

# An Optimal-Control-Based Framework for Trajectory Planning, Threat Assessment, and Semi-Autonomous Control of Passenger Vehicles in Hazard Avoidance Scenarios: Experimental Results

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**Abstract** This paper describes the design of an optimal-control-based active safety framework that performs trajectory planning, threat assessment, and semi-autonomous control of passenger vehicles in hazard avoidance scenarios. This framework allows for multiple actuation modes, diverse trajectory-planning objectives, and varying levels of autonomy. A model predictive controller iteratively plans a best-case vehicle trajectory online as a constrained optimal control problem. The framework then uses this trajectory to assess the threat posed to the vehicle and intervenes in proportion to this threat. This approach minimizes controller intervention while ensuring that the vehicle does not depart from a safe corridor of travel. Experimental results are presented to demonstrate the framework's ability to incorporate multiple threat metrics and configurable intervention laws while sharing control with a human driver. Various maneuvers are tested, including lane-keeping, hazard avoidance, and multiple hazard avoidance.

## Introduction

Recent traffic safety reports from the National Highway Traffic and Safety Administration show that in 2007 alone, over 41,000 people were killed and another 2.5 million injured in motor vehicle accidents in the United States [1]. The long-standing presence of passive safety systems in motor vehicles, combined with the ever-increasing influence of active systems, has contributed to a decline in these

numbers from previous years. Still, the need for improved collision avoidance technologies remains high.

Recent developments in onboard sensing, lane detection, obstacle recognition, and drive-by-wire capabilities have facilitated active safety systems that share steering and/or braking control with the driver [2,3]. These active safety systems operating with a “human in the loop” generally attempt to honor driver intentions, opposing them only when doing otherwise would lead to a collision or loss of control. Such modification of the driver’s intended trajectory requires that these systems assess the threat posed to a vehicle in order determine when and how strongly to intervene. Such systems should honor safe driver inputs and maneuvers while intervening when necessary to correct or override those deemed unsafe.

Among existing proposals for semi-autonomous vehicle navigation, lane-keeping systems using audible warnings [4], haptic alerts [5], steering torque overlays [6], and various combinations of these have been developed with mixed results [7]. In a recent subproject of the European PReVENT consortium, a lane-keeping system was designed to prevent lane departure by perceiving the environment, making heuristic-based trajectory planning decisions based on perceived threat, and implementing warning mechanisms or slight steering torque overlay when the vehicle drifts from the desired trajectory [8].

Many of the semi-autonomous systems developed in previous work address only one piece of the active safety problem. While some use planning algorithms such as rapidly-exploring random trees [3], evolutionary programming [9] or potential fields analysis [10] to plan a safe vehicle path, others simply begin with this path presumed [11]. The threat posed by a particular path is seldom assessed by the controller itself and is often only estimated by a crude threat metric such as lateral vehicle acceleration required to track the path [12]. Finally, hazard avoidance is commonly performed using one or more actuation methods (steering, differential braking, etc.) without explicitly accounting for the effect of driver inputs on the vehicle trajectory [8]. Such controllers selectively replace (rather than assist) the driver in performing the driving task.

Yu addressed this problem in mobility aids for the elderly by designing an adaptive shared controller which allocates control authority between the human user and a controller in proportion to the user’s performance [13]. These metrics and the associated intervention are designed to act on current and past user performance, however, and do not anticipate future states or performance. This reactive approach to semi-autonomy, while sufficient to control low-speed mobility aids, is not well suited for higher-speed applications with significant inertia effects and no pre-planned trajectory.

In this paper, a framework for passenger vehicle active safety is developed that performs vehicle trajectory planning, threat assessment, and hazard avoidance in a unified manner. This framework leverages the predictive and constraint-handling capabilities of MPC to plan trajectories through a pre-selected corridor, assess the threat this path poses to the vehicle, and regulate driver and controller inputs to maintain that threat below a given threshold.

Section II describes the semi-autonomous control framework and its associated trajectory prediction, control law, threat assessment, and intervention law. Section III discusses experimental setup and results. Finally, the paper concludes with general conclusions and recommendations in Section V.

## Framework Description

The framework described below leverages the predictive- and constraint-handling capabilities of MPC to perform trajectory planning, threat assessment, and hazard avoidance. First, an objective function is established to capture desirable performance characteristics of a safe or “optimal” vehicle path. Boundaries tracing the edges of the drivable road surface are assumed to have been derived from forward-looking sensor data and a higher-level corridor planner. These boundaries establish constraints on the vehicle’s projected position. This constraint data, together with a model of the vehicle dynamics is then used to calculate an optimal sequence of inputs and the associated vehicle trajectory. The predicted trajectory is assumed to be a “best-case” scenario and used to establish the minimum threat posed to the vehicle given its current state and a series of best-case inputs. This threat is then used to calculate the necessary intervention required to prevent departure from the safe region of travel and driver/controller inputs are scaled accordingly. Fig. 1 shows a block diagram of this system.

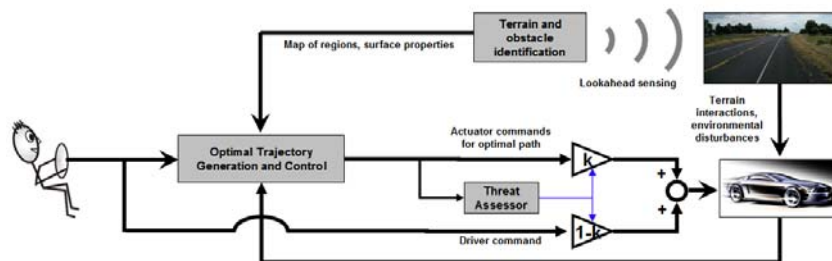


Fig. 1. Diagram of an active safety system.

In this paper it is assumed that road lane data is available and that road hazards have been detected, located, and mapped into a 2-dimensional safe corridor of travel. Existing systems and previous work in onboard sensing and sensor fusion justify this as a reasonable assumption [14]. Radar, LIDAR, and vision-based lane-recognition systems [3,15], along with various sensor fusion approaches [16] have been proposed to provide the lane, position, and environmental information needed by this framework.

Additionally, where multiple corridor options exist (such as cases where the roadway branches or the vehicle must circumnavigate an obstacle in the center of the lane), it is assumed that a high-level path planner has selected a single corridor through which the vehicle should travel.

## Vehicle Path Planning

The best-case (or baseline) path through a given region of the state space is established by a model predictive controller. As described in later sections, metrics from this predicted path will be used to assess threat.

Model Predictive Control is a finite-horizon optimal control scheme that iteratively minimizes a performance objective defined for a forward-simulated plant model subject to performance and input constraints. Stated another way, MPC uses a model of the plant to predict future vehicle state evolution and optimize a set of inputs such that this prediction satisfies constraints and minimizes a user-defined objective function. At each time step,  $t$ , the current plant state is sampled and a cost-minimizing control sequence spanning from time  $t$  to the end of a control horizon of  $n$  sampling intervals,  $t+n\Delta t$ , is computed subject to inequality constraints. The first element in this input sequence is implemented at the current time and the process is repeated at subsequent time steps. The basic MPC problem setup is described in [17].

The vehicle model used in MPC accounts for the kinematics of a 4-wheeled vehicle, along with its lateral and yaw dynamics. Vehicle states include the position of its center of gravity  $[x, y]$ , its yaw angle  $\psi$ , yaw rate  $\dot{\psi}$ , and sideslip angle  $\beta$ , as illustrated in Fig. 2. Table 1 defines and quantifies this model's parameters.

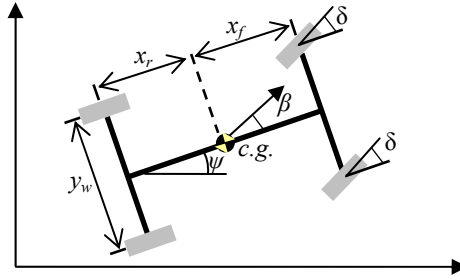


Fig. 2. Vehicle model used in MPC controller.

Table 1. Vehicle model parameters.

Symbol	DESCRIPTION	Value [units]
$m$	Total vehicle mass	2050 [kg]
$I_{zz}$	Yaw moment of inertia	3344 [kg m <sup>2</sup> ]
$x_f$	C.g. distance to front wheels	1.43 [m]
$x_r$	C.g. distance to rear wheels	1.47 [m]
$C_f$	Front cornering stiffness	1433 [N/deg]
$C_r$	Rear cornering stiffness	1433 [N/deg]
$V$	Vehicle speed	5, 10, 14 [m/s]

Tire compliance is included in the model by approximating lateral tire force ( $F_y$ ) as the product of wheel cornering stiffness ( $C$ ) and wheel sideslip ( $\alpha$ ) as shown in (1).

$$F_y = C\alpha \quad (1)$$

Linearized about a constant speed and assuming small slip angles, the equations of motion for this model are

$$\dot{x} = V \quad (2)$$

$$\dot{y} = V(\psi + \beta) \quad (3)$$

$$\dot{\beta} = -\frac{(C_r + C_f)}{mV} \beta + \left( \frac{(C_r x_r - C_f x_f)}{mV^2} - 1 \right) \dot{\psi} + \frac{C_f}{mV} \delta \quad (4)$$

$$\dot{\psi} = \frac{(C_r x_r - C_f x_f)}{I_{zz}} \beta - \frac{(C_r x_r^2 + C_f x_f^2)}{I_{zz} V} \dot{\psi} + \frac{C_f x_f}{I_{zz}} \delta \quad (5)$$

where  $C_f$  and  $C_r$  represent the cornering stiffness of the lumped front wheels and the lumped rear wheels, and  $x_f$  and  $x_r$  are the longitudinal distances from the c.g. of the front and rear wheels, respectively.

### Constraint Setup and Objective Function Description

As mentioned above, this framework assumes that the environment has been delineated previously. The boundaries of the safe road surface at each timestep are then described by the constraint vectors

$$\mathbf{y}^y_{\max}(k) = \begin{bmatrix} y^y_{\max}(k+1) \\ \vdots \\ y^y_{\max}(k+p) \end{bmatrix}, \quad \mathbf{y}^y_{\min}(k) = \begin{bmatrix} y^y_{\min}(k+1) \\ \vdots \\ y^y_{\min}(k+p) \end{bmatrix} \quad (6)$$

In (6),  $\mathbf{y}^y_{\max}$  and  $\mathbf{y}^y_{\min}$  represent the upper and lower limits on the vehicle lateral position ( $y$ ). In order for the constraint space to remain feasible, the following inequality must be satisfied

$$\mathbf{y}^y_{\max} - \mathbf{y}^y_{\min} > 0 \quad (7)$$

By enforcing vehicle position constraints at the boundaries of the safe region of the road surface (i.e. the lane edges on an unobstructed road), the controller forces the MPC-generated path to remain within the constraint-bounded corridor whenever dynamically feasible. Coupling this lateral position constraint with input constraints, input rate constraints, and vehicle dynamic considerations, the safe operating corridor delineated by  $\mathbf{y}^y_{\max}$  and  $\mathbf{y}^y_{\min}$  translates to a safe operating region within the state space.

The controller's projected path through the constraint-imposed tube is shaped by the performance objectives established in the MPC cost function. While many options exist for characterizing desirable vehicle trajectories, here, the total sideslip angle at the front wheels ( $\alpha$ ) was chosen as the trajectory characteristic to be minimized in the objective function. This choice was motivated by the strong influence front wheel sideslip has on the controllability of front-wheel-steered vehi-

cles since cornering friction begins to decrease above critical slip angles. In [18] it is shown that limiting tire slip angle to avoid this strongly nonlinear (and possibly unstable) region of the tire force curve can significantly enhance vehicle stability and performance. Further, the linearized tire compliance model described here does not account for this decrease, motivating the suppression of front wheel slip angles to reduce controller-plant model mismatch. Finally, trajectories that minimize wheel slip also tend to minimize lateral acceleration and yaw rates, leading to a safer and more comfortable ride.

The MPC objective function then takes the form

$$J_k = \sum_{i=k+1}^{k+p} \frac{1}{2} \alpha_i^T R_\alpha \alpha_i + \sum_{i=k}^{k+p-1} \frac{1}{2} \delta_i^T R_\delta \delta_i + \sum_{i=k}^{k+p-1} \frac{1}{2} \Delta \delta_i^T R_{\Delta \delta} \Delta \delta_i + \frac{1}{2} \rho_\varepsilon \varepsilon^2 \quad (8)$$

where  $\varepsilon$  represents constraint violation and was included to soften select position constraints.

In summary, the MPC controller uses vehicle position, input magnitude, and input rate constraints to satisfy safety requirements, while minimizing front wheel slip to maximize controllability.

### Threat Assessment

The vehicle path calculated by the MPC controller is assumed to be the best-case or safest path through the environment. As such, key metrics from this prediction are used to assess instantaneous threat posed to the vehicle. By setting constraint violation weights ( $\rho_\varepsilon$ ) significantly higher than the competing minimization weight ( $R_\alpha$ ) on front wheel sideslip, a hierarchy of objectives is created in order to force the optimal solutions to satisfy corridor constraints before minimizing front wheel sideslip. When constraints are not active, as illustrated by the grey vehicle in Fig. 3, front wheel sideslip – and the corresponding controllability threat – is minimized. When the solution is constrained, predicted front wheel sideslip increases with the severity of the maneuver required to remain within the safe corridor.

The dark vehicle in Fig. 3 illustrates how the MPC-predicted optimal vehicle trajectory might appear as the tire slip angles and corresponding threat increase in the presence of an active constraint. As predicted sideslip approaches tire-cornering-friction-imposed limits, the threat of leaving the safe corridor increases.

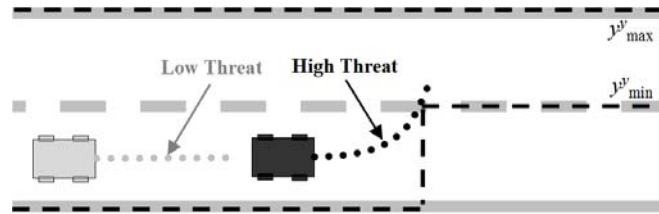


Fig. 3. Obstacle avoidance scenario showing MPC trajectory plans and corresponding threat.

Various approaches are available to reduce the vector  $\mathbf{a}$  to a scalar threat metric  $\Phi$ . In this work,

$$\Phi(k) = \max\left(\alpha_{k+1} \quad \alpha_{k+2} \quad \cdots \quad \alpha_{k+p}\right)^T \quad (9)$$

was chosen for its good empirical performance when used to regulate controller intervention (described in the next section).

### ***Hazard Avoidance***

Given a best-case vehicle path through the environment and a corresponding threat, desired inputs from the driver and controller are blended and applied to the vehicle. This blending is performed based on the threat assessment: a low predicted threat causes more of the driver's input and less of the controller's input to be applied to the vehicle, while high threat allows controller input to dominate that of the driver. This "scaled intervention" may thereby allow for a smooth transition in control authority from driver to controller as threat increases.

Denoting the current driver input by  $u_{dr}$  and the current controller input by  $u_{MPC}$ , the blended input seen by the vehicle,  $u_v$ , is defined as

$$u_v = K(\Phi)u_{MPC} + (1 - K(\Phi))u_{dr} \quad (10)$$

The intervention function  $K$  is used to translate predicted vehicle threat ( $\Phi$ ) into a scalar blending gain. This function is bounded by 0 and 1 and may be linear, piecewise-linear, or nonlinear. Linear and piecewise-linear forms of this function may be described by

$$K = f(\Phi) = \begin{cases} 0 & 0 \leq \Phi \leq \Phi_{eng} \\ \frac{\Phi_{aut} - \Phi}{\Phi_{aut} - \Phi_{eng}} & \Phi_{eng} \leq \Phi \leq \Phi_{aut} \\ 1 & \Phi \geq \Phi_{aut} \end{cases} \quad (11)$$

In (11), the shape of  $K$  is described by the threat level at which the semi-autonomous controller engages ( $\Phi_{eng}$ ) and the level at which it is given full control authority and effectively acts as an autonomous controller ( $\Phi_{aut}$ ).

Using predicted threat ( $\Phi$ ) as calculated in (9) with an appropriate cost function formulation of the form (8) ensures that 1) the threat metric regulating controller intervention is minimized in the path plan (and associated control calculation) and 2) the controller maintains full control authority when constraints are binding.

Increasing  $\Phi_{eng}$  widens the "low threat" band in which the driver's inputs are unaffected by the controller. While this provides greater driver freedom for low-threat situations, this freedom comes at the cost of increasing the rate of controller intervention when  $\Phi_{eng}$  is exceeded. This increased rate of intervention may adversely affect driver experience, as discussed in the results below.

Increasing the value of  $\Phi_{\text{aut}}$ , on the other hand, delays complete controller intervention until more severe maneuvers are predicted. The friction-limited bounds on the linear region of the tire force curve (1) suggest a natural upper limit of  $\Phi \leq 5$  degrees on surfaces with a friction coefficient of 1.0 in order to ensure that by the time the predicted maneuver required to remain within the safe region of the state space reaches this level of severity, the controller has full control authority and can – unless unforeseen constraints dictate otherwise – guide the vehicle to safety.

## Experimental Setup

Experimental testing was performed using a test vehicle and three human drivers. Driver and actuator steering inputs were coupled via an Active Front Steer (AFS) system. An inertial and GPS navigation system was used to measure vehicle position, sideslip, yaw angle, and yaw rate while a 1 GHz dSPACE processor ran controller code and interfaced with steering actuators.

Three common scenarios were used to analyze system performance. In each scenario, obstacles, hazards, and driver targets were represented to the driver by cones and lane markings and to the controller by a constrained corridor (with on-board sensing and constraint mapping assumed to have been performed previously by “virtual sensors” and high-level planners respectively). Only results from multiple-hazard-avoidance tests are shown below. These experiments tested the controller’s ability to navigate more complex road/hazard setups that required maneuvers with appreciable load transfer. In these tests (illustrated in Fig. 4), both lanes of travel were blocked at different locations, forcing the vehicle to change lanes to avoid the first hazard, then change lanes again to avoid the second as in a double-lane-change maneuver.

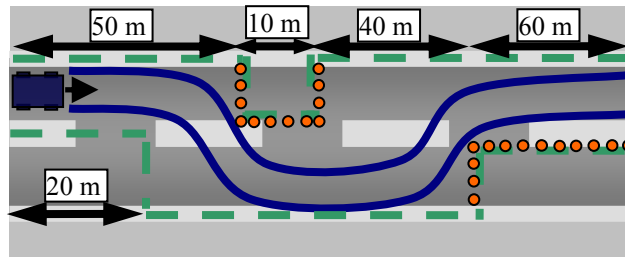


Fig. 4. Multiple hazard avoidance test setup showing hazard cone placement (circles) and lane boundaries (dashed).

Hazard avoidance and multiple hazard avoidance tests were conducted using two different types of driver inputs. Sleeping, inattentive, or otherwise impaired drivers were represented by a constant driver steer input of zero degrees. In these tests, the unassisted driver’s path formed a straight line directly through the obstacle(s). To represent active driver steer inputs, the drivers were asked in separate tests to steer either around or into the obstacles. The urgency of these driver steer

events was varied – sometimes avoiding the obstacle(s) with a smooth input an other times, steering at the last minute.

Table 2 describes and quantifies controller parameters.

Table 2. Controller parameters.

Symbol	DESCRIPTION	Value [units]
$p$	Prediction horizon	35, 40
$n$	Control horizon	18, 20
$R_y^{(a)}$	Weight on front wheel slip	0.2657
$R_u$	Weight on steering input	0.01
$R_{\Delta u}$	Weight on steering input rate ( $\Delta$ per $\Delta t$ )	0.01
$u_{\min/\max}$	Steering input constraints	$\pm 10$ [deg]
$\Delta u_{\min/\max}$	steering input rate (per $\Delta t$ ) constraints	$\pm .75$ [deg] (15 deg/s)
$y_{\min/\max}^y$	Lateral position constraints	Scenario-dependent
$\rho_\epsilon$	Weight on constraint violation	$1 \times 10^5$
$\Phi_{\text{eng/aut}}$	Thresholds for controller intervention	[0 3], [1 3] deg
$V$	Variable constraint relaxation on vehicle position	[1.25, ..., 1.25, 0.01]

## Experimental Results

The semi-autonomous framework proved capable of keeping the vehicle within the safe region of travel for each of the maneuvers tested and with three different human drivers. Each of the maneuvers tested proved feasible for multiple system configurations. Maneuver-specific results for multiple hazard avoidance experiments are shown below.

Fig. 5 compares a semi-autonomous multi-hazard-avoidance maneuver to an autonomous maneuver ( $K=1$ ).

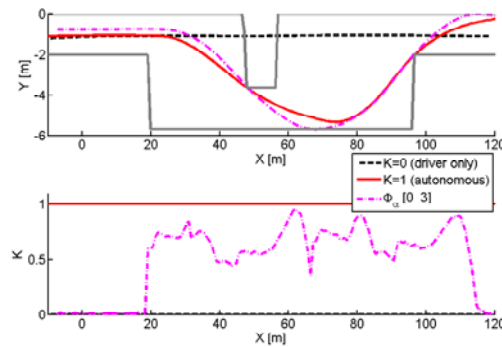


Fig. 5. Multiple hazard avoidance tests showing the similarity between semi-autonomous (dash-dot) and autonomous (solid) vehicle trajectories.

Notice that the semi-autonomous controller delayed intervention until the driver's inputs put the vehicle at risk of leaving the safe road surface. When the framework did intervene, it allocated enough control authority to the controller to avert safe lane departure or loss of control. Even with average controller intervention  $K_{ave}=0.44$ , the ultimate vehicle trajectory using the semi-autonomous controller very closely resembles the "best case" trajectory taken by the autonomous controller. This results from the selective nature of the semi-autonomous system – it intervenes only when necessary, then relinquishes control to the driver once threat to the vehicle has been reduced.

Fig. 6 shows experiments in which the driver was instructed to swerve at the last minute to avoid hazards.

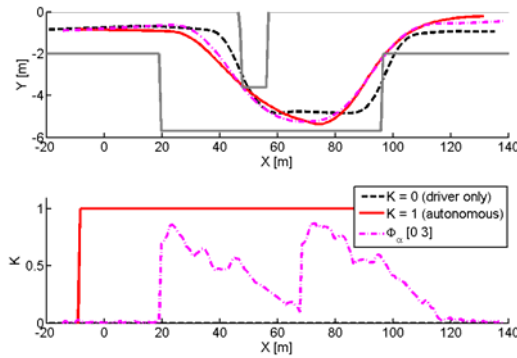


Fig. 6. Multiple hazard avoidance tests showing the vehicle trajectory with an unassisted driver input (dashed) and autonomous controller (solid), and semi-autonomous controller (dash-dot). In each case, the driver swerved to avoid hazards.

Notice that intervention by the semi-autonomous controller slightly preceded an otherwise-late driver reaction. The combined effect of both inputs was then sufficient to avoid both road hazards.

In each of the above experimental results, this shared-adaptive controller behaves as a stable closed-loop system. While this was also true of all of the other simulated and experimental results conducted to date, no rigorous stability proof is presented in this paper.

## Conclusions

This paper presented an optimal-control-based framework that performs trajectory planning, threat assessment, and semi-autonomous control of passenger vehicles in hazard avoidance. This framework has been proven experimentally capable of satisfying position, input, and dynamic vehicle constraints using multiple threat metrics and intervention laws. Additionally, this framework has been shown to provide significant autonomy to a human driver, intervening only as necessary to

keep the vehicle under control and within the safe roadway corridor. Experimental results have also shown this control framework to be stable even in the presence of system-inherent time delays, though a rigorous stability proof is a topic of current investigation.

Finally, while human factors have not been studied in depth here, it is expected that with additional investigation, a best-case, or average driver-preferred intervention law may be described and intervention settings tuned accordingly. Further work is needed before this research is road-ready.

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