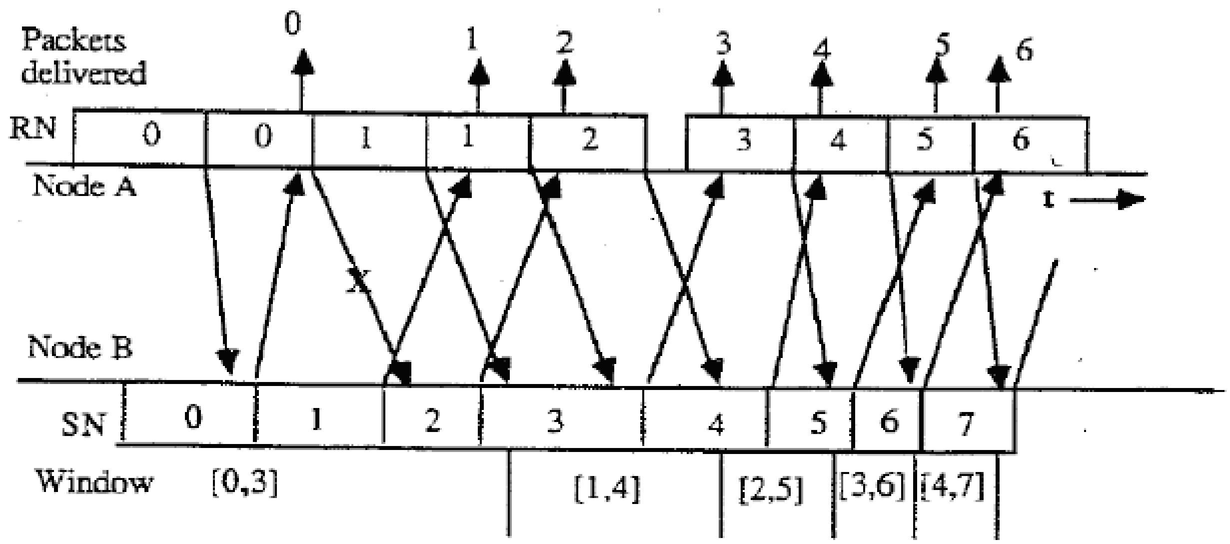
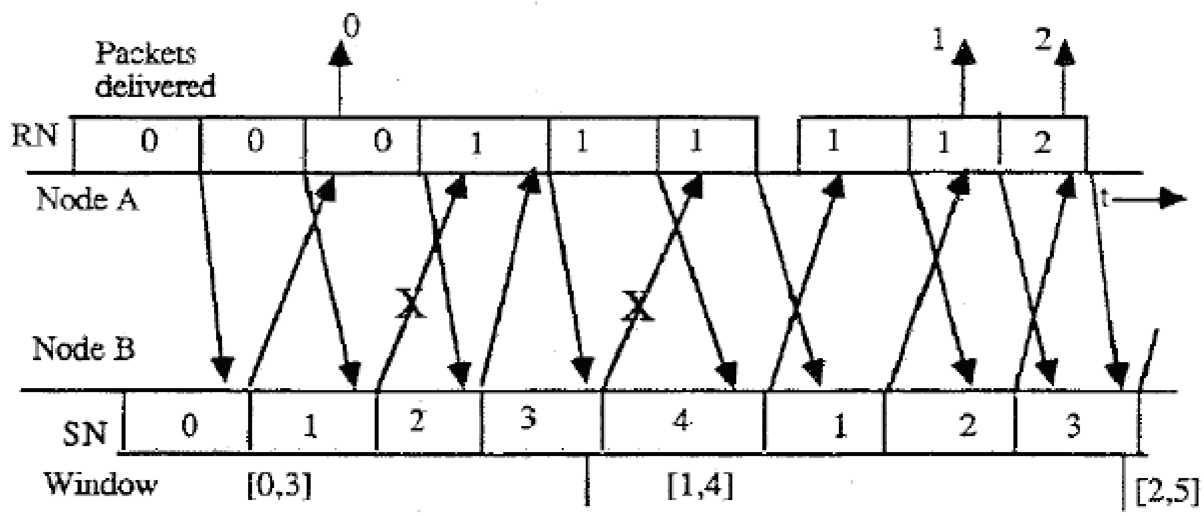
Problem Set 2 Solutions

Problem 1: Text Problem 2.18





Problem 2: Text Problem 2.20

The simplest example for node A to send packets 0 through $n-1$ in the first n frames. In case of delayed acks (i.e. no return packets in the interim), node A goes back and retransmits packet 0. If the other node has received all the packets, it is waiting for packet n, and if the modulus m equals n, this repeat of packet 0 is interpreted as packet n.

The right hand side of Eq. (2.24) is satisfied with equality if $SN = SN_{min}(t_1) + n - 1$. This occurs if node A sends packets 0 through n - 1 in the first n frames with no return packets from node B. The last such frame has $SN = n - 1$, where SN_{min} at that time (say t_1) is 0.

Continuing this scenario, we find an example where the right hand side of Eq. (2.25) is satisfied with equality. If all the frames above are correctly received, then after the last frame, RN becomes equal to n. If another frame is sent from A (now call this time t_1) and if SN_{min} is still 0, then when it is received at B (say at t_2), we have $RN(t_2) = SN_{min}(t_1) + n$.

Problem 3: Text Problem 2.22

a) If the transmitter never has to go back or wait in the absence of errors, then it can send a continuous stream of new packets in the absence of errors. In order for such a continuous stream to be sent, each packet must be acknowledged (i.e. SN_{min} must advance beyond the packet's number) before the next n - 1 frames complete transmission. Thus these n - 1 frame transmission times are in a race with the time, first, for the given packet to propagate over the channel and, second, for the acknowledgement to wait for the feedback frame in progress, then wait to be transmitted in the next feedback frame and propagated back to the original transmitter. In order for the feedback to always win the race, the minimum time for the n - 1 frames to be transmitted must be greater than the maximum time for the feedback, i.e.,

$$(n-1)T_{\min} > 2T_d + 2T_{\max}$$
$$T_{\max} < \left[(n-1)/2\right]T_{\min} - T_d$$

b) If an isolated error occurs in the feedback direction, the feedback could be held up for one additional frame, leading to

$$(n-1)T_{\min} > 2T_d + 3T_{\max}$$
$$T_{\max} < \left[(n-1)/3\right]T_{\min} - (2/3)T_d$$

Problem 4: Text Problem 2.23

After a given packet is transmitted from node A, the second subsequent frame transmission termination from B carries the acknowledgement (recall that the frame transmission in progress from B when A finishes its transmission cannot carry the ack for that transmission) Thus q is the probability of n - 1 frame terminations from A before the second frame termination from B. This can be rephrased as the probability that out of the next n frame terminations from either node, either n - 1 or n come from node A. Since successive frame terminations are equally likely and independently from A or B, this probability is

$$q = \sum_{i=n-1}^{n} \frac{n!}{i!(n-i)!} 2^{-n} = (n+1)2^{-n}$$

Problem 5: Text Problem 2.26

We view the system from the receiver and ask for the expected number of frames, γ, arriving at the receiver starting immediately after a frame containing a packet that is accepted and running until the next frame containing a packet that is accepted. Since the next frame after a packet acceptance must contain the awaited packet, that packet is accepted with probability 1-*p*. With probability *p*, on the other hand, that next frame contains an error. In this case, some number of frames, say *j*, follow this next frame before the awaited packet is again contained in a frame. This new frame might again contain an error, but the expected number of frames until the waited packet is accepted, starting with this new frame, is again γ. Thus, given an error in the frame after a packet acceptance, and given *j* further frames before the awaited packet is repeated, the expected number of frames from one acceptance to the next is $1 + j + \gamma$.

According to the assumption, the expected number of j is β. Combining the events of error and no error on the next frame after a packet acceptance, we have

$$\gamma = (1-p) + p(1+\beta+\gamma) = 1 + p(\beta+\gamma)$$

Solving for $\eta = 1/\gamma$

$$\gamma = (1-p)/(1+p\beta)$$

Problem 6: Text Problem 3.1

The system time of each type of customer is given by:

Carry-out: 5mins
Eat-in: 5mins+20mins = 25mins

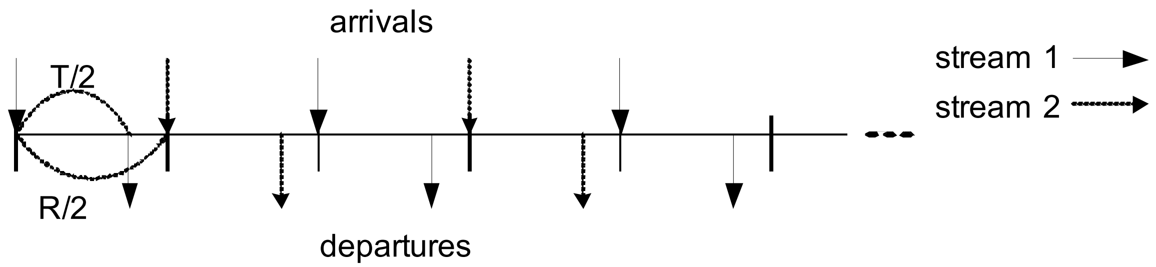
The average system time can be obtained as:

$$T = 0.5\times 5 + 0.5\times 25 = 15\text{mins}$$

By Little's Theorem, we have $N = \lambda T = 5 \text{ persons/min} \times 15\text{mins} = 75 \text{ persons}$.

Problem 7: Text Problem 3.7

Because the inter-arrival time R is larger than the transmission time T, a packet does not experience waiting time and hence, the system time in the former case is T for all packets. In the case of statistical multiplexing, the transmission time is $\frac{T}{2}$, but a packet can experience waiting time W ranging from 0 to $\frac{T}{2}$. The following figure illustrates the best case where there is no waiting time.



Because both of the streams have a fixed inter-arrival time, the worst case will happen when two packets arrive at the same time. In this case, half of the packets will experience the waiting time of $\frac{T}{2}$ while the other half experience no waiting time. Hence, the system time in each case will be

$$\frac{T}{2}+0=\frac{T}{2}: \text{ best case}$$

$$\frac{T}{2}+\frac{1}{2}\cdot 0+\frac{1}{2}\cdot\frac{T}{2}=\frac{3T}{4}: \text{ worst case}$$

In the best case, the variance of W is zero. In the worst case, it is given by

$$E\left[W^2\right]-\left(E\left[W\right]\right)^2=\frac{1}{2}\cdot 0+\frac{1}{2}\left(\frac{T}{2}\right)^2-\left(\frac{T}{4}\right)^2=\frac{T^2}{16},$$

completing the proof.

Problem 8

a) During a successful round with window size of X, X packets are transmitted. Because the window size increases from $\frac{N}{2}$ to N, the number of the packets (including the lost packet) transmitted in a cycle can be written as

$$N_{cycle} = \frac{N}{2} + \left(\frac{N}{2}+1\right) + \left(\frac{N}{2}+2\right) + \cdots + N.$$

Simplifying the above summation yields

$$N_{cycle} = \frac{3}{8}N^2 + \frac{3}{4}N.$$

b) Because the packet losses are independent, the probability that the first lost packet is the *k*-th packet is given by

$$\Pr[\alpha = k] = (1-p)^{k-1} p.$$

$E[\alpha]$ can then be written as

$$\begin{aligned} E[\alpha] &= \sum_{k=1}^{\infty} k \Pr[\alpha = k] \\ &= \sum_{k=1}^{\infty} k(1-p)^{k-1} p \\ &= p\left(1 + 2(1-p) + 3(1-p)^2 + 4(1-p)^3 + \cdots\right) \end{aligned}$$

Multiplying the above equation by $(1-p)$ yields

$$(1-p)E[\alpha] = p\left((1-p) + 2(1-p)^2 + 3(1-p)^3 + 4(1-p)^4 + \cdots\right).$$

Subtracting $(1-p)E[\alpha]$ from $E[\alpha]$ yields

$$\begin{aligned} pE[\alpha] &= p\left(1 + (1-p) + (1-p)^2 + (1-p)^3 + (1-p)^4 + \cdots\right) \\ &= p\frac{\left(1-(1-p)^{\infty}\right)}{1-(1-p)} = 1 \end{aligned},$$

which completes the proof.

c): Because N_{cycle} in a) is equal to $E[\alpha]$ in b), we have the following equation:

$$\frac{3}{8}N^2 + \frac{3}{4}N = \frac{1}{p}.$$

Hence, N is given by $N = \dfrac{-3+\sqrt{9+\frac{24}{p}}}{3}$. Because $p \ll 1$, we have $N \approx \dfrac{2}{3}\sqrt{\dfrac{6}{p}}$. This N is the *peak* window size, and so the average window size is $\frac{1}{2}\left(\frac{N}{2}+N\right)$, giving the desired result.

(This is a famous equation in TCP throughput analysis, and its (more rigorous) details can be found in: J. Padhye et al., “Modeling TCP Throughput: A Simple Model and Its Empirical Validation,” ACM SIGCOMM’98)