Lectures 17: Broadcast routing

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Broadcast Routing

- Route a packet from a source to all nodes in the network

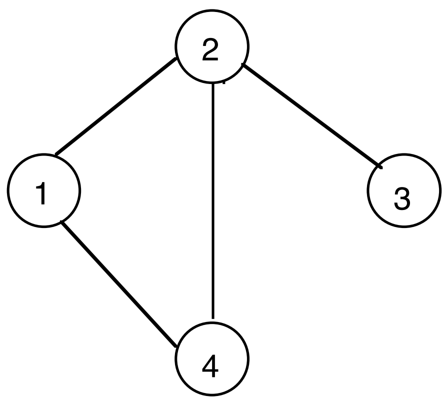
- Possible solutions:

  – Flooding: Each node sends packet on all outgoing links
    Discard packets received a second time

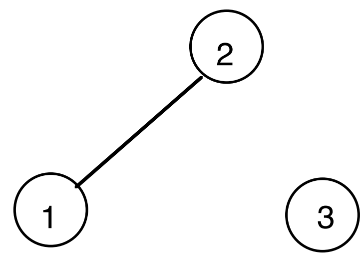
  – Spanning Tree Routing: Send packet along a tree that includes all of the nodes in the network

Graphs

- **A graph G = (N,A) is a finite nonempty set of nodes and a set of node pairs A called arcs (or links or edges)**



N = {1,2,3,4}
A = {(1,2),(2,3),(1,4),(2,4)}

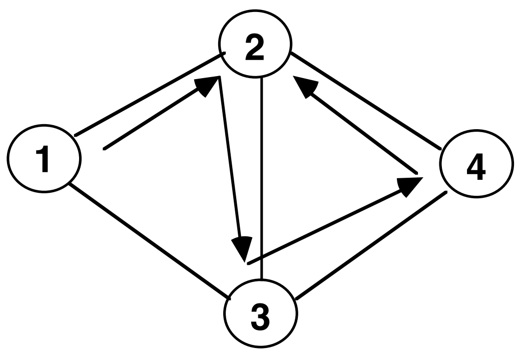


N = {1,2,3}
A = {(1,2)}

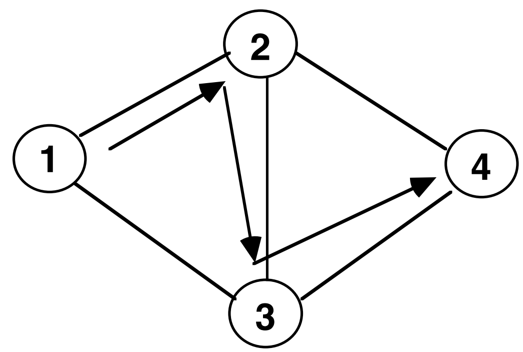
Walks and paths

- A walk is a sequence of nodes $(n_1, n_2, \ldots, n_k)$ in which each adjacent node pair is an arc

- A path is a walk with no repeated nodes



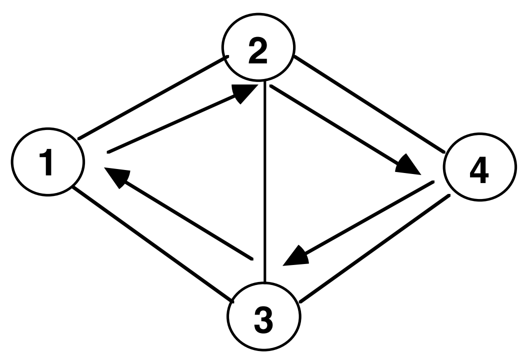
Walk (1,2,3,4,2)



Path (1,2,3,4)

Cycles

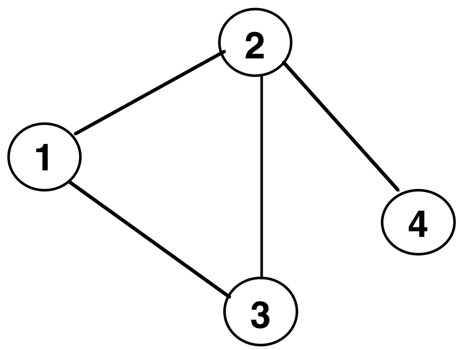
- A cycle is a walk $(n_1, n_2,...,n_k)$ with $n_1 = n_k$, $k > 3$, and with no repeated nodes except $n_1 = n_k$



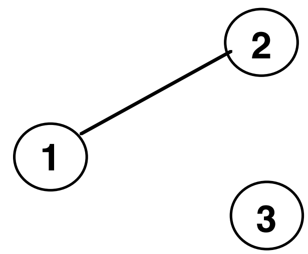
Cycle (1,2,4,3,1)

Connected graph

- A graph is connected if a path exists between each pair of nodes



Connected

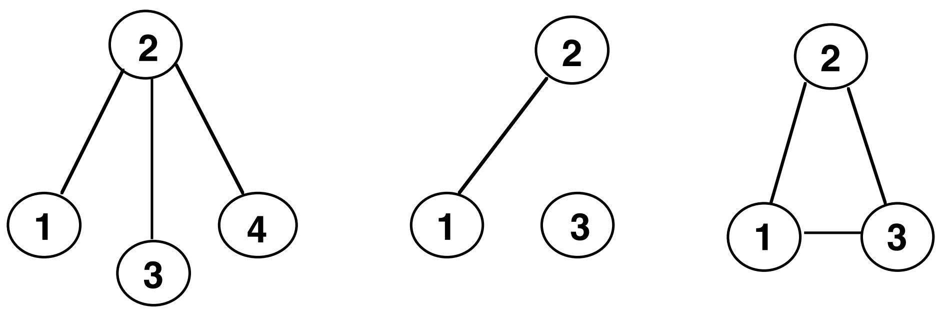


Unconnected

- An unconnected graph can be separated into two or more connected components

Acyclic graphs and trees

- An acyclic graph is a graph with no cycles
- A tree is an acyclic connected graph



Acyclic, connected | unconnected not tree | Cyclic, not tree

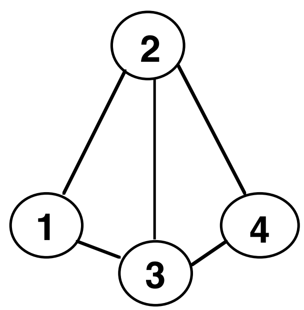
- The number of arcs in a tree is always one less than the number of nodes
  - Proof: start with arbitrary node and each time you add an arc you add a node $\Rightarrow$ N nodes and N-1 links. If you add an arc without adding a node, the arc must go to a node already in the tree and hence form a cycle

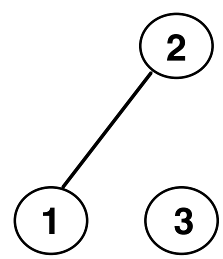
Sub-graphs

- G' = (N',A') is a sub-graph of G = (N,A) if
  - 1) G' is a graph
  - 2) N' is a subset of N
  - 3) A' is a subset of A

- One obtains a sub-graph by deleting nodes and arcs from a graph

  - Note: arcs adjacent to a deleted node must also be deleted

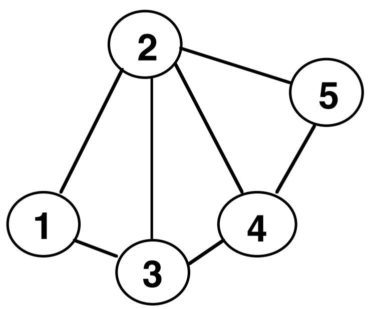




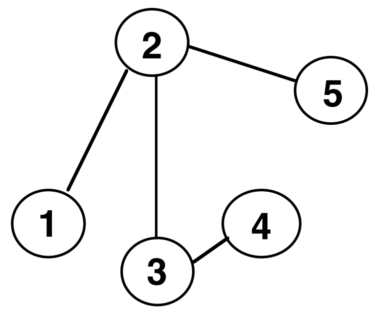
– Graph G Sub-graph G' of G

Spanning trees

- T = (N',A') is a spanning tree of G = (N,A) if
  - T is a sub-graph of G with N' = N and T is a tree



Graph G



Spanning tree of G

Spanning trees

- Spanning trees are useful for disseminating and collecting control information in networks; they are sometimes useful for routing
  – Especially in wireless networks

- To disseminate data from Node n:
  – Node n broadcasts data on all adjacent tree arcs
  – Other nodes relay data on other adjacent tree arcs

- To collect data at node n:
  – All leaves of tree (other than n) send data
  – Other nodes (other than n) wait to receive data on all but one adjacent arc, and then send received plus local data on remaining arc

General construction of a spanning tree

- **Algorithm to construct a spanning tree for a connected graph G = (N,A):**

  1) Select any node n in N; N' = {n}; A' = { }

  2) If N' = N, then stop

  T = (N',A') is a spanning tree

  3) Choose (i,j) $\in$ A, i $\in$ N', j $\notin$ N'

  N' := N'$\cup${j}; A' := A'$\cup${(i,j)}; go to step 2

- **Connectedness of G assures that an arc can be chosen in step 3 as long as N' $\neq$ N**

- **Is spanning tree unique?**
  - **What makes for a good spanning tree?**

Spanning tree algorithm

- The algorithm never forms a cycle, since each new arc goes to a new node
- T = (N',A') is a tree at each step of the algorithm since T is always connected, and each time we add an arc we also add a node

Theorem: If G is a connected graph of n nodes, then

1) G contains at least n-1 arcs
2) G contains a spanning tree
3) if G contains exactly n-1 arcs, G is a spanning tree

Distributed algorithms to find spanning trees

1) A fixed node sends a "start" message on each adjacent arc of the graph

2) Each other node marks the first arc on which a start message was received as a spanning tree arc and then sends a "start" message on each other arc

- This is a distributed implementation of the general spanning tree algorithm

- It has several problems shared by many such algorithms:

  a) Who chooses the starting node?
  b) When does the algorithm terminate?
  c) The resulting tree is somewhat random

Min weight spanning tree

- **Given a graph with weights assigned to each arc, find a spanning tree of minimum total weight (MST)**
- **Define a "fragment" to be a sub-tree of a MST**
- **Theorem:**
  - **Given a fragment F of an MST, Let a(i,j) be a minimum weight outgoing arc from F, where j is not in F**
  - **Then, F extended by arc a(i,j) & node j is a fragment**

    That is, the MST will include the min-weight outgoing arc a(i,j) and node j

- **Proof (by contradiction):**
  - **Let M be the MST that does not include a(i,j) (but does include F)**
  - **Since a(i,j) is not part of M, then adding a(i,j) to M must cause a cycle**

    $\Rightarrow$ There must be some link in the cycle b $\neq$ a which is outgoing from F
  - **Deleting b and adding a creates a new spanning tree**

    Since weight of b cannot be less then weight of a , M' must be a MST

    If weight of a = weight of b, then both are MST's otherwise M could not have been an MST

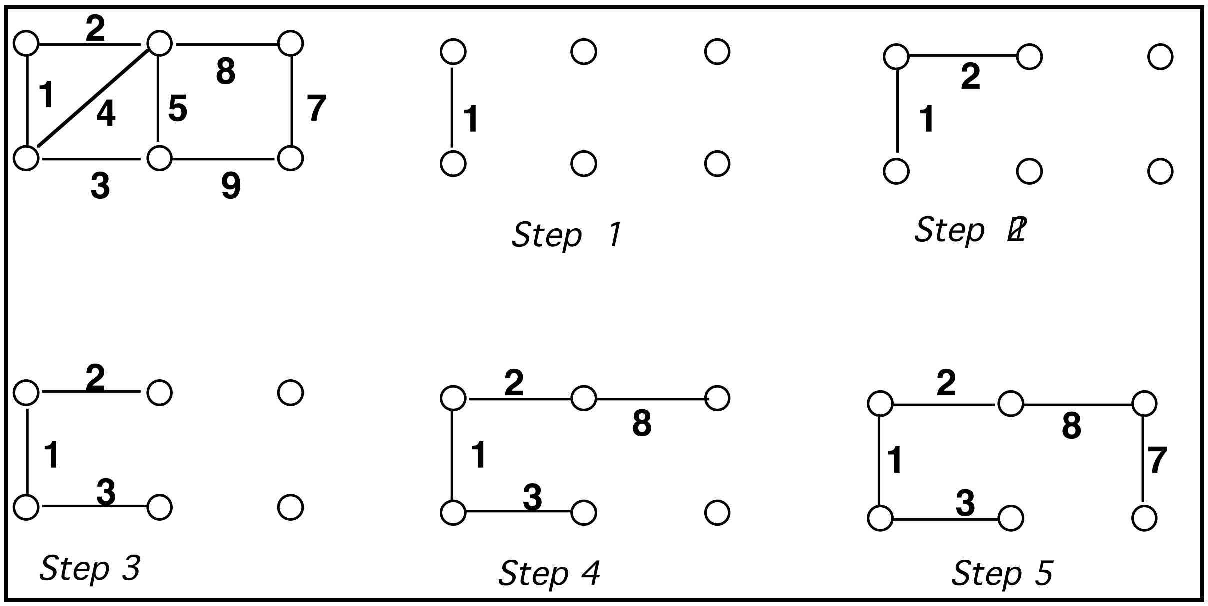
MST algorithms

- **Generic MST algorithm steps:**
  - Given a collection of sub-trees of an MST (called fragments) add a minimum weight outgoing edge to some fragment

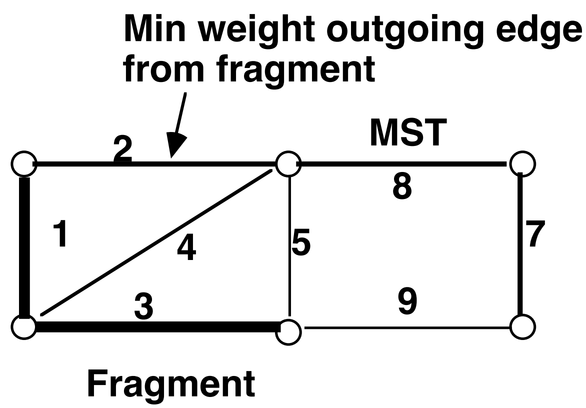
- **Prim-Dijkstra: Start with an arbitrary single node as a fragment**
  - Add minimum weight outgoing edge

- **Kruskal: Start with each node as a fragment;**
  - Add the minimum weight outgoing edge, minimized over all fragments

Prim-Dijkstra Algorithm: example



Kruskal Algorithm: example



- **Suppose the arcs of weight 1 and 3 are a fragment**
  - **Consider any spanning tree using those arcs and the arc of weight 4, say, which is an outgoing arc from the fragment**
  - **Suppose that spanning tree does not use the arc of weight 2**
  - **Removing the arc of weight 4 and adding the arc of weight 2 yields another tree of smaller weight**
  - **Thus an outgoing arc of min weight from fragment must be in MST**