Design of a Highly Maneuverable Wheeled Mobile Robot

Martin Udengaard*, Karl Iagnemma
Massachusetts Institute of Technology, 77 Massachusetts Ave., Cambridge, MA, USA

ABSTRACT

An omnidirectional mobile robot is able, kinematically, to move in any direction regardless of current pose. To date, nearly all designs and analyses of omnidirectional mobile robots have considered the case of motion on flat, smooth terrain. In this paper, an investigation of the design and analysis of an omnidirectional mobile robot for use in rough terrain is presented. Kinematic and geometric properties of the active split offset caster drive mechanism are investigated along with system and subsystem design guidelines. An optimization method is implemented to explore the design space. Use of this method results in a robot that has higher mobility than a robot designed using engineering judgment. A point design generated by the optimization method is shown for a meter-scale mobile robot.

Keywords: Omnidirectional vehicles, rough terrain, isotropy, mobile robots, design, UGV

1. INTRODUCTION

Mobile robots are finding increasing use in military [1], disaster recovery [2], and exploration applications [3]. These applications frequently require operation in rough, unstructured terrain. Currently, most mobile robots designed for these applications are tracked or Ackermann-steered wheeled vehicles. Methods for controlling these types of robots in both smooth and rough terrain have been well studied [4-6]. While these robots can perform well in many scenarios, navigation in cluttered, rocky, or obstacle-dense urban environments can be difficult or impossible. This is partly due to the fact that traditional tracked and wheeled robots must reorient to perform some maneuvers, such as lateral displacement. Omnidirectional mobile robots could potentially navigate faster and more robustly through cluttered urban environments and over rough terrain, due to their ability to track non-smooth motion profiles.

An omnidirectional mobile robot is able, kinematically, to move in any direction regardless of current pose. Previous researchers have proposed and developed omnidirectional mobile robots employing a wide variety of wheel types including roller [7, 8], Mecanum [9, 10], and spherical wheels [11, 12].

Roller wheel designs, as shown in Fig. 1, employ small rollers along the outer edge of a “primary” wheel to allow traction in the wheel’s longitudinal direction and free rolling in the lateral direction. Omnidirectional motion is obtained by orienting several of these wheels in different directions. These wheels are inexpensive, easy to control, and operate well in flat, indoor environments.

![Figure 1. An example of a sliding wheel (from [8]).](image)

Mecanum wheels are similar to roller wheels in that they employ rollers along the outer edge of a wheel; however the rollers are aligned at an angle to produce angular contact forces with the ground. Robots equipped with four Mecanum wheels, as shown in Fig. 2, can produce omnidirectional motion. Again, these wheel types have proved to be simple to control and effective on flat, indoor terrain.

Roller and Mecanum wheels are unsuitable for outdoor environments, where debris can clog the rollers and alter the friction characteristics of the wheels [13]. Also, the (relatively) small rollers on the edge of each primary wheel can be subjected to significant loads, which can lead to high ground pressure and large sinkage in deformable outdoor terrain.

*mru@mit.edu; phone 1 (617) 258-8428
Spherical wheel designs, as shown in Fig. 3, employ frictional drive rollers to allow rolling in any direction. Since the drive rollers rely on friction to transmit energy to the wheel, debris could potentially foul the transmission mechanism in rough, outdoor environments. Due to the two-dimensional curvature of the sphere, the contact patch is smaller than that of a traditional wheel, leading to increased ground pressure given the same ground reaction force.

Near-omnidirectional motion has been achieved using steerable wheels [14]. As shown in Fig. 4, these designs have a wheel mounted to an orthogonal steering actuator. The steering actuator can rotate the wheel to orient it in any planar direction. These wheels can employ standard tires, and have proven effective in outdoor environments. However, they are not truly omnidirectional (i.e., the resulting vehicle kinematics are subject to nonholonomic constraints) since they must undergo wheel slip and/or scrubbing to change direction. This can result in deteriorated path tracking and substantial energy loss. Note that similar designs based on offset caster wheels do allow omnidirectional motion with standard tires [15]. Analysis of this design has been studied extensively for operation on flat ground.

An omnidirectional mobile robot driven by active split offset casters (ASOCs) was initially proposed in [16] for use in structured, indoor environments. ASOC drives employ conventional wheel designs that do not rely on frictional contact, and are thus potentially suitable for use in dirty, outdoor environments. They also can be designed with little constraint on wheel diameter and width, and thus can potentially tolerate large loads with low ground pressure. Finally, ASOC modules can be integrated with suspension systems that allow for traversal of uneven terrain [17]. Therefore ASOC-driven omnidirectional mobile robots hold promise for use in rough, unstructured environments.
In this paper, an investigation of the design and analysis of an ASOC-driven omnidirectional mobile robot for use in rough terrain is presented. This paper is organized as follows: in Section 2, kinematic and geometric properties of the drive mechanism are analyzed, and in Section 3, guidelines for robot design are presented and an optimization method is implemented to explore the design space. These analyses can be used as design guidelines for development of an omnidirectional mobile robot that can operate in unstructured environments. The optimization method is shown to generate design parameters for a robot that has higher mobility than a robot designed using engineering judgment.

2. THE ACTIVE SPLIT OFFSET CASTER

Active split offset caster (ASOC) drive modules possess the ability to achieve omnidirectional motion via a driven wheel pair. Figure 5 shows the ASOC module considered in this study. The assembly consists of a split wheel pair, a connecting axle, and an offset link connecting the wheel pair to the mobile robot body. Each wheel is independently driven about the axis $\theta$. The axle connecting the wheel pair can pivot about the axis $\beta$. The axle pivot can be passive or active, and allows the wheel pair to adapt to terrain unevenness, therefore increasing the likelihood of continuous terrain contact for each wheel even during travel on rough terrain. The wheel pair/axle assembly rotates about axis $\alpha$. As with the axle pivot, the assembly rotation axis can also be active or passive. This axis connects the ASOC module to a robot body or a passive or active suspension element. $L_{\text{offset}}$ is the distance between the axis $\alpha$ and the axis $\theta$. $L_{\text{split}}$ is the distance between the wheels.

By independently controlling each wheel’s velocity, an ASOC module can produce arbitrary (planar) translational velocities at a point along its $\alpha$ axis [16]. Two or more ASOCs attached to a rigid robot body can thus produce arbitrary translational and rotational robot velocities. Therefore, an ASOC-driven omnidirectional robot must minimally employ two ASOC modules, and can employ more to meet other design requirements related to thrust, ground pressure, tip-over stability, etc. Note that passive or active casters can also be used to augment ASOC modules to meet these requirements.

2.1 Isotropy Analysis

Path following in rough terrain may require a robot to quickly change its direction of travel. All holonomic omnidirectional mobile robots are kinematically able to instantaneously move in any planar direction. However, while some omnidirectional mobile robots exhibit preferred directions of travel, others exhibit equal mobility characteristics in all directions. Such robots are said to exhibit “isotropic mobility.” Hence, isotropy is used to quantify the system’s omnidirectional mobility.

Kinematic isotropy is defined as the condition in which a robot possesses a constant input velocity/output velocity ratio for all possible output velocity directions [15]. An isotropy metric is a measure of how near a robot is to the isotropy condition, and increases from 0.0 for a singular configuration (i.e. purely anisotropic, or non-omnidirectional) to 1.0 for kinematic isotropy. Ideally, an omnidirectional mobile robot should possess a metric value of 1.0 for all joint space configurations, and thus not exhibit a preferred direction of travel. This simplifies path planning and navigation by eliminating the effect of robot orientation on movement capability. The output directions considered in this study are two planar translations in the robot body frame, and rotation about the robot body frame $z$ axis.

The isotropy metric for a given robot configuration can be computed as the ratio of the smallest to largest eigenvalues of the Jacobian matrix relating the driving module velocities to the robot body velocities [15]. The isotropy metric can be averaged over the entire configuration space (in this case, the rotation angles between each ASOC and the body, $\alpha$) to yield an average measure of performance that could be used to compare candidate omnidirectional mobile robot designs.
2.2 Effect of ASOC Geometric Parameters on Isotropy

To analyze the effects of ASOC module kinematic parameters on isotropy, variations in the wheel radius, $L_{offset}$ and $L_{split}$ were analyzed over a range of values that represent a practical omnidirectional robot design space. The Jacobian from wheel rotational velocities to $\alpha$ axis translational velocities in the ASOC frame is:

$$
\begin{bmatrix}
\dot{v}_{long} \\
\dot{v}_{lat}
\end{bmatrix} = 
\begin{bmatrix}
\frac{R_{wheel}}{2} & \frac{R_{wheel}}{2} \\
\frac{R_{wheel} \cdot L_{offset}}{L_{split}} & \frac{R_{wheel} \cdot L_{offset}}{L_{split}}
\end{bmatrix}
\begin{bmatrix}
\dot{\theta}_1 \\
\dot{\theta}_2
\end{bmatrix}
$$

(1)

where $v_{long}$ and $v_{lat}$ are the longitudinal (x) and lateral (y) ASOC axis translational velocities, respectively (see Fig. 5). The wheel radius appears in each term exactly once, and cancels out when the ratios of the eigenvalues are computed, thus the module isotropy is independent of the wheel radius.

In Fig. 6, a plot of isotropy is shown as a function of $L_{offset}$ and $L_{split}$. An iso-height exists at an isotropy value of 1.0. This iso-height occurs at $L_{split} / L_{offset} = 2.0$. The sensitivity of isotropy to perturbations in $L_{split}$ and $L_{offset}$ is relatively high; a 10% change in $L_{split}$ or $L_{offset}$ decreases the isotropy metric value by up to 45% for small ASOC module sizes.

![Figure 6. Mean isotropy for a four ASOC omnidirectional robot.](image)

In Fig. 7, a plot of isotropy values over a range of $L_{split} / L_{offset}$ ratios can be seen. There exists a single isotropy value for each $L_{split} / L_{offset}$ ratio, indicating that isotropy is not an independent function of both $L_{split}$ and $L_{offset}$. This is a useful insight for omnidirectional robot design. This also explains the sensitivity of isotropy to changes in $L_{split}$ and $L_{offset}$ for small ASOC modules sizes, since a unit change in $L_{split}$ or $L_{offset}$ results in a relatively large change in $L_{split} / L_{offset}$ for small parameter values. As shown in equation (1), $L_{split}$ and $L_{offset}$ only appear as a ratio, and the Jacobian becomes isotropic (i.e. all eigenvalues are equal) when the ratio of $L_{split}$ to $L_{offset}$ is equal to 2.0.

![Figure 7. Average isotropy for an omnidirectional mobile robot driven by three ASOC modules as a function of $L_{split} / L_{offset}$.](image)

2.3 Effect of ASOC Module Location on Isotropy

The relative location of ASOC modules with respect to one another also affects isotropy. A vehicle with three modules, shown in Fig. 8, was chosen for analysis. A plot of isotropy as a function of relative ASOC angular location is presented in Fig. 9. Each ASOC had an $L_{split} / L_{offset}$ ratio of 2.0. ASOC physical interference was neglected.
Analysis shows that maximum isotropy values (1.0) are obtained when the ASOC modules are evenly spaced and \( \frac{L_{\text{split}}}{L_{\text{offset}}} = 2.0 \). The value drops to 0 for the degenerate case where all ASOC modules coincide. A similar phenomenon is observed for robots with any number of ASOC modules. Thus to maximize isotropy, ASOC modules should be equally spaced.

![Figure 8](image1.png)

**Figure 8.** Top view of representative vehicle for ASOC location analysis.

![Figure 9](image2.png)

**Figure 9.** Isotropy as a function of ASOC module relative location.

### 2.4 Effect of Loss of Wheel Contact on Isotropy

When traversing rough terrain, loss of contact may occur between the wheels and the ground. In this case, system mobility will be decreased. An analysis of the isotropy of robots without full ground contact is presented in Table I. For comparison, robots with two, three, and four ASOC modules are examined. Each ASOC is allowed to possess full, partial (i.e. one wheel on the ground), or no ground contact. It is assumed that the ASOC modules are equally spaced and have \( \frac{L_{\text{split}}}{L_{\text{offset}}} = 2.0 \).

<table>
<thead>
<tr>
<th>Total # ASOCs</th>
<th># no contact ASOCs</th>
<th># partial contact ASOCs</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>0</td>
<td>1.000 0.464 0.000 N/A N/A</td>
</tr>
<tr>
<td>3</td>
<td>0</td>
<td>1.000 0.706 0.504 0.270 N/A</td>
</tr>
<tr>
<td>4</td>
<td>1</td>
<td>0.577 0.367 0.000 N/A N/A</td>
</tr>
<tr>
<td>4</td>
<td>1</td>
<td>1.000 0.791 0.656 0.544 0.399</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>0.707 0.574 0.482 0.259 N/A</td>
</tr>
</tbody>
</table>

As expected, loss of wheel contact causes reduced isotropy due to a loss of full controllability of the ASOC modules. It can be observed that a four ASOC robot with one module that has completely lost terrain contact does not perform as well as a three ASOC vehicle in full contact. This is due to the fact that the three ASOC robot has equally spaced ASOC modules. Also, given an identical number of wheels without terrain contact (e.g., 0 no contact and 2 partial contact vs. 1
no contact and 0 partial contact), a robot generally has higher isotropy when terrain contact is lost on the same ASOC, since more modules remain fully engaged with the ground. The isotropy loss from partial contact ASOC modules reinforces the importance of the $\beta$ axis axle pivot (see Fig. 5).

Finally, a vehicle with a greater number of ASOCs will have a relatively smaller decrease in isotropy for each lost wheel contact, but may have increased difficulty keeping all wheels in contact with the ground due to increased suspension complexity. Introduction of additional modules may also increase mass while decreasing the allowable wheel size and available battery mass given a fixed overall system mass.

2.5 Effect of Terrain Roughness on Isotropy

Isotropy of an omnidirectional robot can also be affected by terrain roughness. Variation in terrain inclination among ASOC modules, or among ASOC module wheel pairs, causes a change in the effective value of $L_{\text{split}}$ with respect to the body frame, which yields a change in $L_{\text{split}} / L_{\text{offset}}$ and thus a change in isotropy (see Fig. 10). Axis $\beta$ allows ASOC wheels to maintain contact during travel on uneven terrain.

In theory, $L_{\text{split}}$ could be modified as a function of terrain inclination via an active, extensible axle to cause the effective $L_{\text{split}} / L_{\text{offset}}$ ratio to always remain near 2.0, thus yielding good isotropy characteristics on rough terrain. In practice, however, such a design would be cumbersome and impractical. Thus it is useful to examine the effects of terrain inclination on robot isotropy.

In Fig. 11, a contour plot is presented of the average isotropy over a range of static robot configurations and terrain angles. The vehicle in this analysis had equally spaced ASOCs. The results are independent of the number of ASOC modules. The terrain angle was varied for each ASOC independently in a full factorial analysis over each terrain angle range. It can be seen that the $L_{\text{split}} / L_{\text{offset}}$ ratio with the largest isotropy value increases with the maximum terrain angle. Larger angles decrease the effective ratio and thus the “true” ratio must increase. Maximum average isotropy also decreases slightly with increasing terrain angle. Table II summarizes these findings.

<table>
<thead>
<tr>
<th>Terrain angle range</th>
<th>Max isotropy</th>
<th>Optimum $L_{\text{split}} / L_{\text{offset}}$ ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>0° (flat)</td>
<td>1.000</td>
<td>2.00</td>
</tr>
<tr>
<td>0-15°</td>
<td>0.987</td>
<td>2.05</td>
</tr>
<tr>
<td>0-30°</td>
<td>0.950</td>
<td>2.27</td>
</tr>
<tr>
<td>0-45°</td>
<td>0.895</td>
<td>2.70</td>
</tr>
</tbody>
</table>
3. DESIGN OF AN OMNIDIRECTIONAL MOBILE ROBOT FOR ROUGH TERRAIN

The class of robots analyzed in this paper is man-portable, battery powered mobile robots with a maximum enclosed envelope of one cubic meter and maximum mass of 65 kg. The primary design objective is to maximize traversable distance over a range of outdoor terrain types while maintaining a high level of mobility. Here, mobility is quantified by the system kinematic isotropy, the ability of an ASOC module to maintain ground contact, the system tipover stability, and the maximum traversable obstacle height. The robot must operate under its own power, and therefore should maximize mass efficiency to increase its battery payload. It should also minimize power loss from motion resistance in deformable terrain. Factors influencing the design space include wheel width, wheel radius, ASOC split and offset lengths, and the number and relative location of ASOC modules. Geometric constraints that bound the allowable design space must also be considered.

Figure 12 shows an illustration of an omnidirectional mobile robot driven by four ASOC modules. This is a representative configuration that will be considered in this work; however the following analysis is general and applies to robots with \( N \) ASOC modules.

3.1 Geometric Constraints

The unique geometry of the ASOC and the large range of motion of each module constrain the size of some mechanical components. Potentially, a control algorithm could utilize the robot’s redundancy to relax these constraints (by ensuring that wheel pairs are never directly oriented towards each other, for example). However, such an algorithm would likely reduce overall system mobility. Therefore, a geometric analysis of the ASOC module workspace is presented here.

3.1.1 ASOC Workspace Analysis

The maximum allowable wheel size that does not risk inter-ASOC interference can be calculated by simple geometric analysis of the module workspace. As seen in Fig. 14, the minimum distance between adjacent ASOC axes, \( d_a \), must be at least twice the maximum radius of the ASOC module workspace, \( r_{workspace} \). This radius is the distance from the vertical axis to the most distal point on the wheel:
\[ r_{\text{wheel envelope}} = \sqrt{(R_{\text{offset}} + r_{\text{wheel}})^2 + (0.5L_{\text{split}} + t_{\text{wheel}})^2}, \]  

where \( L_{\text{offset}} \) and \( L_{\text{split}} \) are the ASOC split and offset lengths, respectively, and \( r_{\text{wheel}} \) and \( t_{\text{wheel}} \) are the wheel radius and width, respectively.

![Figure 13. The circles represent the boundaries of the ASOC module workspace. To avoid ASOC interference, they should not intersect.](image)

### 3.1.2 Maximum Pivot Angle Analysis

In rough terrain, the passive pivot axis (see Fig. 5) allows the ASOC wheels to conform to terrain unevenness. A potential limiting factor of the pivot axis travel is wheel-shaft interference (see Fig. 14).

\[ 0.5L_{\text{split}} \cos \beta = r_{\text{wheel effective}} \sin \beta \]  

where \( \beta \) is the angle of the pivot rotation and \( r_{\text{wheel effective}} \) is the vertical distance from the center of the wheel to the section of the rim that intersects the shaft, as shown in Fig. 14. The value is calculated as

\[ r_{\text{wheel effective}} = \sqrt{r_{\text{wheel}}^2 - L_{\text{offset}}^2}. \]  

Note that when \( L_{\text{offset}} > r_{\text{wheel}} \), the shaft and wheel cannot interfere. However, such a configuration could allow obstacles to collide with the ASOC axis before they contact the wheels, which is undesirable. In a nominal configuration, the maximum value of \( \beta \) is given as

\[ \beta_{\text{max}} = \tan^{-1}\left(\frac{0.5L_{\text{split}}}{\sqrt{r_{\text{wheel}}^2 - L_{\text{offset}}^2}}\right). \]

### 3.2 Design Optimization

A full factorial design optimization was performed using the objectives discussed in Section 2 (system kinematic isotropy, the ability of an ASOC module to maintain ground contact, and the maximum traversable obstacle height) and...
constraints outlined in Section 3.1 (workspace limitations, module interference, and maximum suspension travel). The optimization parameters are the number of ASOC modules, $L_{split}$, $L_{offset}$, $L_{wheel}$, and $w_{wheel}$. An objective function, $J$, is expressed as a sum of the normalized mobility parameters:

$$ J = K + \beta_{max} + \frac{h}{h^*} + \frac{d_{max}}{d^*} + \frac{S}{S^*} $$  \hspace{1cm} (6)$$

where $K$ is the kinematic isotropy, $\beta_{max}$ is the maximum $\beta$ axis pivot angle, $h$ is the maximum traversable obstacle height, $d_{max}$ is the maximum traversable distance, and $S$ is a measure of the robot's stability. The star superscript refers to the maximum value of each parameter in the design space. The optimization consisted of a full factorial analysis over the design space to maximize the value of $J$.

In this analysis, kinematic isotropy and the maximum pivot angle are calculated as described Sections 2.2 and 3.1.2, respectively. The maximum traversable obstacle height is assumed to be a linear function of the wheel radius.

The optimization algorithm estimates maximum traversable distance by first determining the maximum available onboard energy. For the purposes of this study, it is assumed that the vehicle is powered by batteries with an energy density $\rho_{energy}$ of 576 kJ/kg (similar to that of lithium-ion batteries) [18]. The maximum allowable onboard battery mass, $M_{battery}$, is the difference between the non-battery mass (i.e., wheels, structural components, electronics, etc.) and the predetermined total allowable mass. The total allowable mass was chosen as 65 kg. Wheel and ASOC masses are computed as a function of their sizes.

The energy consumed during forward travel is then estimated using an expansion of a semi-empirical formulation for compaction resistance on deformable terrain [19].

$$ CR = \frac{n_{wheels}}{(3-n)^{2n+1} (n+1) \left( \frac{1}{w_{wheel}} + k_c \right) k_c} $$  \hspace{1cm} (7)$$

In (7), $CR$ is the compaction resistance (N), $M$ is the total vehicle mass (kg), $n_{wheels}$ is the number of wheels (i.e., twice the number of ASOC modules), and $n$, $k_c$, and $k_c$ are terrain physical constants (shown in Table III [20, 21]). Note that this estimate holds for straight-line driving and does not consider other resistive forces (such as bulldozing forces) or energy used by other onboard devices.

<table>
<thead>
<tr>
<th>Terrain type</th>
<th>$n$</th>
<th>$k_c$ (kPa/m$^{n+1}$)</th>
<th>$k_c$ (kPa/m$^n$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dry sand</td>
<td>1.1</td>
<td>0.9</td>
<td>1523.4</td>
</tr>
<tr>
<td>Sandy loam</td>
<td>0.7</td>
<td>5.3</td>
<td>1515.0</td>
</tr>
<tr>
<td>Clayey soil</td>
<td>0.5</td>
<td>13.2</td>
<td>692.2</td>
</tr>
<tr>
<td>Snow</td>
<td>1.6</td>
<td>4.4</td>
<td>196.7</td>
</tr>
</tbody>
</table>

The maximum traversable distance is approximated as

$$ d_{max} = \frac{M_{battery} \rho_{energy}}{CR}. $$  \hspace{1cm} (8)$$

Since the optimization compares similar systems, motor and drivetrain efficiencies are assumed identical for all candidate designs and therefore are not considered in the calculations.

The robot’s tipover stability is also considered in the optimization. A stability measure ($S$) is calculated by taking the minimum distance from the center of the robot to the edges of the support polygon over the full joint space of the ASOC modules. Here, the vertices of the support polygon are defined by the points between each ASOC wheel pair, as shown
in Figure 15. This is a conservative estimate, as defining the vertices by each ground contact point will result in a larger polygon.

![Image of robot with labeled parameters](image)

**Figure 15.** The solid gray polygon represents the estimated support polygon used in the optimization. It is bounded inside the true support polygon, shown as a dotted gray polygon.

### 3.3 Design Optimization Results

Table IV compares the values of the optimized mobility parameters of robots with three, four, and five ASOC modules. The robots were optimized for travel over sandy loam. Results are presented relative to the robot with three ASOC modules.

<table>
<thead>
<tr>
<th># ASOCs</th>
<th>(K)</th>
<th>(\beta_{\text{max}})</th>
<th>(h)</th>
<th>(d_{\text{max}})</th>
<th>(S)</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>0%</td>
<td>0%</td>
<td>0%</td>
<td>0%</td>
<td>0%</td>
</tr>
<tr>
<td>4</td>
<td>-0.4%</td>
<td>-1.3%</td>
<td>-6.1%</td>
<td>-38.8%</td>
<td>94.9%</td>
</tr>
<tr>
<td>5</td>
<td>-2.2%</td>
<td>16.2%</td>
<td>-60.9%</td>
<td>-52.9%</td>
<td>161.4%</td>
</tr>
</tbody>
</table>

It can be observed that a robot with four ASOC modules has similar values of kinematic isotropy (\(K\)), maximum \(\beta\) axis pivot angle (\(\beta_{\text{max}}\)), and maximum traversable obstacle height (\(h\)) as a three ASOC robot, however, adding the fourth module decreases available battery mass, and therefore decreases maximum traversable distance (\(d_{\text{max}}\)) but increases the size of the support polygon, and therefore increases the stability measure (\(S\)). Adding a fifth ASOC module results in smaller wheels, resulting in lower maximum traversable obstacle height, but higher maximum \(\beta\) axis pivot angle.

Table V shows the values of the optimized geometric parameters for a three ASOC robot. Optimized values were calculated for each of the four terrain types shown in Table III, assuming travel through randomized rough terrain with an angle range of 0-30°. Table VI shows the change in mobility parameter values for optimized designs compared to a baseline design with parameters determined by engineering judgment (\(L_{\text{offset}}=0.15\) m, \(L_{\text{split}}=0.20\) m, \(r_{\text{wheel}}=0.15\) m, \(w_{\text{wheel}}=0.03\) m).

<table>
<thead>
<tr>
<th>Terrain type</th>
<th>(L_{\text{offset}}) (m)</th>
<th>(L_{\text{split}}) (m)</th>
<th>(r_{\text{wheel}}) (m)</th>
<th>(w_{\text{wheel}}) (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dry sand</td>
<td>0.144</td>
<td>0.325</td>
<td>0.148</td>
<td>0.900</td>
</tr>
<tr>
<td>Sandy loam</td>
<td>0.134</td>
<td>0.306</td>
<td>0.139</td>
<td>0.126</td>
</tr>
<tr>
<td>Clayey soil</td>
<td>0.134</td>
<td>0.306</td>
<td>0.139</td>
<td>0.133</td>
</tr>
<tr>
<td>Snow</td>
<td>0.144</td>
<td>0.325</td>
<td>0.148</td>
<td>0.054</td>
</tr>
</tbody>
</table>
Table VI  Mobility Parameter Increases From Optimization

<table>
<thead>
<tr>
<th>Terrain type</th>
<th>K</th>
<th>$\beta_{\text{max}}$</th>
<th>h</th>
<th>$d_{\text{max}}$</th>
<th>S</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dry sand</td>
<td>13.2%</td>
<td>85.2%</td>
<td>-1.4%</td>
<td>18.1%</td>
<td>1.4%</td>
</tr>
<tr>
<td>Sandy loam</td>
<td>12.8%</td>
<td>82.8%</td>
<td>-7.4%</td>
<td>35.2%</td>
<td>3.6%</td>
</tr>
<tr>
<td>Clayey soil</td>
<td>12.8%</td>
<td>82.8%</td>
<td>-7.4%</td>
<td>31.9%</td>
<td>3.6%</td>
</tr>
<tr>
<td>Snow</td>
<td>13.2%</td>
<td>85.2%</td>
<td>-1.4%</td>
<td>3.3%</td>
<td>1.4%</td>
</tr>
</tbody>
</table>

In all cases, the optimized offset lengths were slightly smaller than the wheel radii, which yielded large allowable $\beta$ tilt angles. The $L_{\text{split}}$ to $L_{\text{offset}}$ ratios were all near 2.27:1, thus maximizing isotropy for the given terrain roughness range.

As presented, the optimized parameter values for the relatively deformable terrains (i.e. dry sand and snow) resulted in wheels with narrower widths compared to those optimized for relatively rigid terrains (i.e. sandy loam and clayey soil). The thinner widths lead to decreased wheel weight. One could also minimize ground pressure by choosing a wider wheel with smaller radius, but for a given a depth of sinkage, a tall, narrow wheel has significantly less compaction resistance than a short, wide one. For the relatively rigid terrains, a wider wheel was preferred as it allowed a greater onboard battery mass, thus increasing maximum traversable distance.

3.5 Point Vehicle Design

This section presents a point robot design with four ASOC modules (Fig. 16). This robot utilizes a four bar linkage suspension that achieves a maximum travel of 0.33 m. The wheels have a 0.163 m radius, the largest allowed given a body length ($L_{\text{body}}$) of 1 m and the workspace constraints outlined in Section 3.1.1. A maximum pivot angle of 30° and a high isotropy is achieved with an $L_{\text{split}}$ of 0.21 m and an $L_{\text{offset}}$ of 0.10 m (see Sections 3.1.2 and 2.5) yielding $L_{\text{split}} / L_{\text{offset}} = 2.1$.

![Figure 16. A four view drawing of a point vehicle design.](image)

4. CONCLUSIONS

In this paper, the design and control of an omnidirectional mobile robot driven by active split offset casters for use in rough terrain has been studied. An isotropy analysis was conducted to determine the optimal geometry and layout of the ASOC modules. This analysis indicates that equally spaced modules with $L_{\text{split}} / L_{\text{offset}} = 2.0$ yield a robot with equal mobility capability in all directions on flat terrain. On rough terrain, a larger ratio is desired, and robot isotropy degrades slightly. It also shows that isotropy is independent of wheel radius, which increases the scalability of the design.
Numerous design considerations for omnidirectional mobile robots were presented. An optimization algorithm was implemented to derive values for ASOC module and wheel geometries. For illustration, a man portable robot was designed, but the geometric constraints and the optimization algorithm are scalable and can be applied to robots of any size. It was shown that the designs suggested by the optimization have improved performance when compared to a non-optimized design. Through deliberate ASOC geometric parameter selection, it was possible to increase estimated traverse distance and mobility versus a baseline design.

5. REFERENCES