= \hat{\mu}^2\sum_{j=1}^{m}(j^2-j(T+m)+(m+1)(T-1)) && \text{(A.34)} \\ &= \hat{\mu}^2\frac{m(m+1)(3T-m-5)}{6} && \text{(A.35)} \end{aligned}$$

and applying the Functional Central Limit Theorem to (A.35) yields:

$$J_3/T^2 \;\Rightarrow\; \sigma^2W^2(1)\frac{(1+m)m(3T-m-5)}{6T^3}\ . \qquad \text{(A.36)}$$

Combining (A.28), (A.32) and (A.36) shows that:

$$I_2 \;\Rightarrow\; \frac{2}{\lambda}\sigma^2\left(\int_0^1 W(r)\big[W(r)-W(\min(r+\lambda,1))\big]dr\;-\right.$$
$$\left.W(1)\int_0^{\lambda}(\lambda-r)(W(1-r)-W(r))dr\right)\;+\;\sigma^2W^2(1)(1-\frac{\lambda^2}{3})-\sigma_\epsilon^2 \quad \text{(A.37)}$$

and summing (A.17) and (A.37) yields the desired result. ∎

A.3 Proof of Proposition 5

Given the weak convergence of $\hat{\bar{\sigma}}^2_{\mathrm{NW}}$ to the functional $f(W)$ in (87), Proposition 5 is an almost trivial consequence of the following well-known result:

Theorem 2 *[Extended Continuous Mapping Theorem]*[46] *Let h_n and h be measurable mappings from* $\mathrm{D}[0,1]$*—the space of all functions on* $[0,1]$ *that are right continuous with left-hand limits—to itself and denote by E the set of $x \in \mathrm{D}[0,1]$ such that $h_n(x_n) \to h(x)$ fails to hold for some sequence x_n converging to x. If $W_n(s) \;\Rightarrow\; W(s)$ and E is of Wiener-measure zero, i.e.* $\mathrm{P}(W \in E) = 0$, *then $h_n(W_n) \;\Rightarrow\; h(W)$.* ∎

A.4 TASS Fund Category Definitions

The following is a list of category descriptions, taken directly from TASS documentation, that define the criteria used by TASS in assigning funds in their database to one of 17 possible categories:

Equity Hedge This directional strategy involves equity-oriented investing on both the long and short sides of the market. The objective is not to be market neutral. Managers have the ability to shift from value to growth, from small to medium to large capitalization stocks, and from a net long position to a net short position. Managers may use futures and options to hedge. The focus may be regional, such as long/short US or European equity, or sector specific, such as long and short technology or healthcare stocks. Long/short equity funds tend to build and hold portfolios that are substantially more concentrated than those of traditional stock funds. US equity Hedge, European equity Hedge, Asian equity Hedge and Global equity Hedge is the regional Focus.

[46] See Billingsley (1968) for a proof.

Dedicated Short Seller Short biased managers take short positions in mostly equities and derivatives. The short bias of a manager's portfolio must be constantly greater than zero to be classified in this category.

Fixed Income Directional This directional strategy involves investing in Fixed Income markets only on a directional basis.

Convertible Arbitrage This strategy is identified by hedge investing in the convertible securities of a company. A typical investment is to be long the convertible bond and short the common stock of the same company. Positions are designed to generate profits from the fixed income security as well as the short sale of stock, while protecting principal from market moves.[47]

Event Driven This strategy is defined as 'special situations' investing designed to capture price movement generated by a significant pending corporate event such as a merger, corporate restructuring, liquidation, bankruptcy or reorganization. There are three popular sub-categories in event-driven strategies: risk (merger) arbitrage, distressed/high yield securities, and Regulation D.

Non Directional/Relative Value This investment strategy is designed to exploit equity and/or fixed income market inefficiencies and usually involves being simultaneously long and short matched market portfolios of the same size within a country. Market neutral portfolios are designed to be either beta or currency neutral, or both.

Global Macro Global macro managers carry long and short positions in any of the world's major capital or derivative markets. These positions reflect their views on overall market direction as influenced by major economic trends and or events. The portfolios of these funds can include stocks, bonds, currencies, and commodities in the form of cash or derivatives instruments. Most funds invest globally in both developed and emerging markets.

Global Opportunity Global macro managers carry long and short positions in any of the world's major capital or derivative markets on an opportunistic basis. These positions reflect their views on overall market direction as influenced by major economic trends and or events. The portfolios of these funds can include stocks, bonds, currencies, and commodities in the form of cash or derivatives instruments. Most funds invest globally in both developed and emerging markets.

Natural Resources This trading strategy has a focus for the natural resources around the world.

Leveraged Currency This strategy invests in currency markets around the world.

Managed Futures This strategy invests in listed financial and commodity futures markets and currency markets around the world. The managers are usually referred to as Commodity Trading Advisors, or CTAs. Trading disciplines are generally systematic or discretionary. Systematic traders tend to use price and market specific information (often technical) to make trading decisions, while discretionary managers use a judgmental approach.

Emerging Markets This strategy involves equity or fixed income investing in emerging markets around the world.

Property The main focus of the investments are property.

Fund of Funds A 'Multi Manager' fund will employ the services of two or more trading advisors or Hedge Funds who will be allocated cash by the Trading Manager to trade on behalf of the fund.

[47]Note that the category closest to this definition in the TASS database is "Convertible Fund (Long Only)", which is related to but different from convertible arbitrage—see footnote 44 for details. TASS recently changed their style classifications, and now defines "Convertible Arbitrage" as a distinct category.

A.5 Supplementary Empirical Results

In Tables A.1–A.7, we provide corresponding empirical results to Tables 7–14 but with Live and Graveyard funds separated so that the effects of survivorship bias can be seen. Of course, since we still apply our five-year minimum returns history filter to both groups, there is still some remaining survivorship bias. Tables A.1 and A.2 contain summary statistics for the two groups of funds, Tables A.3 and A.4 report summary statistics for the maximum likelihood estimates of the smoothing model (21)–(23), Tables A.5 and A.6 report similar statistics for the regression model estimates (62) of the smoothing model, and Table A.7 contains smoothing-adjusted Sharpe ratios for the two groups of funds. Finally, Table A.8 corresponds to the regressions of Table 11 but with dependent variables $\hat{\theta}_0$ and $\hat{\xi}$ estimated by maximum likelihood under the alternate constraint (75).

Category	N	Annual Mean		Annual SD		Skewness		Kurtosis		$\hat{\rho}_1(\%)$		$\hat{\rho}_2(\%)$		$\hat{\rho}_3(\%)$		p-Value(Q)	
		Mean	SD	Mean	SD	Mean	SD	Mean	SD	Mean	SD	Mean	SD	Mean	SD	Mean	SD
Not Categorized	44	12.08	6.04	13.08	10.44	0.05	0.73	2.37	2.54	8.8	15.9	2.3	14.5	1.4	9.8	31.8	32.0
US Equity Hedge	139	23.53	10.92	21.86	11.95	0.20	0.98	3.79	4.89	8.0	13.7	0.7	11.4	−4.1	11.2	33.0	29.0
European Equity Hedge	19	18.75	7.33	15.36	6.53	0.51	0.73	3.10	2.86	13.0	11.1	13.5	9.0	−2.9	8.9	16.4	20.7
Asian Equity Hedge	5	9.16	7.96	21.13	6.64	0.50	0.85	2.32	0.73	11.7	13.0	5.0	5.6	−5.4	13.4	40.2	21.1
Global Equity Hedge	24	15.04	7.67	18.28	6.31	−0.16	1.06	4.17	5.83	10.5	9.9	−0.6	8.8	−4.1	5.4	41.8	26.9
Dedicated Shortseller	6	−0.17	11.15	23.01	14.51	0.44	0.17	2.24	2.63	2.6	7.4	−0.9	9.0	−5.6	7.0	28.3	28.7
Fixed-Income Directional	12	10.17	3.39	9.80	9.08	−1.00	1.33	5.78	7.49	22.8	15.8	14.9	13.4	1.7	14.1	17.6	21.8
Convertible Fund (Long Only)	12	14.05	5.54	11.67	7.39	−0.26	1.88	7.16	10.93	22.9	11.9	5.3	14.9	−1.0	11.9	15.2	22.1
Event Driven	97	15.61	6.42	8.96	6.66	−0.58	1.74	8.10	11.71	20.0	16.6	5.8	13.2	−0.9	12.1	25.2	28.6
Non-Directional/Relative Value	63	13.23	5.91	7.89	4.84	−0.61	1.74	6.08	9.52	18.4	26.0	15.3	14.0	5.4	15.8	14.6	24.0
Global Macro	15	14.92	7.82	17.34	10.46	0.79	1.07	4.67	4.79	5.3	11.8	−1.3	12.2	−0.9	8.4	41.9	24.2
Global Opportunity	1	−17.39	—	31.03	—	−0.48	—	2.80	—	23.1	—	14.1	—	0.0	—	20.8	—
Natural Resources	1	14.49	—	18.42	—	0.30	—	−0.14	—	−10.1	—	−1.6	—	7.8	—	40.6	—
Pure Leveraged Currency	15	10.36	5.23	15.27	6.25	0.81	0.75	3.22	4.04	7.4	8.7	−3.3	9.9	−3.1	7.1	31.2	18.7
Pure Managed Futures	28	10.54	6.19	16.46	7.62	0.42	1.55	5.92	12.20	0.9	13.2	−4.1	11.4	−2.5	8.6	32.4	29.9
Pure Emerging Market	54	9.08	10.21	26.12	11.66	−0.56	1.21	4.97	5.86	18.3	12.0	4.1	11.9	−1.7	9.2	30.7	29.4
Pure Property	1	3.96	—	9.41	—	−1.33	—	5.43	—	−23.1	—	2.3	—	7.3	—	18.9	—
Fund of Funds	115	11.45	4.95	10.16	5.75	−0.29	1.27	4.83	6.17	17.3	14.0	6.2	11.7	−1.2	9.1	26.5	27.9
All Live	651	14.88	9.33	15.05	10.65	−0.15	1.36	5.00	7.63	13.7	16.5	4.6	13.1	−1.2	11.1	28.0	28.3

Table A.1: Means and standard deviations of basic summary statistics for 651 hedge funds in the TASS Hedge Fund Live database with at least five years of returns history during the period from November 1977 to January 2001. The columns 'p-Value(Q)' contain means and standard deviations of p-values for the Box-Pierce Q-statistic for each fund using the first 6 autocorrelations of returns.

Category	N	Annual Mean		Annual SD		Skewness		Kurtosis		$\hat{\rho}_1(\%)$		$\hat{\rho}_2(\%)$		$\hat{\rho}_3(\%)$		p-Value(Q)	
		Mean	SD	Mean	SD	Mean	SD	Mean	SD	Mean	SD	Mean	SD	Mean	SD	Mean	SD
Not Categorized	67	9.92	7.48	17.26	11.52	0.17	1.23	4.53	5.88	3.4	17.5	−0.1	14.9	−0.1	12.4	39.0	29.2
US Equity Hedge	23	16.47	7.82	20.68	9.67	0.03	0.83	3.97	5.90	7.2	12.5	0.4	9.7	−5.3	10.1	38.9	27.6
European Equity Hedge	3	13.33	3.85	10.58	4.17	−0.38	1.06	2.15	1.89	10.5	11.7	−3.7	14.8	3.8	7.6	51.4	9.1
Asian Equity Hedge	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
Global Equity Hedge	3	3.06	7.16	10.41	5.29	−0.88	0.97	2.29	3.48	27.5	12.9	3.2	16.9	7.8	12.4	42.2	36.6
Dedicated Shortseller	1	3.32	—	12.78	—	0.78	—	2.46	—	15.5	—	−13.5	—	−12.0	—	52.4	—
Fixed-Income Directional	1	5.58	—	6.49	—	−0.15	—	0.50	—	7.7	—	4.2	—	4.1	—	24.0	—
Convertible Fund (Long Only)	3	18.55	2.34	8.48	2.77	0.09	0.31	2.05	2.71	20.8	16.5	9.7	4.5	−6.5	16.7	25.2	40.7
Event Driven	12	13.22	8.67	13.98	22.43	−0.66	2.45	9.55	11.85	26.6	17.8	11.4	14.8	5.9	13.1	29.7	37.5
Non-Directional/Relative Value	22	10.58	8.26	9.02	5.67	−1.18	1.61	7.47	9.56	17.8	14.5	3.5	16.3	3.9	10.4	24.4	26.8
Global Macro	10	19.49	7.70	21.29	7.83	0.14	0.86	4.64	9.40	10.1	19.2	1.6	8.7	−0.8	8.0	20.6	13.3
Global Opportunity	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
Natural Resources	2	9.84	5.78	19.15	1.38	0.56	0.59	1.95	1.24	12.5	9.0	13.6	11.9	−1.0	6.7	51.1	61.2
Pure Leveraged Currency	11	7.83	9.18	19.78	11.48	−0.07	1.28	3.90	6.13	2.2	8.2	−10.1	7.8	−6.1	9.3	41.4	34.8
Pure Managed Futures	65	9.19	10.35	24.02	19.41	0.13	1.25	4.51	5.98	−0.6	12.9	−3.7	9.4	−3.9	11.5	36.7	28.9
Pure Emerging Market	18	11.06	13.11	28.05	21.07	−1.08	2.12	9.79	12.04	20.2	11.6	5.1	10.9	−0.7	8.4	38.1	34.0
Pure Property	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
Fund of Funds	17	8.24	5.82	9.50	4.87	−0.04	1.53	4.39	6.25	19.2	14.6	2.2	9.6	2.9	10.2	24.6	24.9
All Graveyard	258	10.80	9.17	18.61	15.21	−0.14	1.45	5.17	7.40	8.2	16.8	0.2	12.7	−1.0	11.4	35.2	29.3

Table A.2: Means and standard deviations of basic summary statistics for 258 hedge funds in the TASS Hedge Fund Graveyard database with at least five years of returns history during the period from November 1977 to January 2001. The columns 'p-Value(Q)' contain means and standard deviations of p-values for the Box-Pierce Q-statistic for each fund using the first 6 autocorrelations of returns.

Category	N	MA(2) with Constrained Sum								MA(2) with Constrained σ_η							
		$\hat\theta_0$		$\hat\theta_1$		$\hat\theta_2$		$\hat\xi$		$\hat\theta_0$		$\hat\theta_1$		$\hat\theta_2$		$\hat\xi$	
		Mean	SD	Mean	SD	Mean	SD	Mean	SD	Mean	SD	Mean	SD	Mean	SD	Mean	SD
Not Categorized	44	0.973	0.273	0.049	0.149	−0.023	0.174	1.075	0.704	0.942	0.269	0.046	0.145	−0.015	0.160	1.006	0.584
US Equity Hedge	139	0.951	0.213	0.065	0.159	−0.016	0.156	1.004	0.562	0.974	0.194	0.071	0.147	−0.008	0.151	1.035	0.416
European Equity Hedge	19	0.790	0.115	0.077	0.088	0.133	0.073	0.673	0.196	0.902	0.170	0.096	0.101	0.156	0.095	0.893	0.317
Asian Equity Hedge	5	0.942	0.296	0.053	0.188	0.005	0.109	0.998	0.712	0.904	0.198	0.064	0.167	0.013	0.096	0.883	0.404
Global Equity Hedge	24	0.926	0.143	0.095	0.091	−0.020	0.099	0.903	0.271	0.994	0.173	0.100	0.098	−0.018	0.100	1.045	0.346
Dedicated Shortseller	6	1.031	0.220	0.008	0.109	−0.039	0.135	1.131	0.528	1.149	0.152	0.024	0.115	−0.031	0.142	1.368	0.363
Fixed-Income Directional	12	0.752	0.177	0.160	0.105	0.088	0.126	0.652	0.280	0.733	0.203	0.157	0.104	0.093	0.128	0.633	0.303
Convertible Fund (Long Only)	12	0.868	0.392	0.189	0.069	−0.057	0.387	1.074	1.516	0.896	0.300	0.202	0.086	−0.027	0.263	0.997	0.650
Event Driven	97	0.816	0.170	0.148	0.128	0.036	0.115	0.747	0.285	0.856	0.240	0.154	0.126	0.038	0.113	0.842	0.439
Non-Directional/Relative Value	63	0.809	0.253	0.077	0.279	0.114	0.133	0.831	0.697	0.764	0.296	0.070	0.275	0.110	0.127	0.777	0.793
Global Macro	15	1.006	0.186	0.046	0.115	−0.053	0.168	1.088	0.422	1.054	0.214	0.052	0.120	−0.049	0.188	1.205	0.525
Global Opportunity	1	0.737	—	0.162	—	0.100	—	0.580	—	0.632	—	0.139	—	0.086	—	0.426	—
Natural Resources	1	1.111	—	−0.113	—	0.003	—	1.246	—	0.861	—	−0.088	—	0.002	—	0.748	—
Pure Leveraged Currency	15	0.970	0.132	0.074	0.091	−0.044	0.121	0.986	0.271	0.926	0.099	0.076	0.084	−0.035	0.115	0.892	0.186
Pure Managed Futures	28	1.138	0.305	−0.034	0.171	−0.104	0.219	1.470	0.904	1.129	0.253	−0.030	0.165	−0.090	0.190	1.406	0.621
Pure Emerging Market	54	0.836	0.165	0.148	0.087	0.016	0.139	0.774	0.328	0.797	0.162	0.141	0.082	0.021	0.136	0.706	0.296
Pure Property	1	1.232	—	−0.306	—	0.074	—	1.618	—	0.951	—	−0.236	—	0.057	—	0.963	—
Fund of Funds	115	0.848	0.220	0.119	0.132	0.034	0.136	0.817	0.522	0.859	0.232	0.122	0.126	0.035	0.133	0.841	0.470
All	651	0.891	0.231	0.093	0.160	0.016	0.158	0.906	0.578	0.899	0.242	0.096	0.155	0.021	0.148	0.923	0.512

Table A.3: Means and standard deviations of maximum likelihood estimates of MA(2) smoothing process $R^o_t = \theta_0 R_t + \theta_1 R_{t-1} + \theta_2 R_{t-2}$, $\xi \equiv \theta_0^2 + \theta_1^2 + \theta_2^2$, for 651 funds in the TASS Hedge Fund Live database with at least five years of returns history during the period from November 1977 to January 2001.

Category	N	MA(2) with Constrained Sum								MA(2) with Constrained σ_η							
		$\hat{\theta}_0$		$\hat{\theta}_1$		$\hat{\theta}_2$		$\hat{\xi}$		$\hat{\theta}_0$		$\hat{\theta}_1$		$\hat{\theta}_2$		$\hat{\xi}$	
		Mean	SD	Mean	SD	Mean	SD	Mean	SD	Mean	SD	Mean	SD	Mean	SD	Mean	SD
Not Categorized	67	1.079	0.353	−0.009	0.194	−0.070	0.267	1.399	1.226	0.993	0.268	−0.007	0.165	−0.049	0.214	1.130	0.613
US Equity Hedge	23	0.945	0.179	0.053	0.113	0.002	0.115	0.951	0.363	0.993	0.212	0.065	0.126	0.000	0.123	1.063	0.471
European Equity Hedge	3	0.984	0.251	0.073	0.107	−0.057	0.147	1.041	0.544	0.859	0.201	0.065	0.094	−0.049	0.127	0.788	0.378
Asian Equity Hedge	0	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
Global Equity Hedge	3	0.790	0.099	0.220	0.043	−0.011	0.057	0.683	0.131	0.670	0.196	0.179	0.003	−0.014	0.044	0.508	0.242
Dedicated Shortseller	1	0.999	—	0.166	—	−0.165	—	1.052	—	1.035	—	0.172	—	−0.171	—	1.131	—
Fixed-Income Directional	1	0.902	—	0.064	—	0.033	—	0.820	—	0.977	—	0.069	—	0.036	—	0.961	—
Convertible Fund (Long Only)	3	0.753	0.121	0.138	0.113	0.109	0.013	0.617	0.166	0.885	0.062	0.182	0.155	0.131	0.033	0.854	0.158
Event Driven	12	0.761	0.174	0.184	0.135	0.055	0.101	0.670	0.286	0.740	0.270	0.166	0.115	0.058	0.099	0.666	0.438
Non-Directional/Relative Value	22	0.844	0.196	0.142	0.099	0.014	0.137	0.796	0.317	0.768	0.203	0.135	0.101	0.014	0.127	0.673	0.332
Global Macro	9	0.869	0.155	0.135	0.061	−0.005	0.126	0.813	0.302	0.818	0.188	0.123	0.050	−0.002	0.124	0.732	0.351
Global Opportunity	0	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
Natural Resources	2	0.811	0.115	0.079	0.030	0.110	0.085	0.687	0.164	0.845	0.050	0.084	0.039	0.119	0.099	0.741	0.055
Pure Leveraged Currency	11	1.272	0.446	−0.024	0.147	−0.248	0.331	1.980	2.016	1.107	0.163	0.000	0.107	−0.183	0.169	1.319	0.424
Pure Managed Futures	65	1.129	0.325	−0.053	0.233	−0.076	0.159	1.467	1.295	1.082	0.244	−0.037	0.193	−0.066	0.144	1.292	0.642
Pure Emerging Market	18	0.817	0.148	0.154	0.085	0.029	0.109	0.730	0.254	0.789	0.181	0.145	0.078	0.028	0.107	0.692	0.313
Pure Property	0	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
Fund of Funds	17	0.845	0.165	0.168	0.103	−0.013	0.107	0.789	0.250	0.823	0.208	0.158	0.090	−0.012	0.106	0.761	0.324
All	257	1.001	0.314	0.042	0.190	−0.042	0.196	1.177	1.064	0.950	0.259	0.046	0.164	−0.031	0.162	1.025	0.569

Table A.4: Means and standard deviations of maximum likelihood estimates of MA(2) smoothing process $R_t^o = \theta_0 R_t + \theta_1 R_{t-1} + \theta_2 R_{t-2}$, $\xi \equiv \theta_0^2 + \theta_1^2 + \theta_2^2$, for 257 funds in the TASS Hedge Fund Graveyard database with at least five years of returns history during the period from November 1977 to January 2001.

Category	N	$\hat{\theta}_0$		$\hat{\theta}_1$		$\hat{\theta}_2$		$\hat{\xi}$		D.W.		$R^2(\%)$	
		Mean	SD	Mean	SD	Mean	SD	Mean	SD	Mean	SD	Mean	SD
Not Categorized	44	1.598	4.141	−0.613	3.212	0.015	2.294	34.919	108.677	1.81	0.32	15.0	13.9
US Equity Hedge	139	0.713	1.557	0.080	0.433	0.207	1.268	4.747	41.449	1.80	0.28	24.8	17.4
European Equity Hedge	19	0.581	0.113	0.344	0.735	0.075	0.756	1.527	4.614	1.76	0.19	18.0	9.3
Asian Equity Hedge	5	0.405	0.252	0.323	0.179	0.272	0.088	0.425	0.086	1.82	0.24	7.4	4.5
Global Equity Hedge	24	0.913	0.323	0.028	0.331	0.059	0.354	1.163	0.956	1.69	0.20	24.3	19.9
Dedicated Shortseller	6	0.886	0.189	0.104	0.106	0.010	0.202	0.869	0.376	1.96	0.17	41.1	15.0
Fixed-Income Directional	12	−1.495	6.678	4.689	15.156	−2.193	8.493	346.600	1198.277	1.61	0.30	14.1	9.7
Convertible Fund (Long Only)	12	0.513	0.239	0.354	0.160	0.133	0.162	0.506	0.152	1.51	0.22	23.8	15.2
Event Driven	97	0.550	0.279	0.330	0.288	0.120	0.232	0.638	0.661	1.64	0.36	18.0	11.4
Non-Directional/Relative Value	63	0.607	2.010	0.762	1.808	−0.369	3.342	19.268	80.833	1.63	0.51	12.7	9.5
Global Macro	15	0.541	0.428	0.182	0.424	0.277	0.161	0.765	0.491	1.84	0.29	12.0	7.0
Global Opportunity	1	0.619	—	0.191	—	0.191	—	0.455	—	1.55	—	30.9	—
Natural Resources	1	0.640	—	0.175	—	0.186	—	0.474	—	2.20	—	2.3	—
Pure Leveraged Currency	15	0.049	1.238	0.596	0.971	0.354	1.241	4.234	10.045	1.85	0.15	4.7	7.4
Pure Managed Futures	28	−0.290	1.930	0.776	2.494	0.514	2.027	14.502	41.277	1.96	0.26	3.8	3.4
Pure Emerging Market	54	0.928	0.916	0.125	0.527	−0.053	0.506	2.226	7.470	1.52	0.28	22.0	10.8
Pure Property	1	1.319	—	−0.024	—	−0.295	—	1.828	—	2.31	—	15.4	—
Fund of Funds	115	0.616	1.287	−0.070	2.694	0.454	3.231	19.776	183.336	1.63	0.30	22.0	14.8
All	651	0.636	1.888	0.256	2.649	0.109	2.284	16.252	184.926	1.71	0.34	19.1	14.7

Table A.5: Means and standard deviations of linear regression estimates of MA(2) smoothing process $R_t^o = \theta_0 R_t + \theta_1 R_{t-1} + \theta_2 R_{t-2}$, $\xi \equiv \theta_0^2 + \theta_1^2 + \theta_2^2$ under the assumption of a linear single-factor model for R_t where the factor is the total return of the S&P 500 Index, for 651 hedge funds in the TASS Hedge Fund Live database with at least five years of returns history during the period from November 1977 to January 2001.

Category	N	$\hat{\theta}_0$		$\hat{\theta}_1$		$\hat{\theta}_2$		$\hat{\xi}$		D.W.		$R^2(\%)$	
		Mean	SD	Mean	SD	Mean	SD	Mean	SD	Mean	SD	Mean	SD
Not Categorized	67	0.042	10.672	1.163	11.752	−0.206	2.251	254.635	1841.566	1.89	0.33	12.6	18.5
US Equity Hedge	23	0.586	1.129	0.182	0.266	0.232	0.900	2.492	8.220	1.81	0.26	33.8	16.6
European Equity Hedge	3	0.434	0.329	0.312	0.173	0.254	0.275	0.493	0.173	1.78	0.17	18.3	21.0
Asian Equity Hedge	—	—	—	—	—	—	—	—	—	—	—	—	—
Global Equity Hedge	3	0.308	0.626	0.294	0.271	0.397	0.361	0.737	0.293	1.40	0.26	4.1	1.3
Dedicated Shortseller	1	1.040	—	0.176	—	−0.216	—	1.159	—	1.84	—	54.7	—
Fixed-Income Directional	1	−0.738	—	0.748	—	0.990	—	2.084	—	1.83	—	1.6	—
Convertible Fund (Long Only)	3	0.636	0.383	0.295	0.127	0.068	0.272	0.655	0.428	1.63	0.32	30.1	16.3
Event Driven	12	0.837	1.223	−0.008	1.244	0.171	0.159	3.543	10.629	1.50	0.38	18.1	11.0
Non-Directional/Relative Value	22	0.748	7.341	1.275	3.420	−1.023	7.574	120.592	461.012	1.59	0.28	10.5	8.2
Global Macro	9	0.587	0.173	0.214	0.139	0.198	0.234	0.522	0.138	1.63	0.22	16.0	9.3
Global Opportunity	—	—	—	—	—	—	—	—	—	—	—	—	—
Natural Resources	2	0.717	0.184	0.072	0.089	0.211	0.095	0.589	0.211	1.71	0.19	2.7	1.2
Pure Leveraged Currency	11	−0.255	1.590	0.037	1.150	1.218	2.690	11.631	36.324	1.95	0.16	4.0	3.5
Pure Managed Futures	65	0.079	1.418	0.662	1.132	0.260	1.406	5.700	15.336	2.00	0.26	6.8	6.5
Pure Emerging Market	18	0.811	0.231	0.256	0.151	−0.067	0.237	0.852	0.460	1.46	0.30	24.9	9.8
Pure Property	—	—	—	—	—	—	—	—	—	—	—	—	—
Fund of Funds	17	1.064	2.613	0.093	1.469	−0.158	1.366	11.379	40.134	1.59	0.30	19.1	17.0
All	257	0.346	5.928	0.643	6.112	0.010	2.680	79.904	950.698	1.80	0.34	14.4	15.6

Table A.6: Means and standard deviations of linear regression estimates of MA(2) smoothing process $R_t^o = \theta_0 R_t + \theta_1 R_{t-1} + \theta_2 R_{t-2}$, $\xi \equiv \theta_0^2 + \theta_1^2 + \theta_2^2$ under the assumption of a linear single-factor model for R_t where the factor is the total return of the S&P 500 Index, for 257 hedge funds in the TASS Hedge Fund Graveyard database with at least five years of returns history during the period from November 1977 to January 2001.

Category	Sharpe Ratios For Live Funds							Sharpe Ratios For Graveyard Funds						
	N	SR		SR*		SR**		N	SR		SR*		SR**	
		Mean	SD	Mean	SD	Mean	SD		Mean	SD	Mean	SD	Mean	SD
Not Categorized	44	1.55	1.34	1.41	1.00	1.33	0.94	67	0.84	0.79	0.83	0.70	0.77	0.65
US Equity Hedge	139	1.31	0.77	1.36	0.76	1.27	0.70	23	0.99	0.54	1.06	0.60	1.00	0.57
European Equity Hedge	19	1.44	0.78	1.46	0.85	1.36	0.78	3	1.36	0.57	1.26	0.49	1.18	0.46
Asian Equity Hedge	5	0.50	0.39	0.52	0.39	0.49	0.37	0	—	—	—	—	—	—
Global Equity Hedge	24	0.95	0.60	1.00	0.66	0.94	0.61	3	0.55	0.75	0.51	0.59	0.48	0.55
Dedicated Shortseller	6	0.28	0.65	0.33	0.70	0.31	0.67	1	0.26	—	0.30	—	0.28	—
Fixed-Income Directional	12	2.11	2.42	1.87	2.31	1.75	2.14	1	0.86	—	0.87	—	0.84	—
Convertible Fund (Long Only)	12	1.67	1.18	1.48	0.72	1.38	0.69	3	2.45	1.26	2.39	1.07	2.23	1.00
Event Driven	97	2.45	1.46	2.31	1.58	2.17	1.47	12	1.61	1.16	1.47	1.39	1.37	1.29
Non-Directional/Relative Value	63	2.36	1.82	2.21	2.52	2.06	2.35	22	1.73	1.93	1.53	1.91	1.42	1.76
Global Macro	15	1.12	0.77	1.30	0.86	1.22	0.82	9	1.02	0.51	0.87	0.34	0.82	0.32
Global Opportunity	1	−0.56	—	−0.39	—	−0.37	—	0	—	—	—	—	—	—
Natural Resources	1	0.79	—	0.75	—	0.69	—	2	0.50	0.27	0.46	0.23	0.43	0.22
Pure Leveraged Currency	15	0.83	0.50	0.83	0.50	0.78	0.47	11	0.35	0.32	0.41	0.39	0.38	0.37
Pure Managed Futures	28	0.81	0.71	0.92	0.66	0.87	0.63	65	0.43	0.43	0.51	0.52	0.48	0.49
Pure Emerging Market	54	0.39	0.48	0.37	0.48	0.34	0.44	18	0.39	0.36	0.34	0.30	0.32	0.27
Pure Property	1	0.42	—	0.45	—	0.41	—	0	—	—	—	—	—	—
Fund of Funds	115	1.48	1.02	1.35	0.90	1.27	0.83	17	1.15	0.98	0.95	0.70	0.89	0.65
All	651	1.50	1.29	1.45	1.35	1.36	1.26	257	0.85	0.95	0.83	0.90	0.78	0.84

Table A.7: Means and standard deviations of Sharpe ratios of 651 funds in the TASS Hedge Funds Live database and 257 funds in the TASS Hedge Fund Graveyard database, both with at least five years of returns history during the period from November 1977 to January 2001. SR is the standard Sharpe ratio, SR* is the smoothing-adjusted Sharpe ratio of Lo (2002), and SR** is the smoothing-adjusted Sharpe ratio using $\hat{\sigma}_{\text{NW}}$. All Sharpe ratios are computed with respect to a 0 benchmark.

Regressor	$\hat{\theta}_0$	$\hat{\xi}$	$\hat{\theta}_0$	$\hat{\xi}$
Constant	1.007	1.152	1.029	1.212
	(42.02)	(21.91)	(35.56)	(19.04)
US Equity Hedge	0.020	−0.009	0.026	−0.004
	(0.69)	(0.15)	(0.85)	(0.07)
European Equity Hedge	−0.063	−0.175	−0.120	−0.281
	(1.19)	(1.50)	(1.97)	(2.10)
Asian Equity Hedge	−0.054	−0.167	−0.030	−0.131
	(0.52)	(0.73)	(0.28)	(0.54)
Global Equity Hedge	0.005	−0.053	0.032	0.006
	(0.11)	(0.49)	(0.64)	(0.05)
Dedicated Shortseller	0.178	0.291	0.184	0.293
	(2.01)	(1.50)	(2.06)	(1.49)
Fixed-Income Directional	−0.203	−0.386	−0.196	−0.375
	(3.04)	(2.63)	(2.92)	(2.54)
Convertible Fund (Long Only)	−0.060	−0.073	−0.064	−0.086
	(0.95)	(0.53)	(1.01)	(0.62)
Event Driven	−0.116	−0.228	−0.124	−0.247
	(3.74)	(3.36)	(3.85)	(3.49)
Non-Directional/Relative Value	−0.194	−0.303	−0.180	−0.281
	(5.88)	(4.18)	(5.26)	(3.73)
Global Macro	0.004	−0.029	0.034	0.026
	(0.09)	(0.26)	(0.63)	(0.22)
Global Opportunity	−0.314	−0.598	−0.309	−0.59
	(1.37)	(1.19)	(1.35)	(1.18)
Natural Resources	−0.096	−0.281	−0.095	−0.284
	(0.72)	(0.96)	(0.72)	(0.97)
Pure Leveraged Currency	0.042	0.019	0.045	0.022
	(0.85)	(0.17)	(0.89)	(0.20)
Pure Managed Futures	0.133	0.267	0.142	0.284
	(4.16)	(3.78)	(4.25)	(3.87)
Pure Emerging Market	−0.161	−0.344	−0.146	−0.311
	(4.63)	(4.50)	(3.53)	(3.42)
Pure Property	−0.056	−0.190	−0.028	−0.146
	(0.25)	(0.38)	(0.12)	(0.29)
Fund of Funds	−0.099	−0.211	−0.092	−0.199
	(3.32)	(3.21)	(3.03)	(2.96)
OPEN	−0.061	−0.128	−0.055	−0.118
	(3.27)	(3.13)	(2.78)	(2.73)
USBASED			0.012	0.022
			(0.68)	(0.58)
GF-GLB			−0.034	−0.078
			(1.69)	(1.78)
GF-USA			−0.043	−0.087
			(2.09)	(1.92)
GF-APC			−0.051	−0.104
			(1.73)	(1.61)
GF-WEU			0.048	0.079
			(1.70)	(1.28)
GF-EEU			−0.058	−0.133
			(1.46)	(1.51)
GF-AFR			0.071	0.137
			(1.27)	(1.12)
Sample Size	908	908	891	891
Adjusted R^2(%)	15.7	11.4	16.9	12.6

Table A.8: Regressions of maximum likelihood estimated smoothing coefficient $\hat{\theta}_0$ and smoothing index $\hat{\xi}$ on indicator variables for 908 hedge funds in the TASS Hedge Fund Combined (Live and Graveyard) database with at least five years of returns history during the period from November 1977 to January 2001, where the maximum likelihood estimator $\hat{\sigma}_\eta$ is constrained to equal a nonparametric estimator $\tilde{\sigma}_\eta$ of the innovation standard deviation. Absolute values of t-statistics are given in parentheses. The indicator variables are OPEN (1 if the fund is open, 0 otherwise); the fund categories (1 if the fund belongs to the category, 0 otherwise); USBASED (1 if the fund is based in the US, 0 otherwise); and geographical focus categories (1 if the geographical focus of the fund is in a given region, 0 otherwise, where the regions are USA, Asia Pacific, Western Europe, Eastern Europe, and Africa, respectively).

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The Life Cycle of Hedge Funds: Fund Flows, Size and Performance

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Abstract

Since the 1980s we have seen a 25% yearly increase in the number of hedge funds, and an annual attrition rate of 7.10% due to liquidation. This paper analyzes the life cycles of hedge funds. Using the TASS database provided by the Tremont Company, it studies industry and fund specific factors that affect the survival probability of hedge funds. The findings show that in general, investors chasing individual fund performance decrease probabilities of hedge funds liquidating. However, if investors follow a category of hedge funds that has performed well, then the probability of hedge funds liquidating in this category increases. We interpret this finding as a result of competition among hedge funds in a category. As competition increases, marginal funds are more likely to be liquidated than funds that deliver superior risk-adjusted returns. We also find that there is a concave relationship between performance and assets under management. The implication of this study is that an optimal asset size can be obtained by balancing out the effects of past returns, fund flows, market impact, competition and favorable category positioning that are modeled in the paper. Hedge funds in illiquid categories are subject to high market impact, have limited investment opportunities, and are more likely to exhibit an optimal size behavior compared to those in more liquid hedge fund categories.

Contents

1 Introduction

The hedge fund industry has grown tremendously since it was founded 50 years ago. Since the late 1980s, the number of hedge funds has risen by more than 25% per year. The value of assets under management has grown as well. In 1990, $39 billion was invested in hedge funds, and in 2003, it was estimated that $650 - $700 billion is managed by 5,000 single-manager hedge funds (Tremont Company). However, alongside the tremendous growth, there has also been a significant attrition in the industry. The annual liquidation rate in the hedge fund industry is 7.10% compared to 1.00% in the mutual fund industry. Despite the increased interest in hedge funds as an asset class, we have only a limited understanding of what drives hedge fund continuation and liquidation. This paper explores the drivers of the life-cycles of hedge funds. Both individual and category [1] related factors are explored in the paper, using the TASS database provided by the Tremont Company.

First, we study the determinants of fund flows. We analyze how net flows into individual funds are affected by past fund performance, current performance, past flows, age, past standard deviation of returns, and past assets. An increase by 10% in a current return increases fund flow by 2%. The relationship between current flow and past return, however, is non-linear. A piecewise linear relationship model between current fund flows and past fund performance is proposed and analyzed. As with private equity funds (Kaplan and Schoar (2003)), the relationship between flows and past returns is positive and concave, so that the top performing funds do not grow proportionally as much as the average fund in the market. An increase in age, assets under management and standard deviation of returns negatively affects fund flows.

Funds flows are also influenced by categories hedge funds belong to. We extend the fund flows-performance analysis to different hedge fund categories. Investors in hedge funds that follow directional strategies, i.e., strategies that follow trends, such as "Global Macro" and "Dedicated Short Seller" are more responsive to past returns. On the other hand, investors in "Market Neutral" and "Event Driven" categories are less responsive to past returns as these categories are more driven by market conditions and events peculiar for that particular

[1] A category is defined as belonging to one of 11 strategies described in Appendix A.2.

time.

Withdrawals due to poor performance can lead to liquidation (Berk and Green (2002) and Vayanos (2003)). Liquidation can appear in two forms: failure of the fund or closure of the fund. Failure can happen due to fraud, forced liquidation due to levered positions that falls below a threshold, or concentrated bets that go against the manager's strategy. Closure can happen if a hedge fund exhausts all opportunities within a category, cannot obtain more capital, or has a bad performance. In the first case, as in bankruptcy, hedge fund managers and investors incur significant costs due to the loss of the capital. In the case of liquidation due to closure, hedge fund investors, rather than managers, incur search costs as they now have to look for new investment opportunities in the hedge fund industry. The new investors might incur higher management and incentive fees in a new fund, as well as being subject to a 2-3 year lock-up period, which is mandatory for new investors. Moreover, liquidation increases survivorship bias which can cause new investors to overestimate potential returns from hedge funds.

Second, this paper studies the determinants of category flows, that is, aggregated fund flows into a category. We find that investors are more likely to invest in categories that have done well. Controlling for fund characteristics, such as returns, age, and assets under management, hedge funds are less likely to be liquidated if they are located in favorable categories. We introduce a favorable positioning metric that determines whether a hedge fund is located in a category that experiences an increased proportional net dollar flow compared to other categories. An increase in the favorable positioning metric decreases the liquidation probability of a hedge fund. However, at the same time, it increases competition due to hedge fund entry into the favorable category. Hedge funds compete for limited opportunities and capital, thus increasing the liquidation probability of a fund. This finding is contrary to what we find by analyzing individual fund flows. As hedge fund investors chase fund returns, thus increasing flows into a fund, the liquidation probability is decreased. However, as hedge fund investors chase category returns, the liquidation probability increases due to competition effects. We also find that as competition increases, marginal funds exit first, and funds that deliver superior returns are left as they are able to withstand competition.

Finally, we look at the relationship between performance and assets under management, both of which affect the life cycles of hedge funds. We find that the relationship between current performance and past asset size is positive and concave. Agarwal, Daniel and Naik (2003), and Goetzmann, Ingersoll, and Ross (2003) find a similar relationship. We confirm this result and extend it to different hedge fund strategies. The performance-asset size relationship takes on different functional forms for different categories. For instance, for illiquid categories such as "Emerging markets" and "Convertible arbitrage" that experience high market impact and are subject to limited opportunities, the relationship is concave and the optimal size can be calculated. The result is opposite for liquid categories such as "Dedicated short bias" and "Equity market neutral" categories. We also propose a model that accounts for fund effects, favorable positioning, competition and market impact. The resulting model produces concave relationship between returns and past assets and an optimal asset size using realistic estimates. The result is important for hedge fund managers and investors because managers of hedge funds with high asset sizes might choose to close the funds to new investors before facing a decrease in returns and an increase in liquidation probabilities. Also, by maximizing returns to hedge fund investors, an optimal asset size can be calculated.

This paper analyzes the effects of fund and category specific factors on life cycles of hedge funds. It introduces new concepts such as competition, favorable positioning and a life cycle in intermediation and hedge fund literatures.

Before describing the empirical analysis of the life cycles in hedge funds, we provide a brief hedge fund overview in Section 2 and a literature review in Section 3. The data are described in Section 4. Performance-fund flow relationship is estimated for different hedge fund styles in Section 5. The effects of category flows, favorable positioning and competition on the liquidation probability of a hedge fund are analyzed in Section 6. The performance-asset size relationship is analyzed in Section 7, and the optimal asset size for different hedge fund categories is calculated. Moreover, a performance model that takes into the account the effects of flows, returns, competition, favorable positioning and market impact is proposed in this section. We conclude in Section 8.

2 Hedge Fund Overview

As of October 2003, the size of the global single-manager hedge fund universe (not including funds of funds) is \$650 - \$700 billion (Tremont Company). There are about 5,000 global single-manager hedge funds in the hedge fund universe. There are about 1,200 - 1,400 funds of funds. There are about 3,000 distinct hedge fund managers that manage both offshore and domestic accounts. In 1990, there were 610 funds managing \$39 billion. Hedge funds differ from mutual funds and other investment vehicles by both internal structure and investment discipline. Hedge fund managers are not restricted to any particular type of investments. Hedge funds can buy (long) or sell (short) securities that they do not own. They are not restricted to common "buy and hold" strategies. Most U.S. hedge funds are limited partnerships, or limited liability companies, established to invest in public securities. However, there is no common definition of a hedge fund. U.S. hedge funds are defined by their freedom from regulatory controls stipulated by the Investment Company Act of 1940. Before 1996, a hedge fund had a 100 investor limit in order to qualify as a limited partnership. However, under the National Securities Markets Improvement Act of 1996, the 100 investor limit was lifted. The minimum new worth requirement for a qualified investor is \$5 million and the minimum institution capital is \$25 million. Companies can also become reporting companies voluntarily by filing with the SEC. Under the Exchange Act, a company must become a reporting company if it has at least 500 shareholders and \$10 million in assets. The Exchange Act contains registration and reporting provisions that may apply to hedge funds.

Depending upon their activities, in addition to complying with the federal securities laws, hedge funds and their advisers may have to comply with other laws including the Commodity Exchange Act ("CEA"), rules promulgated by the National Association of Securities Dealers ("NASD") and/or provisions of the Employment Retirement Income Security Act ("ERISA"). In addition, hedge funds may be subject to certain regulations promulgated by the Department of the Treasury, including rules relating to the prevention of money laundering. Moreover, hedge fund advisers are subject to certain state laws.

Offshore hedge funds are typically corporations registered in a tax haven such as the

British Virgin Islands, the Bahamas, Bermuda, the Cayman Islands, Dublin, or Luxembourg, where tax liabilities to non-U.S. citizens are minimal. In general, the hedge fund industry is not transparent to regulators unlike the mutual funds industry. Like mutual funds, hedge funds are actively managed investment portfolios holding positions in publicly traded securities. However, unlike mutual funds, hedge funds have greater flexibility in the kind of securities they can invest in. Hedge funds can invest in domestic and international debt and derivative securities. They can take undiversified positions, sell short, and lever up their portfolios. These alternative investments mainly attract institutions and wealthy individuals with minimum investments typically in the range of $250,000 - $1 million. Hedge funds are also characterized by a substantial managerial investment and strong managerial incentives. On average, hedge fund managers receive a 1% annual management fee and 20% of the annual profits. Most of funds employ a bonus incentive fee: managers are paid a percentage of the excess of a fund's return over some level, commonly called a "high-water mark." If a hedge fund incurred losses in the past, its managers can be paid in present period only if return in this period exceeds the "high-water mark" plus past losses.

3 Literature Review

Given the availability of public and private hedge fund databases such as TASS, AltVest, Hedge Fund Research (HFR) and Managed Account Reports (MAR), numerous studies have analyzed hedge fund performance. For example, Ackermann, McEnally and Ravenscraft (1999), Agarwal and Naik (2000b, 2000c), Edwards and Caglayan (2001), Fung and Hsieh (1999, 2000, 2001), Kao (2002), and Liang (1999, 2000, 2001) provide empirical studies of hedge fund performance using different hedge fund databases. In comparison, Brown, Harlow and Starks (1996), Chen, Hong, Huang and Kubik (2003), and Chevalier and Ellison (1997) provide empirical performance analysis studies for mutual funds; Kaplan and Schoar (2003) do so for private equity funds.

Performance attribution and style analysis in hedge funds have been studied in the following papers: Agarwal and Naik (2000b, 2000c), Brown and Goetzmann (2001), Brown, Goetzmann and Ibbotson (1999), Brown, Goetzmann, and Park (1997, 2000, 2001), Fung and Hsieh (1997), and Lochoff (2002). Ackermann et al. (1999) and Brown, Goetzmann, Ibbotson, and Ross (1992) find that hedge fund databases can suffer from survivorship biases that can bias both first and second moments in returns. According to Ackermann et al. (1999), termination and self-selection biases are the most powerful data-conditioning biases. Funds that are closed leave the database due to termination. Funds that choose not to be included in the database can voluntarily withdraw from databases. Performance studies of existing hedge funds may be artificially inflated if poorly performing funds are systematically omitted from the database. Brown, Goetzmann and Ibbotson (1999) find that the survivorship bias is about 3 percentage points per year for offshore hedge funds.

Favorable category positioning and competition have not been introduced in the hedge fund literature, and their effects on the life cycles of hedge funds have not been studied. In the Industrial Organization literature, authors find that an increase in competition leads to an increase in the probability of liquidation in a firm (Aghion, Dewatripont and Rey (1995) and Schmidt (1997)). The life cycle of a firm is studied by Mueller (1972).

Khorana and Servaes (1999) find that mutual fund starts are positively related to the level of assets invested in and capital gains embedded in other funds with the same objective. For mutual funds, Chen, Hong, Huang, and Kubik (2003) find that controlling for its size, a fund's performance increases with the asset base increase of other funds in the family that the fund belongs to. For private equity funds, Kaplan and Schoar (2003) find that in the periods of an overall increased entry of funds into the industry, there is a large negative effect on the performance of younger funds compared to the performance of older, more established funds. Unlike mutual funds and private equity funds, hedge funds have very distinct categories with a high barrier to entry. It takes time and managerial talent to set up a hedge fund in a particular category. Therefore, empirical analysis of the effect of favorable positioning of a category and competition among hedge funds within a category can provide insights into understanding the life cycles of hedge funds.

Hedge funds that have higher returns experience higher net flows. Agarwal, Daniel, and Naik (2003) find a convex relationship in hedge fund flow-performance relationship. Ippolito (1992), Chevalier and Ellison (1997), Goetzmann and Peles (1997), Gruber (1996), Sirri and Tufano (1998) and Zheng (1999) look at the determinants of money flows in mutual funds, and find a positive and convex relationship. However, Kaplan and Schoar (2003) find a concave relationship for private equity funds.

The impact of volatility on fund net flows is modeled theoretically by Vayanos (2003) and studied empirically for mutual funds by Chevalier and Ellison (1997). They find that the higher volatility leads to more outflows.

Withdrawals due to poor performance can lead to liquidation (Berk and Green (2002) and Vayanos (2003)). Amin and Kat (2002), Bares, Gibson and Gyger (2001), Brown, Goetzmann and Ibbotson (1999), Brown, Goetzmann and Park (2001), Fung and Hsieh (2000), Gregoriou (2002), and Liang (2000, 2001) focus on hedge fund survival rates. Baquero, Horst, and Verbeek (2002) look at liquidation probabilities of individual hedge funds and find that they are greatly dependent on past performance.

Agarwal, Daniel, and Naik (2003) and Goetzmann, Ingersoll and Ross (2003) find decreasing returns to scale in hedge funds. Perold and Salomon (1991) report decreasing returns to scale in mutual funds and propose an optimal size calculation for mutual funds. Kaplan and Schoar (2003) find decreasing returns to scale in the private equity industry. Our paper suggests that an optimal size can be calculated for different hedge fund categories.

4 Data Description

The TASS database is used for the empirical analysis. As of 2003, the TASS database tracks \$270 billion held by global single-manager hedge funds and \$63 billion held by funds of funds. These numbers exclude money held in separately managed accounts. There are other databases like AltVest, Hedge Fund Research (HFR) and Zurich Capital Markets/Managed Accounts Reports (ZCM/MAR). However, the TASS database is the most comprehensive one. The TASS database consists of 3,928 hedge funds from November 1977 to April 2003.[2] The database is divided into two parts: "Live" and "Graveyard" funds. Funds that are in the "Live" category are considered to be active as of April 2003. Once a hedge fund is considered no longer active, it is transferred into the "Graveyard" category. Hedge funds are in the "Graveyard" category if they stop reporting their performance, are liquidated,

[2]For further information about the TASS database, see http://www.tassresearch.com.

closed to new investment, restructured, or merged with other hedge funds. A hedge fund can be listed in the "Graveyard" database only after being listed in the "Live" database. However, the TASS database is subject to backfill bias: When a fund decides to be listed in the database, all its prior history is incorporated in the TASS database. Also, due to reporting delays, some "Graveyard" funds can be incorrectly listed in the "Live" database. Tremont adopted a policy of transferring funds from the "Live" to the "Graveyard" database if its managers have not heard from hedge funds or were not able to contact the hedge fund managers over a 6-8 month period. Because the "Graveyard" database became active in 1994, thus funds that were dropped from the "Live" database before 1994 were not recorded by TASS, the database is subject to some degree of survivorship bias.[3]

For the analysis in this paper, another sub-database, called "Liquidated" is constructed. This sub-database has funds that are liquidated. To construct the sub-database, we eliminate funds from the "Graveyard" category that are there due to mergers with other hedge funds, that are dormant, or that decided not to list in the database due to their large size. Moreover, we eliminate funds that decided to be closed to new investment, and therefore, not needing advertisement by being listed in the database, got matured, or got reconstructed. In order to compile the "Liquidated" sub-database, we carefully examined the "Notes" section provided by TASS to understand the history of each hedge fund, researched the history of funds, and talked to TASS employees who communicate directly with hedge fund managers. Most hedge funds in the "Graveyard" category with less than $20 million in assets on the last day that the funds were reported are actually liquidated and therefore we put them in the "Liquidated" sub-database. The exception is the funds that merged. There are some hedge funds in the TASS database that stopped reporting their performance to the Tremont between September 2002 and April 2003 and have less than $20 million of assets on the last reporting date. These funds might have characteristics of funds that are liquidated - low returns, and very low assets under management (less than $20 million). However, they are not considered in the "Liquidated" database due to reporting delays. TASS allows up to 8

[3] For studies attempting to quantify the degree and impact of survivorship bias, see Brown, Goetzmann, Ibbotson, and Ross (1992), Brown, Goetzmann, and Ibbotson (1999), Brown, Goetzmann, and Park (1997), Carpenter and Lynch (1999) and Fung and Hsieh (1997b, 2000).

months delay before further investigation and before considering the hedge fund a liquidation, unless, during this 8 month period, it is discovered that the hedge fund is liquidated.

The database is further filtered by considering only hedge funds that report monthly and net-of-fees returns. Hedge funds for which monthly returns or monthly assets are missing are eliminated. Also, to correct for the termination bias, the data from January 1994 until December 2002 is used. In the end, the "Combined" database contains 3,501 hedge funds with continuous net-of-fees monthly returns and monthly assets under management. Out of 3,501, 1,264 funds are in the "Liquidated" sub-database, and 2,237 funds are in the "Successful" subcategory, representing funds still alive. There are 1,387 hedge funds in the "Graveyard" database, out of which 1,264, or 91% are failures; the rest decided not to advertise through TASS, decided to be closed to new investment, or merged with other hedge funds.

The TASS database considers 11 different investment style categories, described in detail in Appendix A.2. The number of hedge funds in each category is presented in Table A.1.

[INSERT TABLE A.1]

The dynamics of annual hedge fund entries and exits are presented in Table A.2. As of 1994, the date when the TASS database started reporting hedge fund liquidations, the average annual attrition rate has been 7.33%. The average attrition rate due only to liquidation is 7.10%.

[INSERT TABLE A.2]

Summary statistics for monthly returns, standard deviation of monthly returns, age, assets under management as well as favorable category positioning and competition metrics for both hedge funds and fund of funds are presented in Table A.3.

[INSERT TABLE A.3]

5 Performance-Fund Flow Relationship

Berk and Green (2002) and Vayanos (2003)propose theoretical models in which fund specific factors such as flows and returns affect liquidation probabilities. This section presents an empirical analysis of the effects of fund specific factors on liquidation probabilities. Also, the fund specific factors are correlated. For example, current fund flows depend on past returns (Agarwal, Daniel and Naik (2003)). The authors find that the relationship between fund flows and performance is positive and non-linear in past performance.

5.1 Hypotheses

Hypothesis 1: As investors chase high fund returns, the liquidation probability of the hedge fund decreases.

5.2 Methodology

The performance-fund flow function is estimated using a piecewise linear relationship between current flows and past returns. A modified methodology proposed by Sirri and Tufano (1998) to study performance fund flow relationship in mutual funds is used. Fractional rank terciles, $Trank_{i,t-1}$ for each time t and fund i are constructed. First, a fractional rank, F_{rank}, is calculated for each fund, from 0 to 1 based on returns in year t-1. Then, $Trank^1$, the bottom tercile rank, $Trank^2$, the middle tercile rank and $Trank^3$, the top tercile rank are calculated as follows: $Trank^1$=$Min(\frac{1}{3}$, $F_{rank})$,
$Trank^2$=$Min(\frac{1}{3}$, F_{rank}-$Trank^1)$,
$Trank^3$=$Min(\frac{1}{3}$, F_{rank}-$Trank^1$-$Trank^2)$,

The following regression is specified to understand the determinants of fund flows:

$$
\begin{aligned}
Flow_{i,t} = \alpha_{i,t} + \sum_{j=1}^{3} \beta_1^j (Trank_{i,t-1}^j) + \beta_2 Size_{i,t-1} + \beta_3 Age_{i,t-1} \\
+\beta_4 Flow_{i,t-1} + \beta_5 Return_{i,t} + \beta_6 \sigma_{i,t-1} \\
+\sum_{k=1}^{10} \beta_7^c I(Category_{i,k}) + \epsilon_{i,t}
\end{aligned}
\tag{1}
$$

Current quarterly flows are regressed on bottom, middle and top terciles of last quarter performance, current fund return, last quarter flow, last quarter asset size, age, and the last quarter standard deviation of returns. The quarterly data from 1994 until 2002 is used.

Empirical studies define net flow of funds as a percentage change in new assets adjusted for return.

$$
F_t = \frac{A_t - A_{t-1}(1 + r_t)}{A_{t-1}} \tag{2}
$$

However, this measure downplays the significance of very large negative returns that lead to a hedge fund liquidation. If a hedge fund is liquidated at time t, then, A_t is 0, and $\frac{A_t - A_{t-1}(1+r_t)}{A_{t-1}} = -(1 + r_t)$. Therefore, when a fund is liquidated, the minimum return will be $r_t = -1$ or -100%. Under the conventional metric of measuring the flow, the response to the flow is 0. Therefore, to correct for this, in the empirical analysis, the flow measure is set to -1 when a fund is liquidated.

To test Hypothesis 1 whether an increase in returns and flows impacts liquidation probabilities of hedge funds, we specify the following logit model.

$$
\begin{aligned}
Liquidation_{i,t} = f(\alpha_{i,t} + \beta_1 Age_{i,t} + \beta_2 r_{i,t-1} \\
+\beta_3 r_{i,t} + \beta_4 Flow_{i,t-1} + \beta_5 Assets_{i,t-1} \\
+\sum_{k=1}^{10} \beta_6^c I(Category_{i,k}) + \sum_{t=1}^{38} \beta_7^t I(Time_{i,t}) + \epsilon_{i,t})
\end{aligned}
\tag{3}
$$

5.3 Results

We find that average net flows into funds in different terciles are as follows: Top: 42.15%, Medium: 5.68%, Bottom: -21.62% per quarter. As predicted, higher returns lead to higher future flows.

The Fama-MacBeth results for the piecewise linear relationship between current flows and past returns are presented in Table A.4.

[INSERT TABLE A.4]

Better performing funds are more likely to attract funds than poorly performing funds. This relationship is concave so that the top performing funds do not grow proportionally as much as the average fund in the market. Top funds might choose not to grow that fast in order not to face diminishing returns. For the analysis of the regions of diminishing returns, see Section 7. The relationship is concave for private equity funds as well (Kaplan and Schoar (2003)). In the mutual fund literature, the relationship is convex, so that the top performing funds increase their share of the overall hedge fund market (Sirri and Tufano (1998)).

For the bottom tercile, the estimate is 0.293 with a t-stat of 5.46. For the middle tercile, the estimate is 0.269 with a t-stat of 5.46. For the top tercile, the estimate is 0.006 with a t-stat of 0.08. Therefore, if in the last quarter a hedge fund was in a top tercile, an increase of return by 10% would lead to a 0.8% increase in flows, and if the hedge fund was in a bottom tercile, an increase of return by 10% would lead to 2.9% increase in flows. In contrast, Agarwal, Daniel and Naik (2003) find that the fund flow-performance relationship is convex for hedge funds. However, unlike this paper, the authors use data from 1994-2000. They also look only at the annual relationship between flows and returns. We find that there is a high degree of volatility in flows and returns on an annual basis. That is why we look at the quarterly data. Also, the authors use the combination of HFR, TASS, ZCM/MAR databases, and we use only the TASS database. The relationship between current returns and current flows is positive and significant with a coefficient of 0.234. Past flows positively affect current flows with a coefficient of 0.048. However, both past size and age negatively affect the future flows with coefficients of -0.041 and -0.001, respectively. Asset size is measured as a natural logarithm of assets under management. The topic of an asset size and the notion of an optimal asset size are covered thoroughly in Section 7. The past standard deviation of returns negatively affects current quarterly flows with a coefficient of -0.002. People are less likely to invest or hold their money in a hedge fund with high volatility, after adjusting for returns. The R^2 for the regression is 5.53%.

To understand whether there are any category effects on the performance-fund flow relationship, we estimate the same model for each fund category and present results in Table A.5.

[INSERT TABLE A.5]

For all categories, the fund flow-performance relationship is positive and concave. By looking at the middle coefficient of past returns, $Trank^2$, we study the average impact of past returns on current flows for different hedge fund categories. We expect that hedge funds that have more directional focus by concentrating on trends have a higher impact of past returns on flows. Funds that follow event oriented and market timing strategies should have a lower impact of past returns on flows. The expectations are closely correlated with the data depicted in Table A.6.

[INSERT TABLE A.6]

Hedge fund categories that have more of a directional focus like “Dedicated Short Bias,” “Fixed Income Arbitrage,” “Global Macro,” “Long/Short Equity” and “Managed Futures” have higher estimates of $Trank^2$. More event oriented strategies and less directional strategies such as "Convertible Arbitrage," "Emerging Markets," "Event Driven" and "Equity Market Neutral" have much smaller coefficients.

To test Hypothesis 1, whether an increase in returns and flows affects liquidation probability of hedge funds, the logit model is specified in equation 3. The results are depicted in Table A.7.

[INSERT TABLE A.7]

The implied probability of liquidation is 0.60% per quarter.

We also adjust coefficients by the mean in the logistic specification so it is possible to directly compare the effects of each coefficient on the liquidation probability. The results are depicted in Table A.8.

[INSERT TABLE A.8]

Both current and past returns and past flows negatively affect liquidation probability. Past asset size also negatively impacts the liquidation probability and has the biggest effect for each percentage deviation from the mean.

Results of the likelihood ratio test and the efficient score test for testing the joint significance of the explanatory variables are included in Table A.9.

[INSERT TABLE A.9]

6 Favorable Positioning and Competition

Different fund characteristics such as fund returns, flows, asset size and age affect the liquidation of hedge funds. Returns are affected by abilities of fund managers, costs, and exogenous shocks to hedge fund investment portfolios. However, even if a manager is able to deliver solid returns, a fund can still be liquidated due to other factors. We propose that given a fund manager's abilities and other fund characteristics, a hedge fund's probability to survive is affected by favorable positioning and competition. The paper is the first to introduce the concepts in the literature on financial intermediation and asset pricing. Competition and its effects on liquidation probability are introduced in the Industrial Organization literature (Aghion, Dewatripont and Ray (1995) and Schmidt (1997)).

A hedge fund experiences favorable positioning if it is located in the right category at the right time. This category experiences a higher proportional increase in net dollar fund flows compared to other categories. A hedge fund experiences competition if it is surrounded by a lot of funds in its category. However, the relationship among favorable positioning, competition, and the probability of liquidation is not obvious and provides surprising results and insights into understanding the life cycles of hedge funds.

Below we list examples of hedge funds in the database that are liquidated even though the returns at the time of liquidation are positive for each of the funds. The liquidation is affected by high competition, low favorable positioning, or both.

Reference 10. Category: Long/Short Equity. The fund is liquidated in March 1999. The return at the time of liquidation was 3.87%. The flow a quarter before the liquidation was -78.96%. The average monthly return during the past year was -0.01%. The favorable positioning metric (FAV) in Q4, 1998 was -0.24. The number of competitors in Q4, 1998 was 497.

Reference 15. Category: Long/Short Equity. The fund is liquidated in April 1995. The return at the time of liquidation was 2.17%. The flow a quarter before the liquidation was 3.20%. The average monthly return during the past year was 0.57%. The favorable positioning metric (FAV) in Q1, 1995 was -0.21. The number of competitors in Q1, 1995 was 192.

Reference 4429. Category: Long/Short Equity. The fund is liquidated in January 2001. The return at the time of liquidation was 31.40%. The flow a quarter before the liquidation was 29.99%. The average monthly return during the past year was 6.93%. The favorable positioning metric (FAV) in Q4, 2000 was 0.2910. The number of competitors in Q4, 2000 was 693.

Reference 560. Category: Managed Futures. The fund is liquidated in December 1995. The return at the time of liquidation was 8.21%. The flow a quarter before the liquidation was 0%. The average monthly return during the past year was -0.40%. The favorable positioning metric (FAV) in Q3, 1995 was -0.05. The number of competitors in Q3, 1995 was 211.

Reference 563. Category: Managed Futures. The fund is liquidated in December 1995. The return at the time of liquidation was 2.16%. The flow a quarter before the liquidation was 3.20%. The average monthly return during the past year was 1.13%. The favorable positioning metric (FAV) in Q3, 1995 was -0.05. The number of competitors in Q3, 1995 was 211.

6.1 Hypotheses

Hypothesis 2: As investors chase category returns, they move money into the favorable category. The favorable category that experiences a higher proportional increase in net dollar fund flows compared to other categories, attracts other hedge funds into the category, thus increasing the liquidation probability of hedge funds in that category. Therefore, as investors chase category returns, by investing in a favorable category, the liquidation probability of hedge funds in that category increases due to competition effects.

Hypothesis 3: As the competitors become smaller, measured by asset value, their impact on liquidation probability reduces.

Hypothesis 4: As competition within a strategy increases, marginal performers leave and only hedge funds with superior risk-adjusted performance survive.

6.2 Methodology

To test Hypothesis 2, we define a Favorable Positioning metric (FAV). FAV measures whether a fund category experiences a higher proportional increase in net dollar fund flows compared to other categories. Hypothesis 2 proposes that if a hedge fund is in the right category at the right time, in other words, in the category with high FAV value, then the liquidation probability of the hedge fund decreases. However, increased FAV leads to an increase in competition which increases the liquidation probability.

In order to define FAV, we need to define fund Dollar Flow at any time t.

$$DollarFlow_{i,t} = A_{i,t} - A_{i,t-1}(1 + r_{i,t}) \tag{4}$$

Equation 4 assumes that fund flows occur at the end of the month, which is true for hedge funds. If an investor chooses to deposit or withdraw money into a hedge fund, he usually has to wait until the end of the month to make the transaction. Of course, for new investors, the average lock-up period is 2 years, which limits the outflow of money from a hedge fund.

Dollar Flows are then aggregated over a quarter to calculate the Quarterly Dollar Flow. Then, as in Chevalier and Ellison (1997), Sirri and Tufano (1998) and Agarwal, Daniel, and Naik (2003), quarterly dollar flows are scaled by the beginning-of-quarter assets under management to measure flows. Flows capture the change in size due to net money flows.

$$Flow_{i,t} = \frac{QuarterDollarFlow_{i,t}}{A_{i,t-1}} \tag{5}$$

FAV is a favorable positioning metric that is measured as follows:

$$FAV_{k,t} = \frac{\sum_{i,i \in k} QuarterDollarFlow_{i,t}}{\sum_{i,i \in POSNET} QuarterDollarFlow_{i,t}} \tag{6}$$

If category k experiences net positive flows during the quarter. POSNET is a set of category flows that are net positive during the quarter.

$$FAV_{k,t} = -\frac{\sum_{i,i \in k} QuarterDollarFlow_{i,t}}{\sum_{i,i \in NEGNET} QuarterDollarFlow_{i,t}} \tag{7}$$

If category k experiences net negative flows during the quarter. NEGNET is a set of category flows that are net negative during the quarter. By construction, FAV metric lies between -1 and 1.

To test whether FAV is affected by previous category return, we run the following regression model:

$$FAV_{k,t} = \alpha_{k,t} + \beta_1 r_{k,t-1} + \epsilon_{k,t} \tag{8}$$

To test the relationship between liquidation probability and FAV, we run the logit model of the probability of liquidation on current age, return during the past quarter, return during the current quarter, past quarter flow, past quarter FAV, and past quarter assets measured as the natural logarithm of assets under management. The model controls for category and time effects.

$$\begin{aligned} Liquidation_{i,t} = f(&\alpha_{i,t} + \beta_1 Age_{i,t} + \beta_2 r_{i,t-1} \\ &+\beta_3 r_{i,t} + \beta_4 FAV_{k,t-1} + \beta_5 Assets_{i,t-1} \\ &+\sum_{k=1}^{10} \beta_6^c I(Category_{i,k}) + \sum_{t=1}^{38} \beta_7^t I(Time_{i,t}) + \epsilon_{i,t}) \end{aligned} \tag{9}$$

To make sure that we are not capturing a trend in variables, the FAV metric is graphed for each category over time, and no consistency in the behavior is found.

EntryFraction for a category k at time t is defined as follows:

$$EntryFraction_{k,t} = \frac{ENTRY_{k,t}}{NUMBER_{k,t-1}} \tag{10}$$

where $ENTRY_{k,t}$ is the number of funds entering a particular category k in a quarter t and $NUMBER_{k,t-1}$ is the number of funds in the category at the end of the quarter t-1.

To measure the effect of FAV on EntryFraction, the following regression is performed:

$$EntryFraction_{k,t} = \alpha_{k,t} + \beta_1 FAV_{k,t-1} + \epsilon_{k,t} \tag{11}$$

However, it takes time for hedge fund managers to perceive the "hotness" of the category as well as to set-up a fund. Therefore, we perform the similar regression on 4 lags of FAV (up to a year).

$$EntryFraction_{k,t} = \alpha_{k,t} + \beta_1 FAV_{k,t-1} + \beta_2 FAV_{k,t-2} + \beta_3 FAV_{k,t-3} + \beta_4 FAV_{k,t-4} + \epsilon_{k,t} \quad (12)$$

To understand the relationship between competition and liquidation probability, we run the logit model of the probability of a hedge fund liquidation on current age, return during the past quarter, return during the current quarter, past quarter flow, the number of hedge funds in the category a year before, and past quarter assets measured as the natural logarithm of assets under management. The model is controlled for category and time effects. The number of hedge funds in the category is a proxy for competition. The reason why a year lag is taken when calculating competition is because it takes some time for hedge funds to enter a category and become competitive. They need to raise awareness in the hedge fund community, communicate their strategy to new investors, build relationships with dealers and bankers who extend credit, and build a track record to attract new investors.

$$\begin{aligned} Liquidation_{i,t} = f(&\alpha_{i,t} + \beta_1 Age_{i,t} + \beta_2 r_{i,t-1} \\ &+ \beta_3 r_{i,t} + \beta_4 Number_{k,t-4} + \beta_5 Assets_{i,t-1} \\ &+ \sum_{k=1}^{10} \beta_6^c I(Category_{i,k}) + \sum_{t=1}^{31} \beta_7^t I(Time_{i,t}) + \epsilon_{i,t}) \end{aligned} \quad (13)$$

The data used in the paper is from 1994 until 2002. The TASS started reporting exits by hedge funds starting in 1994. Therefore, during this time period, FAV and competition metrics are going to be correctly calculated.

To test Hypothesis 2, the joint effect of competition and favorable positioning on liquidation probability is modeled. Also, the estimates are controlled for the mean to allow for direct comparison.

$$
\begin{aligned}
Liquidation_{i,t} &= f(\alpha_{i,t} + \beta_1 Age_{i,t} + \beta_2 r_{i,t-1} \\
+\beta_3 r_{i,t} + \beta_4 Number_{k,t-4} &+ \beta_5 FAV_{i,t-1} + \beta_6 Assets_{i,t-1} \\
+\sum_{k=1}^{10} \beta_7^c I(Category_{i,k}) &+ \sum_{t=1}^{31} \beta_8^t I(Time_{i,t}) + \epsilon_{i,t})
\end{aligned}
\tag{14}
$$

To test Hypothesis 3, we run the logit model for liquidation of hedge funds on age, previous return, current return, previous assets measured in logarithmic quantities, previous FRAC, and previous assets under management measured in logarithmic quantities, where FRAC is defined as follows:

$$
Frac_{k,t} = \frac{Number_{k,t}}{\sum_{i,i\in k} Assets_{i,t}} \tag{15}
$$

FRAC measures the fraction of the number of competitors in a category to the overall asset size in the category.

The logit specification is as follows:

$$
\begin{aligned}
Liquidation_{i,t} &= f(\alpha_{i,t} + \beta_1 Age_{i,t} + \beta_2 r_{i,t-1} \\
+\beta_3 r_{i,t} + \beta_4 Number_{k,t-1} &+ \beta_5 FRAC_{k,t-1} + \beta_6 Assets_{i,t-1} \\
+\sum_{k=1}^{10} \beta_7^c I(Category_{i,k}) &+ \sum_{t=1}^{38} \beta_8^t I(Time_{i,t}) + \epsilon_{i,t})
\end{aligned}
\tag{16}
$$

To test Hypothesis 4, we divide competition metric, NUMBER, into 5 quintiles and run the logistic model in equation 14 five times for each of the quintiles.

6.3 Results

Results for Hypothesis 2

The Fama-MacBeth results of the regression model 8 are presented in Table A.10.

[INSERT TABLE A.10]

When pooled regression is used, the coefficient in front of current FAV is 3.347 with a t-stat of 2.93. When the Fama-MacBeth method is used, the coefficient is 1.358 with a t-stat of 1.85. As can be seen, current FAV significantly depends on a previous category return. Investors chase hedge funds in a category that performed well. The R^2 for the regression is 28.46%. Returns are value-weighted.

The results of logit regression specified in equation 9 are presented in Table A.11. Even if a hedge fund manager has skills and a good track record, the hedge fund can still fail simply due to its belonging to the wrong category at the wrong time.

[INSERT TABLE A.11]

For robustness, the FAV metric is plotted over time for each category to make sure that there is no consistent trend in the measure that can diminish the favorable positioning explanation. FAV for each category is plotted over time in the Figure A.1. According to the figure, FAVs for Global Macro and Long/Short Equity are negatively correlated.

[INSERT FIGURE A.1]

The R^2 for the regression is 13.24%. As predicted, the funds with higher current and past returns have a higher survival probability. Funds with higher assets are less likely to fail. Higher FAV leads to lower liquidation probability. The coefficient in front of the FAV measure is -0.502 and is significant at 1% level. Therefore, for a hedge fund, being in the right category at the right time helps to reduce liquidation probability. Given equation 9, the implied probability of liquidation for a particular hedge fund in any quarter is 0.52%. Changing FAV from -1 standard deviation to +1 standard deviation affects the implied probability by -30.11%. [4]

Results of the likelihood ratio test and the efficient score test for testing the joint significance of the explanatory variables are included in the Table A.12.

[INSERT TABLE A.12]

The result shows that a hedge fund is likely to survive if it is in the right category at the right time. If the category as a whole experiences higher inflows than other categories, a hedge fund in this favorable category has a greater probability of survival after controlling for current and past returns, age, and assets under management in the fund.

However, as the category becomes favorable, other hedge funds are more likely to enter into the same category. The barrier of entry to the hedge fund industry is very low, and over time the cost and time of setting up a new hedge fund has greatly diminished.

The proposition is tested in the equation 11 and the results are as follows. The parameter estimate of the $FAV_{k,t-1}$ is 0.018 with t-statistic of 2.24. Therefore, hedge funds tend to be formed in categories which have increased relative net flows.

It takes time for hedge fund managers to perceive the “hotness” of a category as well as to set-up a fund in this favorable category. Therefore, the EntryFraction is regressed on 4 lags of FAV (up to 1 year). The sum of parameter estimates for all 4 lags is 0.038. The R^2 for the regression is 2.15%.

As EntryFraction into a category increases, competition among hedge funds in a category increases. Table A.13 presents results of the following: logit regression of liquidation probability of a hedge fund at time t on intercept, current age, previous quarterly return on investments, current quarterly return, the number of hedge funds in a particular category in the previous year, and previous quarterly assets under management. Even if a hedge fund manager has skills and a good track record, the hedge fund can still fail due to competition among other hedge funds in a category.

[INSERT TABLE A.13]

[4] Implied Probability=$\frac{exp(\alpha_{i,t}+\sum_{z=1}^{n}\beta_{i,t,z}\mu_{i,t,z})}{1+exp(\alpha_{i,t,z}+\sum_{z=1}^{n}\beta_{i,t,z}\mu_{i,t,z})}$ where $\mu_{i,t,z}$ is a mean value for an independent variable z.

The R^2 for the regression is 12.30%. As suggested by Hypothesis 2, a higher number of hedge funds in a category leads to more competition, thus increasing the liquidation probability. The coefficient on the Number is 0.002 and is significant at 1% level. Therefore, competition increases the liquidation probability. Given equation 13, the implied probability of liquidating a particular hedge fund in any quarter is 0.90%. Changing the number of hedge funds in a category from -1 standard deviation to +1 standard deviation increases the implied probability by +76.97%.

Results of the likelihood ratio test and the efficient score test for testing the joint significance of the explanatory variables are included in Table A.14.

[INSERT TABLE A.14]

The results that show the combined effect of competition and favorable positioning are in Table A.15.

[INSERT TABLE A.15]

The results that show estimates adjusted for the mean are in Table A.16.

[INSERT TABLE A.16]

Results of the likelihood ratio test and the efficient score test for testing the joint significance of the explanatory variables in Tables A.15 and A.16 are included in the Table A.17.

[INSERT TABLE A.17]

If FAV and competition are taken into account, then the annual implied liquidation probability of any hedge fund is 2.72%. If these variables are not taken into the account, then the implied liquidation probability is 1.84%.

Overall, hedge funds are likely to survive if they are in the right category at the right time, even after adjusting for performance. If the category falls out of favor, hedge funds have bigger liquidation probability. Hedge funds are not able to raise new capital and withdrawals can lead to forced liquidation due to levered positions that fall below a threshold. If a category has a favorable positioning, it has a positive effect on survival probability as well as on the number of new hedge fund entrants into the favored category. The increase in the competition among the funds in a category leads to an increased proportional liquidation within the hedge fund category.

The quarterly Spearman correlation coefficients for the current liquidation probability, current age, current returns, past returns, past flows, past FAV, the last year's number of hedge funds in a category and past assets are presented in Table A.18. In Spearman correlation only the order of the data is important, not the level, therefore extreme variations in expression values have less control over the correlation, unlike in the Pearson correlation.

[INSERT TABLE A.18]

The correlation between past returns and current flows is positive and significant: 0.211 for quarterly data. The relationship between flows and returns is thoroughly analyzed in the Section 5.

Results for Hypothesis 3

The results of logistic regression specified in 16 are presented in Table A.19.

[INSERT TABLE A.19]

The coefficient in front of the competition metric, NUMBER, is 0.019, positive and significant. Coefficient in front of FRAC is -0.660, negative and significant. Therefore, as competition increases in a particular category k, more hedge funds in that category are likely to be liquidated. However, as more hedge funds control smaller amounts of assets, it improves success probabilities of hedge funds. Competition from smaller hedge funds is less likely to increase liquidation probabilities than competition from bigger hedge funds. This is consistent with Hypothesis 3.

Results for Hypothesis 4

To test Hypothesis 4, we divide competition metric, NUMBER, into 5 quintiles and run the logistic model in equation 14 five times for each of the quintiles. The results for the effect of the past return on liquidation probabilities are depicted in Table A.20. As can be seen from the table, as competition increases, the effect of the past quarterly return on liquidation probabilities increases.

[INSERT TABLE A.20]

As competition within a strategy increases, more hedge funds are looking for diminishing opportunities, compete for scarce capital and leverage opportunities. Marginal funds that deliver average or below average returns are more affected during the increased competition and will be forced to liquidate. Only hedge funds that can deliver superior returns to their investors stay.

7 Optimal Asset Size

Both performance and assets under management impact the life cycles of hedge funds. The relationship is important as the understanding of the relationship helps investors to optimize future profits and for hedge fund managers to decide when it is appropriate to close the fund to new investments. Agarwal, Daniel and Naik (2003) and Goetzmann, Ingersoll and Ross (2003) find positive and concave relationship between returns and assets. However, they do not analyze the relationship for different hedge fund categories. Categories that hold illiquid assets, have limited market opportunities and high market impact of trades, are more likely to exhibit the concave relationship. Moreover, by optimizing returns, an optimal asset size can be calculated for funds in these categories.

7.1 Hypotheses

Hypothesis 5: The relationship between current performance and past assets is concave. By maximizing returns, an optimal asset size can be obtained for more illiquid categories.

7.2 Methodology

It is important that the dataset we are using does not have survivorship biases. According to Ackermann et al. (1999), termination and self-selection biases are the most powerful data-conditioning biases. Funds that are liquidated leave the database due to termination. Funds that choose not to be included in the database due to mergers, being closed to investors or a decision to discontinue advertising through the database due to sufficient funds, can voluntarily withdraw from the database. This induces a self-selection bias. Performance studies of existing hedge funds may be artificially inflated if poorly performing funds are systematically omitted from the database. Most databases with hedge fund data may contain various forms of conditioning bias. For mutual funds, Elton, Gruber, and Blake (1996), and Malkiel (1995) estimate that the inclusion of discontinued funds reduces the average annual mutual fund return by between 0.2 and 1.4 percentage points. Brown et al. (1999) find that the survivorship bias is about 3 percentage points per year for offshore hedge funds.

Hedge funds that leave due to self-selection are usually those that have raised enough capital and performed well enough that they do not see the need to be listed in the TASS database. One of the major reasons why hedge funds are voluntarily listed in the database is to obtain free advertising. Under Regulation D, that consists of rules governing the limited offer and sale of securities without registration under the Securities Act of 1933, specifically, Reg. 230.502. (c), hedge funds are banned from direct advertising. Hedge funds acquire new investors by using consultants, "word of mouth," or being listed in the database. Therefore, the self-selected hedge funds that left the database are eliminated from the analysis as the data on their returns and assets is not available after they choose to withdraw from the database.

There are two implications of the omission of the self-selected funds. First, if the asset size continues to grow in the future with increased returns, the true relationship might not exhibit the optimal fund size. However, the inclusion of the self-selected funds will negatively bias the optimal fund size. The second implication of the omission of the self-selected funds is that region A of the concave relationship between asset size and performance is omitted. In this case, the self-selection bias undermines the results of a concave relationship between a hedge fund asset size and returns (see Figure 1).

[INSERT FIGURE 1]

If hedge funds are subject to termination bias, then including such funds in the analysis would eliminate regions B and C (see Figure 2).

[INSERT FIGURE 2]

Hedge funds that are liquidated are more likely to be located in the regions B and C, and not adjusting for the termination bias would lead to an incorrect relationship between returns and performance.

In order to eliminate the self-selection bias, we examine all hedge funds in the "Graveyard" database, and eliminate funds that no longer report their performance, are closed to investment, or merged with other hedge funds. Therefore, in this Section we analyze 3,501 combined hedge funds, out of which 2,237 are in the "Successful" sub-database and 1,264 in the "Liquidated" sub-database. In order to eliminate the termination bias, performance and asset size values for hedge funds before 1994 are not considered. TASS started the "Graveyard" database in 1994. Therefore, the "Combined" database that consists of the "Successful" and "Liquidated" sub-databases does not have termination bias starting in 1994.

The relationship between returns and assets is further analyzed using monthly data.

$$r_{i,t} = \alpha_{i,t} + \beta_1 Assets_{i,t-1} + \beta_2 Assets^2_{i,t-1} + \epsilon_{i,t} \tag{17}$$

The returns versus assets are drawn in the Figure 3. The Figure is constructed as follows. First, assets are separated into 20 different bins according to asset sizes with an equal number of hedge funds. Then, for each bin, an average size and an average corresponding return is calculated. The relationship between the average returns and the average asset sizes is depicted in Figure 3.

[INSERT FIGURE 3]

7.3 Results

The results for regression 17 are specified in Table A.21.

[INSERT TABLE A.21]

The relationship between performance and asset size is positive (the coefficient= 0.290, t-stat=2.69); however, when the size squared is taken into account, the relationship is negative (the coefficient=-0.011, t-stat=-3.39). Therefore, the relationship between current returns and past assets is concave, suggesting that it is possible to obtain an optimal size for different hedge fund strategies. Note, when time effects are taken into the account, the relationship is still concave; however, the coefficient on the size squared is not significant at 5% level. This is due to category differences in the functional forms between returns and past assets as well as average assets and returns.

The relationship between returns and assets for all hedge funds is depicted in Figure 3. Volatility of returns versus assets is depicted in the Figure 4.

[INSERT FIGURE 3]

[INSERT FIGURE 4]

The relationship is concave, reaching an optimal size by optimizing returns. The asset size is optimal for a hedge fund investor, as the further increase in the asset size can actually decrease the return. The volatility decays with size according to a power law. Hedge funds that have a smaller asset size, usually younger hedge funds, tend to increase their riskiness in order to obtain high returns. As hedge funds become more mature and bigger in size, hedge fund managers employ less risky strategies. Also, the decrease in volatility can be explained by the diversification hypothesis: As funds become bigger, they have more flexibility of investing in more securities that are not correlated, thus reducing the variance of the overall portfolio.

We expect that the performance-asset relationship is different for various hedge fund strategies. Funds that invest in illiquid securities are more likely to exhibit a concave behavior than hedge funds that invest in liquid strategies. Funds in illiquid categories are also more likely to reach an optimal size in the analysis. These funds have larger market impact costs. Also, hedge funds that employ strategies with limited opportunities, such as

"Convertible arbitrage," "Event driven" and "Emerging market," are more likely to exhibit concave behavior in returns versus assets.

As can be seen in Figures 5 - 14, the performance-asset relationship is concave and reaches the optimal asset size for illiquid categories and categories with limited opportunities, such as "Convertible arbitrage" and "Emerging markets."

"Dedicated short bias", "Equity market neutral" and "Global macro" strategies that involve liquid instruments and have relatively unlimited opportunities, do not exhibit concave relationship between returns and assets under management. Interestingly, according to our analysis, it is possible to calculate an optimal size for a "Managed futures" strategy; however, the result is mainly driven by the outlier.

"Funds of funds" do exhibit a concave relationship between returns and assets; however, the fit is much better for "Convertible Arbitrage," "Emerging Markets," and all hedge funds taken together. "Funds of funds" are less likely to be affected by the diseconomies of scale than individual funds. Agarwal, Daniel, and Naik (2003) came to the same conclusion.

[INSERT FIGURES 5 - 14]

7.4 Performance Model

In order to be assured that the results for the concave relationship between returns and assets and the notion of optimal assets are not mechanical, Monte Carlo simulations of returns are performed. In the beginning of the simulation, we use 3,000 funds. The starting value for each hedge fund is $25 Million, and the cut-off liquidation value is $20 Million. Returns are normally distributed with the monthly μ=0.90% and the $\sigma = 6.58\%$. The simulation is run for 10 years.

Figure 15 shows the relationship between returns and assets using simulated data.

[INSERT FIGURE 15]

The relationship is random around the mean of 0.9%. There is no concave relationship between hedge funds returns and past assets. The proposed model takes into account only the distribution of returns and eliminates hedge funds after they reach a threshold of $20 million in assets. However, the data for the eliminated funds, as in TASS, is used in the analysis.

To make the model more realistic, we use parameters from previous sections of the paper. We adjust the performance for FAV, flows, competition and market impact.

Each month, returns are drawn from the Normal Distribution with $\mu = 0.90$ and $\sigma = 6.58$, $R^0_{i,t} = N(\mu, \sigma)$.

Flows are calculated as follows:

$$F_{i,t} = \begin{cases} 1 - exp(-r_{i,t-1}) & \text{if } r_{i,t-1} \geq 0 \\ -1 + exp(r_{i,t-1}) & \text{if } r_{i,t-1} < 0 \end{cases} \tag{18}$$

Using calculated F_t, DollarFlow, $DollF_t$ is calculated as follows:

$$F_{i,t} = \frac{DollF_{i,t}}{Assets_{i,t-1}} \tag{19}$$

Using calculated DollF and past assets, current assets are calculated using the following formula:

$$DollF_{i,t} = Assets_{i,t} - Assets_{i,t-1}(1 + R_{i,t}) \tag{20}$$

FAV, the favorable positioning metric is calculated as follows:

$$FAV_{k,t} = \frac{\sum_{i,i \in k} DollF_{i,t}}{\sum_{i,i \in POSNET} DollF_{i,t}} \tag{21}$$

POSNET is a subset of flows that are net positive during the time period t.

$$FAV_{k,t} = -\frac{\sum_{i,i \in k} DollF_{i,t}}{\sum_{i,i \in NEGNET} DollF_{i,t}} \tag{22}$$

NEGNET is a subset of flows that are net negative during the time period t. Competition, C, is measured as follows:

$$C_{k,t} = -\mu \exp^{FAV_{i,t-1}-1} \tag{23}$$

New Entry into category k is calculated as follows:

$$ENTRY_{k,t} = INTEGER(\sqrt{NUMBER_{t-1}} FAV_{t-1}) \tag{24}$$

where $NUMBER_{t-1}$ is the number of hedge funds in category k in the previous time period.

Market Impact, M, for each hedge fund i at time t is calculated as follows:

$$M_{i,t} = -0.2 \frac{Assets_{i,t-1}}{\sum_{i,i \in k} Assets_{i,t-1}} \tag{25}$$

Return is given by:

$$R_{i,t} = R^0_{i,t} + C_{k,t} + M_{i,t} \tag{26}$$

Each fund starts with $40 Million. The cut-off for hedge fund liquidation is $20 Million. There are 10 categories, and each category starts off with 100 hedge funds. The model is run for 10 years. The results are depicted in Figure 16.

[INSERT FIGURE 16]

As can be seen from this figure, the relationship between returns and assets is concave and reaches an optimal asset size. The relationship resembles the real data depicted in Figure 3.

The same model is run for 20 years, and the second 10 years of data are taken for the analysis. The results are presented in Figure 17.

[INSERT FIGURE 17]

The same model is run for 50 years, and the last 10 years of data are taken for the analysis. The results are presented in Figure 18.

[INSERT FIGURE 18]

Using these results, we can see the progression of the performance-asset size relationship over time. In the first 10 years, the number of funds is relatively small compared to the number of funds in 50 years. Therefore, the market impact, which is a function of a fraction of total assets in a category, greatly affects the performance of the funds. In 50 years, as more competitors come in, the fraction of asset size for each competitor decreases, reducing market impact. Therefore, in Figure 16 we see negative returns with high assets due to high market impact. That effect diminishes in Figure 18. The concave relationship in all figures is due to competition and market impact effects. As more new hedge funds enter, competitors have to compete for limited opportunities as well as limited capital, thus, reducing returns of the funds in that category.

8 Conclusions

The paper explores the drivers of life cycles of hedge funds. Compared to mutual funds, hedge funds have a very large probability of liquidation. The annual attrition in hedge funds due to liquidation averaged 7.10% between 1994-2002. The paper studies the impact of age, size, returns, flows, favorable positioning and competition on the life cycles of hedge funds. Performance and flows positively affect the survival probability. The relationship between performance and fund flows is studied. The piecewise linear relationship is estimated and applied to different hedge fund categories. The performance-flow relationship is positive and concave. As expected, hedge funds that follow more directional strategies are more likely to have a higher effect of past returns on future flows than funds with more event-driven strategies.

We propose that favorable positioning positively affects the survival probability of a hedge fund. Therefore, being in the right category at the right time can reduce the liquidation probability for a hedge fund, after adjusting for fund characteristics. On the other hand, competition among hedge funds in the same category greatly increases the liquidation probability of an individual hedge fund in that category. As a result, hedge fund managers might choose to stay in the category which experiences favorable positioning and less competition. However, it is shown that as the hedge fund category becomes more favorable, more hedge funds enter such a category, thus increasing the competition. As investors chase category returns, competition among hedge funds within the category increases, thus, liquidation probability of hedge funds in that category increases. Therefore, hedge fund managers should dynamically weigh the risks of being in a particular category at any time and understand the interrelationships between competition and favorable positioning. Smaller hedge funds are less likely to increase liquidation probability. We also find that as competition increases, marginal funds are more likely to be liquidated than funds that deliver superior risk-adjusted returns.

Past asset size also impacts current hedge fund returns that in turn affect the life cycle of hedge funds. The relationship between current performance and past asset size is positive and concave. It is possible to obtain an optimal asset size by optimizing returns. Therefore, hedge fund investors should be wary of hedge fund asset size before investing, and try to invest in a hedge fund that is near its optimal size. Hedge fund managers might be more inclined to increase the asset base, thereby increasing the fees. Therefore, it is in the best interest of an investor to choose hedge fund strategies that do not have asset size higher than the optimum. The performance-asset size relationship takes on different functional forms for different categories. The relationship is concave and the optimal size can be obtained for more illiquid categories such as "Emerging markets" and "Convertible arbitrage." These hedge fund categories experience high market impact and are subject to limited opportunities. 'Funds of funds" are less likely to be affected by the diseconomies of scale than individual funds. Hedge fund managers with high asset sizes might choose to close the fund to new investors before facing a decrease in returns and an increase in liquidation probabilities.

In order to understand the life cycles of hedge funds, it is important to understand the interrelationships of fund characteristics – flows, returns, asset size and age – and industry characteristics – favorable positioning and competition –. For hedge fund managers, the benefit of this approach will be an improved understanding of the effects of survival probabilities. For hedge fund investors, the benefit will be an improved understanding of investment opportunities. Next step would be to propose theoretical models for the optimal dynamic strategies for hedge fund managers and investors given the factors outlined in the paper. This is the focus of our future research.

A Appendix

A.1 Tables and Figures

Code	Category	Number of Funds In:		
		Combined	Successful	Liquidated
1	Convertible Arbitrage	143	117	26
2	Dedicated Short Bias	23	15	8
3	Emerging Markets	219	107	112
4	Equity Market Neutral	190	136	54
5	Event Driven	304	227	77
6	Fixed Income Arbitrage	138	88	50
7	Global Macro	181	84	97
8	Long/Short Equity	1139	779	360
9	Managed Futures	421	164	257
10	Fund of Funds	657	458	199
11	Other	86	62	24
	All	3501	2237	1264

Table A.1: This table presents the number of funds in the TASS Hedge Fund Combined, Successful and Liquidated databases during the period from January 1994 to December 2002.

Year	Existing Funds	New Entries	New Exits	Intrayear Entry/Exit	Total Funds	Attrition Rate (%)
1977	0	3	0	0	3	0.0
1978	3	2	0	0	5	0.0
1979	5	2	0	0	7	0.0
1980	7	3	0	0	10	0.0
1981	10	3	0	0	13	0.0
1982	13	4	0	0	17	0.0
1983	17	8	0	0	25	0.0
1984	25	14	0	0	39	0.0
1985	39	9	0	0	48	0.0
1986	48	21	0	0	69	0.0
1987	69	27	0	0	96	0.0
1988	96	31	0	0	127	0.0
1989	127	43	0	0	170	0.0
1990	170	102	0	0	272	0.0
1991	272	86	0	0	358	0.0
1992	358	155	0	0	513	0.0
1993	513	230	0	0	743	0.0
1994	743	255	19	1	998	2.6
1995	998	289	60	1	1287	6.0
1996	1287	310	119	9	1597	9.3
1997	1597	349	93	6	1946	5.8
1998	1946	325	162	9	2271	8.3
1999	2271	363	183	7	2634	8.1
2000	2634	330	231	9	2964	8.8
2001	2964	355	260	5	3319	8.8
2002	3319	246	275	12	3565	8.3

Table A.2: This table presents the annual number of entries into and exits out of the TASS Hedge Fund Database from January 1994 to December 2002.

Fund Characteristics	Hedge Funds		Funds of Funds	
	Mean	Median	Mean	Median
Monthly Return %	0.90	0.79	0.59	0.59
Standard Deviation of Monthly Returns %	6.58	–	3.68	–
Age (Months)	46.36	35.00	52.53	42.00
Assets (Million US Dollars)	81.77	20.00	67.59	18.39
Favorable Positioning (FAV)(per quarter)	0.07	0.09	0.03	0.13
Standard Deviation of Favorable Positioning (FAV)(per quarter)	0.30	–	0.23	–
NUMBER (per quarter)	307.11	228.00	332.75	369.00
Standard Deviation of NUMBER (per quarter)	223.32	–	64.96	–

Table A.3: This table shows median and average monthly returns, standard deviation of monthly returns, age, assets under management, favorable positioning (FAV) metric, standard deviation for the FAV, the number of competitors in a particular category (NUMBER), and the standard deviation for the number of competitors in a particular category for single-manager hedge funds and for funds of funds. The statistics are calculated using the TASS Database from January 1994 to December 2002.

Parameter	Estimate	t-Statistic
$\alpha_{i,t}$	0.679*	8.05
$Return_{i,t}$	0.234*	5.82
$Trank^1_{i,t-1} - BottomTercile$	0.293*	5.46
$Trank^2_{i,t-1} - MiddleTercile$	0.269*	5.51
$Trank^3_{i,t-1} - TopTercile$	0.006	0.08
$Flow_{i,t-1}$	0.048*	4.56
$Size_{i,t-1}$	−0.041*	−8.32
$Age_{i,t-1}$	−0.001*	−9.71
$\sigma_{i,t-1}$	−0.002	−1.19

Table A.4: This table reports the Fama-MacBeth results of regression of current flows $Flow_{i,t}$ on Bottom, Middle and Top Terciles of past quarter performance, past flows $Flow_{i,t-1}$, last quarter's asset size $Size_{i,t-1}$, past age $Age_{i,t-1}$, last quarter's standard deviation of monthly returns $\sigma_{i,t-1}$ and categories of hedge funds. Funds of funds are omitted from the model. The R^2 for the regression is 5.53%. Quarterly data from 1994 to 2002 is used. The regression is adjusted for category effects. Figures with * are significant at 5% level.

Category	$\alpha_{i,t}$		$Return_{i,t}$		$Trank^1_{i,t-1}$		$Trank^2_{i,t-1}$		$Trank^3_{i,t-1}$		$Flow_{i,t-1}$		$Size_{i,t-1}$		$Age_{i,t-1}$		$\sigma_{i,t-1}$	
	Estimate	SE	Estimate	SE	Estimate	SE	Estimate	SE	Estimate	SE	Estimate	SE	Estimate	SE	Estimate	SE	Estimate	SE
Convertible Arbitrage	0.695	0.341	1.417	0.593	0.350	0.140	0.208	0.169	0.185	0.202	0.056	0.042	−0.042	0.020	−0.002	0.001	0.002	0.015
Dedicated Short Bias	-0.304	0.333	0.383	0.657	0.783	0.628	0.472	0.229	0.189	0.157	0.605	0.183	0.004	0.016	−0.001	0.001	−0.031	0.018
Emerging Markets	0.242	0.172	0.227	0.114	0.406	0.099	−0.013	0.071	0.082	0.087	0.095	0.050	−0.018	0.010	−0.001	0.000	−0.002	0.003
Equity Market Neutral	1.333	0.382	0.024	0.593	0.431	0.222	0.189	0.210	0.609	0.364	0.086	0.057	−0.077	0.021	−0.002	0.001	−0.017	0.012
Event Driven	0.876	0.382	0.165	0.244	0.458	0.138	−0.038	0.078	0.230	0.128	0.076	0.032	−0.051	0.023	−0.001	0.000	−0.011	0.003
Fixed Income Arbitrage	1.041	0.565	0.121	0.256	0.470	0.284	0.357	0.247	0.347	0.370	−0.165	0.260	−0.065	0.032	−0.001	0.000	−0.008	0.011
Global Macro	0.360	0.173	0.132	0.146	0.134	0.114	0.243	0.166	0.174	0.166	0.176	0.063	−0.021	0.009	−0.001	0.000	−0.007	0.005
Long/Short Equity	0.575	0.1112	0.225	0.052	0.297	0.062	0.256	0.086	−0.001	0.092	0.120	0.019	−0.036	0.006	−0.001	0.000	−0.004	0.001
Managed Futures	0.411	0.147	0.186	0.089	0.371	0.242	0.259	0.130	−0.116	0.208	0.136	0.033	−0.034	0.013	−0.001	0.000	0.008	0.006
Fund of Funds	0.262	0.082	0.190	0.094	0.282	0.075	0.049	0.046	0.176	0.053	0.198	0.035	−0.018	0.005	−0.001	0.000	−0.008	0.002
Other	1.308	0.602	0.937	0.747	0.346	0.690	0.031	0.219	0.262	0.242	0.210	0.131	−0.079	0.035	−0.002	0.002	−0.017	0.018

Table A.5: This table reports the Fama-MacBeth results of regression of current flows $Flow_{i,t}$ on Bottom, Middle and Top Terciles of past quarterly performance, past flows $Flow_{i,t-1}$, last quarter's asset size $Size_{i,t-1}$, past age $Age_{i,t-1}$, and last quarter's standard deviation of monthly returns $\sigma_{i,t-1}$ for 11 hedge fund categories.

Code	Category	Estimate
1	Convertible Arbitrage	0.21
2	Dedicated Short Bias	0.47
3	Emerging Markets	-0.01
4	Equity Market Neutral	0.19
5	Event Driven	-0.04
6	Fixed Income Arbitrage	0.36
7	Global Macro	0.24
8	Long/Short Equity	0.26
9	Managed Futures	0.26
10	Fund of Funds	0.05
11	Other	0.03
	All	0.27

Table A.6: This table presents the estimates of the Middle Tercile of past quarterly returns for 11 categories.

Parameter	Estimate	Standard Error	Wald Chi-Square	Pr > Chi-Square
$\alpha_{i,t}$	-1.823^*	0.586	9.670	0.002
$Age_{i,t}$	0.001	0.001	0.028	0.867
$r_{i,t-1}$	-2.677^*	0.356	56.659	< 0.001
$r_{i,t}$	-3.362^*	0.330	103.774	< 0.001
$Flow_{i,t-1}$	-0.790^*	0.192	16.991	< 0.001
$Assets_{i,t-1}$	-0.399^*	0.027	224.442	< 0.001

Table A.7: This table reports the results of logit regression of probability of liquidation of a hedge fund at time t on intercept, current age ($Age_{i,t}$), previous quarterly return on investments ($r_{i,t-1}$), current quarterly return ($r_{i,t}$), previous quarterly flow into the fund ($Flow_{i,t-1}$) and previous assets under management $Assets_{i,t-1}$. The R^2 is 13.62%. The model adjusts for time and category effects. Figures with * are significant to 5% level.

Parameter	Estimate	Standard Error	Wald Chi-Square	Pr > Chi-Square
$\alpha_{i,t}$	-5.103^*	0.411	154.568	< 0.001
$Age_{i,t}$	0.010	0.059	0.028	0.867
$r_{i,t-1}$	-0.076^*	0.010	56.659	< 0.001
$r_{i,t}$	-0.090^*	0.009	103.774	< 0.001
$Flow_{i,t-1}$	-0.105^*	0.025	16.991	< 0.001
$Assets_{i,t-1}$	-6.666^*	0.027	224.442	< 0.001

Table A.8: This table reports the results of logit regression of probability of liquidation of a hedge fund at time t on intercept, current age ($Age_{i,t}$), previous quarterly return on investments ($r_{i,t-1}$), current quarterly return ($r_{i,t}$), previous quarterly flow into the fund ($Flow_{i,t-1}$) and previous assets under management $Assets_{i,t-1}$. All variables are adjusted for the mean. The R^2 is 13.62%. The model adjusts for time and category effects. Figures with * are significant to 5% level.

Test	Chi-Square	Degrees of Freedom	Pr > Chi-Square
Likelihood Ratio	739.795	53	< 0.001
Score	801.630	53	< 0.001
Wald	687.167	53	< 0.001

Table A.9: This table reports the results of a test for global null hypothesis that BETA=0 for logit regression of probability of liquidation of a hedge fund at time t on intercept, current age ($Age_{i,t}$), previous quarterly return on investments ($r_{i,t-1}$), current quarterly return ($r_{i,t}$), previous quarterly flow into the fund ($Flow_{i,t-1}$) and previous assets under management $Assets_{i,t-1}$. The model adjusts for time and category effects.

Parameter	Estimate	t-Statistic
$\alpha_{k,t}$	-0.041	-1.16
$r_{k,t-1}$	1.357	1.85

Table A.10: This table reports Fama-MacBeth regression of favorable positioning (FAV) on past quarter category return $r_{k,t-1}$. The category return is value-weighted. The R^2 for this regression is 28.46%.

Parameter	Estimate	Standard Error	Wald Chi-Square	Pr > Chi-Square
$\alpha_{i,t}$	1.631*	0.590	7.649	0.006
$Age_{i,t}$	0.002	0.001	2.170	0.141
$r_{i,t-1}$	−2.685*	0.357	56.459	< 0.001
$r_{i,t}$	−3.440*	0.331	108.031	< 0.001
$FAV_{k,t-1}$	−0.502*	0.183	7.501	0.006
$Assets_{i,t-1}$	−0.405*	0.026	238.018	< 0.001

Table A.11: This table reports the results of logit regression of probability of liquidation of a hedge fund at time t on intercept, current age ($Age_{i,t}$), previous quarterly return on investments ($r_{i,t-1}$), current quarterly return ($r_{i,t}$), previous favorable positioning of the hedge fund category compared to other categories in the database ($FAV_{k,t-1}$) and previous assets under management $Assets_{i,t-1}$. The R^2 for the regression is 13.24%. The model adjusts for time and category effects. Figures with * are significant to 5% level.

Test	Chi-Square	Degrees of Freedom	Pr > Chi-Square
Likelihood Ratio	727.056	53	< 0.001
Score	811.074	53	< 0.001
Wald	682.541	53	< 0.001

Table A.12: This table reports the results of a test for global null hypothesis that BETA=0 for logit regression of probability of liquidation of a hedge fund at time t on intercept, current age ($Age_{i,t}$), previous quarter return on investments ($r_{i,t-1}$), current quarterly return ($r_{i,t}$), previous favorable positioning of the hedge fund category compared to other categories in the database ($FAV_{k,t-1}$) and previous assets under management $Assets_{i,t-1}$. The model adjusts for time and category effects.

Parameter	Estimate	Standard Error	Wald Chi-Square	Pr > Chi-Square
$\alpha_{i,t}$	1.737*	0.588	8.734	0.003
$Age_{i,t}$	0.002	0.001	2.560	0.110
$r_{i,t-1}$	−2.593*	0.360	51.999	< 0.001
$r_{i,t}$	−3.396*	0.332	104.420	< 0.001
$Number_{k,t-4}$	0.002*	0.001	6.945	0.008
$Assets_{i,t-1}$	−0.412*	0.027	241.442	< 0.001

Table A.13: This table reports the results of logit regression of probability of liquidation of a hedge fund at time t on intercept, current age ($Age_{i,t}$), previous quarterly return on investments ($r_{i,t-1}$), current quarterly return ($r_{i,t}$), the number of hedge funds in a particular category a year before ($Number_{k,t-4}$) and previous assets under management $Assets_{i,t-1}$. The R^2 is 12.30%. The model adjusts for time and category effects. Figures with * are significant to 5% level.

Test	Chi-Square	Degrees of Freedom	Pr > Chi-Square
Likelihood Ratio	653.078	46	< 0.001
Score	754.083	46	< 0.001
Wald	666.474	46	< 0.001

Table A.14: This table reports the results of a test for global null hypothesis that BETA=0 for logit regression of probability of liquidation of a hedge fund at time t on intercept, current age ($Age_{i,t}$), previous quarterly return on investments ($r_{i,t-1}$), current quarterly return ($r_{i,t}$), the number of hedge funds in a particular category a year before ($Number_{k,t-4}$) and previous assets under management $Assets_{i,t-1}$. The model adjusts for time and category effects.

Parameter	Estimate	Standard Error	Wald Chi-Square	Pr > Chi-Square
$\alpha_{i,t}$	1.579*	0.596	7.028	0.008
$Age_{i,t}$	0.002	0.001	2.555	0.110
$r_{i,t-1}$	-2.593*	0.360	51.932	< 0.001
$r_{i,t}$	-3.400*	0.332	104.750	< 0.001
$Number_{k,t-4}$	0.001*	0.000	4.267	0.039
$FAV_{k,t-1}$	-0.416*	0.192	4.674	0.031
$Assets_{i,t-1}$	-0.411*	0.027	240.513	< 0.001

Table A.15: This table reports the results of logit regression of probability of liquidation of a hedge fund at time t on intercept, current age ($Age_{i,t}$), previous quarterly return on investments ($r_{i,t-1}$), current quarterly return ($r_{i,t}$), the number of hedge funds in a particular category a year before ($Number_{k,t-4}$), previous favorable positioning of the hedge fund category compared to other categories in the database ($FAV_{k,t-1}$) and previous assets under management $Assets_{i,t-1}$. The R^2 is 12.38%. The model adjusts for time and category effects. Figures with * are significant to 5% level.

Parameter	Estimate	Standard Error	Wald Chi-Square	Pr > Chi-Square
$\alpha_{i,t}$	-4.979^*	0.447	124.118	< 0.001
$Age_{i,t}$	0.090	0.057	2.555	0.110
$r_{i,t-1}$	-0.074^*	0.010	51.932	< 0.001
$r_{i,t}$	-0.091^*	0.009	104.750	< 0.001
$Number_{k,t-4}$	0.416^*	0.202	4.267	0.039
$FAV_{k,t-1}$	-0.027^*	0.013	4.674	0.031
$Assets_{i,t-1}$	-6.872^*	0.443	240.513	< 0.001

Table A.16: This table reports the results of logit regression of probability of liquidation of a hedge fund at time t on intercept, current age ($Age_{i,t}$), previous quarterly return on investments ($r_{i,t-1}$), current quarterly return ($r_{i,t}$), the number of hedge funds in a particular category a year before ($Number_{k,t-4}$), previous favorable positioning of the hedge fund category compared to other categories in the database ($FAV_{k,t-1}$) and previous assets under management $Assets_{i,t-1}$. All variables are adjusted for the mean. The R^2 is 12.38%. The model adjusts for time and category effects. Figures with * are significant to 5% level.

Test	Chi-Square	Degrees of Freedom	Pr > Chi-Square
Likelihood Ratio	657.	74647	< 0.001
Score	756.373	47	< 0.001
Wald	671.032	47	< 0.001

Table A.17: This table reports the results of a test for global null hypothesis that BETA=0 for logit regression of probability of liquidation of a hedge fund at time t on intercept, current age ($Age_{i,t}$), previous quarterly return on investments ($r_{i,t-1}$), current quarterly return ($r_{i,t}$), the number of hedge funds in a particular category a year before ($Number_{k,t-4}$), previous favorable positioning of the hedge fund category compared to other categories in the database ($FAV_{k,t-1}$) and previous assets under management $Assets_{i,t-1}$. The model adjusts for time and category effects.

Variable	$TOTLIQ_{i,t}$	$Age_{i,t}$	$r_{i,t-1}$	$r_{i,t}$	$Flow_{i,t-1}$	$Flow_{i,t}$	$FAV_{k,t-1}$	$Assets_{i,t-1}$	$Number_{k,t-4}$
$TOTLIQ_{i,t}$	1.000	0.022	−0.082	−0.056	−0.075	−0.049	−0.022	−0.102	0.001
$Age_{i,t}$	0.022	1.000	−0.083	−0.077	−0.249	−0.243	−0.043	0.281	0.026
$r_{i,t-1}$	−0.082	−0.083	1.000	0.078	0.084	0.211	0.010	0.024	−0.047
$r_{i,t}$	−0.056	−0.077	0.098	1.000	0.023	0.083	−0.0145	−0.028	−0.060
$Flow_{i,t-1}$	−0.075	−0.249	0.084	0.023	1.000	0.355	0.151	0.092	0.019
$Flow_{i,t}$	−0.049	−0.243	0.211	0.083	0.355	1.000	0.100	−0.024	0.011
$FAV_{k,t-1}$	−0.022	−0.043	0.010	−0.015	0.151	0.100	1.000	0.104	0.359
$Assets_{i,t-1}$	−0.102	0.281	0.024	−0.028	0.092	−0.024	0.104	1.000	0.012
$Number_{k,t-4}$	0.001	0.026	−0.047	−0.060	0.019	0.011	0.359	0.012	1.000

Table A.18: This table reports the results for quarterly Spearman correlation of $TOTLIQ_{i,t}$, $Age_{i,t}$, $r_{i,t-1}$, $r_{i,t}$, $Flow_{i,t-1}$, $Flow_{i,t}$, $FAV_{k,t-1}$, $Assets_{i,t-1}$ and $Number_{k,t-4}$.

Parameter	Estimate	Standard Error	Wald Chi-Square	Pr > Chi-Square
$\alpha_{i,t}$	2.017*	0.592	11.607	< 0.001
$Age_{i,t}$	0.002	0.001	1.832	0.176
$r_{i,t-1}$	−2.413*	0.362	44.446	< 0.001
$r_{i,t}$	−3.344*	0.339	97.335	< 0.001
$Number_{k,t-1}$	0.019*	0.002	132.944	< 0.001
$Frac_{k,t-1}$	−0.660*	0.047	201.447	< 0.001
$Assets_{i,t-1}$	−0.388*	0.027	209.622	< 0.001

Table A.19: This table reports the results of logit regression of probability of liquidation of a hedge fund at time t on intercept, current age ($Age_{i,t}$), previous quarterly return on investments ($r_{i,t-1}$), current quarterly return ($r_{i,t}$),the number of hedge funds in a particular category a quarter before ($Number_{k,t-1}$), the fraction of the number of hedge funds in a category divided by the total assets under management in that category ($Frac_{k,t-1}$) and previous assets under management $Assets_{i,t-1}$. The R^2 is 18.65%. The model adjusts for time and category effects. Figures with * are significant to 5% level.

Parameter	Estimate	Standard Error	Wald Chi-Square	Pr > Chi-Square
$r_{i,t-1}$ - Comp 1	−0.029	0.026	1.203	0.273
$r_{i,t-1}$ - Comp 2	−0.063*	0.022	8.021	0.005
$r_{i,t-1}$ - Comp 3	−0.066*	0.020	11.416	0.001
$r_{i,t-1}$ - Comp 4	−0.089*	0.026	11.910	0.001
$r_{i,t-1}$ - Comp 5	−0.115*	0.021	31.308	< 0.001
$r_{i,t-1}$ - Comp 1-5	−0.074*	0.010	51.932	< 0.001

Table A.20: This table reports the results of logit regressions of probability of liquidation of a hedge fund at time t on intercept, current age ($Age_{i,t}$), previous quarterly return on investments ($r_{i,t-1}$), current quarterly return ($r_{i,t}$), the number of hedge funds in a particular category a year before ($Number_{k,t-4}$), favorable positioning (FAV) and previous assets under management $Assets_{i,t-1}$ for 5 different terciles of competition, where Comp 1 represents the lowest tercile, and Comp 5 represents the highest tercile. All variables are adjusted for the mean. The coefficients on the past return are shown. The models adjust for time and category effects. Figures with * are significant to 5% level.

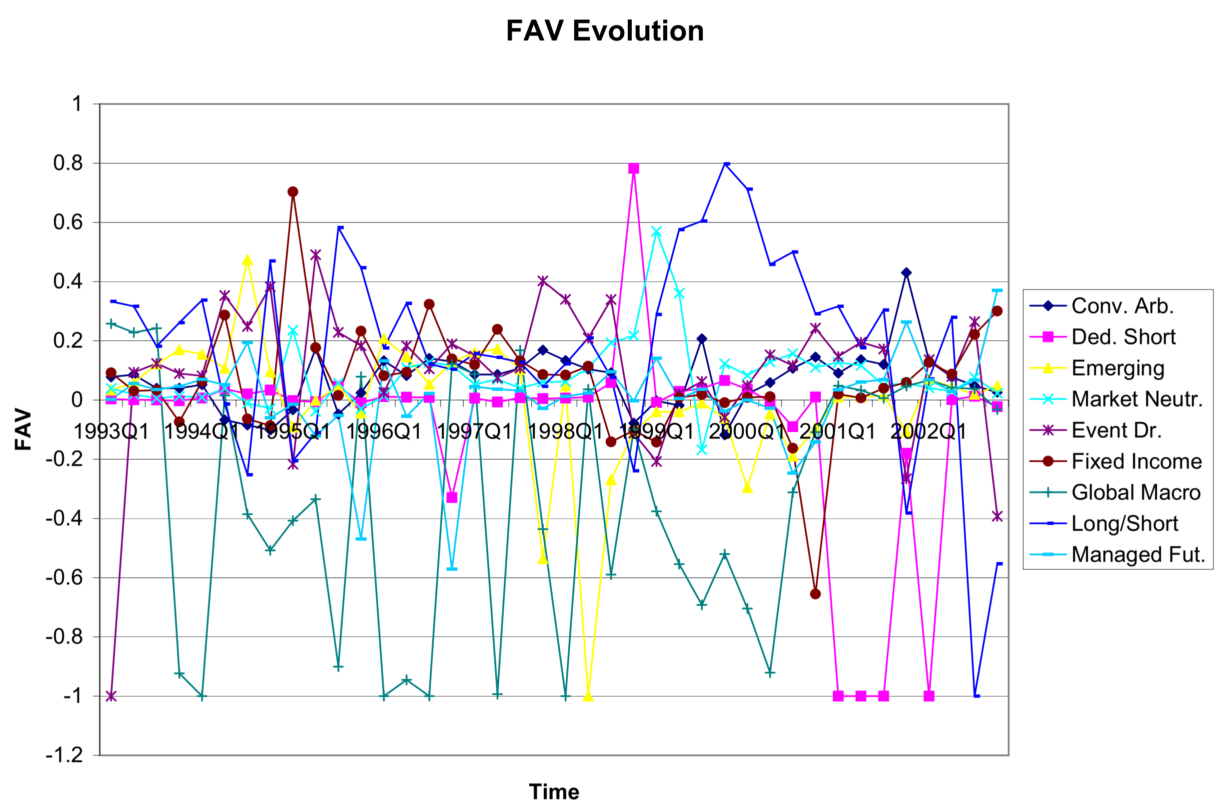

Figure A.1. This figure depicts FAV metric over time for 11 categories.

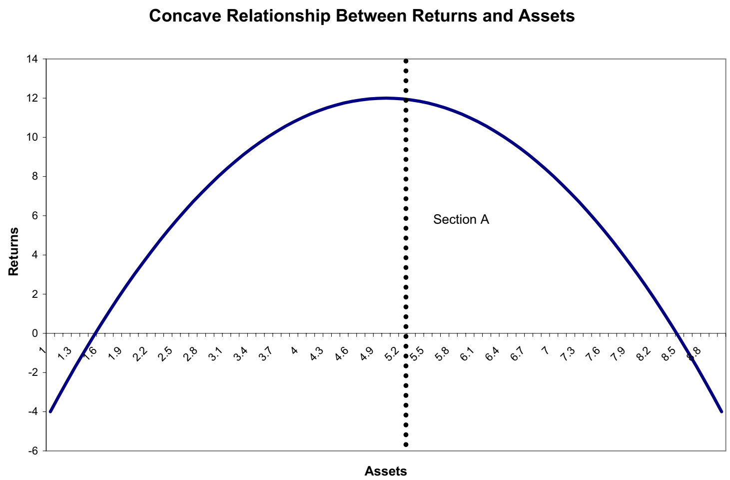

Figure 1: This figure shows that if the data are not adjusted for the self-selection bias, then the section A is going to be eliminated from the analysis.

Parameter	Estimate	t-Statistic
$\alpha_{i,t}$	-0.911	-1.02
$Assets_{i,t-1}$	0.290^{*}	2.69
$Assets^2_{i,t-1}$	-0.011^{*}	-3.39

Table A.21: This table reports the results of linear regression of a monthly return of a hedge fund on $Assets_{i,t-1}$ and $Assets^2_{i,t-1}$. Note, assets are measured in natural logarithm quantities. The R^2 for this regression is 0.05%. Figures with * are significant at 5% level.

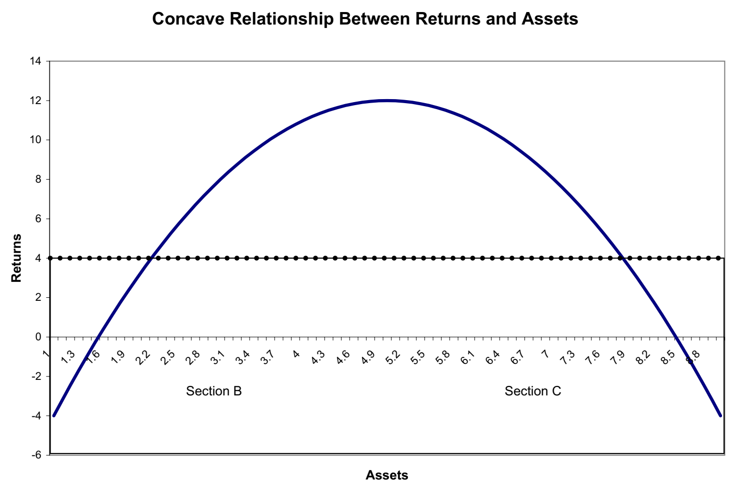

Figure 2: This figure shows that if the data are not adjusted for the termination bias, then the sections B and C are going to be eliminated from the analysis.

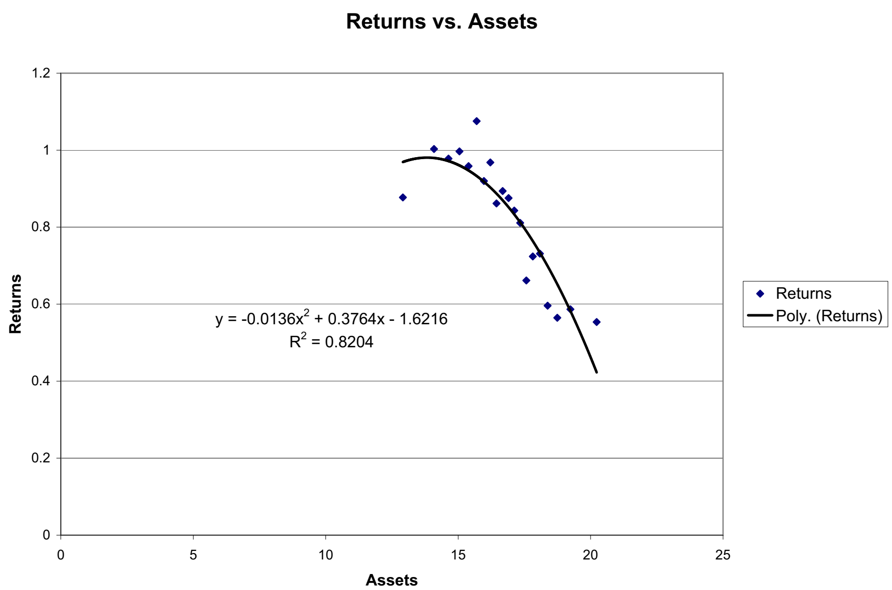

Figure 3: This figure shows the relationship between returns and assets for all hedge funds. The data are adjusted for termination and self-selection biases.

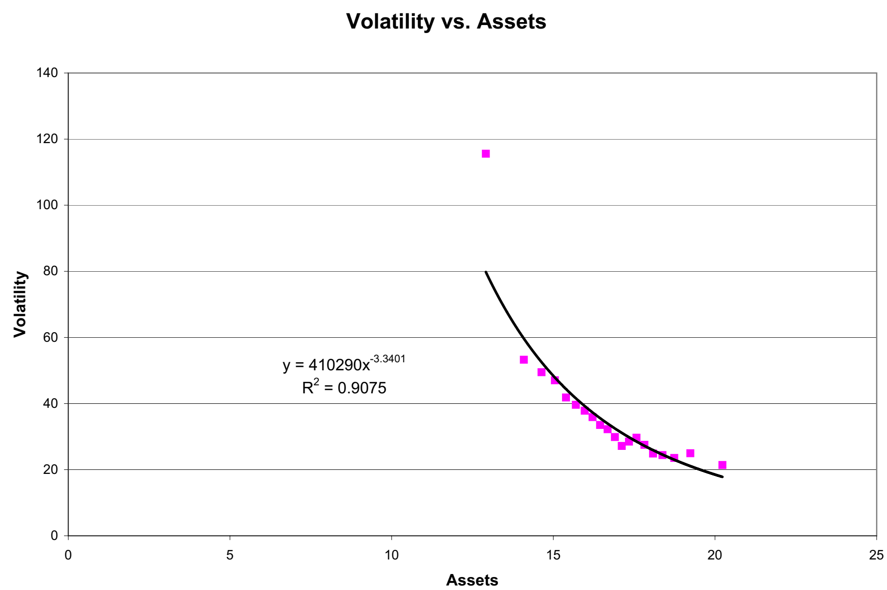

Figure 4: This figure shows the relationship between volatility of returns and assets for all hedge funds. The data are adjusted for termination and self-selection biases.

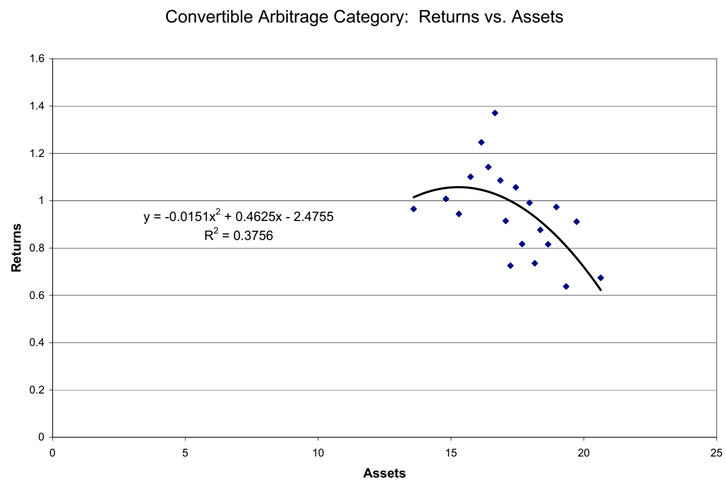

Figure 5: Returns vs. Assets relationship for the Convertible Arbitrage Category.

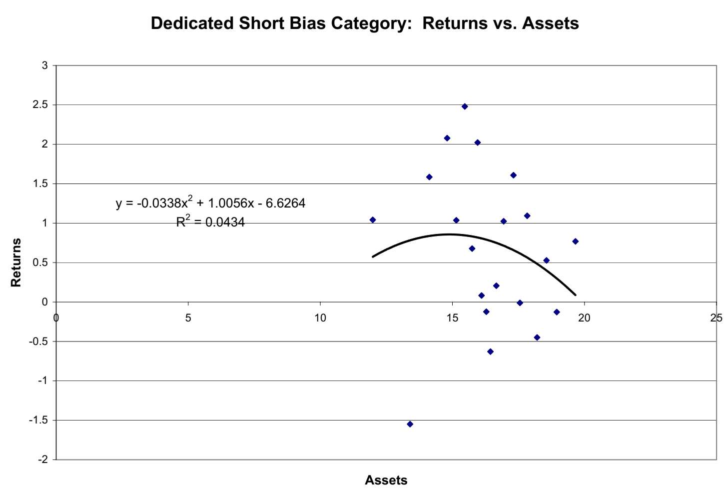

Figure 6: Returns vs. Assets relationship for the Dedicated Short Bias Category.

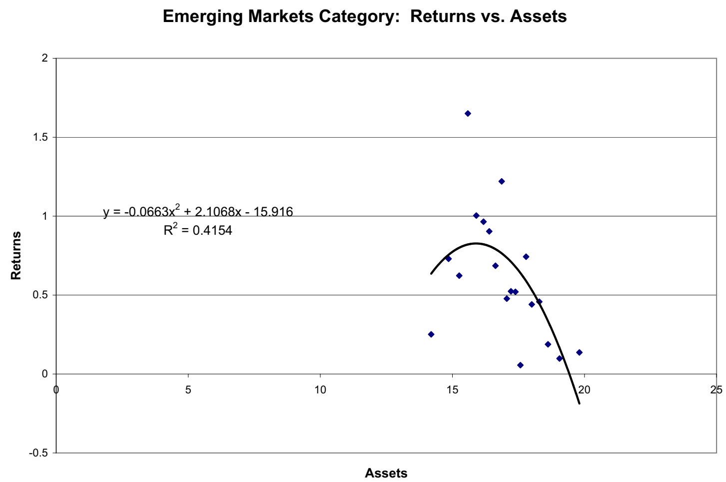

Figure 7: Returns vs. Assets relationship for the Emerging Markets Category. Note, the outlier (Assets=13.16, Returns=2.02%) is taken out of the picture.

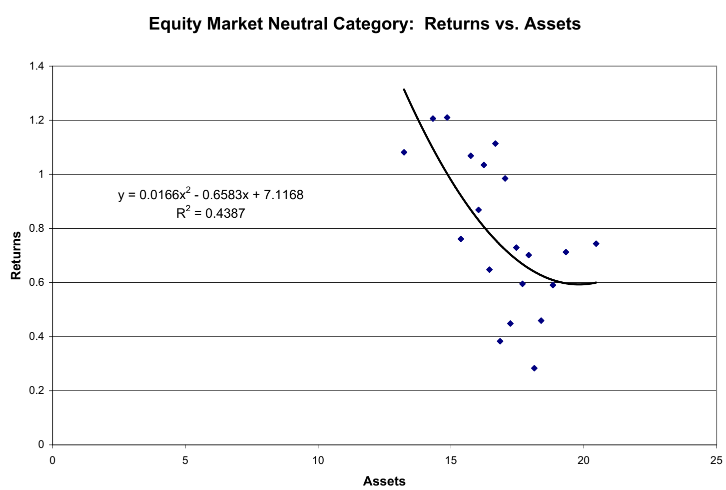

Figure 8: Returns vs. Assets relationship for the Equity Market Neutral Category.

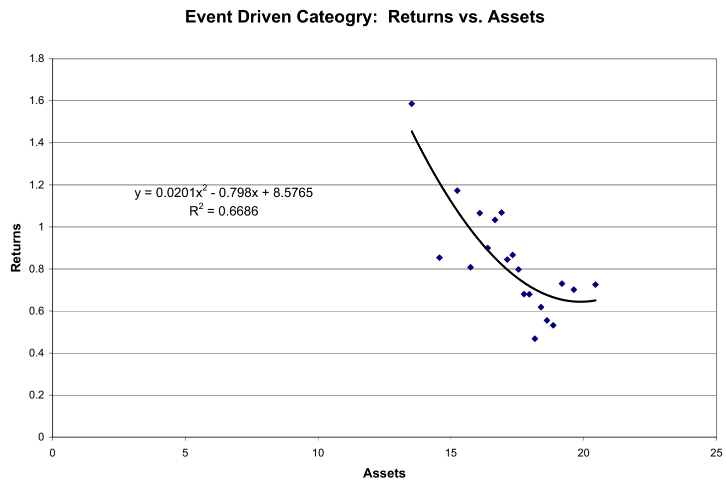

Figure 9: Returns vs. Assets relationship for the Event Driven Category.

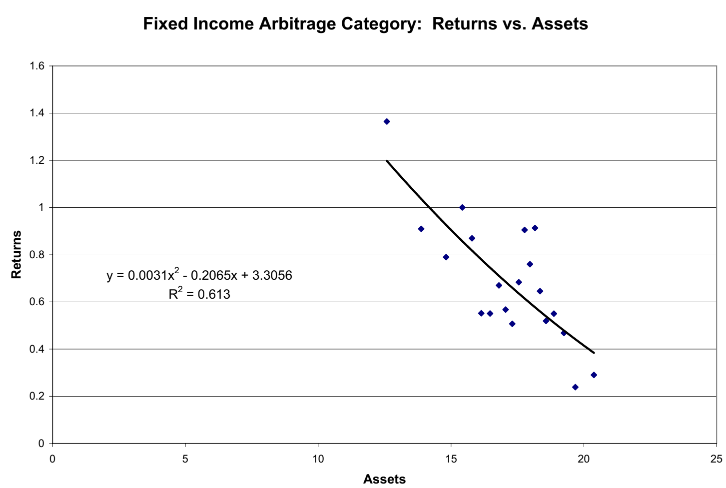

Figure 10: Returns vs. Assets relationship for the Fixed Income Arbitrage Category.

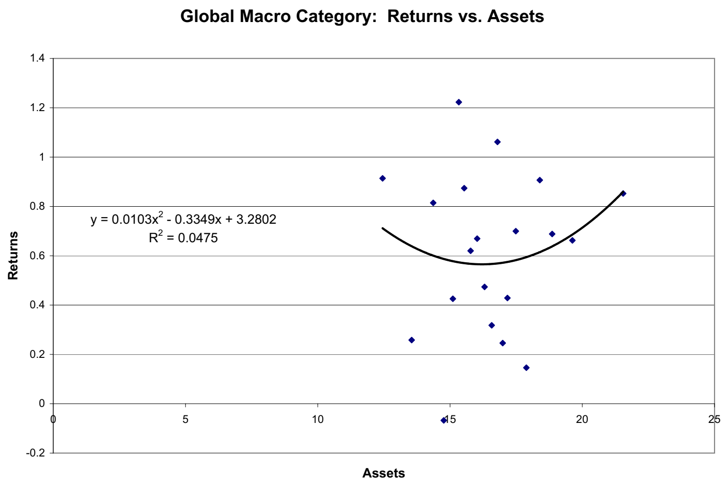

Figure 11: Returns vs. Assets relationship for the Global Macro Category.

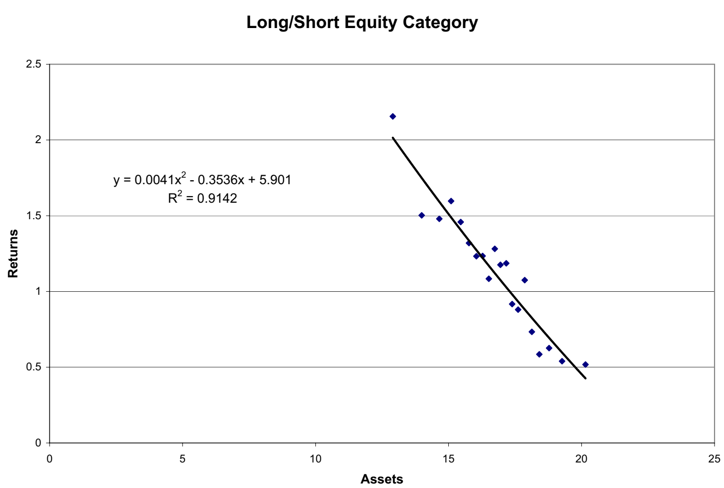

Figure 12: Returns vs. Assets relationship for the Long/Short Equity Category.

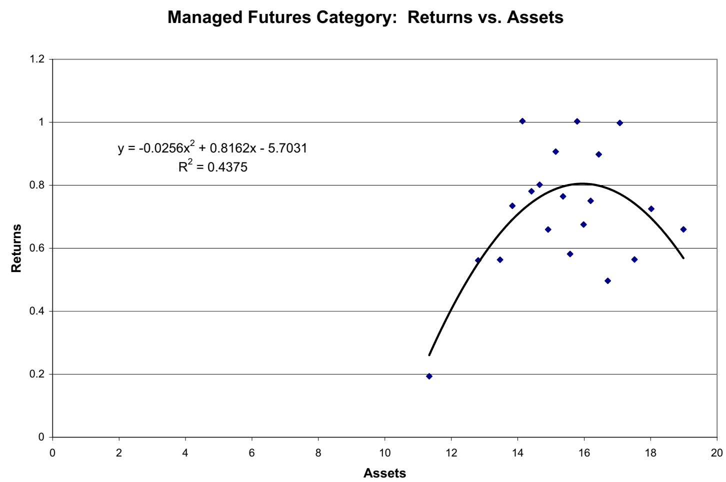

Figure 13: Returns vs. Assets relationship for the Managed Futures Category.

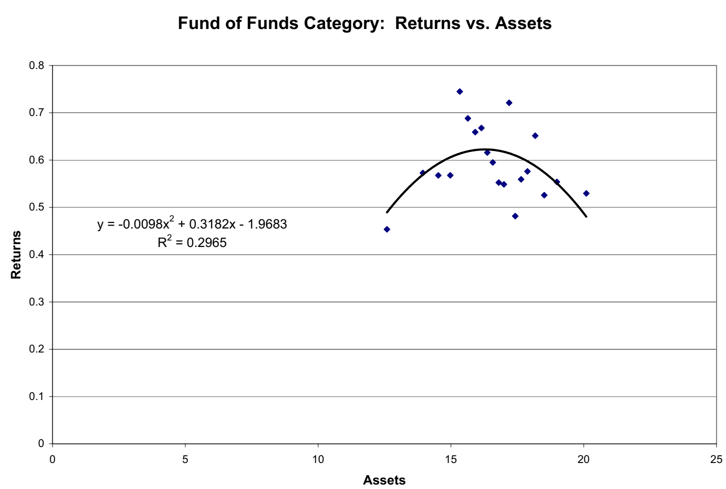

Figure 14: Returns vs. Assets relationship for the Fund of Funds Category.

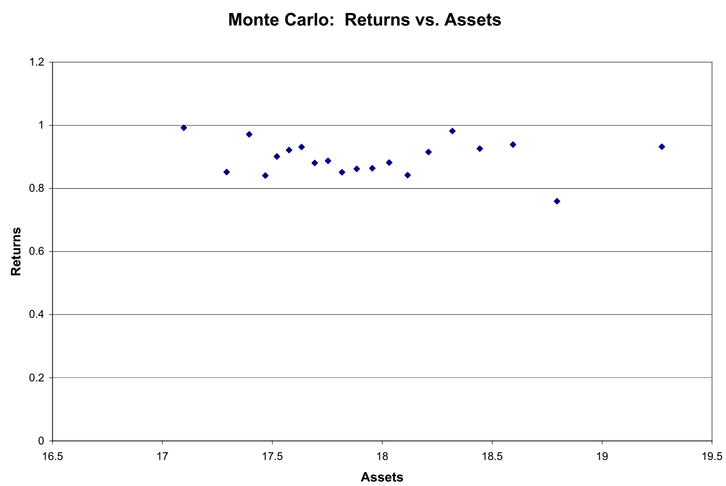

Figure 15: This figure shows the relationship between returns and assets using simulated data. In the beginning of the simulation, 3,000 funds enter. The starting value is \$40 Million, and the cut-off liquidation value is \$20 Million. Returns are normally distributed with the monthly μ=0.90% and the σ = 6.58%. The Monte Carlo simulation is run for 10 years (120 months).

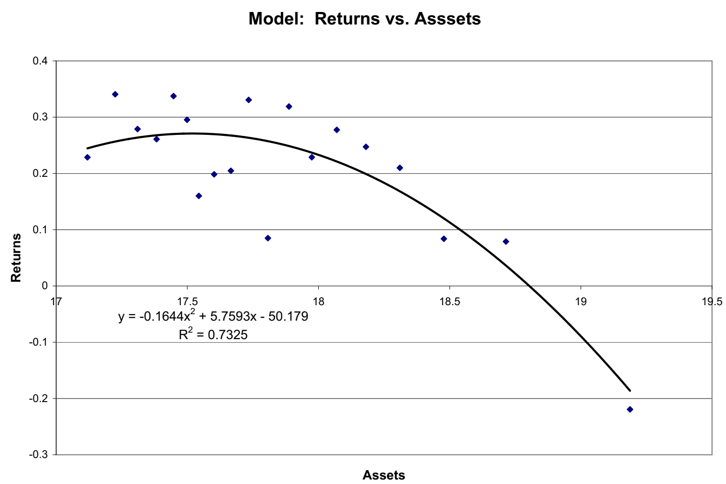

Figure 16: This figure shows the relationship between returns and assets using simulated data. Each month, returns are drawn from the Normal Distribution with the monthly μ=0.90% and the σ = 6.58%. In the beginning of the simulation, 100 funds enter into each of 10 categories. The starting value is \$40 Million, and the cut-off liquidation value is \$20 Million. New funds are allowed to enter during the simulation. Returns are adjusted for competition between categories, favorable positioning and market impact. The simulation is run for 10 years.

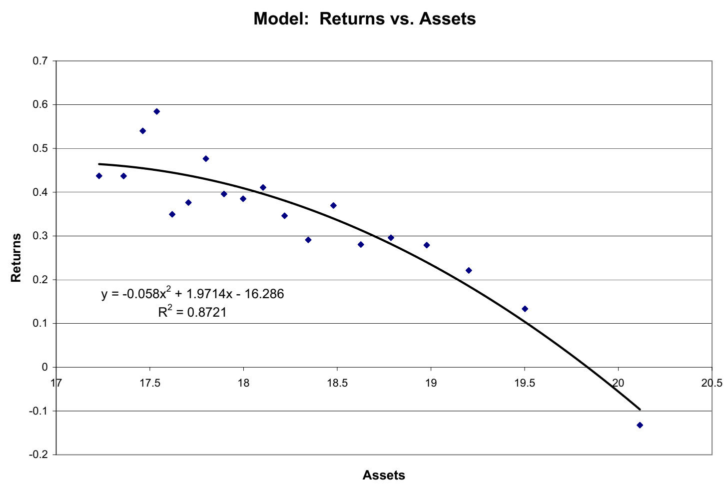

Figure 17: This figure shows the relationship between returns and assets using simulated data. Each month, returns are drawn from the Normal Distribution with μ=0.90% and the σ = 6.58%. In the beginning of the simulation, 100 funds enter into each of 10 categories. The starting value is \$40 Million, and the cut-off liquidation value is \$20 Million. New funds are allowed to enter during the simulation. Returns are adjusted for competition between categories, favorable positioning and market impact. The simulation is run for 20 years, and the last 10 years are taken for the analysis.

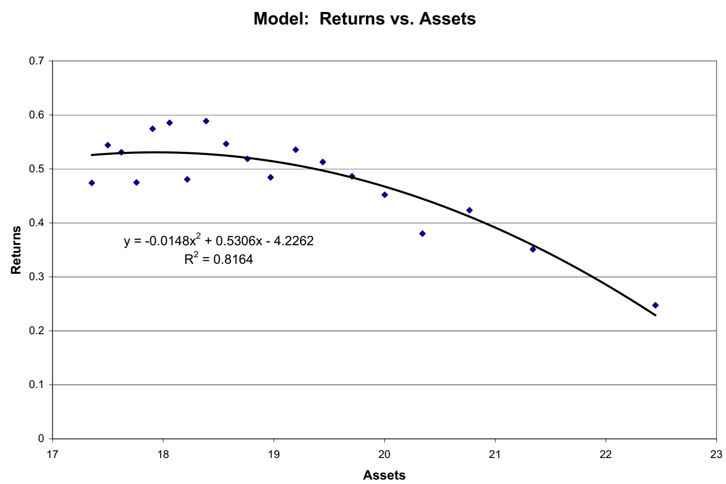

Figure 18: This figure shows the relationship between returns and assets using simulated data. Each month, returns are drawn from the Normal Distribution with μ=0.90% and the σ = 6.58%. In the beginning of the simulation, 100 funds enter into each of 10 categories. The starting value is $40 Million, and the cut-off liquidation value is $20 Million. New funds are allowed to enter during the simulation. Returns are adjusted for competition between categories, favorable positioning and market impact. The simulation is run for 50 years, and the last 10 years are taken for the analysis.

A.2 TASS Fund Category Definitions

The following is a list of category descriptions, taken directly from TASS documentation, that defines the criteria used by TASS in assigning funds in their database to one of 11 possible categories:

Convertible Arbitrage This strategy is identified by hedge investing in the convertible securities of a company. A typical investment is to be long the convertible bond and short the common stock of the same company. Positions are designed to generate profits from the fixed income security as well as the short sale of stock, while protecting principal from market moves.

Dedicated Short Bias Short biased managers take short positions in mostly equities and derivatives. The short bias of a manager's portfolio must be constantly greater than zero to be classified in this category.

Emerging Markets This strategy involves equity or fixed income investing in emerging markets around the world. As many emerging markets do not allow short selling, nor offer viable futures or other derivative products with which to hedge, emerging market investing often employs a long-only strategy.

Equity Market Neutral This investment strategy is designed to exploit equity and/or fixed income market inefficiencies and usually involves being simultaneously long and short matched market portfolios of the same size within a country. Market neutral portfolios are designed to be either beta or currency neutral, or both.

Event Driven This strategy is defined as 'special situations' investing designed to capture price movement generated by a significant pending corporate event such as a merger, corporate restructuring, liquidation, bankruptcy or reorganization. There are three popular sub-categories in event-driven strategies: risk (merger) arbitrage, distressed/high yield securities, and Regulation D.

Risk Arbitrage - This strategy is identified by managers investing simultaneously in long and short positions in both companies involved in a merger or acquisitions. Merger arbitrageurs are typically long the stock of the company being acquired and short the stock of the acquirer. The principal risk is deal risk, should the merger or acquisition fail to close.

Distressed Securities - Fund managers invest in the debt, equity or trade claims of companies in financial distress and generally bankrupt. The securities of companies in need of legal action or restructuring to revive financial stability typically trade at substantial discounts to par value and thereby attract investments when managers perceive that a turn-around will materialize.

High Yield - Often called junk bonds, this strategy refers to investing in low-grade fixed-income securities of companies that show significant upside potential. Managers generally buy and hold high yield debt.

Regulation D - This strategy refers to investments in micro and small capitalization public companies that are raising money in private capital markets. Investments usually take the form of a convertible security with an exercise price that floats or is subject to a look-back provision that insulates the investor from a decline in the price of the underlying stock.

Fixed Income Arbitrage Funds that attempt to limit volatility and generate profits from price anomalies between related fixed income securities. Most managers trade globally with a goal of generating steady returns with low volatility. This category includes interest rate swap arbitrage, United States and non-United States government bond arbitrage, forward yield curve arbitrage and mortgage-backed securities arbitrage. The mortgage-backed market is primarily United States-based and over-the-counter.

Fund of Funds A 'Multi Manager' fund will employ the services of two or more trading advisors or Hedge Funds who will be allocated cash by the Trading Manager to trade on behalf of the fund.

Global Macro Global macro managers carry long and short positions in any of the world's major capital or derivative markets. These positions reflect their views on overall market direction as influenced by major economic trends and or events. The portfolios of these funds can include stocks, bonds, currencies, and commodities in the form of cash or derivatives instruments. Most funds invest globally in both developed and emerging markets.

Long/Short Equity This directional strategy involves equity-oriented investing on both the long and short sides of the market. The objective is not to be market neutral. Managers have the ability to shift from value to growth, from small to medium to large capitalization stocks, and from a net long position to a net short position. Managers may use futures and options to hedge. The focus may be regional, such as long/short US or European equity, or sector specific, such as long and short technology or healthcare stocks. Long/short equity funds tend to build and hold portfolios that are substantially more concentrated than those of traditional stock funds.

Managed Futures This strategy invests in listed financial and commodity futures markets and currency markets around the world. The managers are usually referred to as Commodity Trading Advisors, or CTAs. Trading disciplines are generally systematic or discretionary. Systematic traders tend to use price and market specific information (often technical) to make trading decisions, while discretionary managers use a judgmental approach.

Other This strategy describes hedge funds that cannot be classified in one of the ten listed categories.

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EDUCATION

Ph.D. MIT Sloan School of Management, Management, April 2004 (expected)
B.S. MIT, Chemical Engineering Major and Economics Minor, 1998. Tau Beta Pi. GPA 4.8/5.0

APPOINTMENTS

Post-Doctoral Fellow, Laboratory for Financial Engineering, Department of Finance, MIT Sloan School of Management, 5/04-9/04

ACADEMIC INTERESTS

Empirical Asset Pricing, Financial Institutions, Investments, Financial Econometrics, System Dynamics, Liquidity, Financial Crises, Hedge Funds

PUBLICATIONS

An Econometric Model of Serial Correlation and Illiquidity in Hedge Fund Returns,
Co-authored with Andrew W. Lo and Igor Makarov, 2003 (Forthcoming, Journal of Financial Economics)

RESEARCH PAPERS

The Life Cycle of Hedge Funds: Fund Flows, Size and Performance, 2003 (Job Market Paper)

Limits to Arbitrage: Understanding How Hedge Funds Fail, 2003

The Dynamics of Global Financial Crises,
Co-authored with Kevin Amonlidviman, Andrew W. Lo, and Rishi Kumar, 2003

An Overview of Major Hedge Fund Collapses,
Co-authored with Andrew W. Lo, 2003

Extrapolating Expectations: An Explanation for Excess Volatility and Overreaction,
Co-authored with Jannette Papastaikoudi, 2002

RESEARCH AND TEACHING EXPERIENCE

2/01-Present Research Assistant for Professor Andrew W. Lo - Hedge Fund Performance

- Analyzed performance for different investment strategies, developed models that explain high correlation in hedge fund returns, and investigated hedge fund failures. See abstracts below.

2/02-6/02 Teaching Assistant for Professor James H. Hines - Applications of System Dynamics

- Held weekly recitation sections, conducted weekly modeling exercises.

2/01-6/01 Teaching Assistant for Professor Andrew W. Lo - Investments

- Held weekly recitation sections, graded exams and problems sets, conducted computer exercises.

6/00-8/00 Research Intern, Deutsche Asset Management Quantitative Research Group, New York, NY

- Devised a model for measuring market impact of large trades and wrote a software program to be used by traders in the Deutsche Asset Management Division for managing indexed investment portfolios.

1/00-2/01 Instructor – Business Dynamics

- Developed and taught the course to undergraduate, MBAs and doctoral students.
- Organized exercises and invited speakers from academia and industry.

9/99-2/01 Research Assistant for Professor Andrew W. Lo - Global Financial Crises

- Analyzed the "Hall of Shame," financial institutions that failed. Developed dynamic models for failures of a generic bank and a hedge fund. Also, developed a model for transmission of financial crises. See abstracts below.

6/98-8/98 Research Assistant for Professors Charles Fine and Nelson Repenning - Strategic Decisions within a Firm

- Developed a dynamic model of vertical and horizontal integrations within a firm. Used the model to explore optimal strategic decisions for company integrations. Research cited in "Clockspeed: Winning Industry Control in the Age of Temporary Advantage," Reading, Massachusetts: Perseus Books, 1998.

PROFESSIONAL ACTIVITIES

Referee

Journal of Risk, System Dynamics Review

Presentations at Professional Meetings

2003 Annual Meeting of the Western Finance Association (presenter)
2003 Annual Meeting of the Financial Management Association (presenter and discussant)
2003 MIT Batterymarch Finance Seminar (presenter)
2003 MIT Laboratory for Financial Engineering Seminar Series (presenter)
2003 International System Dynamics Conference, New York, NY (presenter)
2002 MIT-SUNY, Albany PhD Colloquium (presenter)
2002 International System Dynamics Conference, Palermo, Italy (presenter and session chair)
2001 International System Dynamics Conference, Atlanta, GA (presenter)
2000 Methodology Conference, University of Texas at Austin (presenter)

Invited Presentations

Boston University (2003)
University of Massachusetts, Amherst (2003)
Temple University (2003)

Memberships

American Finance Association
Financial Management Association
Western Finance Association

HONORS, SCHOLARSHIPS AND AWARDS

MIT Sloan School of Management Fellowship, 2003
National Science Foundation Graduate Fellowship, 1998-2000, 2002-2003
Tau Beta Pi, 1998

INTERVIEWS AND MEDIA APPEARANCE

Wall Street Journal, March 25, 2003
CMI Gala, Invitee and Presenter, London, England, 2003
Making Money Work Edition of Open Door, MIT Alumni Association, 2002

EXTRACURRICULAR ACTIVITIES

Finalist in MIT $50K Entrepreneurship Competition, 2003
MIT Russia Business and Technology Initiative, President and Founder, 2002

THESIS COMMITTEE AND REFERENCES

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ABSTRACTS OF PUBLICATIONS AND WORKING PAPERS
MILA GETMANSKY

PUBLICATIONS

An Econometric Model of Serial Correlation and Illiquidity in Hedge Fund Returns,
Co-authored with Andrew W. Lo and Igor Makarov, 2003 (Forthcoming, Journal of Financial Economics)

Abstract: The returns to hedge funds and other alternative investments often have high serial correlation, in sharp contrast to the returns of more traditional investment vehicles such as long-only equity portfolios and mutual funds. In this paper, we explore several sources of such serial correlation and show that the most likely explanation is illiquidity exposure, i.e., investments in securities that are not actively traded and for which market prices are not always readily available. For portfolios of illiquid securities, reported returns will tend to be smoother than true economic returns, which will understate volatility and increase risk-adjusted performance measures such as the Sharpe ratio. We propose an econometric model of illiquidity exposure and develop estimators for the smoothing profile as well as a smoothing-adjusted Sharpe ratio. For a sample of 908 hedge funds drawn from the TASS database, we show that our estimated smoothing coefficients vary considerably across hedge-fund style categories and may be a useful proxy for quantifying illiquidity exposure.

RESEARCH PAPERS

Life Cycle of Hedge Funds: Fund Flows, Size and Performance, 2003

Abstract: Since the 1980s we have seen a 25% yearly increase in the number of hedge funds, and an annual attrition rate of 7.10% due to liquidation. This paper analyzes the life cycles of hedge funds. Using the TASS database provided by the Tremont Company, it studies industry and fund specific factors that affect the survival probability of hedge funds. The findings show that in general, investors chasing individual fund performance decrease probabilities of hedge funds liquidating. However, if investors follow a category of hedge funds that has performed well, then the probability of hedge funds liquidating in this category increases. We interpret this finding as a result of competition among hedge funds in a category. As competition increases, marginal funds are more likely to be liquidated than funds that deliver superior risk-adjusted returns. We also find that there is a concave relationship between performance and assets under management. The implication of this study is that an optimal asset size can be obtained by balancing out the effects of past returns, fund flows, market impact, competition and favorable category positioning that are modeled in the paper. Hedge funds in illiquid categories are subject to high market impact, have limited investment opportunities, and are more likely to exhibit an optimal size behavior compared to those in more liquid hedge fund categories.

Limits to Arbitrage: Understanding How Hedge Funds Fail, 2003

Abstract: Even if arbitrage opportunities can be found in statistical sense, they might not be exploitable. This paper models such limits to arbitrage in the framework of a hedge fund. In particular, the paper explores how hedge funds fail given arbitrage opportunities. Dynamic relationships between a hedge fund, dealers, a bank, and market are modeled. As a case study, Long Term Capital Management is studied. The model explores a phenomenon that a fund manager who engages in arbitrage and uses high leverage might lose all his money before realizing positions at a profit. As assets go down in value, the firm has to post more collateral. If it is unavailable, this often leads to a hedge fund collapse. However, given that positions are well diversified and not closely correlated, leverage by itself does not lead to collapse of a fund. Correlated positions in the absence of leverage might lead to a loss, but are not subject to collateral collapse. However, the superimposition of both leverage and induced high correlation between assets can lead to collapse. The paper explores these “flight to quality” and “collateral collapse” dynamics.

The Dynamics of Global Financial Crises,
Co-authored with Kevin Amonlidviman, Andrew W. Lo, and Rishi Kumar, 2003

Abstract: This paper presents a Markov chain model of the transmission of financial crises. Using bilateral trade data and a measure of exchange market pressure, a method to determine a set of transition probabilities that describes the crisis transmission dynamics is developed. The dynamics are characterized by one-month conditional crisis probabilities and the probability of a crisis occurring within one year. The framework allows for modeling and comparing various channels of contagion, such as investments and bilateral trade. Using macroeconomic data on 45 countries, the model predicts and gives insights into all of the financial crises that we studied: Mexico (1994), Asia (1997), Russia (1998), Brazil (1999), Turkey (2001), and Argentina (2002).

An Overview of Major Hedge Fund Collapses,
Co-authored with Andrew W. Lo, 2003

Abstract: This paper studies structural and statistical properties of major hedge fund collapses. Several variables such as investment and accounting strategy, crisis outcome, internal dynamics, fee structure, performance, leverage, asset types, geographical location of investments, transparency, personal characteristics of a hedge fund manager and relationships with brokers are analyzed.

Extrapolating Expectations: An Explanation for Excess Volatility and Overreaction,
Co-authored with Jannette Papastaikoudi, 2002

Abstract: In this paper, we explain excess market volatility by means of momentum and acting on analysts' forecasts. We show excessive price movements with respect to fundamentals can be caused either by "irrational" trend chasing behavior of investors, or by trading too often based on experienced analysts' forecasts (in case of continuous earnings). Price volatility depends on the prevalent investor type and on the type of analysts an investor listens to. Within our market framework, price setting mechanisms are introduced based on demand/supply balance and on trading strategies. In forming their demand, investors consider three factors: their beliefs about the intrinsic value of the marketed assets, past stock performance, and predictions of financial analysts of assets' price targets.