A Stochastic Response Surface Approach to Statistical Prediction of Mobile Robot Mobility

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Abstract — **The ability of autonomous or semi-autonomous mobile robots to rapidly and accurately predict their mobility characteristics is an important requirement for their use in unstructured environments. Most methods for mobility prediction, however, assume precise knowledge of environmental (i.e. terrain) properties. In practical conditions, significant uncertainty is associated with terrain parameter estimation from robotic sensors, and this uncertainty must be considered in a mobility prediction algorithm. Here a method for efficient mobility prediction based on the stochastic response surface approach is presented that explicitly considers terrain parameter uncertainty. The method is compared to a Monte Carlo-based method and simulations show that the stochastic response surface approach can be used for efficient, accurate prediction of mobile robot mobility.**

I. Introduction

MOBILE robots are increasingly required to operate in unstructured environments. A fundamental requirement for systems in such environments is the capacity to quickly and accurately predict their ability to negotiate rugged terrain regions and surmount obstacles. This "mobility prediction" capability is often integrated into motion planning algorithm, and is critical to the safety and efficient operation of the robotic systems.

There has been little research that explicitly addresses the challenge of autonomously assessing the traversability over a given terrain region or obstacle. Previous work by the authors has focused on stochastic performance prediction of mobile robots using classical Monte Carlo simulation methods [1]. Another recent approach to mobility prediction of small robots relies on analysis of system performance over obstacle "primitives" such as single rocks and well-characterized rock fields; however it is unclear how these results can be generalized to complex terrain profiles [2]. In addition, the issue of mobility through terrain with non-geometric hazards (such as highly deformable or slippery regions) is not addressed in this paradigm. Related work has developed a stochastic analysis of terrain profiles and wheel-terrain interaction [3]. This work, while related to the method proposed here, does not explicitly address the mobility prediction problem.

Manuscript received February 22, 2008. Support for this work was provided by the Army Research Office under award number W912HZ-07-P-0288.

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Most other related work has attempted to designate terrain regions as "traversable" or "non-traversable" based solely on remotely-sensed terrain geometry. One such approach for outdoor robots is described in [4]. An extension to the work, described in [5] attempts to characterize the nature of various obstacles; however this work focuses solely on identifying obstacles that are likely to be traversable despite their geometry (e.g. tall grass, which may possess an obstacle-like geometric profile but is often traversable due to its compliant nature). Another approach is presented in [6] to detect obstacles from color and LIDAR data. A novel terrain classification component is used to distinguish vegetation from the underlying terrain. This improves the estimate of the location of the load-bearing surface in thick vegetation; however it is not employed for mobility prediction.

Various statistical methods for mobility prediction have been developed by the U.S. Army over the past 50 years [7]-[9]. These mobility prediction methodologies include the NATO Reference Mobility Model (NRMM), NRMM II, and others. However, these are numerical algorithms for predicting cross-country vehicle movement at length scales of several meters to several kilometers. These methods were developed for vehicles of 500 kg and larger based on empirical results drawn from resource-intensive experimental testing. Thus, these techniques are generally inapplicable to small mobile robots, for which extensive empirical test data does not yet exist. In addition, the mobility prediction problem considered in this paper is concerned with movement over particular vehicle-sized terrain regions and obstacles, rather than gross (i.e. km-scale) mobility characteristics.

In summary, most previous methods for mobility prediction of mobile robots rely on deterministic analysis that assumes accurate knowledge of vehicle and terrain parameters. In field conditions, however, mobile robots frequently have access to only sparse and uncertain terrain parameter estimates drawn from "standard" robotic sensors such as LIDAR and vision. Moreover, vehicle parameters may be uncertain and/or time-varying due to, for example, fuel consumption, mechanical wear, and cargo and passenger loading.

There exists a vast body of literature on techniques to estimate the probability distributions of processes that are subject to uncertainty. Such techniques can be applied to the mobility prediction problem, by first modeling the uncertainty in terrain parameters, then defining a range for their probable values, and finally analyzing the performance of an analytical or numerical robot model over that parameter space, as in [1]. The result would be a prediction of the ability of a robot to successfully traverse a given route that rigorously considers vehicle and terrain parameter uncertainty. This analysis can be performed using a variety of techniques such as interval mathematics, probabilistic methods and fuzzy set theory among others [10]-[12].

A traditional method for estimating the probability density function of a system's output response from known or estimated input distributions is the Monte Carlo method [13], [14]. This approach involves the random selection of a value for each uncertain parameter from its uncertainty range, weighted by its probability of occurrence, followed by model simulation using this parameter set. This process is repeated many times to obtain the probability distribution of an output metric. Since parameter values are selected randomly, a large number of simulation runs is generally required to obtain reasonable results, leading to a (usually) high computational cost. Structured sampling techniques such as Latin hypercube sampling, importance sampling, and others can be used to improve computational efficiency; however these gains may be modest for complex problems [15], [16].

More recent approaches to stochastic simulation include the polynomial chaos approach, which is based on Wiener's theory of homogeneous chaos. Since the introduction of the spectral stochastic finite element method [17], polynomial chaos has been successfully applied to represent uncertainty in various structural and fluid mechanics problems. As noted above, researchers have recently applied this technique to the dynamic simulation of a 7 DOF vehicle [3]. However, the collocation approach employed therein has been noted to be inherently unstable and exhibit convergence problems [18]. Moreover, different combinations of collocation points may lead to considerably different output estimates, or they may not correspond to high probability regions of the input parameter space.

In this paper a method for efficient mobility prediction based on the stochastic response surface (SRSM) approach is presented that explicitly considers uncertainty in terrain parameters. This method follows an implementation described in [19] to develop a regression based approach to obtain an equivalent reduced-order model for robot drawbar pull (i.e. net wheel/track thrust), which can then be used for computationally inexpensive analysis of robot mobility. This method is here applied to the case of a simplified scenario involving traversability of highly deformable uncertain natural terrain. Simulation results for the SRSM approach are compared to results from a classical Monte Carlo analysis.

This paper is organized as follows. In Section 2, the Monte Carlo and response surface methods are briefly introduced, and their applicability to the study of robot dynamics and mobility analysis is discussed. This is followed by a description of a simplified mobile robot terrain traversal scenario in Section 3. The effect of terrain physical parameter uncertainty on vehicle mobility is analyzed. The simulation results obtained using Monte Carlo and SRSM approaches are compared in Section 4. It can be seen that accurate, efficient statistical mobility prediction can be achieved using the proposed technique.

II. Uncertainty Analysis Techniques

A. Monte Carlo Method

Monte Carlo methods involve the analysis of a (usually) large number of simulation runs of an analytical or numerical system model with various combinations of model parameters. The model parameters (known as "input parameters") are randomly sampled from their respective probability distributions. These distributions are assumed to be known (or can be estimated) a priori. From this analysis, an estimate of the probability distribution of a user-defined output metric can be estimated.

Various methods have been proposed for efficient sampling from the input parameter probability distributions, including stratified, importance and Latin Hypercube sampling (among others) [20], [21]. Generally, these methods attempt to ensure that samples are generated from the entire range of the input parameter space while reducing computational costs.

In this paper, uncertain terrain parameters are considered as the input parameters in the mobility prediction scenario. A fundamental assumption of the proposed approach is that while the terrain parameters may not be precisely known, engineering estimates of their distributions are available. This is a reasonable assumption since many methods exist for coarsely classifying terrain from standard robotic sensors such as LIDAR and vision [22]-[24].

Algorithmic implementation

For completeness, a procedure for Monte Carlo analysis is here reviewed, in the context of mobile robot mobility prediction. This analysis considers functions of the form:

$$\mathbf{Y} = \mathbf{g}(\mathbf{X}) \tag{1}$$

where $\mathbf{g}$ represents the model under consideration, $\mathbf{X}$ is a vector of uncertain input variables and $\mathbf{Y}$ represents a vector of estimated outputs.

A general procedure for Monte Carlo analysis is as follows:

a) Define a vector $\mathbf{X}$ consisting of n relevant terrain parameters. To characterize the uncertainty in the elements of $\mathbf{X}$, define the probability distribution for each input parameter, based on engineering estimates. This defines the input parameter space.

While many forms of the input parameter distribution are possible, in this paper, parameter values are assumed to be distributed normally and to be independent of one another.

b) Generate a sample value for each of the n input variables from its corresponding probability distribution. More specifically, a sample set:

$$\mathbf{X}_j = \left[x_{j1}, x_{j2}, \ldots, x_{jn} \right] \tag{2}$$

is generated from the input parameter space. This set may be generated randomly or using structured sampling techniques such as stratified sampling, importance sampling or Latin hypercube sampling.

In the "standard" Monte Carlo approach, random sampling of the input parameter distributions is performed. As an attempt to ensure representation of the entire parameter range, a large number of simulations must often be performed. Stratified sampling, on the other hand, partitions the sample space into strata, with each stratum having a specified probability of occurrence. Random samples are then drawn from each stratum. While this ensures dense coverage of the parameter space, it requires definition of the strata and calculation of their probabilities. Latin hypercube sampling can ensure dense coverage of the range of each input variable while avoiding difficulties associated with stratified sampling. It achieves this by exhaustively dividing each input parameter's range into disjoint intervals of equal probability, then randomly sampling a parameter value from each interval. This ensures representation from the entire range of each variable. This is illustrated in Fig. 1.

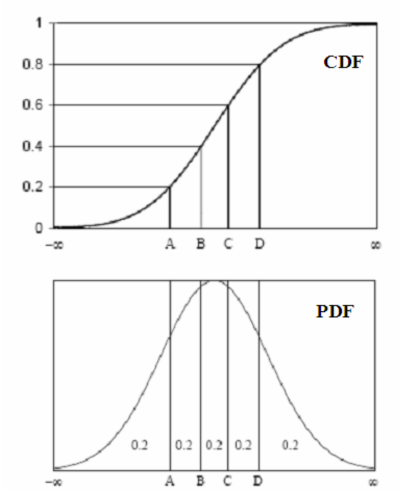

Fig. 1. Illustration of sampling using the Latin Hypercube Method.

c) Evaluate the output response from the analytical or numerical system model under analysis using the values from the input parameter set $\mathbf{X_j}$ as model parameter values.

d) Repeat steps b) and c) to generate a distribution for the output metric. The number of simulations (*N*) is chosen to be large enough such that the output distribution converges to a stable value. The probability distribution of the output metric can then be determined, as well as various statistics such as its estimated expectation, μ, or variance, σ^2, as follows:

$$\mu = \frac{1}{N}\sum_{j=1}^{N} g(X_j) \tag{3}$$

$$\sigma^2 = \frac{1}{N}\sum_{j=1}^{N}(g(X_j) - \mu)^2 \tag{4}$$

Fig. 2 represents schematically the general Monte Carlo approach for uncertainty analysis:

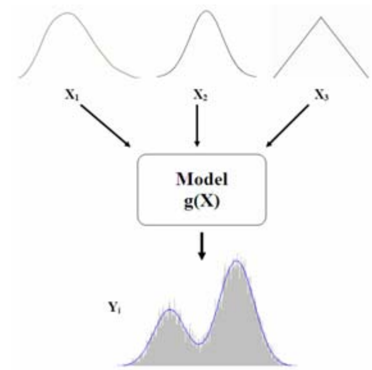

Fig. 2. Illustration of uncertainty analysis using the Monte Carlo method.

B. *Stochastic Response Surface Method*

The stochastic response surface method (SRSM) represents inputs and outputs of a system under consideration via series approximations using standard random variables. This can result in a computationally efficient means for uncertainty propagation through computational models.

In SRSM, inputs are represented as functions of normal random variables, each having zero mean and unit variance. The same set of random variables which that is used to represent input stochasticity can then be used for representation of outputs.

An equivalent reduced model for an output is expressed in the form of a series expansion consisting of multi-dimensional Hermite polynomials of normal random variables, as:

$$y = a_0 + \sum_{i_1=1}^{n} a_{i_1}\Gamma_1(\xi_{i_1}) + \sum_{i_1=1}^{n}\sum_{i_2=1}^{i_1} a_{i_1 i_2}\Gamma_2(\xi_{i_1}, \xi_{i_2}) + \ldots \tag{5}$$

where y refers to an output metric, a_{i1}, a_{i2},… are coefficients to be determined, ξ_{i1}, ξ_{i2},… are i.i.d. normal random variables, and $\Gamma_q(\xi_{i1}, \xi_{i2},\ldots, \xi_{iq})$ is the Hermite polynomial of degree *q*, given as:

$$\Gamma_q(\xi_{i_1}, \xi_{i_2}, \ldots, \xi_{i_q}) = (-1)^q e^{\frac{1}{2}\boldsymbol{\xi}^{\mathbf{T}}\boldsymbol{\xi}} . \frac{\partial^q}{\partial\xi_{i_1}\ldots\ldots\partial\xi_{i_q}} . e^{-\frac{1}{2}\boldsymbol{\xi}^{\mathbf{T}}\boldsymbol{\xi}} \tag{6}$$

For notational simplicity, the series may be written as:

$$y = \sum_{j=0}^{N} y_j \Phi_j(\xi) \tag{7}$$

where the series is truncated to a finite number of terms and there exists a correspondence between $\Gamma_q(\xi_{i1}, \xi_{i2},\ldots, \xi_{iq})$ and $\Phi(\xi)$, and their corresponding coefficients.

The series expansion contains unknown coefficient values that can be estimated from a limited number of model simulations to generate an approximate reduced model. This is achieved by choosing a set of sample points from high probability regions and generating model output at these points [19], [26]. A regression based approach is then utilized to obtain the values for the unknown coefficients [25]. Once the (statistically equivalent) reduced model is formulated, it can be used to facilitate analysis of the system under uncertainty.

This procedure thus results in a reduction in the number of model simulations (and, therefore, a reduction in computational cost) required for estimation of output uncertainty, as compared to the conventional probabilistic methods such as Monte Carlo methods.

Algorithmic implementation

Here a summary of the SRSM procedure is presented as it will be applied in our analysis. Further details can be found in [25].

a) Represent uncertain input parameters in terms of standard random variables (here Gaussian variables). A terrain parameter X_j can then be written as:

$$X_j = \mu_j + \sigma_j \xi \tag{8}$$

where μ_j is the mean, σ_j represents the standard deviation and ξ is a standard normal random variable.

b) Express the model output under consideration in terms of the same set of random variables. While for Gaussian variables Hermite polynomials are used, different orthogonal polynomial basis functions are used corresponding to the probability distributions of other non-Gaussian variables. This is shown in Table 1.

TABLE 1
POLYNOMIAL BASIS FUNCTIONS AND CORRESPONDING RANDOM VARIABLES

Random Variable	Polynomial Function
Gaussian	Hermite
Gamma	Laguerre
Beta	Jacobi
Uniform	Legendre

c) Estimate the unknown coefficients of the approximating series expansion. This is accomplished via a regression based approach, by computing the model output at a set of collocation points [19], [26]. These points are selected such that each standard random variable takes a value of either zero or a root of the Hermite polynomial of a higher order. This ensures that points from high probability regions are represented. Taking the number of collocation points to be nearly twice in number to the number of coefficients has been shown to yield robust coefficient estimates [25], [26]. Calculation of the model output at these points results in set of equations with the number of equations exceeding the number of unknown coefficients. This system of equations is then solved using the singular value decomposition technique.

d) Estimate the statistics of the output metric, modeled as a stochastic response surface in terms of a chaos expansion, using an efficient Monte Carlo method.

The advantage of the SRSM technique is that the number of model simulations is greatly reduced relative to more conventional methods, thus improving computational efficiency. Further, the accuracy of the computational model can often be increased by increasing the order of the polynomial chaos expansion.

III. Mobility Prediction Scenario

Here an analysis of a simplified mobile robot terrain traversal scenario is presented using the SRSM technique. Due to space limitations this analysis is simplified, however, it should be noted that the proposed approach can be applied to systems with a larger number of uncertain parameters.

To demonstrate the application of the technique to mobility analysis, the scenario considered here is that of a mobile robot traveling on flat, firm outdoor terrain (here modeled as heavy clay), then attempting to navigate up an inclined region of highly deformable terrain (here modeled as dry sand). This is illustrated in Fig. 3. It is assumed that significant uncertainty is associated with several important terrain physical parameters. The robot's mobility is analyzed using a baseline "standard" Monte Carlo approach (SMC), a Latin hypercube Monte Carlo approach (LHSMC) and SRSM techniques.

A. Scenario Description

A simple description of robot mobility in the proposed scenario is defined as the probability that, for a given initial velocity (u_o) at robot initial position (A) (See Fig. 3), the robot will have a positive velocity at point (B), after traversing the sandy incline. For various uncertain terrain parameters, this metric can be presented as a distribution of traversal probability versus initial velocity. This distribution can then be used to predict for which velocities the robot will be able to traverse the deformable terrain region with a reasonably high probability.

For online implementation, the first step would be autonomous classification of terrain patches, which would then be associated with a priori defined ranges of terrain parameters corresponding to each class. These data would then be supplied to the SRSM algorithm as inputs.

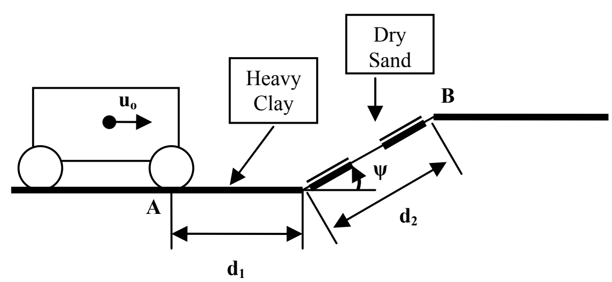

Fig. 3. Simplified scenario for mobility prediction under uncertainty

B. Reduced SRSM Model for Drawbar Pull

A classical Bekker-type wheel terrain interaction model is used to calculate the drawbar pull (i.e. net longitudinal wheel thrust) for the above analysis [27]-[29]. This model assumes quasi-static motion, and that the robot wheel is stiff relative to the terrain. An equivalent model is then formulated using the approach.

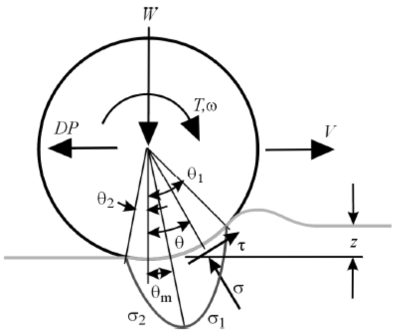

Fig. 4. Wheel-terrain interaction model for rigid wheel on deformable terrain.

For the vehicle soil interaction model shown in Fig. 4, drawbar pull is given by:

$$DP = rb\left[\int_{\theta_1}^{\theta_2} \tau(\theta)\cos\theta \, d\theta - \int_{\theta_1}^{\theta_2} \sigma(\theta)\sin\theta \, d\theta\right] \quad (10)$$

where *σ(θ)* and *τ(θ)* represent respectively, the normal stress and the shear stress at the wheel-terrain interface (divided into two regions in Fig. 4 to more clearly represent the stress distribution), and are given by:

$$\sigma_1(\theta) = (\frac{k_c}{b} + k_\phi)\left(r(\cos\theta - \cos\theta_1)\right)^n \quad (11)$$

$$\sigma_2(\theta) = (\frac{k_c}{b} + k_\phi)\left[r\left((\cos\left(\theta_1 - \theta\frac{(\theta_1 - \theta_m)}{\theta_m}\right) - \cos\theta_1)\right)\right]^n \quad (12)$$

$$\tau(\theta) = (c + \sigma(\theta)\tan\phi)\left(1 - e^{-\frac{r}{k}[\theta_1 - \theta - (1-i)(\sin\theta_1 - \sin\theta)]}\right) \quad (13)$$

The drawbar pull can hence be written as:

$$DP = rb\left(\begin{array}{l}\int_{0}^{\theta_m} \tau_2(\theta)\cos\theta \, d\theta + \int_{\theta_m}^{\theta_1} \tau_1(\theta)\cos\theta \, d\theta \\ -\int_{0}^{\theta_m} \sigma_2(\theta)\sin\theta \, d\theta - \int_{\theta_m}^{\theta_1} \sigma_1(\theta)\sin\theta \, d\theta\end{array}\right) \quad (14)$$

The parameters employed in (10)-(14) are defined in Table 2.

TABLE 2
PARAMETERS INVOLVED IN DRAWBAR PULL CALCULATION

Symbol	Quantity
r	Wheel radius
b	Wheel width
θ_1	Angle corresponding to start of contact
θ_2	Angle corresponding to loss of contact
θ_m	Maximum stress angle
c	Cohesion
φ	Internal friction angle
i	Wheel slip
n	Sinkage exponent
k_c, k_φ	Pressure sinkage moduli
k	Shear Deformation Modulus

The governing equation of motion for the mobility prediction scenario can now be written as:

$$\ddot{x} = \frac{DP}{m} - g\sin\psi \quad (15)$$

where *m* is the vehicle mass, *g* represents the acceleration due to gravity and ψ is the angle of the incline w.r.t. the horizontal.

For the SRSM approach, a reduced stochastic model is developed for the drawbar pull considering *c* and *φ* as the uncertain parameters. These are represented as:

$$c = \mu_c + \xi_c\sigma_c \quad (16)$$

$$\varphi = \mu_\varphi + \xi_\varphi\sigma_\varphi \quad (17)$$

where ξ_c and ξ_φ are standard normal random variables. The drawbar pull is now expressed as:

$$DP = a_0 + a_1\xi_c + a_2\xi_\varphi + a_3(\xi_c^2 - 1) + a_4(\xi_\varphi^2 - 1) + a_5\xi_c\xi_\varphi \quad (18)$$

The parameters *c* and *φ* were chosen since they exhibit significant influence on terrain thrust. The parameters are assumed to be normally distributed, though other possible probability distributions (such as uniform or beta distribution) can be considered as well. The corresponding values for *c* and *φ* used in this analysis can be found below in Table 3.

TABLE 3
PROBABILITY DISTRIBUTION INFORMATION FOR UNCERTAIN TERRAIN PARAMETERS (c, φ)

Parameter	Distribution Function	Mean	Std. Dev.
c (Heavy Clay)	Gaussian	69 kPa	8.50 kPa
φ (Heavy Clay)	Gaussian	34 deg	2.10 deg
c (Dry Sand)	Gaussian	1.04 kPa	0.125 kPa
φ (Dry Sand)	Gaussian	28 deg	1.75 deg

C. Results

Here results from the analysis of the mobility prediction scenario using SMC, LHSMC and SRSM methods are presented for inclination angle (ψ) equal to 6º and 15º. (See Fig. 5).

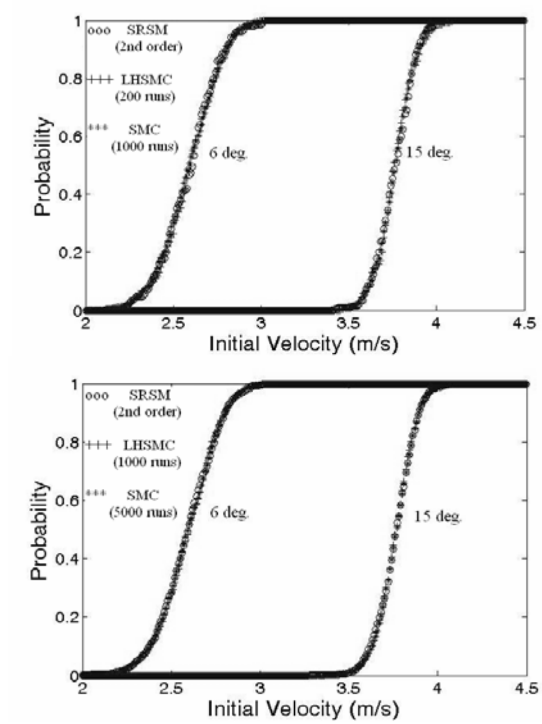

Fig. 5. Probability plots for mobility prediction scenario.

The coefficient values obtained for a 2nd order expansion of drawbar pull (18) for the sandy slope (ψ = 15º) were: a_0 = -1.4260, a_1 = 0.2981, a_2 = 0.5586, a_3 = 0.0000, a_4 = 0.0091, a_5 = 0.0000.

D. Discussion

The SRSM method is compared to LHSMC with respect to computational efficiency, while considering uncertainties in terrain parameters. The relative times taken (for the case when inclination angle is 10 degrees) are given in Table 4.

TABLE 4
TIME TAKEN FOR GENERATING PROBABILITY PLOTS

Method	Simulation Runs	Time Taken (sec)
SMC	5000	53.4060
	20000	210.9840
LHSMC	1000	10.9220
	10000	106.4530
SRSM (2nd order)		0.5940

The scenario discussed above clearly highlights a potential application of stochastic methods in mobility analysis, which can prove to be a computationally inexpensive method for mobility prediction.

IV. CONCLUSION AND FUTURE WORK

This paper has presented an approach to mobile robot mobility prediction based on the stochastic response surface method. This approach explicitly considers uncertainty present in terrain physical parameter estimates. Simulation results of a simplified mobility prediction scenario have shown that the proposed method represents a significant improvement over conventional Monte Carlo methods in terms of computational efficiency, and thus can be used for robustly and efficiently predicting the traversability of mobile robots in unstructured environments.

Current work in this area is focused on the application of statistical modeling methods to various aspects of mobile robot mobility. Stochastic analysis could also be performed to get the temporal response of a suitable output metric that would relate to the vehicular mobility.

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