Asymptotic analysis of the Boltzmann-BGK equation for oscillatory gas flows with applications to nanoscale devices

Jason Nassios*
Department of Mathematics and Statistics
The University of Melbourne
VIC 3010

The Navier-Stokes equations and the no-slip boundary condition provide a rigorous mathematical description of the dynamics of many important flow phenomena. The validity of these equations is contingent on the continuum approximation, which is violated when operating at low gas densities or in miniaturised systems. In contrast, kinetic theory provides a rigorous foundation for calculating the dynamics of gas flow at arbitrary degrees of rarefaction. Concomitant with the generality of this approach is its analytical intractability in many cases of practical interest. Importantly, the near-continuum regime has been examined analytically using a model equation and asymptotic techniques. This asymptotic analysis assumed steady flow, for which analytical slip models were derived. Recently, developments in nanoscale fabrication have stimulated research into oscillatory non-equilibrium flows, drawing into question the applicability of the steady flow assumption. In this talk, I will discuss some key findings of a formal asymptotic analysis of the unsteady linearized Boltzmann-BGK equation, where inter molecular collisions are modelled via a relaxation process. This generalizes existing theory to the oscillatory (time-varying) case. We consider the near-continuum limit where the mean free path and oscillation frequency are small. The complete set of hydrodynamic equations and associated boundary conditions are derived for arbitrary Stokes number, and to second-order in the Knudsen number. The first-order steady boundary conditions for the velocity and temperature are found to be unaffected by oscillatory flow. In contrast, the second-order boundary conditions are modified, except for the velocity component tangential to the solid wall. Implications of this theory are explored for the oscillatory thermal creep problem, where temperature gradients along adjacent walls generate a rarefied flow.

*Email for correspondence: jnassios@student.unimelb.edu.au