**Problem 2.4: Energy-conserving Dynamic Systems**

Dynamic modeling of MEMS devices will require us to perform time integrations of vector functions. In this problem, and in related exercises in later problems, we will develop some of the nitty-gritty details needed to do these integrations successfully. It will be necessary to become familiar with MATLAB functions defined by M-files, and with the integration functions `ode23` and `ode45`. There are some syntactical issues that must be handled correctly, because an appropriate function to use with these integration routines must yield a column vector output when given a column vector input. This means, among other things that multiplications, divisions, and exponentiations may have to be done on an element-by-element basis, using array arithmetic operations. It happens that these issues are not present in this first problem, but may surface in later problems.

(**a**) Consider a dynamical system described by the pair of state variables $x_1$ and $x_2$, governed by the following state equations.

$$\begin{aligned} \frac{dx_1}{dt} &= x_2 \\ \frac{dx_2}{dt} &= -x_1 \end{aligned}$$

If the *stored energy* in this system is defined as

$$W = x_1^2 + x_2^2$$

show that the energy in this system is strictly conserved.

(**b**) Use the MATLAB command `ode23` to integrate these equations forward in time from an initial state $x_1 = 1$, $x_2 = 0$. Is energy conserved in this numerical simulation? Try modifying the parameters for `ode23` to see if you can get a more ideal answer. Repeat with `ode45`.

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