**11.124: Introduction to Teaching and Learning Math and Science**

# Design a Math Game – Math Rummy

# Math Rummy Student Handout

Description: In "Math Rummy", a fast-paced and fun game for ages 9 and up, players compete to build hands of playing cards combined with the basic arithmetic operations (addition, subtraction, multiplication, division) to reach a target number.

**Instructions for Basic Game**
Setup: The "Math Rummy" deck is a normal deck of 52 cards with the 10, Jack, Queen, and King removed from each suit, leaving 36 cards (the Ace-9 of each suit). The Ace counts as the number 1. Choose a dealer. The dealer distributes 4 cards to each player, places the remaining cards face down in one pile, then turns over the top two cards and places them side-by-side above the deck. This two-digit number is now the "target." Play begins to the dealer's left.

On Your Turn: At the start of your turn, you must take a card from either the top of the deck or the top of the discard pile. With five cards in your hand, see if you can hit the "target" with any combination of adding, subtracting, multiplying, and dividing using at least two cards together. If you can hit the target, you have won the round! Show your winning hand to the other players. If you cannot hit the target, pick one of your cards and place it face up on top of the discard pile. It is now the player to your left's turn.

Once a player wins a round, the winner picks new cards so that s/he has four cards again. The dealer turns over the next two cards for a new target, and play moves to the person on the left of the winner.

Winning the Game: The first player to win six rounds has won the game!

**Variants –** To make the game more challenging, try these variations!
- "More is Better": Score each winning hand as follows – 1 point for using two cards; 3 points for using three cards; 6 points for using four cards; 10 points for using five cards.
- "Time's Ticking": After the target is shown, set a timer for 1 minute. During that time, everyone tries to hit the target using two to all four cards that s/he is dealt. Students may trade in up to four cards to get new cards from the dealer during that time. At the end of the one minute, everyone who shows a hand (2-4 cards) that successfully hit the target gets a score according to the "More is Better" rules.
- "All Four": In order to win a round, you must use exactly four cards to hit the target.
- "Double Jeopardy": During the setup, give eight cards to each player (use two decks if you have more than three players!) and play with two targets instead of one. To win, you must hit both targets in the same turn by making two hands of 2-6 cards.
- "Inside-Out": Deal each player a target, and put four cards in the middle. Each player must use this same set of cards to reach their own target using all 4 cards. Any player can call out a winning hand at any time! If no one can identify a winning hand, a fifth card is added.
- "Two Brains": Play the Basic Game or any Variant with a partner.

**Hints**
- If you can't make your target on your turn, what cards do you need to make a winning hand? Which cards don't you need?
- Is the target number prime, or can it be factored? Are the factors single-digit or double-digit numbers? How does this change your strategy?
- Can you get close to your target with 2 or 3 cards? How does this simplify your strategy?
- When is each operation (addition, subtraction, multiplication, division) most useful?
- Can you develop a strategy based around how to use the different operations?

**Questions**
1. What is the largest target that can be reached by just addition? Explain your thinking. Use examples to support your reasoning.

2. Which operation (+, -, * or /) lets you to get high numbers more easily? Explain. Give some examples.

3. Which operations let you increase your numbers by small bits, and why is this useful? Explain.

# Math Rummy Teacher Lesson Plan

**Materials**
Math Rummy requires 1-2 decks of cards.  Refer to the Student Activity Sheet for setup and gameplay instructions

**Lesson Objectives and Relevant NCTM Standards**
Math Rummy is primarily designed to increase fluency with basic arithmetic, but high levels of play also requires players to sort through a long list of choices to find a winning move and develop strategies to quickly pare away bad choices to focus on card combinations more likely to win.  While the game conforms to NCTM standards set for 3$^{rd}$-5$^{th}$ grade, in our experience students in grades 6-8 often need to shore up their arithmetic skills, and Math Rummy provides several variations for increasing the challenge as students become more adept at the game.  The following is a table comparing NCTM standards, expectations, and how they are addressed in Math Rummy:

| i.  Number and Operations Standard for Grades 3–5 | | |
| --- | --- | --- |
| **Standard** | **Expectation** | **How it is addressed in Math Rummy** |
| Understand numbers, ways of representing numbers, relationships among numbers, and number systems | Understand the place-value structure of the base-ten number system and be able to represent and compare whole numbers and decimals | This is more of a pre-requisite for this game. However, Math Rummy (MR) can solidify this standard if it is not concrete in the student's mind. |
| | Describe classes of numbers according to characteristics such as the nature of their factors | With more playing time, sturdents will get an intuition of which numbers can be reach with which of  its factors through multiplication. |
| Understand meanings of operations and how they relate to one another | Understand meanings of operations and how they relate to one another | Multiplication and division are essential in MR to reach targets which are beyond 36 (=9x4) and will most like be needed for even smaller numbers. |
| Compute fluently and make reasonable estimates | Develop fluency with basic number combinations for multiplication and division and use these combinations to mentally compute related problems, such as 30 x50 | A skill learned with increased experience in playing MR |
| | Develop fluency in adding, subtracting, multiplying, and dividing whole numbers | This is the core skill learned in MR. |

| | Select appropriate methods and tools for computing with whole numbers from among mental computation, estimation, calculators, and paper and pencil according to the context and nature of the computation and use the selected method or tools. | We noticed when we tested the game that there was an interplay between mental and pencil-and-paper calculations, which players used to be the most efficient in reaching a result. |
|---|---|---|
| **ii. Algebra Standard for Grades 3–5** | | |
| **Standard** | **Expectation** | **How it is addressed in Math Rummy** |
| Understand patterns, relations, and functions | Describe, extend, and make generalizations about geometric and numeric patterns | Numerical patterns are explored by players of MR |
| **iii. Problem Solving Standard for Grades 3–5** | | |
| Apply and adapt a variety of appropriate strategies to solve problems | | There is a large space of possibilities (of numbers and operations) available for players to reach a certain target. A skill developed in MR |
| **iv. Communication Standard for Grades 3–5** | | |
| Communicate their mathematical thinking coherently and clearly to peers, teachers, and others | | In variants of MR where teamwork is incorporated, communication becomes vital between members of the same team. |
| Analyze and evaluate the mathematical thinking and strategies of others; | | |
| **v. Connections Standard for Grades 3–5** | | |
| Analyze and evaluate the mathematical thinking and strategies of others; | | The context here is a game, and mathematics is the tool necessary to play it. |

**Gameplay Analysis**

There is no "winning" strategy to Math Rummy – the game depends upon luck as much as skill. However, due to the combinatorial nature of the game, one target can always be hit with many hands, and one hand can produce many targets (see question 1 of the assessment). Inexperienced players will often find that they possessed a winning hand, but they just could not "see" the solution among so many options. Accordingly,

there are some strategies for managing the complexity of the problem space to better find solutions.

Strategy 1:  Target Oriented
        Given a target, before receiving any cards, players can begin to assess what sorts of hands will be valuable:

        *If the number is composite, the player's first action should be to factor the number.  They will find that not all composite numbers are equally easy.  A number like 24, which is divisible by 2, 3, 4, 6, 8, and 12, gives them many more options to work with than 35, which is only divisible by 5 and 7.
        *If the number is prime, the player should look at the composite numbers nearby.  For example, 37 is prime, but 36 and 40 both have many factors.  An ace or a three would open up their hand to many more options.
        *Players should, in fact, become comfortable looking at all of the numbers within one card of their target for factors, as this will offer them many options for hitting their target.

        Of course, one could spend all day searching for all of the hands that fulfill their target, but some strategies are not very likely to work.  If it is not easy to reach the target through direct multiplication, a very effective strategy is to use two cards to hone in on the target with multiplication, and two cards to make up the difference with addition, subtraction, or multiplication.  Some general factors affecting this strategy:

        *If the number is high, greater than 50 for example, they may have only two cards in their hand (the two highest) that can get them within the ballpark, or they may even have to use three cards.  This can lead to building hands that get close to the target, but then require exactly one card to be picked up on the draw to complete the hand.  High targets are most prone to creating longer games.
        *If the number is very low, less than 20, players may find multiplication much less useful.  Low target games offer the greatest number of different winning hands, so it is often best to take extra time searching for a combination that has been overlooked that could offer a win.

Strategy 2:  Hand Oriented
        Once the players receive their hands, they must try to figure out whether they can win with the cards they have.  There are too many possibilities of arranging the four cards to compute for every hand (see question 1), so the best way to proceed is by trying to fit the four cards to the targets found with strategy 1.  For example, with a target of 72, the following are winning hands:
8 9 _ _                identify two factors of 72
8 4 5 _                identify one factor of 72 and two numbers which can be added to
                       make the other factor
        With a target of 73, the following are winning hands:
8 9 1 _ or 8 9 5 4     get close to the target through multiplication and make up the
                       difference with addition and subtraction

**Answers to Activity Sheet**
Q. What is the largest target that can be reached by just addition?
A. 36
Q. Which operation (+, -, * or /) lets you to get high numbers more easily?
A. Multiplication allows you to reach high numbers with few cards.

Q. Which operations let you increase your numbers by small bits, and why is this useful?
A. You can often get close to your target by multiplying just two numbers. If you have the right small card in your hand, you can add or subtract by it to hit your target. More often, you can use the difference or quotient of two cards to create just the right number needed to hit your target.

**Questions to Facilitate Gameplay**
Is the number prime, or composite? What are its factors?
What numbers can you create with any two cards from your hand? Can you do anything with that number and the remaining two cards?
Can you use two or three cards from your hand to get close to the target? What card or cards could you draw to get the rest of the way there?
**Wrap-Up**
At the end of the game, the players have hopefully enjoyed themselves and gotten practice performing a lot of arithmetic. To explore the deeper strategies involved in playing, have students talk about what strategies they used to hit the targets, and see if they can generate the strategies discussed above. You can also pose the assessment questions to the entire class, to generate the answers together.
**Assessment**
Present your students these three game scenarios to see how well they have mastered the different skills involved in Math Rummy.

Q1.    Exhaustion – How many different two-digit numbers can you find using all of your cards?
        Your Cards:  2 3 5 6

| A1. | | | | | |
|---|---|---|---|---|---|
| 10* | 2+5+6-3 | 2*6+3-5 | (3+5)*2-6 | (5-3)*6-2 | |
| 11 | 3*5+2-6 | 6/2+3+5 | (5+6)*(3-2) | 6+5*(3-2) | 6*(3-2)+5 |
| 12 | 3+5+6-2 | 2+5*6/3 | 3*5-6/2 | (6/2*5-3) | |
| 13 | 2*5+6-3 | (3+6)*2-5 | | | |
| 14 | 2*6+5-3 | 6/2*3+5 | (5-3)*6+2 | 2*(5+6/3) | |
| 15 | 3*6+2-5 | (2+5)*3-6 | (5-2)*6-3 | (5-2)*3+6 | |
| 16 | 2+3+5+6 | (5-3)*(2+6) | 2*(5+6-3) | | |
| 17 | 2*3+5+6 | | | | |
| 18 | 6/2+3*5 | 3+6/2*5 | | | |
| 19 | 2*5+3+6 | 3*5+6-2 | (2+3)*5-6 | (2+6)*3-5 | (5+6)*2-3 |
| 20* | 2*6+3+5 | 2*5*6/3 | (2+6/3)*5 | | |
| 21 | 3*6+5-2 | (5-2)*6+3 | | | |
| 22 | (3+5)*2+6 | | | | |
| 23 | 3*5+2+6 | (3+6)*2+5 | | | |

| 24 | 5*6-3*2 | (5-3)*2*6 | (6/2+5)*3 | | |
| 25 | 3*6+2+5 | 5*6-3-2 | (2+3)*6-5 | (5+6)*2+3 | 5*(6/2+3) |
| | 5*(2+6-3) | | | | |
| 27 | 3*5+6*2 | (2+5)*3+6 | 3*(5+6-2) | | |
| 28 | 6*3+2*5 | 2*(3+5+6) | | | |
| 29 | 5*6+2-3 | (2+6)*3+5 | | | |
| 30* | (6-3)*5*2 | 6*5*(3-2) | (6/2+3)*5 | | |
| 31 | 5*6+3-2 | (2+3)*5+6 | (5+6)*3-2 | | |
| 32 | (3+5)*(6-2) | | | | |
| 35 | 5*6+2+3 | (2+3)*6+5 | (5+6)*3+2 | 5*(3+6-2) | |
| 36 | 5*6+2*3 | 6*(3+5-2) | | | |
| 37 | (2+6)*5-3 | | | | |
| 39 | (2+5)*6-3 | 3*(2+5+6) | | | |
| 43 | (2+6)*5+3 | (3+6)*5-2 | | | |
| 45 | 3*5*6/2 | (2+5)*6+3 | | | |
| 46 | (3+5)*6-2 | | | | |
| 47 | (3+6)*5+2 | | | | |
| 50 | (3+5)*6+2 | | | | |
| 54 | (5-2)*3*6 | | | | |
| 55 | 5*(2+3+6) | (2+3)*(5+6) | | | |
| 57 | 5*6*2-3 | | | | |
| 60* | (6-2)*3*5 | 6*(2+3+5) | | | |
| 63 | 5*6*2+3 | (6+3)*(5+2) | | | |
| 64 | (5+3)*(6+2) | | | | |
| 88 | 5*6*3-2) | | | | |
| 92 | 5*6*3+2 | | | | |

Since list is so long, you can let students work on it for a short period, and then compile the entire class' answers. Even then, it is unlikely that they will produce every one of the combinations listed above. You can also list all of the numbers that can be made in the left hand column with blanks where each answer appears in this list, and ask the students to try to generate all of the answers. Be careful, as forcing them to sit down and do this all at once can become tedious. The exhaustive list should demonstrate both how many different targets can be reached with one hand, and the different strategies that can be used to hit these targets.

*[Note – 10 cannot come up as a target because game has no '0']

[Also note - although I tried to be as thorough as possible, it is possible that I missed some possibilities]

Q2.    Negation – Can you hit the target with your hand?  If yes, show how.  If no, explain how you know that you cannot.

        Target: 91
        Your Cards: 9 8 5 2

A2.    Students could go through the same exercise as question 1, but because the target is large and prime, they can greatly reduce the number of combinations they examine. First, they multiply the highest two numbers in their hand:

8*9=72

With the remaining two cards in their hand, they still cannot make it to 91.

8*9+5+2=79   8*9+5*2=82

Therefore they need to either multiply three numbers together or add together two numbers before multiplying.  This leaves them with:

2*5*8=80 with 9 remaining

2*5*9=90 with 8 remaining

2*8*9,5*8*9 are much too high

(2+5)*9=63 too low

(2+8)*9=90 with 5 remaining

(2*9)*8=88 with 5 remaining

(8+5)*9=117 too high

Since none of these combinations allows them to reach the target, they know they cannot make 91.  Notice how many fewer cases they must examine, compared to the exhaustive list of question 1.


Q3.    For the win – Can you find what cards can fill in your blank to hit the target?
       Target:  66
       Your cards:  2 4 8 _

A3.    3      (2+6)*(8+3)
       4      2+(4+4)*8
       7      7*(8+2)-4
       8      8*8+4-2
       9      8*9-4-2