A Proposal for the Calculation of Wear

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Abstract
Finite element analysis usually neglects the contributions of wear and the changes in the surface due to wear. However, wear may be important in any structure subjected to repeated loadings and may be critical for certain tribological applications including the prediction of the sealing potential of surfaces. In this paper, a procedure is proposed whereby the effects of wear may be calculated and included in the overall analysis of the structure. The Archard equation is used as the basis for calculating wear strain which is used to modify the elastic strain in an element in an explicit manner. Extensions of the theory are also proposed and an example using explicit creep for the wear adjustments is included.

Introduction
Historically, wear and the calculation of wear between surfaces in contact has not been a concern in the finite element realm. Initially, finite element analysis was viewed primarily as a means for calculating the loads needed for design analysis. Once the load distribution was determined from the finite element analysis, those loads would be used to perform a detailed component evaluation of the structure. Although fatigue was often calculated to estimate the life of the structure, the affects of wear on life were generally assumed to be small and thus had no effect on the ability of the product to perform its design function. Where surface imperfection could be a problem, as with journal bearing for example, special manufacturing processes were employed to reduce wear and any potential problems it could create. As a result, the nominal dimensions of the component were usually sufficient for design purposes.

Over time, the mechanics and impact of wear are becoming better understood and the calculation and reduction of wear is becoming critical for some applications, including biomedical applications like joint replacements. Just as the application of the fundamental equations of structural mechanics to complicated geometries led to the use of finite element analysis decades ago, the increasingly complex geometries for tribological systems is leading some to consider the use of finite element analysis for the calculation of wear. The purpose of this paper is to initiate a discussion of a procedure for the calculation of wear in ANSYS.

Mechanisms of Wear
For the purposes of this paper, wear is defined as the removal of material (mass) from the surface of an object through contact with another surface and not merely the deformation or dislocation of that material to a different part of the object. Some of the various types of wear and wear mechanisms are listed in Table 1.

Wear can be split into two majority categories: wear dominated by the mechanical behavior of materials and wear dominated by the chemical behavior of materials. Since finite element programs typically do not consider the chemical interactions between bodies or surfaces, these types of wear will not be discussed in this paper.

There are seven mechanical wear mechanisms listed, however there are only three types of surface to surface interaction that can cause them: sliding (one surface sliding relative to another over long distances), fretting (one surface oscillates over minute distances relative to the other) and erosion (solid particles
impinging on a single surface from an external source). For this paper, only dry (non-lubricated) sliding wear will be considered.

The actual wear mechanisms for dry sliding wear depends on a number of variables including: surface finish, surface geometry, orientation, sliding speed, relative hardness (of one surface relative to the other or relative to the abrasive particles between the surfaces), material microstructure, and more. From these variables, it can be seen that wear rate is not a pure material property and does not always occur uniformly.

<table>
<thead>
<tr>
<th>Classification</th>
<th>Wear Mechanisms</th>
<th>Wear coefficient K (range)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wear dominated by mechanical behavior of materials</td>
<td>1. Asperity deformation and removal</td>
<td>10^{-4}</td>
</tr>
<tr>
<td></td>
<td>2. Wear caused by plowing</td>
<td>10^{-4}</td>
</tr>
<tr>
<td></td>
<td>3. Delamination wear</td>
<td>10^{-4}</td>
</tr>
<tr>
<td></td>
<td>4. Adhesive wear</td>
<td>10^{-4}</td>
</tr>
<tr>
<td></td>
<td>5. Abrasive wear</td>
<td>10^{-2} to 10^{-1}</td>
</tr>
<tr>
<td></td>
<td>6. Fretting wear</td>
<td>10^{-8} to 10^{-4}</td>
</tr>
<tr>
<td></td>
<td>7. Wear by solid particle impingement</td>
<td>-</td>
</tr>
<tr>
<td>Wear dominated by chemical behavior of Materials</td>
<td>1. Solution wear</td>
<td></td>
</tr>
<tr>
<td></td>
<td>2. Oxidation wear</td>
<td></td>
</tr>
<tr>
<td></td>
<td>3. Diffusion wear</td>
<td></td>
</tr>
<tr>
<td></td>
<td>4. Wear by melting of the surface layer</td>
<td></td>
</tr>
<tr>
<td></td>
<td>5. Adhesive wear at high temperatures</td>
<td></td>
</tr>
</tbody>
</table>

Finite element modeling of dry sliding wear can be accomplished one of two ways. First, the details of the surface interaction, including surface finish, can be included and calculated in the model. If that approach is taken, it would require that individual finite elements be removed from the model to simulate the gouging or plowing. This in turn requires that the size of the finite element be of the same size as the particles being removed since there are currently no options for removing part of an element (in ANSYS the element is either dead or alive). And as the particles being removed are on the size of molecules, the mesh density, at least near the wearing surface, would also need to be on the size of molecules. This is not a theoretical problem since finite elements are being used to analyze MEMS size devices but it is a practical problem since small size implies a large number of elements which requires large amounts of memory and disk space for the storage of the data generated by a finite element program.

The second approach would be to ignore the details of what is going on at the micro or nano scale and take a macro scale approach to the problem. One the macro scale, the size of the elements would be much larger than the anticipated changes due to wear and the calculations could be performed within the element rather than relying on the birth and death procedure needed at the surface level.

The Archard Equation

The starting point for any discussion of wear on the macro scale is the Archard Equation, which states that:

\[ W = K \times s \times P \]

where \( W \) is the worn volume, \( s \) is the sliding distance, \( P \) is the applied load and \( K \) is the wear per unit load per sliding distance. Archard says “[\( K \)] may be described as the coefficient of wear and, in a series of experiments with the same combination of materials, changes in \([K]\) denote changes in surface conditions.” The Archard equation assumes that the wear rate is independent of apparent area of contact. However, it makes no assumptions about the surface topography (surface roughness effects are encompassed by the experimental wear coefficient) and it also makes no assumptions about variations with time. It must also be stated that although it is widely used, the Archard equation only provides for an order of magnitude estimate and is a true calculation of wear.
One of the more common methods for determining the value for K is to press a stationary pin using a pre-load of P into the surface of a rotating disk. The load P is known and the sliding distance S can be determined from the rotational speed of the disk and time that the disk has rotated. The amount of wear on the pin is determined by change in mass (weight) of the pin and the constant K calculated.

![Figure 1. Pin on Disk Configuration for Measuring Wear Coefficient](image)

This method for determining a constant wear coefficient for a given pair of surfaces has limitations. It ignores changes in apparent area of contact with time, also known as “running in” effects. It assumes that the direction of the load is constant which may not be the case in real conditions. And, it assumes that the surface topography of the experimental surfaces accurately represent the surfaces of interest. Despite these limitations, we will assume that the values of wear coefficients determined by the pin-on-disk method or by other methods are accurate enough to use in engineering analysis.

In engineering applications, the loss of volume (and thus, loss of mass) may not be as important as the change of a given dimension at a given location on the device or structure. For example, we may be interested in the change of diameter for a radial bearing after a long period of time in use. The change of a single dimension can be calculated from the change in volume by dividing by the apparent area of contact assuming that the apparent area of contact is constant. When the contact area changes with time, a more sophisticated calculation must be performed to determine the change in desired dimension over time.

This method for generating wear is not universally applicable. First it ignores the details of the surface and assumes that the full surface is in contact with the disk. This is similar to the usual assumption for contact in finite elements where the surface is assumed to be smooth. It also assumes that the direction of the load is constant and that the load is unchanging which may not be the case in real conditions. And if the state of stress at the surface is considered, the magnitude and direction of those surface stresses would also remain constant. A common type of wear is that generated by the repetitive application of a load on the same surface. For example, a crankshaft that is rotating at a constant velocity will have the load at any particular position around the bearing alternating from zero to maximum at a rate equal to the rotational speed. The Archard equation would also imply that the particles would be removed from the surface in a uniform manner and that the surface would maintain the same general shape. That is that particles that exist in the valleys of the surface would be eroded at the same rate as particles at the peaks of the surface.

Kauzlarich et al. have proposed analytical models to correct for changes in apparent area over time for a variety of common wear configurations (spherically ended pin sliding against a flat, a pair of crossed cylinders, radially loaded journal bearing and radially loaded spherical bearing). Their approach is based on known geometry and cannot be applied to arbitrary surfaces.

Hegadekatte et al. presented a calculation of wear in the finite element domain, using the Archard equation and the ABAQUS program which could also correct for changes in apparent area of contact. Their
approach was to post process the results at the individual nodes obtained from an analysis using the Archard equation in an attempt to back out the wear. Ignoring that the Archard approach is a system approach and Hegadekatte’s approach is for individual nodes, the primary objection to the Hegadekatte approach is that it is after the fact of the analysis and does not take into account that wear will change the model and that the results obtained at the individual nodes must include the history of the loading at those nodes.

Podra and Andersson\textsuperscript{5} presented a method for calculating wear that is similar to Hegadekatte but did allow for updating the geometry. The procedure was to perform an analysis load step in solution, post processes the solution results to determine the wear accumulated in that load step, update the nodal coordinates and repeat the simulation. The authors do not identify the method for updating the geometry and it appears that each solution is considered a separate run and that the time integration of the wear is done external to the analysis, perhaps using APDL. This method is an improvement on that proposed by Hegadekatte but would be very time consuming compared to an in-solution procedure.

Proposed Theory

As an alternative, we propose a method for calculating wear for inclusion into a finite element program where wear will be calculated in the solution processor instead of in the post processor, thus eliminating some of the issues associated with the Hegadekatte model.

Consider a modified form of the Archard equation:

\[
W = K \cdot S^{C2} \cdot R^{C3}
\]

where \( W \) is the change in volume, \( K \), \( C2 \) and \( C3 \) are equation constants to account for such things as the materials in contact, \( S \) is the stress created by the contacting pair and \( R \) is the number of repetitions of the load (one sliding pass, for example.) If \( W \) represents the change in volume of the element due to wear, then we can define wear strain as the change in volume divided by the original volume and rewrite the wear equation as

\[
e_{wr} = C1 \cdot S^{C2} \cdot R^{C3}
\]

where \( C1 \) is equal to \( K \) divided by the volume. This strain is similar to volumetric strain which has the form

\[
e = \frac{\Delta V}{V} = \frac{1}{3} (e_1 + e_2 + e_3).
\]

Often only one of these strains is present for wear as it is expected that wear will occur perpendicular to the surface of the component. But that is not a requirement and wear strain may be a vector quantity in a manner similar to any other type of strain. Loads that are applied oblique to the surface may generate wear that is not perpendicular to that surface and provisions should be made for including this type of wear. The principal difference between wear strain and any other strain quantity is that wear strain represents material that is removed from the system.

Wear strain as proposed here is different from wear as proposed by Archard. The Archard equation is a systems approach where the applied load is assumed to be distributed over the entire loading area. Wear would be expected to occur uniformly over the entire surface. The wear strain proposed here is a function stress and load repetitions. This implies that where load is applied to the surface, wear will occur and that parts of the surface which are (currently) unloaded will not experience change due to wear. This definition of wear strain also considers the local effect of stress and permits wear to be different at different locations on the surface. This does not change the fact that the approach presented here is only a systems level estimate of the wear and that detailed calculations at specific locations on a surface should not be relied upon.

Wear is not currently included as a calculable quantity in ANSYS and there are no variables available for storing this data. And, the effect of this type of calculation will need to be investigated to insure that problems do not arise in other locations of the program.
An Interim Approach

This form of the wear equation is similar to creep equations that have a material constant (C1), a stress contribution (S and C2) and a third factor, creep strain or time in the case of creep equation and repetitions in the case wear. This suggests that creep may be used to simulate wear until such time as wear is directly calculable in ANSYS. Explicit creep is used since the plan is to calculate the wear strain based upon the final configuration of the surface at the end of the load step. In ANSYS, the explicit creep calculation is performed after the elastic and plastic calculations are completed; this is the approach used for wear. The strain hardening creep equation that is programmed into ANSYS has the form

\[ \frac{d\varepsilon_{cr}}{dt} = C1 \times \text{stress}^{C2} \times \varepsilon_{cr}^{C3} \times \exp(-C4/T) \]

where C1, C2, C3 and C4 are constants that are supplied by the user. For each time increment, the incremental creep strain is calculated using this equation, then incremental creep strain is multiplied by the incremental time and added to the previous creep strain. A similar procedure can be used to calculate wear.

The incremental wear strain can be calculated in a similar manner. For each load step, the incremental wear strain is calculated multiplied the load step time and added to the previous wear strain. The advantage of using explicit creep for the wear (creep) calculation is a correction to the elastic strain and that correction is done after the plasticity calculation. As a result, plasticity may be included in the analysis.

Example

An example problem was created to demonstrate this method. The model consists of two bodies: the lower body is a rectangular block with two small rounded ridges on the top surface and the top body is a rigid contact surface. This rigid contact surface is used to apply the load to the random surface.

The ridges on the top of the lower body are shown in Figure 2. The model block consists of two materials: a base material and a second surface material with the same mechanical properties as the base material but with the wear (creep) option. Figure 3 shows the block with the top layer that includes the wearing material shown in dark blue. The entire model is shown in Figure 4.
Figure 2. Rough surface consisting of two 0.045 ridges

Figure 3. Finite element model showing wearing material at the surface

Figure 4. Finite element model including the rigid loading surface
The creep law used is strain hardening primary creep that:

1. Includes a material constant as C1
2. Includes stress raised to some power. In this case use C2 = 1 so that wear is a linear function of stress.
3. Removes the contribution of creep strain by setting C3 = 0.
4. Ignores the Arrhenius effect by setting C4 = 0.

For the purposes of this analysis, the wear assumptions are:

1. An imperfect surface consisting of two parallel ridges separated by some distance. The ridge height is 0.045 above the surface and the ridges are separated by a distance of 1.0. This ridge geometry may represent tool marks in a machined surface but are intended here for demonstration purposes only.
2. The load will be applied using a rigid surface and a pilot node on the rigid surface.
3. The wear strain will be based upon 10 million repetitions of the load and a stress level of 100,000 psi. Thus, the wear will equal 0.015/0.2 = 0.075 (or equal to the average variation in the surface). From this information C1 = 0.075/100000/1e8 = 7.5e-14.
4. C2 = 1 so that stress is a linear function for the problem.
5. Each substep will be for 1 million repetitions of the load. Since the number of repetitions is not included in the creep equation, the repetition value will be included in C1. Then C1 = 7.5e-14 * 1e7 = 7.5e-7.
6. C3 is set to zero so that the contribution of the creep strain in the strain hardening law will be removed.

The analysis is performed in three load step groups: the first load step is a displacement to generate the contact; the second load step converts the displacement to load after contact has been defined; and the third load step (static) is a group of load steps, each with an incremental time of 1 that represents 1 million repetitions of the load to calculate wear over time. For this analysis, the time was run to 99 time units (99 million repetitions.)

The initial results for the wear analysis are shown in Figure 5. Examination shows that the surface is smoother than the original but that the sides of the block have bulged. This bulging is consistent with creep but is not consistent with wear and will need to be eliminated.
For this simple analysis, the bulging can be eliminated by applying boundary conditions along the sides of the block. Figure 6 shows the displacement plot at the end of time. In this figure, the surface is more uniform indication wear and the sides of the block have not bulged.
Since we assume that wear is dependent upon stress and the stress values are changing over time due to the wear and the changing contact, then wear at any particular location is changing over time. This can be seen from an examination of the contact pressure. Figure 7 shows the contact pressure at different times representing different amounts of wear. Window 1 (top left) shows the contact pressure at the beginning of the analysis before wear has taken place. Window 2 (top right) shows the contact pressure at time 30 representing 30 million repetitions. Window 3 (bottom left) shows the contact pressure at time 60 representing 60 million repetitions. And Window 4 (bottom right) shows the contact pressure at time 100 representing 100 million repetitions.
Analysis Results & Discussion

The example problem demonstrates that wear can be simulated and included in the evaluation of products. The use of creep as a mechanism for the calculation of wear is workable in the short term but a more permanent method needs to be added if more complex wear calculations are to be included. This is especially important if true creep is to be included in the model in addition to wear.

The above example assumes that the load applied is constant in magnitude and direction. That is not a requirement. In the current configuration, wear is calculated as an update to the state of strain at the end of each substep. The incremental wear strain would be calculated and would be added to the previously calculated wear strain. The one problem with using strain hardening creep is that additional boundary conditions are needed to insure that the wear calculated will be unidirectional. Creep, as implemented in ANSYS, is a three-dimensional concept and creep strains in all applicable directions will be calculated when loads are applied. Changing loads, and particularly, changing the direction of the load will require the changing the boundary conditions. The alternative is to use the user explicit creep routine, USERCR, where the individual components of the wear may be calculated. USERCR would also permit the input of data for the number of repetitions which would permit a different wear calculation for different loading conditions.

Input included at the end since the results are best seen as an animation.

Conclusion

Wear on a macro scale is doable.

The approach presented here is a system level that will predict wear on a average basis in a manner consistent with Archard. The use of stress in the wear equation may lead users to conclude that details of
the surface can be calculated. Users are caution to interpret the results correctly as more work needs to be performed on the theoretical basis for wear before more accurate simulations can be implemented.

What is needed from ANSYS to make this a viable method for calculating wear in real analyses:

1. A variable within the program to hold the wear strains that are calculated. At the present time, such a variable does not exist. This conclusion is drawn from a review of the Fortran coding for user element UEL101 which is virtually identical to LINK8. Provisions are provided to calling plasticity routine, creep routines and the swelling routine but no other variable except user versions of the above material laws. Likely this is only practical in the new generation elements.

2. Routines needed to calculate the wear strain, including a user routine for wear mechanism equation, that are programmed into ANSYS.

3. It would be nice to be able to return strain vectors from the UserMAT routine and to be able to plot those results. At the present time, UserMAT has the ability to return user calculated variables for storage on the results file but only individual values and not a vector of values, such as might be required for wear. The individual values are passed back and forth in the STATE variable vector. Contour plotting is very difficult.

References


Appendix 1 – Input Listing

/BATCH
/input,menust,tmp,'',,,,,,,,,,,,,,,,1

/filnam,wear2

/GRA,POWER
/GST,ON
/PLO,INFO,3
/GRO,CURL,ON
/CPLANE,1
/REPLOT,RESIZE
WPSTYLE,,,,,,,,0
/VIEW,1,1,1,1
/vup,,2

asf=0.045 !!!! height of ridge above surface

/prep7

k,1 !!!! keypoints for first plane
k,2,,1
k,3,,2,1
k,4,,2

k,11,.5 (first ridge) !!!! keypoints for second plane
k,12,.5,1+asf
k,13,.5,2,1+asf
k,14,.5,2
k,15,1,,1.1 !!!! end of first block
k,16,1,2,1.1

k,21,1.5 (second ridge) !!!! keypoints for fourth plane
k,22,1.5,,1+asf
k,23,1.5,2,1+asf
k,24,1.5,2

k,31,2 !!!! keypoints for fifth plane
k,32,2,,1
k,33,2,2,1
k,34,2,2

larc,12,22,15,10
larc,13,23,16,10

v,1,2,12,11,4,3,13,14 !!!! create volumes
*rep,3,10,10,10,10,10,10,10,10

et,1,45 !!!! elements for model
et,11,174 !!!! target elements
et,12,170 !!!! contact elements
esize,.2
vmesh, all
nsel, s, loc, z, 1, 1.1
esln, s
emodif, all, mat, 10
      !!!! change top layer to creep material
    type, 11
    esurf
      !!!! add contact

mp, ex, 1, 1e7
mp, nuxy, 1, .3
mp, ex, 10, 1e7
mp, nuxy, 10, .3
ki = .015/.2/1e6

k1 = .015/.2/1e6

      !!!! use explicit creep, use strain hardening
tb, creep, 10

      !!!! equation is C1*sig^C2*epcr^C3
      !!!! for this case, C2 = 1, C3 = 0
      !!!! want strain = 0.015/.2 in 1e7

repetitions
asel, none
nsel, none
esel, none

wpooff, , , 1.1
rect, , , 2
*get, alod, area, , , num, max
  type, 12
tshap, quad
esize, , , 1
amesh, all

      !!!! create target surface
esurf, , , reverse
*get, nmax, node, , , num, min
tshap, pilot
e, nmax

allsel

finish

/solu
tunif, 100
nsel, s, loc, z, 0
      !!!! apply constraints
d, all, uz, 0
nsel, s, loc, y, 0
d, all, uy, 0
nsel, s, loc, x, 0
d, all, ux, 0
nsel, s, loc, y, 2
d, all, uy, 0
nsel, s, loc, x, 2
d, all, ux, 0
nsel, all
d, nmax, uz, -0.06
      !!!! move target surface into contact
allsel
time, le-6
solve
ddele,nmax,uz  
!!! replace displacement with force
f,nmax,fz,-1e5
time,2e-6
solve

*do,ii,3,99,3  
!!! march out time
time,ii
solve
*endo

finish

/post1
set,3
esel,s,type,,1

/USER, 1
pldi
/VIEW, 1, 0.709203341790 , 0.692018981620 , 0.134686113137
/ANG, 1, -1.68111192131
!*  
-999,-999,
ANDATA,0.5, ,0,3,35,1,0,1
!*  
/VIEW,1,1,1,1
/ANG,1
/REP,FAST
esel,s,type,,11
plns,cont,pres
!*  
-999,-999,
ANDATA,0.5, ,2,3,35,1,0,1
!*