Direct shear behaviour of dry, granular soils for low normal stress with application to lightweight robotic vehicle modelling

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Abstract

Modelling of soil shearing behaviour under wheeled or tracked vehicles requires the knowledge of three soil properties: cohesion, angle of internal friction, and shear modulus. For lightweight robots it is necessary to characterize the soil for small normal stress ($<$15kPa) while most of the data collected in the literature regards higher stress testing conditions. Soil failure at low stress may diverge from Mohr-Coulomb envelope invalidating the fundamental assumptions behind classical terramechanics approach. Through the analysis of direct shearing performance of a dry, granular, soil, this paper address several issues related to off-road traction mechanics (not necessarily limited to low stress cases). We present an improved approach for shear modulus calculation that overcome the inaccuracies introduced by Wong method. We analyze the importance of density in modifying terrain response. Moreover, we show how erroneous estimation of soil cohesion and angle of internal friction may limit the applicability of Bekker/Wong theory to lightweight tracked vehicles.

Keywords: Direct Shear Test, Mojave Mars Simulant, Shear modulus, Low stress, Lightweight robot

1. Introduction

Traction generation on deformable soil is intimately dependent on soil shearing properties. Wheels and tracks provide traction by inducing shear failure in

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the mass of soil beneath them. [14, 16] have visually shown that flow of soil under running gears follows failure envelope predicted by theory of plastic equilibrium. Several studies by Wong [12] highlighted that good correlation exists between measured shear stress under a track and direct shear tests results. Soil shearing fundamental properties are cohesion and angle of internal friction. These can be obtained through three main testing methodologies: tri-axial compression tests, direct (or ring) shear test, vane shear tests. Each methodology has its own strengths and weakness and it is beyond the scope of this study to discuss them. Direct shear test is chosen here because it is the testing methodology that closely mimic soil failure behaviour under a running gear. Mohr-Coulomb failure criterion is typically employed to describe soil shear strength. This consists in a linear envelope that bounds admissible shear-normal stress configurations [3]. However, this criterion provides a condition for soil failure but it does not provide any information regarding shear vs. displacement trend. Different shearing behaviours can be summarized under three main categories (also shown in figure 1):

- A Shear stress asymptotically reaches residual state.
- B Shear stress rises to a maximum, then decreases to reach steady residual state.
- C Shear stress rises to a maximum, then monotonically decreases without reaching steady residual stat.e

A and B behaviours have been observed for a wide range of mineral terrains while C behaviour is typical of organic soils (i.e., muskeg) and won't be considered in this study. Janosi and Hanamoto [4] proposed an empirical function to describe shear vs. displacement for A behaviour. The model is based on Mohr-Coulomb failure criterion augmented with an additional parameter k, referred as shear modulus. Bekker [2], Wong [13], and Oida [6] have introduced other fitting curves for type-B curves; these implementations are again based on Mohr-Coulomb envelope with the addition of some empirically tuned scaling function.

A part from shear stress trend it should be noted that the vast majority of data available in the literature regards data collected for large normal stress. In the last 15 years, classical terramechanics models have been employed for mobility analysis of lightweight rovers. However, discrepancies noticed by several researchers warrant for new models dedicated to lightweight vehicles.

The paper will present an improved method for shear modulus calculation in 2, a discussion of type A and B behaviour for granular soils in 3 , and a case study of lightweight tracked vehicle modelling in 4.

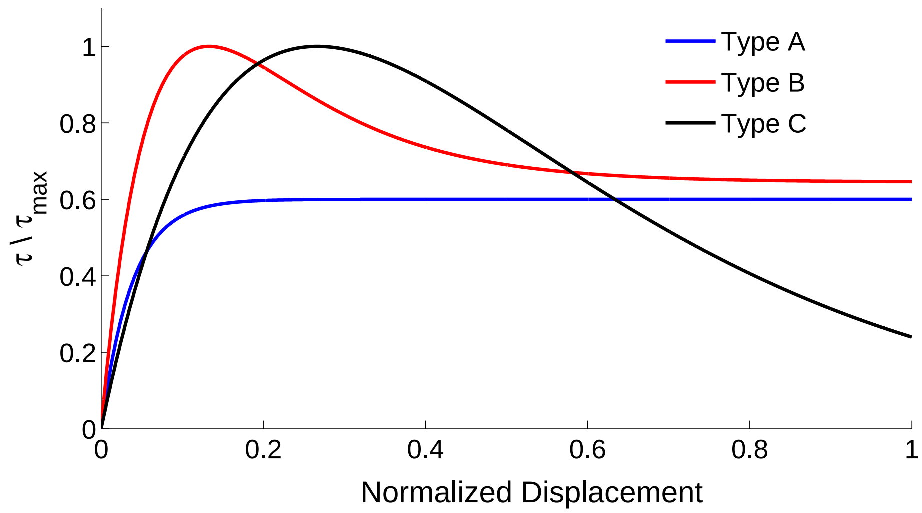

Figure 1: Idealized shear displacement curves for type A, B, and C behaviour.

2. Shear Modulus Calculation

Wong proposed an algorithm for automated calculation of shear modulus through direct (or ring) shear tests [15]. Method is based on least square minimization and it was developed for type A, B, and C. Notwithstanding its diffusion among researchers [12, 7], the algorithm introduced some simplifications that ultimately invalidated fit accuracy at low displacement. This section will focus on type A behaviour as modeled by Janosi and Hanamoto:

$$\tau = \tau_{max}\left(1 - e^{\frac{-j}{k}}\right) \tag{1}$$

where j is shear displacement and k is the fitting constant labeled shear modulus (τ_{max} will be discussed later). Wong rearranged equation 1 and, through least square minimization, came up with a closed form equation for k:

$$k = -\frac{\sum_{i=1}^{n}\left(1 - \frac{\tau_i}{\tau_{max}}\right)^2 j_i^2}{\sum_{i=1}^{n}\left(1 - \frac{\tau_i}{\tau_{max}}\right)^2 j_i \log\left(1 - \frac{\tau_i}{\tau_{max}}\right)} \tag{2}$$

where subscript i refers to the experimental data points, n is the number of samples collected, and τ_{max} is maximum shear measured. Disappointedly, the simplifications introduced by Wong severely deteriorate estimation accuracy. The algorithm was developed in an era when computational power was limited and model simplification was necessary to make it run quickly. Such approximations can be avoided now that computers' computational power is not a limitation. In particular, equation 2 is sensitive to the chosen shear displacement range. This is shown in figure 2 where an experimental direct shear dataset is compared with equation 1 fitted with Wong method and exact least square method. It is clear that Wong approach introduces inaccuracies that will prove to be critical in section 4.

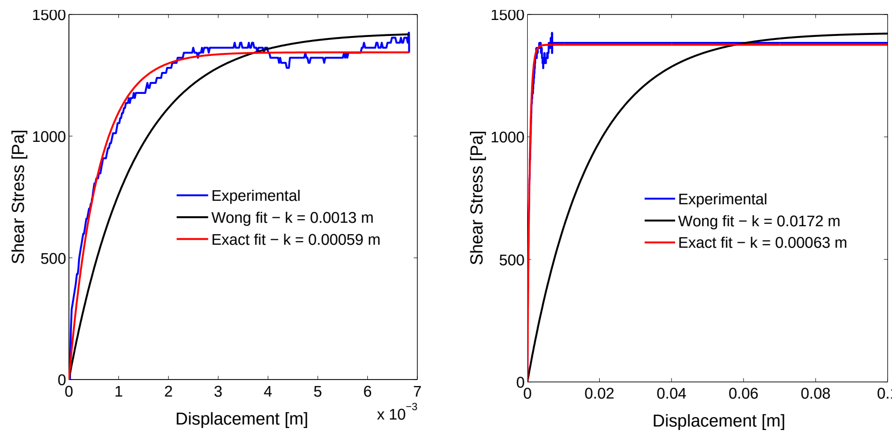

Figure 2: Quality of fit of Janosi-Hanamoto equation using Wong approximation and exact least square approach. Experimental data has been obtained through direct shear testing of a Mars Mojave Simulant [1, 8] subjected to a normal pressure equal to 2.08kPa.

Problem can be fixed by removing the simplifying assumptions. The objective function (least square approach) becomes:

$$\varepsilon(k,\tau_{max}) = \sum_{i=1}^{n}\left(\tau_i - \tau_{max}\left(1-e^{\frac{-j_i}{k}}\right)\right)^2 \tag{3}$$

Parameters k and τ_{max} can be found through standard minimization procedure:

$$\frac{\partial\varepsilon(k,\tau_{max})}{\partial k} = 0 \tag{4}$$

$$\frac{\partial\varepsilon(k,\tau_{max})}{\partial\tau_{max}} = 0 \tag{5}$$

3. Shear vs. Displacement

Terramechanics is a multi-disciplinary field that combines (principally) mechanical engineering and soil mechanics. This sometimes creates a *disconnect* between fundamental knowledge of soil behaviour and its application to applied problems. In particular here we highlight how in the terramechanics community has not been clearly recognized that different shear vs. displacement behaviour is not function of soil type but it is rather a function of soil state (at least for dry, granular, cohesionless, soils). Behaviour A or B is function of density: same material may exhibit A or B trend purely depending on soil state. This is a well know phenomenon to granular physicists and geotechnical engineers but it is often overlooked by terramechanics investigators. In figure 3 direct shear test results for Mars Mojave Simulant [1, 8] are presented. Data clearly show that type A or B behaviour purely depends on soil density.

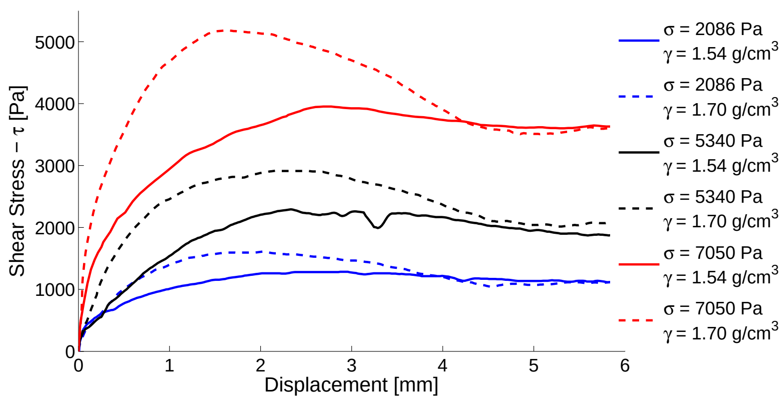

Figure 3: Type A and B behaviour for Mojave Mars soil simulant. Same material, at different density, presents distinct different shearing behaviour.

In section 2, shear modulus calculation for type A behaviour has been discussed. However, it has been shown that type B behaviour may occur for the same material if density is varied. Bekker [2], Oida [6], and Wong [13] proposed different methods for description of type B behaviour but none of these methods was able to capture at the same time type A and B behaviour [5, 13]. Wan and Guo [11] introduced a constitutive model based on Rowe's work [9] which predicted fairly well the behaviour of a granular media under different density levels. However, the model has only been tested for high stress levels and it requires

11 parameters in order to be initialized. Sela [10] hypothesized that shear displacement curve may be described by the sum of two exponential functions, one accounting for the cohesive component and the other for the frictional component. Although the distinction between cohesive and frictional behaviour is difficult to be verified experimentally and it is probably not the correct interpretation of the phenomena under study, the equation suggested by Sela is an appealing extension of Janosi-Hanamoto for terramechanics analysis. Here we propose a model for type A and B characterization based on Sela's work, that includes the influence of density. Total shear stress is calculated as the summation of two components:

$$\tau = \tau_{res}\left[\frac{k_\gamma j}{k}e^{1-\frac{j}{k}} + \left(1 - e^{-\frac{j}{k}}\right)\right] \tag{6}$$

First term introduces the parameter k_γ that controls density influence while second term governs the residual behaviour. Residual stress τ_{res} is in fact constant for a given stress level (see figure 3) and thus k_γ can be used as a tuning parameter to account for density dependence. Figure 4 shows how equation 6 fits experimental results. Holding τ_{max} and k constant it is possible to model type A and B behaviour with a single equation. It is interesting to note that the first term in the modified Sela's equation 6 closely follow the rate of dilation measured during the test. This suggests that k_γ is indeed related to density and equation 6 may represent a reduced order approximation of the model introduced by Wan and Guo [11]. However, this approach has not been fully validated yet and further investigations are underway.

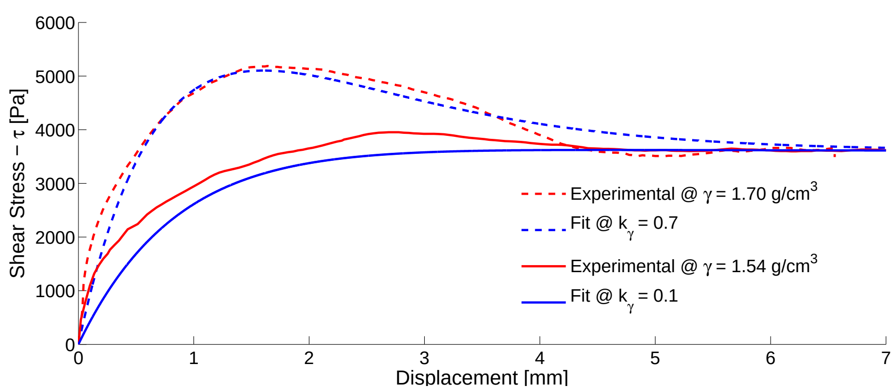

Figure 4: Type A and B behaviour for Mojave Mars soil simulant tested at σ = 7050 Pa and fitted through equation 6. Parameters k and τ_{res} are held constant for both cases.

4. Lightweight Tracked Vehicles Case Study

In this section it will be discussed how the aforementioned problems affect lightweight off-road vehicles modelling. For a given material, shear vs normal stresses at failure lay on a straight line in the $\tau - \sigma$ plane, named failure envelope. However, definition of failure, for a granular material, can be vague. In general, failure is considered to occur when large deformations occur. For type B behaviour it is common to define failure at maximum shear stress while for type A there is not clear presence of a maximum shear stress. For this reason two envelopes can be defined: one at maximum shear stress and one at residual shear stress. In soil mechanics the failure envelope is traditionally known as Mohr-Coulomb failure criterion and expressed as:

$$\tau_f = c + \sigma \tan\phi \tag{7}$$

This failure criterion is obtained from a series of direct shear tests (or triaxial tests) where maximum or residual stress are plotted against normal stress and then a failure envelope is calculated. Figure 5 presents a Mohr-Coulomb envelope obtained from direct shear tests of Mars Mojave Simulant [1, 8]. Each experimental point correspond to a peak in curves for 1.7 g/cm^3 presented in figure 3. Slope and intersect of the linear envelope correspond to angle of internal friction ϕ and cohesion c respectively. From this procedure it is clear that (apparent) cohesion is calculated through a minimization procedure that is only loosely related to true material cohesion. In fact it is not uncommon for the envelope to cross the y-axis at negative values: in such conditions cohesion is typically assumed to be zero.

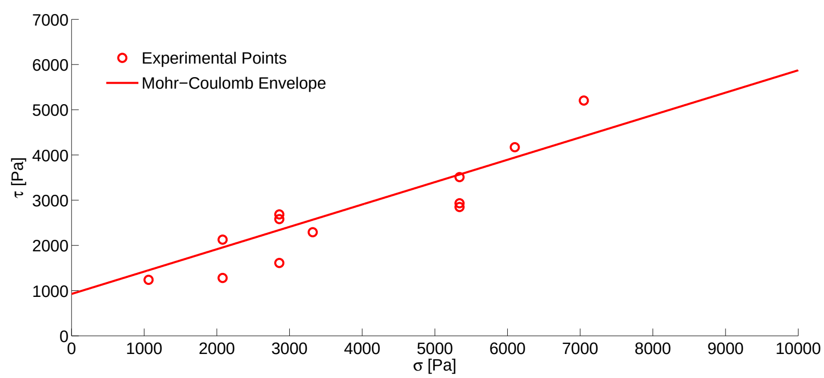

Figure 5: An example of Mohr-Coulomb envelope obtained from direct shear tests of Mars Mojave Simulant at 1.7 g/cm^3.

These sort of approximations have been accepted by geotechnical engineer and large vehicles designers because they usually worked well for high normal stress applications. However, when vehicle mass is modest, average ground pressure is reduced. This creates a situation where σ and c are of the same order of magnitude and thus, even for low-cohesion soils, it may be an unreasonable approximation to set $c = 0$. Apparent cohesion values for dry, granular soils, usually ranges between 0.5 and 2kPa. For a small rover weighting 15 kg, with 45 cm tracks of 12 cm width (typical dimensions of a iRobot 510 Packbot ®) , average ground pressure would be around 1.5 kPa. Clearly, in this case c and σ are of the same order of magnitude and crudely assuming $c = 0$ may invalidate the results. The concept is visualized in figure 6. Starting from a set of terrain parameters taken from Wong [12], cohesion is taken equal to zero and equal to nominal, tabulated value, for a dry sand. Shear prediction is obtained through Janosi-Hanamoto's equation.

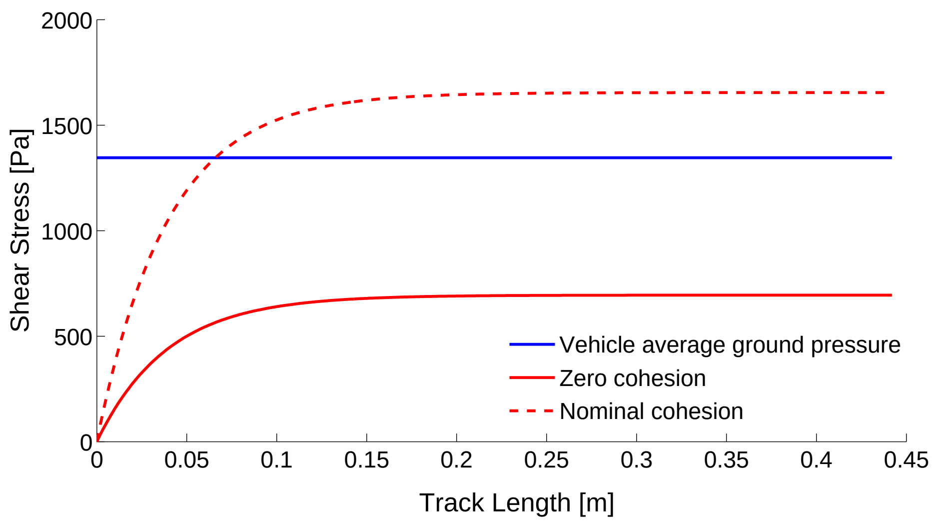

Figure 6: Shear stress profile under the track of a small rover as predicted through Janosi-Hanamoto's equation. Slip is assumed to be 30% and cohesion is set either to zero or nominal value (960Pa, from Wong [12]).

In 2 and 3 we have shown how shear modulus is typically overestimated and how density plays a crucial role in defining shearing properties of granular soils. However, for large vehicles, shear deformation is usually quite large and these effects may not really influence traction performance estimation. For instance, a tank with 5 meters long tracks, slipping a 10%, would produce a shear displacement of 50 cm, far beyond the few millimeters investigated by direct shear tests (see figure 1). On the other hand, a 45cm track, slipping ad 10% only displaces 4cm of soil. If any discontinuity occurs (rocks, track compliance, terrain unevenness), soil deformation can easily go down to the order of 1cm. Thus, for this class of vehicles, what happens in the very first millimeter of soil deformation can significantly change performance predictions. The concept is presented in figure 7 where experimental shear stress curves are presented over a domain of 1 cm (a) and 45 cm (b).

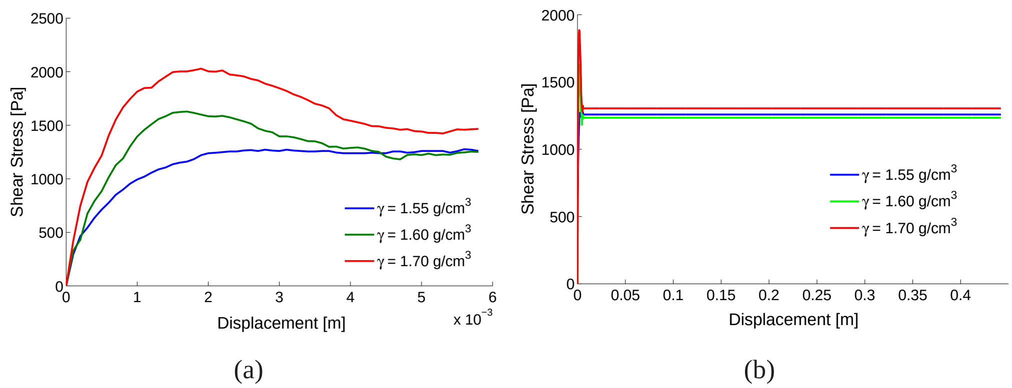

Figure 7: (a) Shows shear stress curves at three different densities for 2.86 kPa of normal stress. (b) presents the same results extended over a 45 cm length. It is obvious that shear modulus and shear behaviour A, B, and C, have less influence when shear deformation is large.

Figure 8 presents predicted thrust variations for a 45 cm long track that exerts 2.86kPa average ground pressure. 1 point of contact means that the whole track uniformly distribute its pressure on the soil. 10 points of contact means that the track looses contact with the soil for 10 times because of uneven terrain profile, suspension mechanism, rocks, etc. The plot shows that soil density significantly influences thrust prediction.

5. Conclusions

We have shown that it is possible to calculate with more accuracy shear modulus of granular materials. Moreover, the influence of density on soil shearing behaviour has been clarified. These aspects have been discussed in details to highlight the limitations of Bekker/Wong models when applied to lightweight vehicles. It should be remembered that Bekker clearly stated [2] that his models were scale dependent. However, in the last 15 years, researchers in need of a

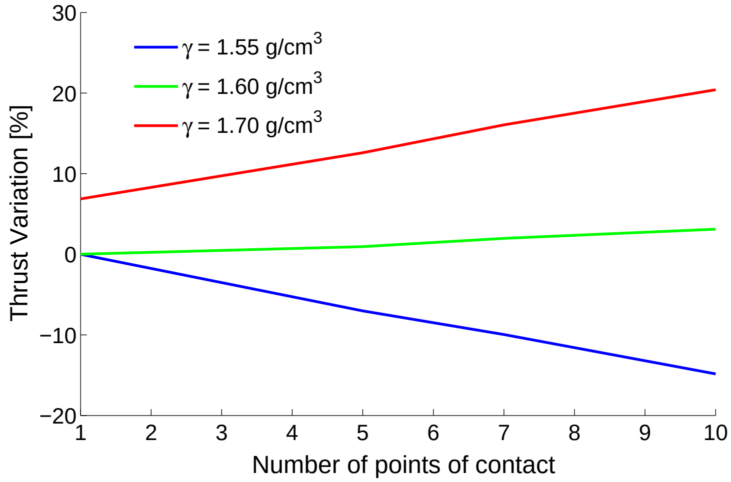

Figure 8: Thrust variation for a 45 cm tracked vehicles that travels over the same soil compacted at different densities. 1 point of contact means that the whole track uniformly distribute its pressure on the soil. 10 points of contact means that the track looses contact with the soil for 10 times because of uneven terrain profile, suspension mechanism, rocks, etc.

model lightweight vehicles have often overlooked that limitations. For lightweight robots it is necessary to characterize the soil for small normal stress (<15kPa) in order to have more accurate measure of cohesion and it is crucial to understand the influence of density on soil behaviour. A unified model for shear stress vs. displacement curve that combine A and B behaviour has been introduced.

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